CBSE Sample Question Paper Term 1

Class – XI (Session : 2021 - 22)

SUBJECT- MATHEMATICS 041 - TEST - 01

Class 11 - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections - A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

Section A

Attempt any 16 questions

1. Let A = $\{x : x \notin R, x \geqslant 4\}$and B = $\{x : x \notin R, x < 5\}$ then $A \cap B$ is **[1]**

   a) {5, 4} b) {4, 5}

   c) {4} d) $\{x : x \in R,\ 4 \leq x < 5\}$

2. If f (x) = sin $\left[x^2\right] x + \sin\left[-\pi^2\right]$ x. where x denotes the greatest integer less than or equal to x them **[1]**

   a) None of these b) f $(\pi/2)$ = 1

   c) f (π) = 2 d) f $(\pi/4)$ = -1

3. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$,then **[1]**

   a) Im (z) = 0 b) none of these

   c) Re (z) $>$0 d) Im (z) $<$ 0

4. If the sum of n terms of an A.P. is $2n^2 + 5n$, then its nth term is: **[1]**

   a) 4n + 3 b) 3n + 4

   c) 3n - 4 d) 4n - 3

5. The distance between the parallel lines $x^2 + 2xy + y^2 - 6x - 6y + 8 = 0$ is **[1]**

   a) 2 b) 1

   c) $\sqrt{2}$ d) 3

6. $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-1}{x}$ is equal to: **[1]**

   a) 0 b) 1

c) $\frac{1}{2}$ d) 2

7. Let $x_1, x_2, ..., x_n$ be n observations and $\bar{x}$ be their arithmetic mean. The formula for the standard deviation is given by **[1]**

a) $\frac{(x_i-\bar{x})^2}{n}$ b) $\sqrt{\frac{(x_i-\bar{x})^2}{n}}$

c) $(x_i - \bar{x})^2$ d) $\sqrt{\frac{x_i^2}{n} + \bar{x}^2}$

8. If A, B, C be any three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then **[1]**

a) B = C b) A = B = C

c) A = C d) A = B

9. If f(x) = $\frac{x}{x-1}$ then $\frac{f(a)}{f(a+1)}$ = **[1]**

a) $f\left(-\frac{a}{a-1}\right)$ b) $f(a^2)$

c) f(- a) d) $f\left(\frac{1}{a}\right)$

10. If (x + iy) = $\left(\frac{a+ib}{c+id}\right)$ then $(x^2 + y^2)$ = ? **[1]**

a) None of these b) $\frac{(a^2+b^2)}{(c^2+d^2)}$

c) $\frac{(a^2-b^2)}{(c^2+d^2)}$ d) $\frac{(a^2+b^2)}{(c^2-d^2)}$

11. The sum of the infinite GP $\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right)$ is: **[1]**

a) $\frac{3}{2}$ b) $\frac{4}{9}$

c) $\frac{5}{9}$ d) $\frac{2}{3}$

12. The lines lx + my + n = 0, mx + ny + l = 0 and nx + ly + m = 0 are concurrent if **[1]**

a) l + m – n = 0 b) l + m + n = 0

c) l – m – n = 0 d) I – m + n = 0

13. $\lim_{x\to 0} \frac{|\sin x|}{x}$ is **[1]**

a) None of these b) -1

c) 1 d) 0

14. Following are the marks obtained by 9 students in a mathematics test: **[1]**
50, 69, 20, 33, 53, 39, 40, 65, 59
The mean deviation from the median is:

a) 9 b) 14.76

c) 10.5 d) 12.67

15. The number of subsets of a set containing n elements is **[1]**

a) 2^n-1 b) 2^n - 2

c) 2^n d) n

16. If f : [1, ∞∞) → [2, ∞∞) is given by f(x) = x + $\frac{1}{x}$ then $f^{-1}(x)$ equals] **[1]**

a) $\frac{x\sqrt{x^2-4}}{2}$

b) $\frac{x+\sqrt{x^2-4}}{2}$

c) $1+\sqrt{x^2-4}$

d) $\frac{x}{1+x^2}$

17. If z $=\frac{-2}{1+i\sqrt{3}}$, then the value of arg (z) is **[1]**

a) $\frac{2\pi}{3}$

b) π

c) $\frac{\pi}{3}$

d) $\frac{\pi}{4}$

18. In a GP, the ratio between the sum of first 3 terms and the sum of first 6 terms is 125 : 152. The common ratio is **[1]**

a) $\frac{1}{2}$

b) $\frac{5}{6}$

c) $\frac{2}{3}$

d) $\frac{3}{5}$

19. A point equidistant from the lines 4x + 3y + 10 = 0, 5x – 12y + 26 = 0 and 7x + 24y – 50 = 0 is **[1]**

a) (0, 0)

b) (1, -1)

c) (1, 1)

d) (0, 1)

20. $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$ is equal to **[1]**

a) $\frac{1}{2}$

b) 2

c) 0

d) -1

Section B

Attempt any 16 questions

21. For a normal distribution, we have **[1]**

a) mean = median

b) mean = mode

c) mean = median = mode

d) median = mode

22. Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second set. Then, the values of m and n are: **[1]**

a) 7, 6

b) 6, 4

c) 6, 3

d) 7, 4

23. The domain of definition of f(x) $=\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$ is **[1]**

a) $(4,\infty)$

b) $(-\infty,4]$

c) $[4,\infty)$

d) $(-\infty,4)$

24. If the roots of x^2 - bx + c = 0 are two consecutive integers, then b^2 - 4c is **[1]**

a) 2

b) 0

c) 1

d) None of these

25. If in an A.P., S_n = qn^2 and S_m = qm^2, where S_r denotes the sum of r terms of the A.P., then S_q equals to: **[1]**

a) $\frac{q^3}{2}$

b) mnq

c) $(m + n)q^2$ | d) q^3

26. $\lim_{x \to 0} \frac{\sin x^0}{x}$ is equal to **[1]**

a) x | b) $\frac{\pi}{180}$
c) 1 | d) π

27. If the mean of the squares of first n natural numbers be 11, then n is equal to **[1]**

a) 5 | b) $\frac{-13}{2}$
c) 11 | d) 13

28. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to **[1]**

a) $F_2 \cap F_3$ | b) $F_3 \cap F_4$
c) $F_2 \cup F_5$ | d) $F_2 \cup F_3 \cup F_4 \cup F_1$

29. Range of $f(x) = \frac{1}{1-2\cos x}$ is **[1]**

a) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$ | b) $\left[-1, \frac{1}{3}\right]$
c) $\left[\frac{1}{3}, 1\right]$ | d) $\left[-\frac{1}{3}, 1\right]$

30. The value of $\left(\frac{1+\omega}{\omega^2}\right)^3$ is **[1]**

a) 1 | b) -1
c) none of these | d) 0

31. If the sum of n terms of a progression be a quadratic expression in n then it is **[1]**

a) a GP | b) None of these
c) an AP | d) an HP

32. $\lim_{x \to 0} \frac{x^2 \cos x}{1-\cos x}$ is equal to **[1]**

a) 2 | b) -3/2
c) 3/2 | d) 1

33. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, the variance of the numbers so obtained is **[1]**

a) 3.87 | b) 8.25
c) 2.87 | d) 6.5

34. $(z+1)(\bar{z}+1)$ can be expressed as **[1]**

a) $|z|^2 + 1$ | b) $|z|^2 + 2$
c) none of these | d) $|z + 1|^2$

35. In a G.P. the ratio of the sum of first three terms to the sum of first six terms is 125 : 152. The common ratio of the G.P. is **[1]**

a) none of these | b) 3.5

c) $\frac{5}{3}$ d) $\frac{3}{5}$

36. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are **[1]**

a) 7, 4 b) 6, 4

c) 3, 3 d) 6, 3

37. If f(x) = cos (log x) then $f\left(x^2\right) f\left(y^2\right)-\frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right)+f\left(x^2 y^2\right)\right\}$ has the value **[1]**

a) -1 b) -2

c) None of these d) 1/2

38. $\frac{1+2i+3i^2}{1-2i+3i^2}$ equals. **[1]**

a) 4 b) -1

c) -i d) i

39. The ratio of the 7^{th} to the (n - 1)th mean between 1 and 31, when n arithmetic means are inserted between them, is 5 : 9. The value of n is **[1]**

a) 15 b) 12

c) 13 d) 14

40. The 6th term from the end of the GP 8, 4, 2, ... $\frac{1}{1024}$ is **[1]**

a) $\frac{1}{64}$ b) $\frac{1}{128}$

c) $\frac{1}{16}$ d) $\frac{1}{32}$

Section C

Attempt any 8 questions

41. Let R be set of points inside a rectangle of sides a and b (a, b > 1) with two sides along the positive direction of x-axis and y-axis. Then **[1]**

a) R = {(x, y) : 0 $\leq$ x $\leq$ a, 0 $\leq$ y $\leq$ b} b) R = {(x, y) : 0 $\leq$ x < a, 0 $\leq$ y $\leq$ b}

c) R = {(x, y) : 0 < x < a, 0 < y < b} d) R = {(x, y) : 0 $\leq$ x $\leq$ a, 0 < y < b}

42. The function f : $R \rightarrow R$ is defined by f (x) = $\sin^4$ x - $\sin^2$ x +1 f,then R(f) = **[1]**

a) (3 / 4 ,1) b) [3 /4 ,1]

c) [3 /4 ,1) d) (3 / 4 ,1]

43. Mark the correct answer for: arg$\left(\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}\right)$ = ? **[1]**

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$

c) $\frac{2\pi}{3}$ d) π

44. If second term of a G.P. is 2 and the sum of its infinite terms is 8, then its first term is **[1]**

a) $\frac{1}{4}$ b) 2

c) $\frac{1}{2}$ d) 4

45. If the mode of a data is 18 and the mean is 24, then median is **[1]**

a) 21

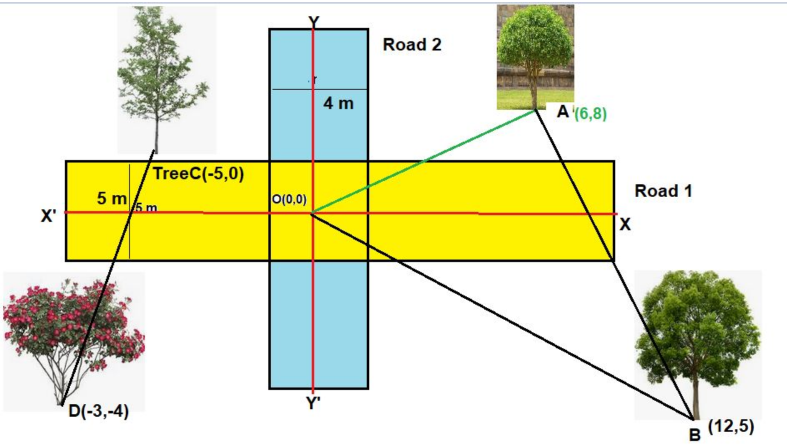
b) 22

c) 24

d) 18

Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:

In a park Road 1 and road 2 of width 5 m and 4 m are crossing at centre point O(0, 0). As shown in the following figure:



For trees A, B, C and D are situated in four quadrants of the Cartesian system of coordinate. The coordinates of the trees A, B, C and D are (6, 8), (12, 5), (-5, 0) and (-3, -4) respectively.

46. What is the distance of Tree C from the Origin? **[1]**

a) 10 m

b) 25 m

c) 5 m

d) 15 m

47. What is the equation of line AB? **[1]**

a) x - 2y = -6

b) x + 2y = 6

c) x + 2y - 22 = 0

d) 2x + y = 22

48. What is the slope of line CD? **[1]**

a) $\frac{3}{2}$

b) $\frac{-2}{1}$

c) $\frac{-1}{2}$

d) $\frac{2}{1}$

49. What is the slope of line OA? **[1]**

a) 1

b) $\frac{6}{8}$

c) $\frac{4}{3}$

d) $\frac{3}{4}$

50. What is the distance of point B from the origin? **[1]**

a) 5 m

b) 12 m

c) 13 m

d) 15 m

Solution

SUBJECT- MATHEMATICS 041 - TEST - 01

Class 11 - Mathematics

Section A

1. **(d)** $\{x : x \in R,\ 4 \leq x < 5\}$
   Explanation: Set A represents the elements which are greater or equals to 4 and the elements are real no. A$[4, \infty)$
   Set B represents the elements which are less than 5 and are real no. B$(-\infty, 5)$
   So if we represent these two in number line we can see the common region is between 4(included) and 5(excluded).
2. **(b)** f $(\pi/2)$ = 1
   Explanation: f $\left(\frac{\pi}{2}\right) = \sin 9\left(\frac{\pi}{2}\right) - \sin 10\left(\frac{\pi}{2}\right)$

   = 1-0
   =1
3. **(a)** Im (z) = 0
   Explanation: $\frac{\sqrt{3}+i}{2} = r(cos\theta + isin\theta) \Rightarrow rcos\theta = \frac{\sqrt{3}}{2}, rsin\theta = \frac{1}{2}$
   $\therefore r^2(\cos^2\theta + \sin^2\theta) = \frac{3+1}{4} = 1 \Rightarrow r^2 = 1 \Rightarrow r = 1$
   $cos\theta = \frac{\sqrt{3}}{2}, \quad sin\theta = \frac{1}{2}$
   $\therefore Amplitude = \theta = \frac{\Pi}{6}$
   $\frac{\sqrt{3}+i}{2} = 1\left(cos\frac{\Pi}{6} + isin\frac{\Pi}{6}\right) = e^{\frac{i\Pi}{6}} \ldots\ldots\ldots\ldots(i)$
   Since $\frac{\sqrt{3}-i}{2}$ lies in the fourth quadrant,the amplitude=$= -\theta = -\frac{\Pi}{6}$
   $\frac{\sqrt{3}-i}{2} = 1\left(cos\frac{-\Pi}{6} + isin\frac{-\Pi}{6}\right) = e^{\frac{-i\Pi}{6}} \ldots\ldots\ldots\ldots(i)$
   $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$
   $\Rightarrow z = \left(e^{\frac{i\Pi}{6}}\right)^5 + \left(e^{\frac{-i\Pi}{6}}\right)^5 = e^{\frac{i5\Pi}{6}} + e^{-\frac{i5\Pi}{6}} = cos\left(\frac{5\Pi}{6}\right) + isin\left(\frac{5\Pi}{6}\right) + cos\left(\frac{-5\Pi}{6}\right) + isin\left(\frac{-5\Pi}{6}\right) = 2cos\left(\frac{5\Pi}{6}\right)$
   which is purely real [imaginary part of z=0]
4. **(a)** 4n + 3
   Explanation: It is given in the question,
   $S_n = 2n^2 + 5n$
   $S_1 = 2.1^2 + 5.1 = 7$
   $\therefore a_1 = 7$
   $S_n = 2.2^2 + 5.2 = 18$
   $\therefore a_1 + a_2 = 18$
   $\Rightarrow a_2 = 11$
   Common difference, d = 11 - 7 = 4
   a_n = a + (n - 1) d
   =7 + (n - 1) 4
   = 4n + 3
5. **(c)** $\sqrt{2}$
   Explanation: Consider the equation $x^2 + 2xy + y^2 = 0$
   On factorizing we get,
   (x + y)(x + y) = 0
   Hence the equation of the parallel lines is x + y + l = 0 and x + y + m = 0
   Now equating the coefficents of like terms for x and y with the combined equation
   l + m = -6 and lm = 8
   l + $(\frac{8}{l})$ = -6
   $l^2 + 6l + 8 = 0$
   on solving we get
   l = -4 or l = 2
   Therefore m = -2 or 4

Hence the distance between these two parallel lines is

$\frac{|4-2|}{\sqrt{1^2+1^2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

6. **(c)** $\frac{1}{2}$

Explanation: $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

$= \lim_{x\to 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)x}$

$= \lim_{x\to 0} \frac{1+x-1}{x\sqrt{1+x+1}}$

$= \lim_{x\to 0} \frac{x}{x(\sqrt{1+x}+1)}$

$= \frac{1}{2}$

7. **(b)** $\sqrt{\frac{(x_i-\bar{x})^2}{n}}$

Explanation: We know, standard deviation for $x_1, x_2, \ldots, x_n$ observations can be written as

$\sigma = \sqrt{\frac{1}{n}\sum_{i=0}^{n}(x_i-\bar{x})^2}$

Where $\bar{x}$ is the arithmetic mean

8. **(a)** B = C

Explanation: A ∪ B = A ∪ C

⇒ (A ∪ B) ∩ C = (A ∪ C) ∩ C

⇒ (A ∩ C) ∪ (B ∩ C) = C .....(i)

Now again A ∪ B = A ∪ C

⇒ (A ∪ B) ∩ B = (A ∪ C) ∩ B

⇒ B = (A ∩ B) ∪ (C ∩ B)

⇒ B = (A ∩ C) ∪ (C ∩ B), Since (A ∩ B) = (A ∩ C)

⇒ (A ∩ C) ∪ (B ∩ C) = B .....(ii)

Now from (i) and (ii) we get B = C

9. **(b)** $f(a^2)$

Explanation: $\frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a+1}{a+1-1}} = \frac{\frac{a}{a-1}}{\frac{a+1}{a}}$

$= \frac{a}{a-1} \times \frac{a}{a+1} = \frac{a^2}{a^2-1}$

$f(a^2) = \frac{a^2}{a^2-1}$

$\therefore \frac{f(a)}{f(a+1)} = f(a^2)$

10. **(b)** $\frac{(a^2+b^2)}{(c^2+d^2)}$

Explanation: $(x + iy) = \left(\frac{a+ib}{c+id}\right) \Rightarrow |x + iy| = \left|\frac{a+ib}{c+id}\right| = \frac{|a+ib|}{|c+id|}$

$\Rightarrow |x + iy|^2 = \frac{|a+ib|^2}{|c+id|^2} \Rightarrow (x^2 + y^2) = \frac{(a^2+b^2)}{(c^2+d^2)}$

11. **(a)** $\frac{3}{2}$

Explanation: Here, we have $a_1 = 1$ and $a_2 = \frac{1}{3}$ and $r = \frac{a_2}{a_1} = \frac{1}{3}$

$\therefore S_\infty = \frac{a}{(1-r)} = \frac{1}{\left(1-\frac{1}{3}\right)} = \frac{3}{2}.$

12. **(b)** l + m + n = 0

Explanation: The required condition for concurrency is $a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$

Here $a_1 = l$, $a_2 = m$, $a_3 = n$ and $b_1 = m$, $b_2 = n$, $b_3 = l$ and $c_1 = n$, $c_2 = l$ and $c_3 = m$

Substituting the values we get

$n(ml - n^2) + l(nm - l^2) + m(ln - m^2) = 0$

This implies $l^3 + m^3 + n^3 - 3lmn = 0$

That is $(l + m + n)(l^2 + m^2 + n^2 - lm - mn - nl) = 0$

This implies l + m + n = 0

13. **(a)** None of these

Explanation: We have,

$$|\sin x| = \begin{cases} \sin x, & 0 \le x \le \frac{\pi}{2} \\ -\sin x, & -\frac{\pi}{2} \le x < 0 \end{cases}$$

Now,

$$\lim_{x \to 0^-} \frac{|\sin x|}{x} = \lim_{x \to 0} \frac{-\sin x}{x} = -\lim_{x \to 0} \frac{\sin x}{x} = -1$$

$$\lim_{x \to 0^+} \frac{|\sin x|}{x} = \lim_{x \to 0} \frac{\sin x}{x} = 1$$

Clearly,

$$\lim_{x \to 0^-} \frac{|\sin x|}{x} \neq \lim_{x \to 0^+} \frac{|\sin x|}{x}$$

$\therefore \lim_{x \to 0} \frac{|\sin x|}{x}$ does not exist.

14. **(d)** 12.67

Explanation: Given the marks obtained by 9 students in a mathematics test are 50, 69, 20, 33, 53, 39, 40, 65, 59

As number of students = 9, which is odd.

So median will be $\frac{9+1}{2}$ = 5^{th} term.

Arranging these in ascending order, we get

20, 33, 39, 40, 50, 53, 59, 65, 69

So the 5^{th} term after arranging is 50,

So median is 50.

This can be written in table form as,

Marks (x_i)	**d_i = |x_i = median|**
20	= |20 - 50| = 30
33	= |33 - 50| = 17
39	= |39 - 50| = 11
40	= |40 - 50| = 10
50	= |50 - 50| = 0
53	= |53 - 50| = 3
59	= |59 - 50| = 9
65	= |65 - 50| = 15
69	= |69 - 50| = 19
Total	$\sum d_i$ = 114

Hence Mean Deviation becomes,

M.D = $\frac{\sum d_i}{n} = \frac{114}{5}$ = 12.67

Therefore, the mean deviation about the median of the marks of 9 subjects is 12.67

15. **(c)** 2^n

Explanation: 2^n

The total number of subsets of a finite set consisting of n elements is 2^n.

16. **(b)** $\frac{x+\sqrt{x^2-4}}{2}$

Explanation: Let y = f(x), then

y = x + $\frac{1}{x} \Rightarrow$ y = $\frac{x^2+1}{x}$

$\Rightarrow x^2 + 1 = xy \Rightarrow x^2 - xy + 1 = 0$

$\Rightarrow$ x = $\frac{y \pm \sqrt{y^2-4}}{2}$

$\Rightarrow f^{-1}(y) = \frac{y \pm \sqrt{y^2-4}}{2}$ (negative sign is rejected)

$f^{-1}(x) = \frac{x+\sqrt{x^2-4}}{2}$

17. **(a)** $\frac{2\pi}{3}$

Explanation: $\frac{2\pi}{3}$

$z = \frac{-2}{1+i\sqrt{3}}$

Rationalising z, we get

$z = \frac{-2}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$
$\Rightarrow z = \frac{-2+i2\sqrt{3}}{1+3}$
$\Rightarrow z = \frac{-1+i\sqrt{3}}{2}$
$\Rightarrow z = \frac{-1}{2} + \frac{i\sqrt{3}}{2}$
$\tan\alpha = \left|\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right|$
$= \sqrt{3}$
$\Rightarrow \alpha = \frac{\pi}{3}$
since, z lies in the second quadrant.
Therefore, arg (z) $= \pi - \frac{\pi}{3}$
$= \frac{2\pi}{3}$

18. **(d)** $\frac{3}{5}$
Explanation: $S_3 = \frac{a(r^3-1)}{(r-1)}$ and $S_6 = \frac{a(r^6-1)}{(r-1)}$.
$\therefore \frac{S_3}{S_6} = \frac{125}{152} \Rightarrow \frac{a(r^3-1)}{(r-1)} \times \frac{(r-1)}{a(r^6-1)} = \frac{125}{152}$
$\Rightarrow \frac{1}{(r^3+1)} = \frac{125}{152}$
$\Rightarrow$ 125r^3 + 125 = 152 $\Rightarrow$ 125r^3 = (152 -125) = 27
$\Rightarrow r^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3 \Rightarrow r = \frac{3}{5}$
$\therefore$ the required common ratio is $\frac{3}{5}$.

19. **(a)** (0, 0)
Explanation: We note that distance of each of three lines from (0, 0) is 2 units

20. **(a)** $\frac{1}{2}$
Explanation: Substitute x = $\frac{1}{t}$
$\Rightarrow \lim_{t\to 0} \frac{\sqrt{t^2+t+1}-1}{t}$
Using L' Hospital
$\lim_{t\to 0} \frac{\frac{2t+1}{2\sqrt{t^2+t+1}}}{1}$
= $\frac{1}{2}$

Section B

21. **(c)** mean = median = mode
Explanation: As in normal distribution, the curve is symmetric and unimodal. So, mean is at peak, mode is also at peak and median as well.

22. **(b)** 6, 4
Explanation: Let A and B be the set which contain m and n elements respectively.
Then n (P(A)) = 2^m and n (P(B)) = 2^n
Also given that, n (P(A)) = n (P(B)) +48
$\Rightarrow$ 2^m = 2^n + 48
Therefore, Above equation is only true when m = 6 and n = 4

23. **(c)** $[4, \infty)$
Explanation: Here, $x - 3 - 2\sqrt{x-4} \geq 0$
$(\sqrt{x-4})^2 + 1 - 2\sqrt{x-4} \geq 0$
$(\sqrt{x-4} - 1)^2 \geq 0$
$x - 4 \geq 0; x \geq 4$
$x - 3 + 2\sqrt{x-4} \geq 0$
$(\sqrt{x-4})^2 + 1 + 2\sqrt{x-4} \geq 0$
$(\sqrt{x-4} + 1)^2 \geq 0$
$x \geq 4$

24. **(c)** 1
Explanation: Given equation:
x^2 - bx + c = 0
Let α and $\alpha + 1$ be the two consective roots of the equation.
Sum of the roots = $\alpha + \alpha + 1$ = $2\alpha + 1$

Product of the roots = $\alpha(\alpha+1) = \alpha(\alpha+1)$

So, sum of the roots = $2\alpha+1 = \frac{-\text{ Coeffecient of } x}{\text{Coeffecient of } x^2} = \frac{b}{1} = b$

Product of the roots = $\alpha^2+\alpha = \frac{\text{Constant term}}{\text{Coeffecient of } x^2} = \frac{c}{1} = c$

Now, $b^2 - 4c = (2\alpha+1)^2 - 4\left(\alpha^2+\alpha\right) = 4\alpha^2+4\alpha+1-4\alpha^2-4\alpha = 1$

25. **(d)** q^3

Explanation: The given series is A.P whose first term is ‘a’ and common difference is ‘d’.

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow qn^2 = \frac{n}{2}[2a + (n - 1)d]$ [$\because S_n = qn^2$]

$\Rightarrow 2qn = 2a + (n – 1)d$

$\Rightarrow 2qn – (n – 1)d = 2a$ ...(i)

and $S_m = \frac{m}{2}[2a + (m - 1)d]$

$\Rightarrow qm = \frac{m}{2}\{2\}[2a + (m - 1)d]$ [$\because S_m = qm^2$]

$\Rightarrow 2qm = 2a + (m - 1) d$

$\Rightarrow 2qm – (m – 1)d = 2a$ ...(ii)

Solving eq. (i) and (ii), we get

$2qn – (n – 1)d = 2qm – (m – 1)d$

$\Rightarrow 2qn – 2qm = (n – 1)d – (m – 1)d$

$\Rightarrow 2q(n – m) = d[n – 1 – (m – 1)]$

$\Rightarrow 2q(n – m) = d[n – 1 – m + 1]$

$\Rightarrow 2q(n – m) = d(n – m)$

$\Rightarrow 2q = d$

Putting the value of d in eq. (i), we obtain

$2qn – (n – 1)(2q) = 2a$

$\Rightarrow 2qn – 2qn + 2q = 2a$

$\Rightarrow 2q = 2a$

$\Rightarrow q = a$

$\therefore$ a = q and d = 2q. So,

$S_q = \frac{q}{2}[2a + (q - 1)d]$

$\Rightarrow S_q = \frac{q}{2}[2q + (q - 1)2q]$

$\Rightarrow S_q = \frac{2q^2}{2} + \frac{2q^2(q-1)}{2}$

$\Rightarrow S_q = q^2 + q^2(q – 1)$

$\Rightarrow S_q = q^2 + q^3 – q^2$

$\Rightarrow S_q = q^3$

Therefore, the correct option is q^3.

26. **(b)** $\frac{\pi}{180}$

Explanation: $\lim\limits_{x\to0} \frac{\sin x^0}{x}$

$= \lim\limits_{x\to0} \frac{\sin \frac{\pi}{180} x}{x}$

$= \lim\limits_{x\to0} \frac{\sin\left(\frac{\pi}{180} x\right)}{\left(\frac{\pi}{180} x\right)} \times \frac{\pi}{180}$

$= \frac{\pi}{180} \times 1 = \frac{\pi}{180}$

27. **(a)** 5

Explanation: Mean = $\frac{\frac{n(n+1)(2n+1)}{6}}{n} = \frac{(n+1)(2n+1)}{6}$

$\Rightarrow 11 = \frac{(n+1)(2n+1)}{6}$

$\Rightarrow 66 = (n + 1)(2n + 1)$

$\Rightarrow 2n^2 + 3n - 65 = 0$

$\Rightarrow 2n^2 + 13n - 10n - 65 = 0$

$\Rightarrow (2n + 13)(n - 5) = 0$

$\Rightarrow n = 5, \frac{-13}{2}$

So, n = 5

28. **(d)** $F_2 \cup F_3 \cup F_4 \cup F_1$

Explanation: We know that
Every rectangle, square and rhombus is a parallelogram
But, no trapezium is a paralleogrm
Thus, $F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$

29. **(a)** $(-\infty,-1] \cup \left[\frac{1}{3},\infty\right)$

Explanation: We know that, $-1 \leq \cos x \leq 1$
$\Rightarrow -1 \leq -\cos x \leq 1$
$\Rightarrow -2 \leq -2\cos x \leq 2$
$\Rightarrow -1 \leq 1-2\cos x \leq 3$
Now f(x) $= \frac{1}{1-2\cos x}$ is defined if
$-1 \leq 1-2\cos x < 0$ or $0 < 1-2\cos x \leq 3$
$\Rightarrow -1 \geq \frac{1}{1-2\cos x} > -\infty$ or $\infty > \frac{1}{1-2\cos x} \geq \frac{1}{3}$
$\Rightarrow \frac{1}{1-2\cos x} \in (-\infty,-1] \cup \left[\frac{1}{3},\infty\right)$

30. **(b)** -1

Explanation: $\left(\frac{1+\omega}{\omega^2}\right)^3 = \left(\frac{-\omega^2}{\omega^2}\right)^3 = (-1)^3 = -1$ [$\because 1+\omega+\omega^2 = 0$]

31. **(c)** an AP

Explanation: Let $S_n = an^2 + bn + c$. Then,
$S_{n-1} = a(n-1)^2 + b(n-1) + c$
$T_n = (S_n - S_{n-1}) = a[n^2 - (n-1)^2] + b[n-(n-1)] = a(2n-1) + b$
$= 2an + (b-a)$, which is a linear expression in n.
Therefore, the given progression is an AP.

32. **(a)** 2

Explanation: Given $\lim\limits_{x\to 0}\frac{x^2\cos x}{1-\cos x} = \lim\limits_{x\to 0}\frac{x^2\cos x}{2\sin^2\frac{x}{2}}\left[\because 1-\cos x = 2\sin^2\frac{x}{2}\right]$

$= \lim\limits_{x\to 0}\frac{\frac{x^2}{4}\times 4\cos x}{2\sin^2\frac{x}{2}} = \lim\limits_{\frac{x}{2}\to 0}\frac{\left(\frac{x}{2}\right)^2\cdot 2\cos x}{\sin^2\frac{x}{2}}$

$= \lim\limits_{\frac{x}{2}\to 0}\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2\cdot 2\cos x$

$= 2\cos 0 = 2 \times 1 = 2$ $\left[\because \lim\limits_{x\to 0}\frac{x}{\sin x} = 1\right]$

33. **(b)** 8.25

Explanation: First 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
on multiplying each number by – 1, we get
- 1, -2, -3, -4, -5, -6, -7, -8, -9, -10
on adding 1 to each of the number, we get
0, - 1, -2, -3, -4, -5, -6, -7, -8, -9
$\therefore \sum x_i = 0 -1 -2 -3 -4 -5 -6 -7 -8 -9 = -45$
and
$\sum x_i^2 = 0^2 + (-1)^2 + (-2)^2 + (-3)^2 + (-4)^2 + \ldots + (-9)^2$
But we know $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$, so the above equation on applying this formula when n = 9, we get
$\Sigma x_i^2 = \frac{9(9+1)(2(9)+1)}{6} = \frac{9\times 10\times 19}{6} = 285$
Now we know,
$\sigma = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$
Substituting the corresponding values, we get
$\sigma = \sqrt{\frac{285}{10} - \left(\frac{-45}{10}\right)^2}$
$\sigma = \sqrt{28.5 - 20.25}$
$\sigma = \sqrt{28.5 - 20.25}$
$\sigma = \sqrt{8.25}$
Now for variance we will square on both sides, we get

σ^2 = 8.25
Hence the variance of the numbers so obtained is 8.25

34. **(d)** $|z + 1|^2$
Explanation: We have $z\bar{z} = |z|^2$
Now $(z+1)(\bar{z}+1) = (z+1)\left(\overline{z+1}\right)$ = $|z + 1|^2$

35. **(d)** $\frac{3}{5}$
Explanation: Given $\frac{S_3}{S_6} = \frac{125}{152}$
$\Rightarrow \frac{\frac{a\left(r^3-1\right)}{r-1}}{\frac{a\left(r^6-1\right)}{r-1}} = \frac{125}{152}, r-1 \neq 0$
$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152}$
$\Rightarrow$ $152r^3 - 15^2 = 125r^6 - 125$
$\Rightarrow$ $125r^6 - 152r^3 + 27 = 0$
$\Rightarrow$ $125r^6 - 125r^3 - 27r^3 + 27 = 0$
$\Rightarrow$ $125r^3(r^3 - 1) - 27(r^3 - 1) = 0$
$\Rightarrow$ $(125r^3 - 27)(r^3 - 1) = 0$
$\Rightarrow$ $r^3 = \frac{27}{125}$ or $r^3 = 1$
Since $r - 1 \neq 0$, r cannot be 1
$\Rightarrow$ $r = \frac{3}{5}$

36. **(d)** 6, 3
Explanation: Let A and B be two sets having m and n elements respectively. Then,
Number of subsets of A = 2^m, Number of subsets of B = 2^n
It is given that $2^m - 2^n = 56$
So, $2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$
n = 3 and m - n = 3 $\Rightarrow$ n = 3 and m = 6.

37. **(c)** None of these
Explanation: f(x) = cos (log x)
Now, $f\left(x^2\right) f\left(y^2\right) - \frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right) + f\left(x^2y^2\right)\right\}$
= cos (log x^2) cos (logy^2) - $\frac{1}{2}\left\{\cos\left(\log\left(\frac{x^2}{y^2}\right)\right) + \cos\left(\log x^2y^2\right)\right\}$
= $\cos(2\log x)\cos(2\log y) - \frac{1}{2}\left\{\cos\left(\log x^2 - \log y^2\right) + \cos\left(\log x^2 + \log y^2\right)\right\}$
$= \cos(2\log x)\cos(2\log y) - \frac{1}{2}$\{ cos(2 log x - 2 log y) + cos (2 log x + log y)\}
using cosx cos y = 1/2 cos (x + y) + cos (x - y)
= cos (2 log x) cos (2 log y) - cos (2 log x) cos(2 log y)
= 0

38. **(c)** -i
Explanation: -i
Let z $= \frac{1+2i+3i^2}{1-2i+3i^2}$
$\Rightarrow z = \frac{1+2i-3}{1-2i-3}$
$\Rightarrow z = \frac{-2+2i}{-2-2i} \times \frac{-2+2i}{-2+2i}$
$\Rightarrow z = \frac{(-2+2i)^2}{(-2)^2-(2i)^2}$
$\Rightarrow z = \frac{4+4i^2-8i}{4+4}$
$\Rightarrow z = \frac{4-4-8i}{8}$
$\Rightarrow z = \frac{-8i}{8}$
$\Rightarrow$ z = -i

39. **(d)** 14
Explanation: Let $A_1, A_2, A_3, A_4 ..., A_n$ be the n arithmetic means inserted between 1 and 31.
The we have 1, $A_1, A_2, A_3, A_4 ..., A_n$, 31 is an A.P with a = 1, T_n = 31 and number of terms = n + 2
Now, $T_n = 31 \Rightarrow 1 + [(n + 2) - 1]d = 31$

$\Rightarrow$ d = $\frac{30}{n+1}$

Hence we get T_7 = a + 7d = 1 + 7$\left[\frac{30}{n+1}\right]$ ...(i)

And T_{n-1} = a(n - 1) + d = 1 + (n - 1)$\left[\frac{30}{n+1}\right]$ ...(ii)

Given $\frac{T_7}{T_{n-1}} = \frac{5}{9}$

$\Rightarrow \frac{1+7\left[\frac{30}{n+1}\right]}{1+(n-1)\left[\frac{30}{n+1}\right]} = \frac{5}{9}$

$\Rightarrow \frac{\frac{n+1+210}{n+1}}{\frac{31n-29}{n+1}} = \frac{5}{9}$

$\Rightarrow$ 9n + 1899 = 155n - 145

$\Rightarrow$ 2044 = 146 n

$\Rightarrow$ n = $\frac{2044}{146}$ = 14

40. **(d)** $\frac{1}{32}$

Explanation: Given GP is 8, 4, 2 , ... , $\frac{1}{1024}$

Here, we have r = $\frac{4}{8}$ = $\frac{1}{2}$ and l = $\frac{1}{1024}$

$\therefore$ 6th term from the end = $\frac{l}{r^{(6-1)}} = \frac{l}{r^5} = \frac{1}{1024} \cdot \frac{1}{\left(\frac{1}{2}\right)^5} = \frac{2^5}{1024} = \frac{32}{1024} = \frac{1}{32}.$

Section C

41. **(c)** R = {(x, y) : 0 < x < a, 0 < y < b}

Explanation: We have, R be set of points inside a rectangle of sides a and b

Since, a, b > 1

a and b cannot be equal to 0

Thus,R = {(x, y) : 0 < x < a, 0 < y < b}

42. **(b)** [3 /4 ,1]

Explanation: f (x) = $\sin^4$ x +1 - $\sin^2$ x

$f(x) = \sin^4 x - \sin^2 x + \frac{1}{4} - \frac{1}{4} + 1$

$f(x) = \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\left(\sin^2 x - \frac{1}{2}\right)^2 \geq 0$

Minimum value of f(x) = 3/4

0 < $\sin^2$ x < 1

So, maximum value of f(x) = $\left(1 - \frac{1}{2}\right)^2 + \frac{3}{4}$

$= \frac{1}{4} + \frac{3}{4}$

= 1

R(f) =[3/4,1]

43. **(a)** $\frac{\pi}{3}$

Explanation: $\left(\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}\right)$ = $\frac{(2+6\sqrt{3}i)}{(5+\sqrt{3}i)} \times \frac{(5-\sqrt{3}i)}{(5-\sqrt{3}i)}$ = $\frac{(2+6\sqrt{3}i)(5-\sqrt{3}i)}{(5+\sqrt{3}i)(5-\sqrt{3}i)}$

= $\frac{(28+28\sqrt{3}i)}{28}$ = $\frac{28(1+\sqrt{3}i)}{28}$ = (1 +$\sqrt{3}$) = $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\therefore$ arg$\left(\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}\right)$ = $\frac{\pi}{3}$

44. **(d)** 4

Explanation: According to the question , we can write,

a^2 = 2

$\therefore$ ar = 2 .... (i)

Also, S_∞ = 8

$\Rightarrow \frac{a}{(1-r)}$ = 8

$\Rightarrow \frac{a}{\left(1-\frac{2}{a}\right)}$ = 8 [Using (i)]

$\Rightarrow a^2$ = 8(a - 2)

$\Rightarrow a^2$ - 8a + 16 = 0

$\Rightarrow (a - 4)^2$ = 0

$\Rightarrow$ a = 4

45. **(b)** 22

Explanation: Using the formula, 3 median = mode + 2 mean

Median = $\frac{18+2(24)}{3}$ = $\frac{66}{3}$ = 22

46. **(c)** 5 m

Explanation: 5 m

47. **(c)** x + 2y - 22 = 0

Explanation: x + 2y - 22 = 0

48. **(b)** $\frac{-2}{1}$

Explanation: $\frac{-2}{1}$

49. **(c)** $\frac{4}{3}$

Explanation: $\frac{4}{3}$

50. **(c)** 13 m

Explanation: 13 m