SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)#	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)*	–	**3(9)**
7.	Integrals	1(1)*	1(2)	1(3)	–	**3(6)**
8.	Application of Integrals	1(1)	1(2)	1(3)	–	**3(6)**
9.	Differential Equations	1(1)*	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	3(3)	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)*	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2) + 1(4)	1(2)*	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.

2. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.

3. Both Part-A and Part-B have internal choices.

Part - A :

1. It consists of two Sections-I and II.

2. Section-I comprises of 16 very short answer type questions.

3. Section-II contains 2 case study-based questions.

Part - B :

1. It consists of three Sections-III, IV and V.

2. Section-III comprises of 10 questions of 2 marks each.

3. Section-IV comprises of 7 questions of 3 marks each.

4. Section-V comprises of 3 questions of 5 marks each.

5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section - I

1. If the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, then find x.

OR

What positive value of x makes the following pair of determinants equal ?

$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

2. If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that $2A + X = 5B$.

3. Determine the order and degree of differential equation $\dfrac{d^3x}{dt^3} + \dfrac{d^2x}{dt^2} + \left(\dfrac{dx}{dt}\right)^2 = e^t$.

OR

What is the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} + 4 - \frac{3dy}{dx} = 0$?

4. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, then find the range of R.

5. If a line makes angles 90°, 60° and 30° with the positive directions of x, y and z-axis respectively, then find its direction cosines.

OR

Find the direction cosines of the line passing through two points (2, 1, 0) and (1, –2, 3).

6. Find the area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant.

7. Evaluate: $\int xe^{x^2}\, dx$

OR

Evaluate : $\int \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3} dx$

8. If α, β, γ are the angles made by a line with the co-ordinate axes. Then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

9. Check whether the relation $R = \{(1, 1), (2, 2)\ (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is an equivalence relation or not.

OR

Find the domain of the function $f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}}$ where $\{\cdot\}$ denotes fractional part.

10. Find the value of $(\vec{a}\cdot\hat{i})^2 + (\vec{a}\cdot\hat{j})^2 + (\vec{a}\cdot\hat{k})^2$.

11. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find $P(A \cap B)$.

12. Find the value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

13. Let $f : N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$. Find whether the function f is bijective or not.

14. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cup B) = \frac{12}{13}$, then evaluate $P(A|B)$.

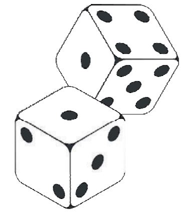
15. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then write the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

16. Write a 3 × 2 matrix whose elements in the i^{th} row and j^{th} column are given by $a_{ij} = \frac{(2i - j)}{2}$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Arun and Richa decided to play with dice to keep themselves busy at home as their schools are closed due to coronavirus pandemic. Arun throw a dice repeatedly until a six is obtained. He denote the number of throws required by X. Based on this information, answer the following questions.



(i) The probability that $X = 3$ equals

(a) $\frac{1}{6}$ (b) $\frac{5^2}{6^3}$ (c) $\frac{5}{3^6}$ (d) $\frac{1}{6^3}$

(ii) The probability that $X = 5$ equals

(a) $\frac{1}{6^4}$ (b) $\frac{1}{6^6}$ (c) $\frac{5^4}{6^5}$ (d) $\frac{5}{6^4}$

(iii) The probability that $X \geq 3$ equals

(a) $\frac{25}{216}$ (b) $\frac{1}{36}$ (c) $\frac{5}{36}$ (d) $\frac{25}{36}$

(iv) The value of $P(X > 3) + P(X \geq 6)$ is

(a) $\frac{5^3}{6^5}$ (b) $1 - \frac{5^3}{6^5}$ (c) $\frac{5^3 \times 61}{6^5}$ (d) $\frac{5^3}{6^4}$

(v) The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{36}{25}$ (b) $\frac{5^2}{6^2}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

18. Peter's father wants to construct a rectangular garden using a rock wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 100 ft of wire fencing. Based on the above information, answer the following questions.

(i) To construct a garden using 100 ft of fencing, we need to maximise its

(a) volume (b) area (c) perimeter (d) length of the side

(ii) If x denote the length of side of garden perpendicular to rock wall and y denote the length of side parallel to rock wall, then find the relation representing total amount of fencing wall.

(a) $x + 2y = 100$ (b) $x + 2y = 50$ (c) $y + 2x = 100$ (d) $y + 2x = 50$

(iii) Area of the garden as a function of x i.e., $A(x)$ can be represented as

(a) $100 + 2x^2$ (b) $x - 2x^2$ (c) $100x - 2x^2$ (d) $100 - x^2$

(iv) Maximum value of $A(x)$ occurs at x equals

(a) 25 ft (b) 30 ft (c) 26 ft (d) 31 ft

(v) Maximum area of garden will be

(a) 1200 sq. ft (b) 1000 sq. ft (c) 1250 sq. ft (d) 1500 sq. ft

PART - B

Section - III

19. Evaluate : $\int_{2}^{4} \frac{(x^2+x)}{\sqrt{2x+1}} dx$

20. The equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

OR

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

21. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

22. Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

23. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

24. Determine the value of 'k' for which the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

25. Solve the differential equation :

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

26. Find $(AB)^{-1}$, if $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

27. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

28. Prove that the points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$.

OR

If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then find the projection of $\vec{b}$ on $\vec{a}$.

Section - IV

29. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto.

30. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

31. Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$.

OR

Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing (b) strictly decreasing

32. Evaluate : $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

33. For what value of a is the function f defined by

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0 \\ \frac{\tan x-\sin x}{x^3}, & x>0 \end{cases}$$

is continuous at $x = 0$?

34. Solve the following differential equation : $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0$

OR

Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

35. Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

Section - V

36. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$x + 2y - 3z = 6,\ 3x + 2y - 2z = 3$

$2x - y + z = 2$

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$

37. A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinates axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

OR

Find the distance between the lines l_1 and l_2 given by

$l_1 : \vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

$l_2 : \vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k}).$

38. Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints :

$2x + 4y \le 8,\ 3x + y \le 6,\ x + y \le 4;\ x \ge 0,\ y \ge 0$

OR

Maximise $z = 8x + 9y$ subject to the constraints :

$2x + 3y \le 6,\ 3x - 2y \le 6,\ y \le 1;\ x, y \ge 0$

SOLUTIONS

1. Given that matrix A is singular $\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow 1(-6-2) + 2(-3-x) + 3(2-2x) = 0$
$\Rightarrow -8 - 6 - 2x + 6 - 6x = 0$
$\Rightarrow -8x - 8 = 0 \Rightarrow x = -1$

OR

We have, $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

$\Rightarrow 2x^2 - 15 = 32 - 15 \Rightarrow 2x^2 = 32 \Rightarrow x^2 = 16$
$\Rightarrow x = 4 \; [\because x > 0]$

2. We have, $2A + X = 5B$
$\Rightarrow X = 5B - 2A$

$$\Rightarrow X = \left(5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 10-16 & -10-0 \\ 20-8 & 10+4 \\ -25-6 & 5-12 \end{bmatrix} = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

3. Given differential equation is of order 3 and degree 1.

OR

We have, $\left(\dfrac{d^2y}{dx^2}\right)^{2/3} = 3\dfrac{dy}{dx} - 4$

$\Rightarrow \left(\dfrac{d^2y}{dx^2}\right)^2 = \left(3\dfrac{dy}{dx} - 4\right)^3$ (On cubing both sides)

$\therefore$ Degree is 2.

4. Here, $R = \{(x, y) : x + 2y = 8\}$, where $x, y \in N$.
For $x = 1, 3, 5, \ldots$; $x + 2y = 8$ has no solution in N.
For $x = 2$, we have $2 + 2y = 8 \Rightarrow y = 3$
For $x = 4$, we have $4 + 2y = 8 \Rightarrow y = 2$
For $x = 6$, we have $6 + 2y = 8 \Rightarrow y = 1$
For $x = 8, 10, \ldots$; $x + 2y = 8$ has no solution in N.
$\therefore$ Range of $R = \{1, 2, 3\}$

5. Let the direction cosines of the line be l, m, n. Then,

$l = \cos 90° = 0,\; m = \cos 60° = \dfrac{1}{2}$ and $n = \cos 30° = \dfrac{\sqrt{3}}{2}$

So, direction cosines are $< 0, \dfrac{1}{2}, \dfrac{\sqrt{3}}{2} >$.

OR

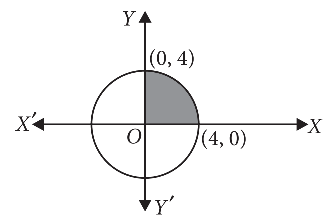
Here, $P(2, 1, 0)$ and $Q(1, -2, 3)$

So, $PQ = \sqrt{(1-2)^2 + (-2-1)^2 + (3-0)^2}$

$= \sqrt{1+9+9} = \sqrt{19}$

Thus, the direction cosines of the line joining two points are $< \dfrac{1-2}{\sqrt{19}}, \dfrac{-2-1}{\sqrt{19}}, \dfrac{3-0}{\sqrt{19}} >$ i.e., $< \dfrac{-1}{\sqrt{19}}, \dfrac{-3}{\sqrt{19}}, \dfrac{3}{\sqrt{19}} >$

6. Given curve is a circle with centre (0, 0) and radius 4.



$\therefore$ Required area

$= \displaystyle\int_0^4 \sqrt{16 - x^2}\, dx$

$= \left[\dfrac{x}{2}\sqrt{16-x^2} + \dfrac{16}{2}\sin^{-1}\dfrac{x}{4}\right]_0^4 = 4\pi$ sq. units

7. Let $I = \int x e^{x^2} dx$

Put $x^2 = t \Rightarrow 2x dx = dt \Rightarrow x\,dx = \dfrac{dt}{2}$

$\therefore \; I = \dfrac{1}{2}\int e^t dt = \dfrac{e^t}{2} + C = \dfrac{e^{x^2}}{2} + C$

OR

Let $I = \displaystyle\int \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3} dx = \int \frac{\cos^2(x/2) - \sin^2(x/2)}{\{\cos(x/2) + \sin(x/2)\}^3} dx$

$= \displaystyle\int \frac{\cos(x/2) - \sin(x/2)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} dx$

Put $t = \cos\dfrac{x}{2} + \sin\dfrac{x}{2} \Rightarrow dt = \dfrac{1}{2}\left(-\sin\dfrac{x}{2} + \cos\dfrac{x}{2}\right)dx$

$\therefore I = 2\displaystyle\int \frac{dt}{t^2} = \frac{-2}{t} + C = \frac{-2}{\cos(x/2) + \sin(x/2)} + C$

8. $\because \alpha, \beta$ and γ are the angles made by line with the co-ordinate axes.
$\therefore \; \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

9. Reflexive : $(1, 1), (2, 2), (3, 3) \in R$, R is reflexive
Symmetric : $(1, 2) \in R$ but $(2, 1) \notin R$, R is not symmetric.
Transitive : $(1, 2) \in R$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$, R is transitive.

Since, R is not symmetric. So, R is not an equivalence relation.

OR

$$f(x)=\frac{1}{\sqrt{\{\sin x\}+\{\sin(\pi+x)\}}}=\frac{1}{\sqrt{\{\sin x\}+\{(-\sin x)\}}}$$

Now, $\{\sin x\}+\{-\sin x\}=\begin{cases}0, \sin x \text{ is integer}\\ 1, \sin x \text{ is not integer}\end{cases}$

For $f(x)$ to be defined, $\{\sin x\}+\{-\sin x\}\neq 0$

$\Rightarrow$ $\sin x\neq$ integer $\Rightarrow$ $\sin x\neq \pm 1, 0$

$\Rightarrow$ $x\neq \frac{n\pi}{2}$

Hence, domain is $R-\left\{\frac{n\pi}{2}, n\in I\right\}$.

10. Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ $\Rightarrow$ $(\vec{a}\cdot\hat{i})^2=x^2$

Similarly, $(\vec{a}\cdot\hat{j})^2=y^2$ and $(\vec{a}\cdot\hat{k})^2=z^2$

$\therefore$ $(\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2=x^2+y^2+z^2=|\vec{a}|^2$

11. We have, $S=\{1, 2, 3, 4, 5, 6\}$

Let A be the event that number is even $=\{2, 4, 6\}$

and B be the event that number is red $=\{1, 2, 3\}$

Now, $A\cap B=\{2\}$

$\therefore$ $P(A\cap B)=\frac{1}{6}$

12. Let $\vec{a}=(\hat{i}+\hat{j}+\hat{k})$

So, unit vector of $\vec{a}=\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{1+1+1}}=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

$\therefore$ The value of p is $\frac{1}{\sqrt{3}}$.

13. Here, $f(1)=\frac{1+1}{2}=1, f(2)=\frac{2}{2}=1,$

$f(3)=\frac{3+1}{2}=2,\ f(4)=\frac{4}{2}=2$

Thus, $f(2k-1)=\frac{(2k-1)+1}{2}=k$ and

$f(2k)=\frac{2k}{2}=k$

$\Rightarrow$ $f(2k-1)=f(2k)$, where $k\in N$

But, $2k-1\neq 2k \Rightarrow f$ is not one-one.

Hence, f is not bijective.

14. Given,

$P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cup B)=\frac{12}{13}$

$\therefore$ $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\frac{7}{13}+\frac{9}{13}-\frac{12}{13}$ $\Rightarrow P(A\cap B)=\frac{4}{13}$

$\therefore$ $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{4/13}{9/13}=\frac{4}{9}$

15. We have $\vec{a},\vec{b},\vec{c}$ are unit vectors.

Therefore, $|\vec{a}|=1, |\vec{b}|=1$ and $|\vec{c}|=1$

Also, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (given)

$\Rightarrow$ $|\vec{a}+\vec{b}+\vec{c}|^2=0$

$\Rightarrow$ $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow$ $1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow$ $3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow$ $(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-\frac{3}{2}$

16. $a_{ij}=\frac{2i-j}{2}$, $i=1, 2, 3$ and $j=1, 2$

$a_{11}=\frac{2-1}{2}=\frac{1}{2},\ a_{12}=\frac{2-2}{2}=0,\ a_{21}=\frac{2(2)-1}{2}=\frac{3}{2},$

$a_{22}=\frac{2(2)-2}{2}=\frac{2}{2}=1,\ a_{31}=\frac{2(3)-1}{2}=\frac{5}{2},$

$a_{32}=\frac{2(3)-2}{2}=\frac{4}{2}=2$

$\therefore$ Required matrix $=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{3}{2} & 1\\ \frac{5}{2} & 2\end{bmatrix}$

17. (i) (b) : $P(X=3)=$ (Probability of not getting six at first chance) $\times$ (Probability of not getting six at second chance) $\times$ (Probability of getting six at third chance)

$=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{5^2}{6^3}$

(ii) (c) : $P(X=5)=\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{5^4}{6^5}$

(iii) (d) : $P(X\geq 3)=1-P(X<3)$

$=1-[P(X=1)+P(X=2)]$

$=1-\left[\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}\right]=1-\frac{11}{36}=\frac{25}{36}$

(iv) (c) : $P(X\geq 6)=\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\left(\frac{5}{6}\right)^6\times\frac{1}{6}+\ldots\infty$

$=\frac{5^5}{6^6}\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+\ldots+\infty\right]$

$=\frac{5^5}{6^6}\left[\frac{1}{1-\frac{5}{6}}\right]=\left(\frac{5}{6}\right)^5$

$P(X>3)=\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+\ldots\infty$

$$=\left(\frac{5}{6}\right)^3\times\frac{1}{6}\left[1+\frac{5}{6}+...\right]$$

$$=\frac{5^3}{6^4}\times\left[\frac{1}{1-\frac{5}{6}}\right]=\left(\frac{5}{6}\right)^3$$

$$\therefore\quad P(X>3)+P(X\geq 6)=\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^5=\left(\frac{5}{6}\right)^3\left[\frac{5^2}{6^2}+1\right]$$

$$=\frac{5^3\times 61}{6^5}$$

(v) (b) : Required conditional probability $=\dfrac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\dfrac{5^2}{6^2}$

18. (i) (b) : To create a garden using 100 ft fencing, we need to maximise its area.

(ii) (c) : Required relation is given by $2x + y = 100$.

(iii) (c) : Area of garden as a function of x can be represented as

$A(x) = x\cdot y = x(100 - 2x) = 100x - 2x^2$

(iv) (a) : $A(x) = 100x - 2x^2 \Rightarrow A'(x) = 100 - 4x$

For the area to be maximum

$A'(x) = 0$

$\Rightarrow$ $100 - 4x = 0 \Rightarrow x = 25$ ft.

(v) (c) : Maximum area of the garden

$= 100(25) - 2(25)^2$

$= 2500 - 1250 = 1250$ sq. ft

19. Using integration by parts, we get

$$\int_2^4\frac{(x^2+x)}{\sqrt{2x+1}}dx=\left[(x^2+x)\cdot\sqrt{2x+1}\right]_2^4-\int_2^4(2x+1)\cdot\sqrt{2x+1}\,dx$$

$$=(60-6\sqrt{5})-\int_2^4(2x+1)^{3/2}dx$$

$$=(60-6\sqrt{5})-\frac{1}{5}\cdot\left[(2x+1)^{5/2}\right]_2^4$$

$$=(60-6\sqrt{5})-\left(\frac{243}{5}-5\sqrt{5}\right)$$

$$=\left(\frac{57}{5}-\sqrt{5}\right)=\left(\frac{57-5\sqrt{5}}{5}\right)$$

20. The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$$\Rightarrow\quad\frac{x-\frac{3}{5}}{\frac{1}{5}}=\frac{y+\frac{7}{15}}{\frac{1}{15}}=\frac{z-\frac{3}{10}}{-\frac{1}{10}}$$

Its direction ratios are $\frac{1}{5},\frac{1}{15},-\frac{1}{10}$

i.e., its direction ratios are proportional to 6, 2, –3.

Now, $\sqrt{6^2+2^2+(-3)^2}=7$

$\therefore$ Its direction cosines are $\frac{6}{7},\frac{2}{7},-\frac{3}{7}$.

OR

Any point on the line

$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=r\text{ (say)}\quad\text{...(i)}$$

is $(3r - 1, 5r - 3, 7r - 5)$.

Any point on the line

$$\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=k\text{ (say)}\quad\text{...(ii)}$$

is $(k + 2, 3k + 4, 5k + 6)$

For lines (i) and (ii) to intersect, we must have

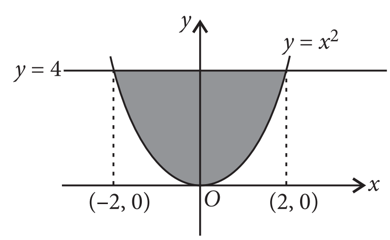
$3r - 1 = k + 2,\ 5r - 3 = 3k + 4,\ 7r - 5 = 5k + 6$

On solving these, we get $r=\frac{1}{2}$, $k=-\frac{3}{2}$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$

21. We have, $y = x^2$

and $y = 4$



Required area = area of shaded region

$$=2\int_0^2(4-x^2)dx=2\left(4x-\frac{x^3}{3}\right)\Bigg|_0^2=\frac{32}{3}\text{sq. units}$$

22. Put $\sin^{-1}x = \theta$. Then $x = \sin\theta$

Now, $\sin 3\theta = (3\sin\theta - 4\sin^3\theta) = (3x - 4x^3)$

$\Rightarrow$ $3\theta = \sin^{-1}(3x - 4x^3)$

$\Rightarrow$ $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ $[\because \theta = \sin^{-1}x]$

Hence, $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

23. If all the 36 elementary events of the experiment are considered to be equally likely, then we have

$$P(A)=\frac{18}{36}=\frac{1}{2}\text{ and }P(B)=\frac{18}{36}=\frac{1}{2}$$

Also, $P(A\cap B) = P$(odd number on both throws)

$$=\frac{9}{36}=\frac{1}{4}$$

Now, $P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Clearly, $P(A\cap B) = P(A)\times P(B)$

Thus, A and B are independent events.

OR

Consider the following events.
E : Two balls drawn are white
A : There are 2 white balls in the bag
B : There are 3 white balls in the bag
C : There are 4 white balls in the bag

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(E/A) = \frac{^2C_2}{^4C_2} = \frac{1}{6},\ P(E/B) = \frac{^3C_2}{^4C_2} = \frac{3}{6} = \frac{1}{2}$$

$$P(E/C) = \frac{^4C_2}{^4C_2} = 1$$

$$\therefore\ P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{3}{5}$$

24. Given, $f(x)$ is continuous at $x = 3$.

So, $\lim\limits_{x\to 3} f(x) = f(3) \Rightarrow \lim\limits_{x\to 3} \dfrac{(x+3)^2 - 36}{x-3} = k$

$$\Rightarrow \lim_{x\to 3} \frac{(x+3)^2 - 6^2}{x-3} = k$$

$$\Rightarrow \lim_{x\to 3} \frac{(x+3+6)\,(x+3-6)}{x-3} = k$$

$\Rightarrow 3 + 3 + 6 = k$
$\Rightarrow k = 12$

25. We have, $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$$\therefore \frac{dy}{dx} = 1 + x^2 + y^2(1+x^2) = (1+x^2)\,.\,(1+y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2)\,dx$$

Integrating both sides, we get $\tan^{-1} y = x + \dfrac{x^3}{3} + C$
When $x = 0$, $y = 1$

$$\tan^{-1}1 = 0 + 0 + C \Rightarrow C = \frac{\pi}{4}$$

$\therefore\ \tan^{-1} y = x + \dfrac{1}{3}x^3 + \dfrac{\pi}{4}$ is the required solution.

26. Given, $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 0 \\ -4 & 2 \end{vmatrix} = 2$ and adj $A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$

$$\therefore\ A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}A^{-1}$

$$= \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1/2 \\ 9 & 1 \end{bmatrix}$$

27. We have, $f(x) = x^3 - 3x^2 + 6x - 100$ ...(i)
Differentiating (i) w.r.t. x, we get

$$f'(x) = 3x^2 - 6x + 6 = 3(x^2 - 2x + 1) + 3$$
$$= 3(x-1)^2 + 3 > 0$$

$\because$ For all values of x, $(x-1)^2$ is always positive
$\therefore\ f'(x) > 0$
So, $f(x)$ is an increasing function on R.

28. The points A, B and C are collinear

$\Leftrightarrow \overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel vectors.

$\Leftrightarrow \overrightarrow{AB} \times \overrightarrow{BC} = \vec{0}$

$\Leftrightarrow (\vec{b} - \vec{a}) \times (\vec{c} - \vec{b}) = \vec{0} \Leftrightarrow (\vec{b} - \vec{a}) \times \vec{c} - (\vec{b} - \vec{a}) \times \vec{b} = \vec{0}$

$\Leftrightarrow (\vec{b} \times \vec{c} - \vec{a} \times \vec{c}) - (\vec{b} \times \vec{b} - \vec{a} \times \vec{b}) = \vec{0}$

$\Leftrightarrow (\vec{b} \times \vec{c} + \vec{c} \times \vec{a}) - (\vec{0} - \vec{a} \times \vec{b}) = \vec{0}$ $[\because \vec{a} \times \vec{c} = -(\vec{c} \times \vec{a})]$

$\Leftrightarrow \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

OR

Given, $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore$ Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

$$= \frac{14 + 6 - 12}{\sqrt{49 + 1 + 16}} = \frac{8}{\sqrt{66}}$$

29. Here, $f : A \to B$ is given by $f(x) = \dfrac{x-1}{x-2}$,

where $A = R - \{2\}$ and $B = R - \{1\}$
Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2$, $x_2 \neq 2$)

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$
$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$
$\Rightarrow -2x_1 - x_2 = -x_1 - 2x_2$
$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.
Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$
such that $f(x) = y$

$$\Leftrightarrow \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x - 1$$

$\Leftrightarrow xy - 2y = x - 1 \Leftrightarrow x(y - 1) = 2y - 1$

$$\Leftrightarrow x = \frac{2y-1}{y-1}$$

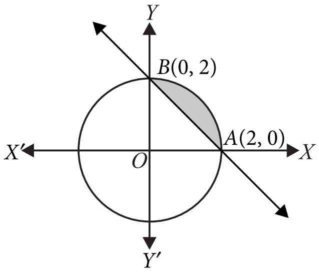
$\therefore \; f(x) = y$ when $x = \dfrac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one–one and onto.

30. The given curves are

$x^2 + y^2 = 4$ and $x + y = 2$



$\therefore$ Required area = area of shaded region

$$= \int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$$

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - 2x + \frac{x^2}{2}\right]_0^2$$

$= 0 + 2\sin^{-1}(1) - 4 + 2 - 0$

$$= 2\cdot\frac{\pi}{2} - 2 = (\pi - 2) \text{ sq.units.}$$

31. The given curve is $x = \sin 3t;\; y = \cos 2t$

$$\Rightarrow \frac{dx}{dt} = 3\cos 3t\,;\; \frac{dy}{dt} = -2\sin 2t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\frac{2\sin 2t}{3\cos 3t}$$

At $t = \dfrac{\pi}{4}$, $x = \sin\dfrac{3\pi}{4} = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

$$y = \cos\frac{\pi}{2} = 0$$

and $$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = -\frac{2\sin\frac{\pi}{2}}{3\cos\frac{3\pi}{4}} = -\frac{2\cdot 1}{3\left(-\frac{1}{\sqrt{2}}\right)} = \frac{2\sqrt{2}}{3}$$

$\therefore$ Equation of the tangent to the given curve at $t = \dfrac{\pi}{4}$ is

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right) \Rightarrow 3y = 2\sqrt{2}x - 2$$

OR

We have, $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$

$\Rightarrow f'(x) = 12x(x + 1)(x - 2)$

Now, $f'(x) = 0$

$\Rightarrow 12x(x + 1)(x - 2) = 0$

$\Rightarrow x = -1, x = 0$ or $x = 2$

Hence these points divide the whole real line into four disjoint open intervals namely $(-\infty, -1)$, $(-1, 0)$, $(0, 2)$ and $(2, \infty)$

Interval	Sign of $f'(x)$	Nature of function
$(-\infty, -1)$	$(-)(-)(-) < 0$	Strictly decreasing
$(-1, 0)$	$(-)(+)(-) > 0$	Strictly increasing
$(0, 2)$	$(+)(+)(-) < 0$	Strictly decreasing
$(2, \infty)$	$(+)(+)(+) > 0$	Strictly increasing

(a) $f(x)$ is strictly increasing in $(-1, 0) \cup (2, \infty)$

(b) $f(x)$ is strictly decreasing in $(-\infty, -1) \cup (0, 2)$

32. Let $$I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\dfrac{\sin x}{\cos x}}}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{...(i)}$$

By the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2}\left[\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right]dx$$

$$= \int_0^{\pi/2} 1\cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

33. For $f(x)$ to be continuous at $x = 0$, we have

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) \quad \text{...(i)}$$

Here, $f(0) = a\sin\dfrac{\pi}{2} = a$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} a\sin\left(\frac{\pi}{2}(-h+1)\right)$$

$$= \lim_{h\to 0} a\sin\left(\frac{\pi}{2} - h\frac{\pi}{2}\right) = \lim_{h\to 0} a\cos\left(\frac{\pi}{2}h\right) = a$$

Now, $$\lim_{x\to 0^+} f(x) = \lim_{h\to 0}\frac{\tan h - \sin h}{h^3}$$

$$= \lim_{h\to 0} \frac{\frac{\sin h}{\cos h} - \sin h}{h^3} = \lim_{h\to 0} \frac{\sin h\left(\frac{1}{\cos h} - 1\right)}{h^3}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} \times \lim_{h\to 0} \frac{1-\cos h}{\cos h \cdot h^2}$$

$$= 1 \times \lim_{h\to 0} \frac{1}{\cos h} \times \lim_{h\to 0} \frac{2\sin^2\left(\frac{h}{2}\right)}{4\times\left(\frac{h}{2}\right)^2} = 1 \times \frac{1}{2} = \frac{1}{2}$$

$\therefore$ From (i), we get $a = 1/2$

Hence, $f(x)$ is continuous at $x = 0$, if $a = 1/2$.

34. We have, $[x\sin^2\left(\frac{y}{x}\right) - y]\,dx + x\,dy = 0$

$$\Rightarrow \sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0 \qquad \text{... (i)}$$

This is a linear homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$\sin^2 v - v + v + x\frac{dv}{dx} = 0$$

$$\Rightarrow x\frac{dv}{dx} + \sin^2 v = 0 \Rightarrow \operatorname{cosec}^2 v\,dv + \frac{dx}{x} = 0$$

Integrating both sides, we get

$$-\cot v + \log x = C \Rightarrow -\cot\left(\frac{y}{x}\right) + \log x = C$$

is the required solution.

OR

We have, $x\frac{dy}{dx} + y = x\cos x + \sin x$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$

It is a linear differential equation.

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$\therefore \quad y\cdot x = \int x\left(\cos x + \frac{\sin x}{x}\right)dx + c$$

$$= \int [x\cos x + \sin x]dx + c$$

$$= x\sin x - \int \sin x\,dx + \int \sin x\,dx + c = x\sin x + c$$

$$\Rightarrow \quad y = \sin x + \frac{c}{x}$$

Given that, $y = 1$ when $x = \frac{\pi}{2}$

$$\therefore \quad 1 = 1 + \frac{c}{\pi/2} \quad \Rightarrow \quad c = 0$$

$\therefore$ $y = \sin x$ is the required solution.

35. The given function is $f(x) = |x-1| + |x+1|$

$$= \begin{cases} -(x-1)-(x+1), & x<-1 \\ -(x-1)+x+1, & -1\le x<1 \\ x-1+x+1, & x\ge 1 \end{cases} = \begin{cases} -2x, & x<-1 \\ 2, & -1\le x<1 \\ 2x, & x\ge 1 \end{cases}$$

At $x = 1$,

$$f'(1^-) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{2-2}{-h} = 0$$

$$f'(1^+) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{2(1+h)-2}{h} = \lim_{h\to 0}\frac{2h}{h} = 2$$

$\therefore$ $f'(1^-) \ne f'(1^+) \Rightarrow f$ is not differentiable at $x = 1$.

At $x = -1$,

$$f'(-1^-) = \lim_{h\to 0}\frac{f(-1-h)-f(-1)}{-h}$$

$$= \lim_{h\to 0}\frac{-2(-1-h)-(2)}{-h} = \lim_{h\to 0}\frac{2h}{-h} = -2$$

$$f'(-1^+) = \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0}\frac{2-2}{h} = 0$$

$\therefore$ $f'(-1^-) \ne f'(-1^+)$

$\Rightarrow$ f is not differentiable at $x = -1$.

36. Given, $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(2-2) - 2(3+4) - 3(-3-4)$$

$$= -14 + 21 = 7 \ne 0$$

$\therefore$ A^{-1} exists

Now, $A_{11} = 0, A_{12} = -7, A_{13} = -7, A_{21} = 1, A_{22} = 7,$
$A_{23} = 5, A_{31} = 2, A_{32} = -7, A_{33} = -4$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is

$x + 2y - 3z = 6$
$3x + 2y - 2z = 3$
$2x - y + z = 2$

The system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

$\because$ A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$

OR

We have, $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$\Rightarrow \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

The given system of equations is
$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$ and it can be written as $AX = B$
where,

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Here, $|A| = 1(-6 + 2) + 1(3 + 4) + 1(1 + 4)$
$= -4 + 7 + 5 = 8 \neq 0$

So, the given system of equations has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = -1$$

37. Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{...(i)}$$

where, a, b, c are variables.
This meets X, Y and Z axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.
Let (α, β, γ) be the coordinates of the centroid of triangle ABC. Then,

$$\alpha = \frac{a+0+0}{3} = \frac{a}{3}, \beta = \frac{0+b+0}{3} = \frac{b}{3},$$

$$\gamma = \frac{0+0+c}{3} = \frac{c}{3} \quad \text{...(ii)}$$

The plane (i) is at a distance $3p$ from the origin.
$\therefore$ $3p$ = Length of perpendicular from (0, 0, 0) to the plane (i)

$$\Rightarrow 3p = \frac{\left|\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow 3p = \frac{1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \frac{1}{9p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \quad \text{...(iii)}$$

From (ii), we have
$a = 3\alpha$, $b = 3\beta$ and $c = 3\gamma$
Substituting the values of a, b, c in (iii), we get

$$\frac{1}{9p^2} = \frac{1}{9\alpha^2} + \frac{1}{9\beta^2} + \frac{1}{9\gamma^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$$

So, the locus of (α, β, γ) is $\frac{1}{p^2} = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$

OR

Given lines are

$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$

$l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

$\therefore$ We have $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b}_2 = 4\hat{i} + 6\hat{j} + 12\hat{k}$

So, $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$

Also, $\vec{b}_2 = 4\hat{i} + 6\hat{j} + 12\hat{k} = 2\vec{b}_1 \Rightarrow \vec{b}_1 \parallel \vec{b}_2$

Hence l_1 and l_2 are parallel lines.
Shortest distance between two parallel lines is,

$$d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$$

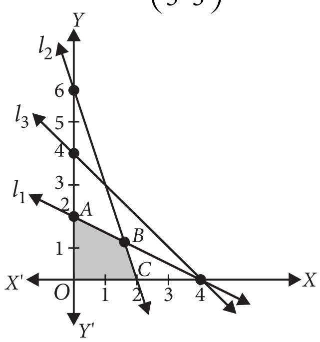
$$\Rightarrow \quad d = \left| \frac{(2\hat{i}+3\hat{j}+6\hat{k}) \times (2\hat{i}+\hat{j}-\hat{k})}{\sqrt{2^2+3^2+6^2}} \right|$$

$$\Rightarrow \quad d = \left| \frac{-9\hat{i}+14\hat{j}-4\hat{k}}{7} \right|$$

$$\Rightarrow \quad d = \frac{\sqrt{(-9)^2+14^2+(-4)^2}}{7} = \frac{\sqrt{293}}{7} \text{ units.}$$

38. Let $l_1 : 2x + 4y = 8$, $l_2 : 3x + y = 6$, $l_3 : x + y = 4$; $x = 0, y = 0$

Solving l_1 and l_2, we get $B\left(\frac{8}{5}, \frac{6}{5}\right)$



Shaded portion $OABC$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 2)$, $B\left(\frac{8}{5}, \frac{6}{5}\right)$, $C(2, 0)$

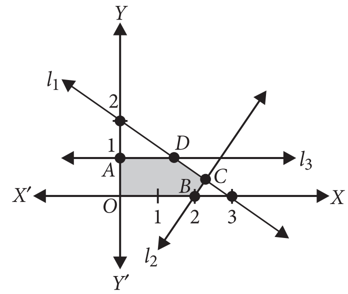
The value of objective function at these points are :

Corner points	**Value of the objective function $z = 2x + 5y$**
$O(0, 0)$	$2 \times 0 + 5 \times 0 = 0$
$A(0, 2)$	$2 \times 0 + 5 \times 2 = 10$ (Maximum)
$B\left(\frac{8}{5}, \frac{6}{5}\right)$	$2 \times \frac{8}{5} + 5 \times \frac{6}{5} = 9.2$
$C(2, 0)$	$2 \times 2 + 5 \times 0 = 4$

$\therefore$ The maximum value of z is 10, which is at $A(0, 2)$.

OR

Let $l_1 : 2x + 3y = 6$, $l_2 : 3x - 2y = 6$, $l_3 : y = 1$; $x = 0, y = 0$



Solving l_1 and l_3, we get D (1.5, 1)

Solving l_1 and l_2, we get $C\left(\frac{30}{13}, \frac{6}{13}\right)$

Shaded portion $OADCB$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 1)$, $D(1.5, 1)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, $B(2, 0)$.

The value of the objective function at these points are :

Corner points	**Value of the objective function $z = 8x + 9y$**
O (0, 0)	$8 \times 0 + 9 \times 0 = 0$
A (0, 1)	$8 \times 0 + 9 \times 1 = 9$
D (1.5, 1)	$8 \times 1.5 + 9 \times 1 = 21$
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$8 \times \frac{30}{13} + 9 \times \frac{6}{13} = 22.6$ (Maximum)
B (2, 0)	$8 \times 2 + 9 \times 0 = 16$

The maximum value of z is 22.6, which is at $C\left(\frac{30}{13}, \frac{6}{13}\right)$

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