

Ex 14.1

Differentials Errors and Approximation Ex 14.1 Q1

$$\text{Let } x = \frac{\pi}{2}, \quad x + \Delta x = \frac{22}{14}$$

$$\Delta x = \frac{22}{14} - x$$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2} \right)$$

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx} \right)_{x=\frac{\pi}{2}} = \frac{\cos \pi}{2}$$

$$\left(\frac{dy}{dx} \right)_{x=\frac{\pi}{2}} = 0$$

$$\therefore \Delta y = \left(\frac{dy}{dx} \right)_{x=\frac{\pi}{2}} \times \Delta x$$

$$= 0 \times \left(\frac{22}{14} - \frac{\pi}{2} \right)$$

$$\Delta y = 0$$

So, there is no change in y .

Differentials Errors and Approximation Ex 14.1 Q2

$$\text{Let } x = 10, x + \Delta x = 9.8$$

$$\begin{aligned}\Delta x &= 9.8 - x \\ &= 9.8 - 10 \\ \Delta x &= -0.2\end{aligned}$$

$$y = \frac{4}{3}\pi x^3 \quad [\text{volume of sphere}]$$

$$\frac{dy}{dx} = 4\pi x^2$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 4\pi (10)^2$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 400\pi \text{ cm}^2$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= 400\pi \times (-0.2)$$

$$\Delta y = -80\pi \text{ cm}^3$$

So, approximate decrease in volume is $80\pi \text{ cm}^3$.

Differentials Errors and Approximation Ex 14.1 Q3

$$\text{Let } x = 10, x + \Delta x = 10 + \frac{k}{100} \times 10$$

$$x + \Delta x = 10 + 0.k$$

$$\begin{aligned}\Rightarrow \Delta x &= 10 + 0.k - 10 \\ \Delta x &= 0.k\end{aligned}$$

$$y = \pi r^2$$

$$\frac{dy}{dx} = 2\pi r$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 2\pi (10)$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 20\pi \text{ cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= (20\pi) \times (0.k)$$

$$\Delta y = 2k\pi \text{ cm}^2$$

Area of the plate increases by $2k\pi \text{ cm}^2$.

Differentials Errors and Approximation Ex 14.1 Q4

$$\text{Let length}(L) = x$$

$$x + \Delta x = x + \frac{x}{100}$$

$$\Delta x = 0.01x$$

Now,

$$y = 6x^2$$

$$\frac{dy}{dx} = 12x \text{ cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$

$$= (12x)(0.01x)$$

$$\Delta y = 0.12x^2 \text{ cm}^2$$

$$= 6(0.02)x^2$$

$$= 2\% \text{ of } 6x^2$$

Percentage error in area is 2%.

Differentials Errors and Approximation Ex 14.1 Q5

Let x be the radius of sphere,

$$\Delta x = 0.1\% \text{ of } x$$

$$\Delta x = 0.001x$$

Now,

Let y = volume of sphere

$$y = \frac{4}{3} \pi x^3$$

$$\frac{dy}{dx} = 4\pi x^2$$

$$\Delta y = \left(\frac{dy}{dx} \right) \times \Delta x$$

$$= 4\pi x^2 \times 0.001x$$

$$= \frac{4}{3} \pi x^3 (0.003)$$

$$= \frac{0.3}{100} \times y$$

$$\Delta y = 0.3\% \text{ of } y$$

So, percentage error in volume of error = 0.3%.

Differentials Errors and Approximation Ex 14.1 Q6

$$\text{Given, } \Delta v = -\frac{1}{2}\%$$

$$= -0.5\%$$

$$\Delta v = -0.005$$

Here,

$$pv^{1.4} = k$$

Taking log on both the sides,

$$\log(pv^{1.4}) = \log k$$

$$\log p + 1.4 \log v = \log k$$

Differentiate it with respect to v ,

$$\frac{1}{p} \frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\frac{dp}{dv} = -\frac{1.4}{v} p$$

$$\Delta p = \left(\frac{dp}{dv} \right) \Delta v$$

$$= -\frac{1.4p}{v} \times (-0.005)$$

$$\Delta p = \frac{1.4p(0.005)}{v}$$

$$\Delta p \text{ in } \% = \frac{\Delta p}{p} \times 100$$

$$= \frac{1.4p(0.005)}{p} \times 100$$

$$= 0.7\%$$

So, percentage error in p = 0.7%.

Differentials Errors and Approximation Ex 14.1 Q7

Let h be the height of the cone, and α be the semivertide angle.

Here vertgide angle α is fixed.

$$\begin{aligned}\Delta h &= k\% \text{ of } h \\ &= \frac{k}{100} \times h \\ \Delta h &= (0.0k)h\end{aligned}$$

$$\begin{aligned}\text{(i)} \quad A &= \pi r (r + l) \\ &= \pi (r^2 + rl) \\ &= \pi (r^2) + r\sqrt{h^2 + r^2} \quad \left[\text{Since, in a cone } l^2 = h^2 + r^2 \right]\end{aligned}$$

$$r = h \tan \alpha \quad \left[\text{from figure} \right]$$

$$\begin{aligned}A &= \pi \left[h^2 \tan^2 \alpha + h \tan \alpha \sqrt{h^2 + h^2 \tan^2 \alpha} \right] \\ &= \pi \left[h^2 \tan^2 \alpha + h \tan \alpha \sqrt{h^2 (1 + \tan^2 \alpha)} \right] \\ &= \pi \left[h^2 \tan^2 \alpha + h \tan \alpha \times h \sec \alpha \right] \\ &= \pi h^2 \left[\frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\sin \alpha}{\cos^2 \alpha} \right] \\ A &= \pi h^2 \frac{\sin \alpha (\sin \alpha + 1)}{\cos^2 \alpha}\end{aligned}$$

Differentiating with respect to h as α is fixed.

$$\frac{dA}{dh} = 2\pi h \frac{\sin \alpha (\sin \alpha + 1)}{\cos^2 \alpha}$$

So,

$$\begin{aligned}\Delta A &= \frac{dA}{dh} \times \Delta h \\ \Delta A &= \frac{2\pi h (0.0kh) \sin \alpha (\sin \alpha + 1)}{\cos^2 \alpha} \\ \Delta A \text{ in \% of } A &= \frac{2\pi h (0.0kh) \sin \alpha (\sin \alpha + 1)}{\cos^2 \alpha} \times \frac{100}{A} \\ &= \frac{2\pi kh^2 \times \sin \alpha (\sin \alpha + 1)}{\cos^2 \alpha} \times \frac{\cos^2 \alpha}{\pi h^2 \sin \alpha (\sin \alpha + 1)} \\ &= 2k\%\end{aligned}$$

So, percentage increase in area = $2k\%$.

(ii)

Let v = volume of cone

$$\begin{aligned}&= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (h \tan \alpha)^2 h \\ v &= \frac{\pi}{3} \tan^2 \alpha h^2\end{aligned}$$

Differentiating it with respect to h treating α as constant,

$$\begin{aligned}\frac{dv}{dh} &= \pi \tan^2 \alpha \times h \\ \Delta v &= \left(\frac{dv}{dh} \right) \Delta h \\ &= \pi \tan^2 \alpha h^2 \times (0.0kh) \\ \Delta v &= 0.0k \pi h^3 \tan^2 \alpha\end{aligned}$$

$$\begin{aligned}\text{Percentage increase in } v &= \frac{\Delta v \times 100}{v} \\ &= \frac{0.0k \pi h^3 \tan^2 \alpha \times 100}{\frac{\pi}{3} \tan^2 \alpha \times h^3} \\ &= 3k\%\end{aligned}$$

So, percentage increase in volume = $3k\%$.

Differentials Errors and Approximation Ex 14.1 Q8

Let error in radius (r) = $x\%$ of r

$$\Delta r = 0.0x r$$

Let v = volume of sphere

$$v = \frac{4}{3} \pi r^3$$

Differentiating it with respect to r ,

$$\frac{dv}{dr} = 4\pi r^2$$

So,

$$\begin{aligned}\Delta v &= \left(\frac{dv}{dr} \right) \times \Delta r \\ &= (4\pi r^2) (0.0x) r \\ \Delta v &= 0.0x \times 4\pi r^3\end{aligned}$$

$$\begin{aligned}\text{Percentage of error in volume} &= \frac{\Delta v \times 100}{v} \\ &= \frac{(0.0x) 4\pi r^3 \times 100}{\frac{4}{3} \pi r^3} \\ &= 3x\%\end{aligned}$$

Percentage of error in volume = 3 (percentage of error in radius).

Differentials Errors and Approximation Ex 14.1 Q9(i)

Let $x = 25$, $x + \Delta x = 25.02$

$$\Delta x = 25.02 - 25$$

$$\Delta x = 0.02$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx} \right)_{x=25} = \frac{1}{2\sqrt{25}}$$

$$\left(\frac{dy}{dx} \right)_{x=25} = \frac{1}{10}$$

Now,

$$\Delta y = \left(\frac{dy}{dx} \right)_{x=25} \times \Delta x$$

$$= \frac{1}{10} (0.02)$$

$$\Delta y = 0.002$$

$$\sqrt{25.02} = y + \Delta y$$

$$= \sqrt{25} + 0.002$$

$$= 5 + 0.002$$

$$\sqrt{25.02} = 5.002$$

Differentials Errors and Approximation Ex 14.1 Q9(ii)

$$\begin{aligned}\text{Let } x &= 0.008, \quad x + \Delta x = 0.009 \\ \Delta x &= 0.009 - 0.008 \\ \Delta x &= 0.001\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3x^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=0.008} &= \frac{1}{3(0.008)^{\frac{2}{3}}} \\ &= \frac{1}{3(0.04)} \\ &= \frac{100}{12} \\ &= 8.333\end{aligned}$$

So,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x \\ &= (8.333)(0.001) \\ \Delta y &= 0.008333 \\ (0.009)^{\frac{1}{3}} &= y + \Delta y \\ &= (x)^{\frac{1}{3}} + 0.008333 \\ &= (0.008)^{\frac{1}{3}} + 0.008333 \\ &= 0.52 + 0.008333\end{aligned}$$

$$(0.009)^{\frac{1}{3}} = 0.208333$$

Differentials Errors and Approximation Ex 14.1 Q9(iii)

$$\begin{aligned}\text{Let } x &= 0.008, \quad x + \Delta x = 0.007 \\ \Delta x &= 0.007 - 0.008 \\ \Delta x &= -0.001\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3(x)^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=0.008} &= \frac{1}{3(0.008)^{\frac{2}{3}}} \\ &= \frac{100}{12} \\ \left(\frac{dy}{dx}\right)_{x=0.008} &= 8.333\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x \\ &= (8.333)(-0.001) \\ \Delta y &= -0.008333 \\ (0.007)^{\frac{1}{3}} &= y + \Delta y \\ &= x^{\frac{1}{3}} - 0.008333 \\ &= (0.008)^{\frac{1}{3}} - 0.008333 \\ &= 0.2 - 0.008333\end{aligned}$$

$$(0.007)^{\frac{1}{3}} = 0.191667$$

Differentials Errors and Approximation Ex 14.1 Q9(iv)

$$\begin{aligned}\text{Let } x &= 400, x + \Delta x = 401 \\ \Delta x &= 401 - 400 \\ \Delta x &= 1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=400} &= \frac{1}{2\sqrt{400}} \\ &= \frac{1}{40} \\ \left(\frac{dy}{dx}\right)_{x=400} &= 0.025\end{aligned}$$

So,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=400} \times \Delta x \\ &= (0.025)(1) \\ &= 0.025 \\ \sqrt{401} &= y + \Delta y \\ &= \sqrt{x} + 0.025 \\ &= \sqrt{400} + 0.025 \\ &= 20 + 0.025\end{aligned}$$

$$\sqrt{401} = 20.025$$

Differentials Errors and Approximation Ex 14.1 Q9(v)

$$\begin{aligned}\text{Let } x &= 16, x + \Delta x = 15 \\ \Delta x &= 15 - 16 \\ \Delta x &= -1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{4}} \\ \frac{dy}{dx} &= \frac{1}{4x^{\frac{3}{4}}} \\ \left(\frac{dy}{dx}\right)_{x=16} &= \frac{1}{4(16)^{\frac{3}{4}}} \\ &= \frac{1}{32} \\ &= 0.03125\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=16} \times \Delta x \\ &= (0.03125)(-1) \\ \Delta y &= -0.03125 \\ (15)^{\frac{1}{4}} &= y + \Delta y \\ &= (x)^{\frac{1}{4}} - 0.03125 \\ &= (16)^{\frac{1}{4}} - 0.03125 \\ &= 2 - 0.03125\end{aligned}$$

$$(15)^{\frac{1}{4}} = 1.96875$$

Differentials Errors and Approximation Ex 14.1 Q9(vi)

$$\begin{aligned}\text{Let } x &= 256, \quad x + \Delta x = 255 \\ \Delta x &= 255 - 256 \\ \Delta x &= -1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{4}} \\ \frac{dy}{dx} &= \frac{1}{4x^{\frac{3}{4}}} \\ \left(\frac{dy}{dx}\right)_{x=256} &= \frac{1}{4(256)^{\frac{3}{4}}} \\ &= \frac{1}{256} \\ &= 0.00391\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=256} \times \Delta x \\ &= (0.00391)(-1) \\ \Delta y &= -0.00391 \\ (255)^{\frac{1}{4}} &= y + \Delta y \\ &= (x)^{\frac{1}{4}} + (-0.00391) \\ &= (256)^{\frac{1}{4}} - 0.00391 \\ &= 4 - 0.00391\end{aligned}$$

$$(255)^{\frac{1}{4}} = 3.99609$$

Differentials Errors and Approximation Ex 14.1 Q9(vii)

$$\begin{aligned}\text{Let } x &= 2, \quad x + \Delta x = 2.002 \\ \Delta x &= 2.002 - 2 \\ \Delta x &= 0.002\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \frac{1}{x^2} \\ \frac{dy}{dx} &= -\frac{2}{x^3} \\ \left(\frac{dy}{dx}\right)_{x=2} &= -\frac{2}{8} \\ &= -0.25\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=2} \times \Delta x \\ &= (-0.25)(0.002) \\ \Delta y &= -0.0005\end{aligned}$$

Now,

$$\begin{aligned}\frac{1}{(2.002)^3} &= y + \Delta y \\ &= \frac{1}{x^2} + (-0.0005) \\ &= \frac{1}{4} - 0.0005 \\ &= 0.25 - 0.0005\end{aligned}$$

$$\frac{1}{(2.002)^3} = 0.2495$$

Differentials Errors and Approximation Ex 14.1 Q9(viii)

$$\begin{aligned}\text{Let } x &= 4, x + \Delta x = 4.04 \\ \Delta x &= 4.04 - 4 \\ \Delta x &= 0.04\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \log x \\ \frac{dy}{dx} &= \frac{1}{x} \\ \left(\frac{dy}{dx}\right)_{x=4} &= \frac{1}{4} \\ &= 0.25\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=4} \times \Delta x \\ &= (0.25)(0.04) \\ \Delta y &= 0.01\end{aligned}$$

$$\begin{aligned}\log_e 4.04 &= y + \Delta y \\ &= \log x + (0.01) \\ &= \log_e 4 + 0.01 \\ &= \frac{\log_e 4}{\log_{10} e} + 0.01 \\ &= \frac{0.6021}{0.4343} + 0.01 \\ &= 1.38637 + 0.01\end{aligned}$$

$$\left[\text{Since, } \log_a b = \frac{\log_c b}{\log_c a} \right]$$

$$\log_e 4.04 = 1.39637$$

Differentials Errors and Approximation Ex 14.1 Q9(ix)

$$\begin{aligned}\text{Let } x &= 10, x + \Delta x = 10.02 \\ \Delta x &= 10.02 - 10 \\ \Delta x &= 0.02\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \log_e x \\ \frac{dy}{dx} &= \frac{1}{x} \\ \left(\frac{dy}{dx}\right)_{x=10} &= \frac{1}{10} \\ \left(\frac{dy}{dx}\right)_{x=10} &= 0.1\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x \\ &= (0.1)(0.02) \\ \Delta y &= 0.002\end{aligned}$$

$$\begin{aligned}\log_e (10.02) &= y + \Delta y \\ &= \log_e x + 0.002 \\ &= \log_e 10 + 0.002 \\ &= 2.3026 + 0.002\end{aligned}$$

$$\log_e (10.02) = 2.3046$$

Differentials Errors and Approximation Ex 14.1 Q9(x)

$$\begin{aligned}\text{Let } x &= 10, \quad x + \Delta x = 10.1 \\ \Delta x &= 10.1 - 10 \\ \Delta x &= 0.1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \log_{10} x \\ &= \frac{\log_e x}{\log_e 10} & \left[\text{Since, } \log_a b = \frac{\log_c a}{\log_c b} \right] \\ \left(\frac{dy}{dx} \right) &= \frac{1}{x \log_e 10}\end{aligned}$$

$$\left(\frac{dy}{dx} \right)_{x=10} = \frac{1}{10 \log_e 10}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx} \right)_{x=10} \times \Delta x \\ &= \frac{1}{10 (\log_e 10)} \times 0.1\end{aligned}$$

$$\Delta y = \frac{0.01}{(\log_e 10)}$$

$$\begin{aligned}\log_{10}(10.1) &= y + \Delta y \\ &= \log_{10} x + \frac{0.01}{\log_e 10} \\ &= \log_{10} 10 + 0.01 \log_{10} e \\ &= 1 + (0.01)(0.4343)\end{aligned}$$

$$\left[\text{Since, } \log_a b = \frac{1}{\log_b a} \right]$$

$$\log_{10}(10.1) = 1.004343$$

Differentials Errors and Approximation Ex 14.1 Q9(xi)

$$\begin{aligned}\text{Let } x &= 60^\circ, \quad x + \Delta x = 61^\circ \\ \Delta x &= 61^\circ - 60^\circ \\ \Delta x &= 1^\circ = \frac{\pi}{18^\circ} = 0.01745\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \cos x \\ \frac{dy}{dx} &= -\sin x \\ \left(\frac{dy}{dx} \right)_{x=60^\circ} &= -\sin(60^\circ) \\ &= -\frac{\sqrt{3}}{2} \\ &= -0.866 \\ \Delta y &= \left(\frac{dy}{dx} \right)_{x=60^\circ} \times (\Delta x) \\ &= (-0.866)(0.01745) \\ &= -0.01511\end{aligned}$$

So,

$$\begin{aligned}\cos 61^\circ &= y + \Delta y \\ &= \cos 60^\circ - 0.01511 \\ &= \frac{1}{2} - 0.01511 \\ &= 0.5 - 0.01511\end{aligned}$$

$$\cos 61^\circ = 0.48489$$

Differentials Errors and Approximation Ex 14.1 Q9(xii)

$$\begin{aligned}\text{Let } x &= 25, \quad x + \Delta x = 25.1 \\ \Delta x &= 25.1 - 25 \\ \Delta x &= 0.1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \frac{1}{\sqrt{x}} \\ \frac{dy}{dx} &= \frac{2}{2x^{\frac{3}{2}}} \\ \left(\frac{dy}{dx}\right)_{x=25} &= -\frac{1}{2(25)^{\frac{3}{2}}} \\ &= -\frac{1}{250} \\ &= -0.004\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x) \\ &= (-0.004)(0.1) \\ &= -0.0004\end{aligned}$$

$$\begin{aligned}\frac{1}{\sqrt{25.1}} &= y + \Delta y \\ &= \frac{1}{\sqrt{x}} + (-0.0004) \\ &= \frac{1}{\sqrt{25}} - 0.0004 \\ &= \frac{1}{5} - 0.0004 \\ &= 0.2 - 0.0004\end{aligned}$$

$$\frac{1}{\sqrt{25.1}} = 0.1996$$

Differentials Errors and Approximation Ex 14.1 Q9(xiii)

$$\begin{aligned}\text{Let } x &= \frac{\pi}{2}, \quad x + \Delta x = \frac{22}{14} \\ \Delta x &= \left(\frac{22}{14} - \frac{\pi}{2}\right) \\ \Delta x &= \sin x\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sin x \\ \frac{dy}{dx} &= \cos x \\ \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} &= \cos \frac{\pi}{2} \\ \left(\frac{dy}{dx}\right)_{\frac{\pi}{2}} &= 0 \\ \Delta y &= \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times (\Delta x) \\ &= 0 \times \left(\frac{22}{14} - \frac{\pi}{2}\right) \\ &= 0\end{aligned}$$

So,

$$\begin{aligned}\sin\left(\frac{22}{14}\right) &= y + \Delta y \\ &= \sin x + 0 \\ &= \sin\left(\frac{\pi}{2}\right)\end{aligned}$$

$$\sin\left(\frac{22}{14}\right) = 1$$

Differentials Errors and Approximation Ex 14.1 Q9(xiv)

$$\begin{aligned}
 \text{Let } x &= \frac{\pi}{3}, \quad x + \Delta x = \frac{11\pi}{36} \\
 \Delta x &= \frac{11\pi}{36} - \frac{\pi}{3} \\
 &= -\frac{\pi}{36} \\
 &= -\frac{22}{7 \times 36} \\
 &= -0.0873
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } y &= \cos x \\
 \frac{dy}{dx} &= -\sin x \\
 \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} &= -\sin \frac{\pi}{3} \\
 &= -\frac{\sqrt{3}}{2} \\
 &= -0.866 \\
 \Delta y &= \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} \times (\Delta x) \\
 &= (-0.866)(-0.0873) \\
 &= 0.0756
 \end{aligned}$$

$$\begin{aligned}
 \cos\left(\frac{11\pi}{36}\right) &= y + \Delta y \\
 &= \cos x + (0.0756) \\
 &= \cos \frac{\pi}{3} + 0.0756 \\
 &= \frac{1}{2} + 0.0756 \\
 &= 0.5 + 0.0756
 \end{aligned}$$

$$\cos \frac{11\pi}{36} = 0.7546$$

Differentials Errors and Approximation Ex 14.1 Q9(xv)

$$\begin{aligned}
 \text{Let } x &= 36, \quad x + \Delta x = 37 \\
 \Delta x &= 37 - 36 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } y &= \sqrt{x} \\
 \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\
 \left(\frac{dy}{dx}\right)_{x=36} &= \frac{1}{2\sqrt{36}} \\
 &= \frac{1}{12} \\
 &= 0.0833
 \end{aligned}$$

$$\begin{aligned}
 \Delta y &= \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x) \\
 &= (0.0833)(1) \\
 &= 0.0833
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{37} &= y + \Delta y \\
 &= \sqrt{x} + 0.0833 \\
 &= \sqrt{36} + 0.0833
 \end{aligned}$$

$$\sqrt{37} = 6.0833$$

Differentials Errors and Approximation Ex 14.1 Q9(xvi)

$$\begin{aligned}\text{Let } x &= 81, x + \Delta x = 80 \\ \Delta x &= 80 - 81 \\ &= -1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{4}} \\ \frac{dy}{dx} &= \frac{1}{4(81)^{\frac{3}{4}}} \\ &= \frac{1}{108} \\ &= 0.00926 \\ \Delta y &= \left(\frac{dy}{dx}\right)_{x=81} \times (\Delta x) \\ &= (0.00926)(-1) \\ &= -0.00926\end{aligned}$$

$$\begin{aligned}(80)^{\frac{1}{4}} &= y + \Delta y \\ &= x^{\frac{1}{4}} - 0.00926 \\ &= (81)^{\frac{1}{4}} - 0.00926 \\ &= 3 - 0.00926\end{aligned}$$

$$(80)^{\frac{1}{4}} = 2.99074$$

Differentials Errors and Approximation Ex 14.1 Q9(xvii)

$$\begin{aligned}\text{Let } x &= 27, x + \Delta x = 29 \\ \Delta x &= 29 - 27 \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3(x)^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=27} &= \frac{1}{3(27)^{\frac{2}{3}}} \\ &= \frac{1}{27} \\ &= 0.03704\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x) \\ &= (0.03704)(2) \\ \Delta y &= 0.07408\end{aligned}$$

$$\begin{aligned}(28)^{\frac{1}{3}} &= y + \Delta y \\ &= x^{\frac{1}{3}} + 0.07408 \\ &= (27)^{\frac{1}{3}} + 0.07408 \\ &= 3 + 0.07408\end{aligned}$$

$$(28)^{\frac{1}{3}} = 3.07408$$

Differentials Errors and Approximation Ex 14.1 Q9(xviii)

$$\begin{aligned}\text{Let } x &= 64, x + \Delta x = 66 \\ \Delta x &= 66 - 64 \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3(x)^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=64} &= \frac{1}{3(64)^{\frac{2}{3}}} \\ &= \frac{1}{48} \\ &= 0.020833\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=64} \times (\Delta x) \\ &= (0.020833)(2) \\ &= 0.041666\end{aligned}$$

$$\begin{aligned}(66)^{\frac{1}{3}} &= y + \Delta y \\ &= x^{\frac{1}{3}} + 0.041666 \\ &= (64)^{\frac{1}{3}} + 0.041666 \\ &= 4 + 0.041666\end{aligned}$$

$$(66)^{\frac{1}{3}} = 4.041666$$

Differentials Errors and Approximation Ex 14.1 Q9(xix)

$$\begin{aligned}\text{Let } x &= 25, x + \Delta x = 26 \\ \Delta x &= 26 - 25 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=25} &= \frac{1}{2\sqrt{25}} \\ &= \frac{1}{10} \\ &= 0.1\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x) \\ &= (0.1)(1) \\ &= 0.1\end{aligned}$$

$$\begin{aligned}\sqrt{26} &= y + \Delta y \\ &= \sqrt{x} + 0.1 \\ &= \sqrt{25} + 0.1\end{aligned}$$

$$\sqrt{26} = 5.1$$

Differentials Errors and Approximation Ex 14.1 Q9(xx)

$$\begin{aligned}\text{Let } x &= 0.49, \quad x + \Delta x = 0.487 \\ \Delta x &= 0.48 - 0.49 \\ &= -0.01\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=0.49} &= \frac{1}{2\sqrt{0.49}} \\ &= \frac{1}{1.4} \\ &= 0.71428\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=0.49} \times (\Delta x) \\ &= (0.71428)(-0.01) \\ \Delta y &= -0.0071428\end{aligned}$$

$$\begin{aligned}\sqrt{0.48} &= y + \Delta y \\ &= \sqrt{0.49} - 0.0071428 \\ &= 0.7 - 0.0071428\end{aligned}$$

$$\sqrt{0.48} = 0.6928572$$

Differentials Errors and Approximation Ex 14.1 Q9(xxi)

$$\begin{aligned}\text{Let } x &= 81, \quad x + \Delta x = 82 \\ \Delta x &= 82 - 81 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{4}} \\ \frac{dy}{dx} &= \frac{1}{4x^{\frac{3}{4}}} \\ \left(\frac{dy}{dx}\right)_{x=81} &= \frac{1}{4(81)^{\frac{3}{4}}} \\ &= \frac{1}{108} \\ &= 0.009259\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=81} \times (\Delta x) \\ &= (0.009259)(1) \\ \Delta y &= 0.009259\end{aligned}$$

$$\begin{aligned}(82)^{\frac{1}{4}} &= y + \Delta y \\ &= x^{\frac{1}{4}} + 0.009259 \\ &= (81)^{\frac{1}{4}} + 0.009259\end{aligned}$$

$$(82)^{\frac{1}{4}} = 3.009259$$

Differentials Errors and Approximation Ex 14.1 Q9(xxii)

$$\begin{aligned}\text{Let } x &= \frac{16}{81}, \quad x + \Delta x = \frac{17}{81} \\ \Delta x &= \frac{17}{81} - \frac{16}{81} \\ &= \frac{1}{81}\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{4}} \\ \frac{dy}{dx} &= \frac{1}{4x^{\frac{3}{4}}} \\ \left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} &= \frac{1}{4\left(\frac{16}{81}\right)^{\frac{3}{4}}} \\ &= \frac{27}{32} \\ &= 0.84375\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} \times (\Delta x) \\ &= (0.84375) \left(\frac{1}{81}\right) \\ &= 0.01041\end{aligned}$$

$$\begin{aligned}\left(\frac{17}{81}\right)^{\frac{1}{4}} &= y + \Delta y \\ &= \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.01041 \\ &= 0.6666 + 0.01041\end{aligned}$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = 0.67707$$

Differentials Errors and Approximation Ex 14.1 Q9(xxiii)

$$\begin{aligned}\text{Let } x &= 32, \quad x + \Delta x = 33 \\ \Delta x &= 33 - 32 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{5}} \\ \frac{dy}{dx} &= \frac{1}{5x^{\frac{4}{5}}} \\ \left(\frac{dy}{dx}\right)_{x=32} &= \frac{1}{5(32)^{\frac{4}{5}}} \\ &= \frac{1}{80} \\ &= 0.0125\end{aligned}$$

$$\begin{aligned}\therefore \Delta y &= \left(\frac{dy}{dx}\right)_{x=32} \times (\Delta x) \\ &= (0.0125)(1) \\ \Delta y &= 0.0125\end{aligned}$$

$$\begin{aligned}(33)^{\frac{1}{5}} &= y + \Delta y \\ &= x^{\frac{1}{5}} + 0.0125 \\ &= (32)^{\frac{1}{5}} + 0.0125\end{aligned}$$

$$(33)^{\frac{1}{5}} = 2.0125$$

Differentials Errors and Approximation Ex 14.1 Q9(xxiv)

$$\begin{aligned}\text{Let } x &= 36, \quad x + \Delta x = 36.6 \\ \Delta x &= 36.6 - 36 \\ &= 0.6\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=36} &= \frac{1}{2\sqrt{36}} \\ &= \frac{1}{12} \\ &= 0.0833\end{aligned}$$

$$\begin{aligned}\therefore \Delta y &= \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x) \\ &= (0.0833)(0.6) \\ &= 0.04998\end{aligned}$$

$$\begin{aligned}\sqrt{36.6} &= y + \Delta y \\ &= \sqrt{x} + 0.04998 \\ &= \sqrt{36} + 0.04998\end{aligned}$$

$$\sqrt{36.6} = 6.04998$$

Differentials Errors and Approximation Ex 14.1 Q9(xxv)

$$\begin{aligned}\text{Let } x &= 27, \quad x + \Delta x = 25 \\ \Delta x &= 25 - 27 \\ &= -2\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3x^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=27} &= \frac{1}{3(27)^{\frac{2}{3}}} \\ &= \frac{1}{27} \\ &= 0.037\end{aligned}$$

$$\begin{aligned}\therefore \Delta y &= \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x) \\ &= (0.037)(-2) \\ &= -0.074\end{aligned}$$

$$\begin{aligned}(25)^{\frac{1}{3}} &= y + \Delta y \\ &= x^{\frac{1}{3}} + (-0.074) \\ &= (27)^{\frac{1}{3}} - 0.074 \\ &= 3 - 0.074\end{aligned}$$

$$(25)^{\frac{1}{3}} = 2.926$$

Differentials Errors and Approximation Ex 14.1 Q9(xxvi)

Let $y = f(x) = \sqrt{x}$, $x = 49$ and $x + \Delta x = 49.5$

Then $\Delta x = 0.5$

For $x = 49$ we have

$$y = \sqrt{49} = 7$$

$$dx = \Delta x = 0.5$$

$$y = \sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=49} = \frac{1}{2 \times 7} = \frac{1}{14}$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = \frac{1}{14}(0.5) = \frac{5}{140}$$

$$\Rightarrow \Delta y = \frac{1}{28}$$

Hence,

$$\sqrt{49.5} = y + \Delta y = 7 + \frac{1}{28} = 7 + 0.0357 = 7.0357$$

Differentials Errors and Approximation Ex 14.1 Q9(xxvii)

Define a function $y = x^{3/2}$

For $x = 4$, $y = 8$

$$x + \Delta x = 3.968 \Rightarrow \Delta x = 3.968 - 4 = -0.032$$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$\Rightarrow dy = \left(\frac{3}{2}x^{1/2}\right)dx$$

$$\Rightarrow \Delta y|_{x=4} \approx (3)\Delta x$$

$$\Rightarrow \Delta y|_{x=4} \approx 3 \times (-0.032) = -0.096$$

$$\begin{aligned}(3.968)^{3/2} &= y + \Delta y = 8 - 0.096 \\ &= 7.904\end{aligned}$$

Differentials Errors and Approximation Ex 14.1 Q9(xxviii)

Let $y = f(x) = x^5$, $x = 2$ and $x + \Delta x = 1.999$

Then $\Delta x = -0.001$

For $x = 2$ we have

$$y = (2)^5 = 32$$

$$dx = \Delta x = -0.001$$

$$y = x^5$$

$$\Rightarrow \frac{dy}{dx} = 5x^4$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 5(2)^4 = 80$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = 80(-0.001) = -0.080$$

$$\Rightarrow \Delta y = -0.080$$

Hence,

$$(1.999)^5 = y + \Delta y = 32 - 0.080 = 31.920$$

Differentials Errors and Approximation Ex 14.1 Q9(xxix)

Let $y = f(x) = \sqrt{x}$, $x = 0.09$ and $x + \Delta x = 0.082$

Then $\Delta x = -0.008$

For $x = 0.09$ we have

$$y = \sqrt{0.09} = 0.3$$

$$dx = \Delta x = -0.008$$

$$y = \sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{x=1} = \frac{1}{2 \times \sqrt{0.09}} = \frac{1}{2 \times 0.3} = \frac{1}{0.6}$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = \frac{1}{0.6} (-0.008)$$

$$\Rightarrow \Delta y = -\frac{8}{600}$$

Hence,

$$\sqrt{0.082} = y + \Delta y = 0.3 - \frac{8}{600} = 0.3 - 0.0133 = 0.2867$$

Differentials Errors and Approximation Ex 14.1 Q10

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(2.01) &\approx (4x^2 + 5x + 2) + (8x + 5)\Delta x \\ &= [4(2)^2 + 5(2) + 2] + [8(2) + 5](0.01) \quad [x = 2, \Delta x = 0.01] \\ &= (16 + 10 + 2) + (16 + 5)(0.01) \\ &= 28 + (21)(0.01) \\ &= 28 + 0.21 \\ &= 28.21 \end{aligned}$$

Hence, the approximate value of $f(2.01)$ is 28.21.

Differentials Errors and Approximation Ex 14.1 Q11

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(5.001) &\approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \\ &= [(5)^3 - 7(5)^2 + 15] + [3(5)^2 - 14(5)](0.001) \quad [x = 5, \Delta x = 0.001] \\ &= (125 - 175 + 15) + (75 - 70)(0.001) \\ &= -35 + (5)(0.001) \\ &= -35 + 0.005 \\ &= -34.995 \end{aligned}$$

Hence, the approximate value of $f(5.001)$ is -34.995.

Differentials Errors and Approximation Ex 14.1 Q12

$$\begin{aligned}\text{Let } x &= 1000, x + \Delta x = 1005 \\ \Delta x &= 1005 - 1000 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \log_{10} x \\ \frac{dy}{dx} &= \frac{\log_e x}{\log_e 10} & \left[\because \log_a b = \frac{\log_e b}{\log_e a} \right] \\ \frac{dy}{dx} &= \frac{1}{x \log_e 10}\end{aligned}$$

$$\begin{aligned}\left(\frac{dy}{dx} \right)_{x=1000} &= \frac{\log_{10} e}{1000} & \left[\because \log_a b = \frac{1}{\log_b a} \right] \\ &= \frac{0.4343}{1000} \\ &= (0.0004343)\end{aligned}$$

$$\begin{aligned}\therefore \Delta y &= \left(\frac{dy}{dx} \right)_{x=1000} \times (\Delta x) \\ &= (0.0004343)(5) \\ &= 0.0021715\end{aligned}$$

$$\begin{aligned}\log_{10} 1005 &= y + \Delta y \\ &= \log_{10} x + 0.0021715 \\ &= \log_{10} 1000 + 0.0021715 \\ &= \log_{10} 10^3 + 0.0021715 \\ &= 3 \log_{10} 10 + 0.0021715\end{aligned}$$

$$\log_{10} 1005 = 3.0021715$$

Differentials Errors and Approximation Ex 14.1 Q13

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 9 \text{ m and } \Delta r = 0.03 \text{ m}$$

Now, the surface area of the sphere (S) is given by,

$$S = 4\pi r^2$$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\begin{aligned}\therefore dS &= \left(\frac{dS}{dr} \right) \Delta r \\ &= (8\pi r) \Delta r \\ &= 8\pi(9)(0.03) \text{ m}^2 \\ &= 2.16\pi \text{ m}^2\end{aligned}$$

Hence, the approximate error in calculating the surface area is $2.16\pi \text{ m}^2$.

Differentials Errors and Approximation Ex 14.1 Q14

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\begin{aligned}\therefore \frac{dS}{dx} &= \left(\frac{dS}{dx} \right) \Delta x \\ &= (12x) \Delta x \\ &= (12x)(0.01x) & [\text{as } 1\% \text{ of } x \text{ is } 0.01x] \\ &= 0.12x^2\end{aligned}$$

Hence, the approximate change in the surface area of the cube is $0.12x^2 \text{ m}^2$.

Differentials Errors and Approximation Ex 14.1 Q15

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 7 \text{ m and } \Delta r = 0.02 \text{ m}$$

Now, the volume V of the sphere is given by,

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \therefore \frac{dV}{dr} &= 4\pi r^2 \\ \therefore dV &= \left(\frac{dV}{dr}\right)\Delta r \\ &= (4\pi r^2)\Delta r \\ &= 4\pi(7)^2(0.02) \text{ m}^3 = 3.92\pi \text{ m}^3 \end{aligned}$$

Hence, the approximate error in calculating the volume is $3.92 \pi \text{ m}^3$.

Differentials Errors and Approximation Ex 14.1 Q16

The volume of a cube (V) of side x is given by $V = x^3$.

$$\begin{aligned} \therefore dV &= \left(\frac{dV}{dx}\right)\Delta x \\ &= (3x^2)\Delta x \\ &= (3x^2)(0.01x) \quad \text{[as 1% of } x \text{ is } 0.01x\text{]} \\ &= 0.03x^3 \end{aligned}$$

Hence, the approximate change in the volume of the cube is $0.03x^3 \text{ m}^3$.