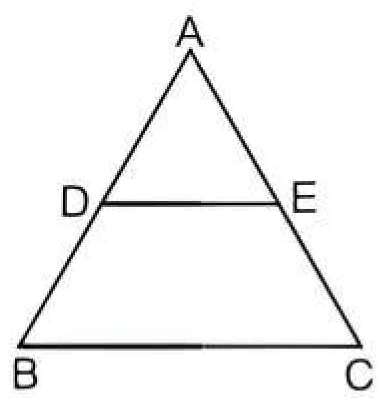
6 Triangles

Fastrack Revision

- **Similar Figures:** Two figures having the same shapes (and not necessarily the same size) are called similar figures.
- **Similar Triangles:** Two triangles are said to be similar if:
  1. their corresponding angles are equal.
  2. their corresponding sides are in the same ratio (or proportional).

  Note: Suppose ΔABC is similar to ΔPQR, we write as $\Delta ABC \sim \Delta PQR$. But we do not write as $\Delta ABC \sim \Delta QRP$ or $\Delta BAC \sim \Delta PQR$.
- **Equiangular Triangles:** If corresponding angles of two triangles are equal, then they are equiangular triangles. The ratio of any two corresponding sides of each pair in two equiangular triangles is always the same.
- **Basic Proportionality Theorem—BPT (Thales' Theorem):** In a triangle, a line drawn parallel to one side, to intersect the other two sides at distinct points, divides the two sides in the same ratio.



  In Δ ABC, DE || BC, then $\frac{AD}{DB} = \frac{AE}{EC}$,

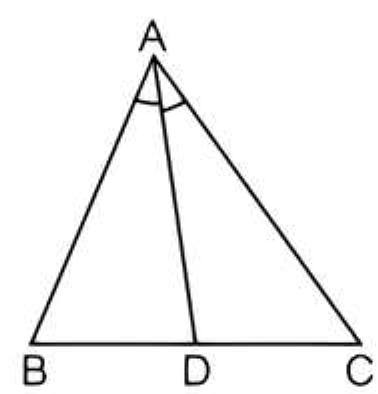
  $$\frac{AD}{AB} = \frac{AE}{AC} \text{ and } \frac{AB}{DB} = \frac{AC}{EC}$$
- **Converse of Basic Proportionality Theorem:** If a line divides any two sides of a ΔABC in the same ratio, *i.e.*, $\frac{AD}{DB} = \frac{AE}{EC}$, then the line must be parallel to the third side, *i.e.*, DE || BC.
- **Criterion for Similarity of Triangles:** There are three criteria for similarity of triangles :
  1. **AAA Similarity:** In two triangles, if three angles of one triangle are respectively equal to the three angles of the other triangle, then the two triangles are similar. If two of their angles are equal, then the third angle must also be equal, because sum of angles of a triangle always make 180°. So, AA could also be called similarity.
  2. **SSS Similarity:** In two triangles, if the corresponding sides are proportional, then they are similar.

     Or

     In two triangles, if sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
  3. **SAS Similarity:** In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then the two triangles are similar.

Knowledge BOOSTER

1. *All congruent triangles are similar but the similar triangles need not be congruent.*
2. ***Mid-point Theorem:*** *The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.*
3. ***Angle Bisector Theorem:*** *The internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.*



   In ΔABC, $\frac{AB}{AC} = \frac{BD}{DC}$
4. *If two triangles are similar, then their corresponding sides, medians and altitudes are proportional.*

Practice Exercise

Multiple Choice Questions

Q 1. Two polygons have same number of sides are similar, if:

a. their corresponding sides are proportional

b. their corresponding angles are equal

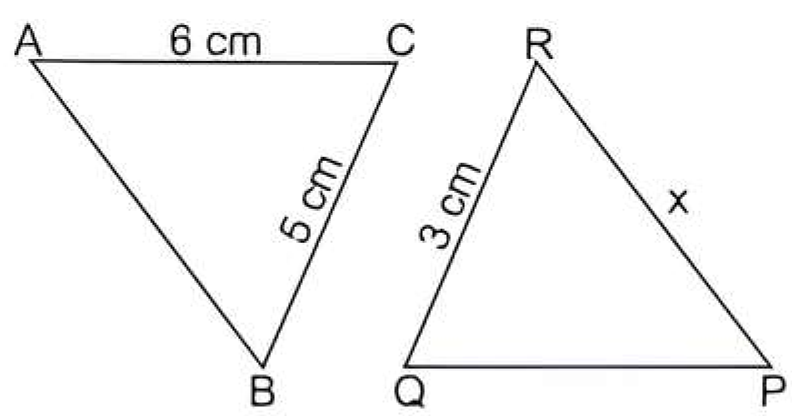
c. Both a. and b.

d. None of the above

Q 2. If $\Delta ABC \sim \Delta PQR$ with $\angle A = 32°$ and $\angle R = 65°$, then the measures of $\angle$ B is: *[CBSE 2023]*

a. 32° b. 65° c. 83° d. 97°

Q 3.



In the given figure, $\Delta ABC \sim \Delta QPR$. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x, then the value of x is: *[CBSE 2023]*

a. 3.6 cm b. 2.5 cm c. 10 cm d. 3.2 cm

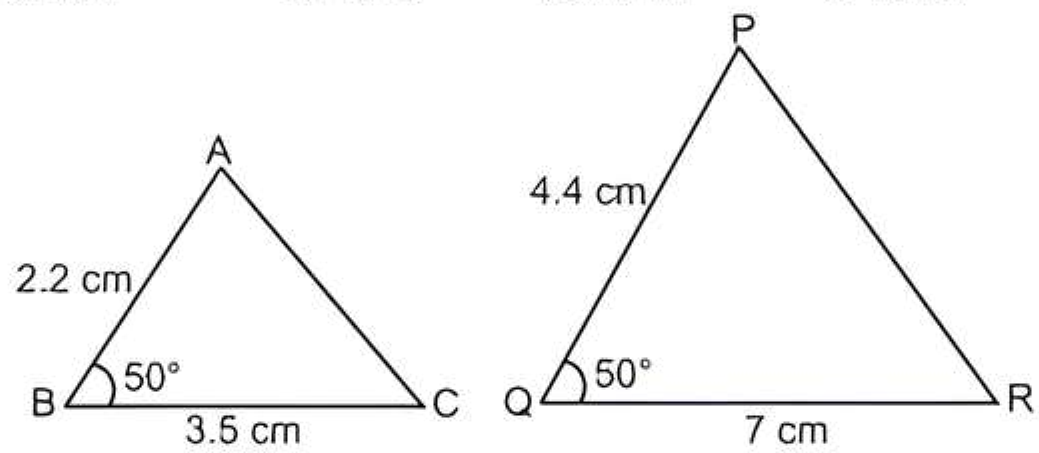
Q 4. If in $\triangle ABC$ and $\triangle PQR$, we have $\frac{AB}{QR}=\frac{BC}{PR}=\frac{CA}{PQ}$ then: *[CBSE SQP 2023-24, NCERT EXEMPLAR]*

a. $\triangle PQR \sim \triangle CAB$
b. $\triangle PQR \sim \triangle ABC$
c. $\triangle CBA \sim \triangle PQR$
d. $\triangle BCA \sim \triangle PQR$

Q 5. Which of the following is NOT a similarity criterion? *[CBSE SQP 2023-24]*

a. AA b. SAS c. AAA d. RHS

Q 6.



In the above figure, the criterion of similarity by which $\triangle ABC \sim \triangle PQR$ is: *[CBSE 2023]*

a. SSA (Side-Side-Angle) similarity
b. ASA (Angle-Side-Angle) similarity
c. SAS (Side-Angle-Side) similarity
d. AA (Angle-Angle) similarity

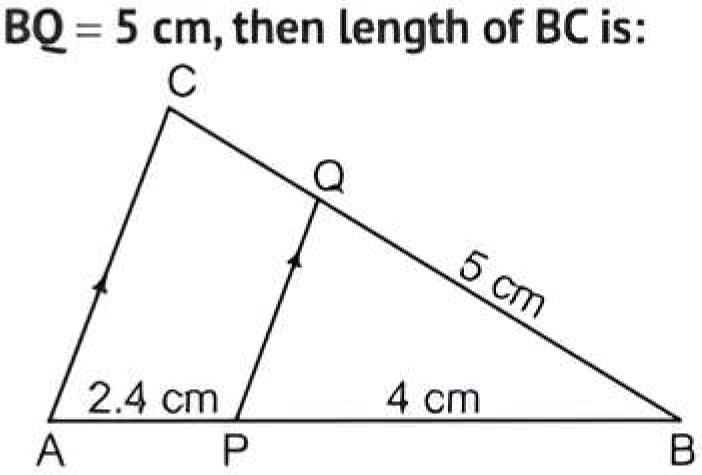
Q 7. In a $\triangle ABC$, it is given that $AB = 6$ cm, $AC = 8$ cm and AD is the bisector of $\angle A$. Then, BD : DC =

a. 3 : 4 b. 9 : 16 c. 4 : 3 d. $\sqrt{3}:2$

Q 8. $\triangle ABC \sim \triangle PQR$. If AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively and $AB^2 : PQ^2 = 4 : 9$, then AM : PN = *[CBSE SQP 2021 Term-I]*

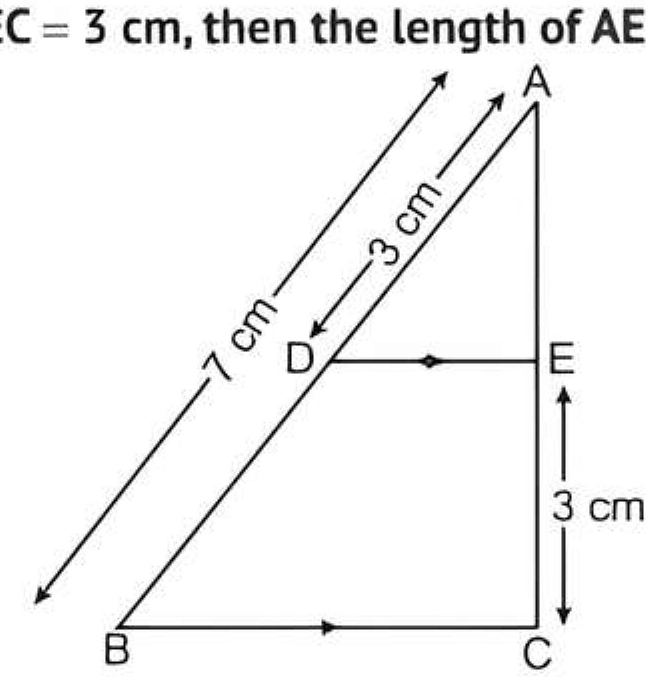
a. 16 : 81 b. 4 : 9 c. 3 : 2 d. 2 : 3

Q 9. In the given figure, PQ || AC. If BP = 4 cm, AP = 2.4 cm and BQ = 5 cm, then length of BC is: *[CBSE 2023]*



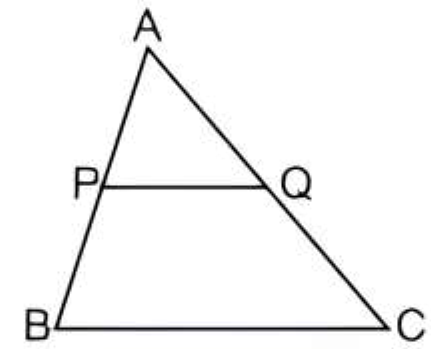
a. 8 cm
b. 3 cm
c. 0.3 cm
d. $\frac{25}{3}$ cm

Q 10. In the given figure, DE || BC. If AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is: *[CBSE 2023]*



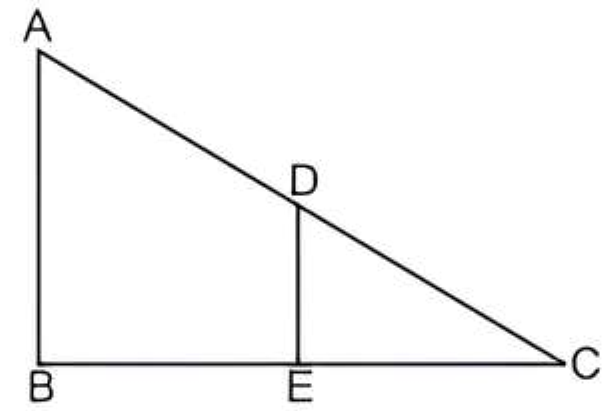
a. 2 cm b. 2.25 cm c. 3.5 cm d. 4 cm

Q 11. In $\triangle ABC$, PQ || BC. If PB = 6 cm, AP = 4 cm, AQ = 8 cm, find the length of AC. *[CBSE 2023]*



a. 12 cm
b. 20 cm
c. 6 cm
d. 14 cm

Q 12. In $\triangle ABC$, DE || AB. If $AB = a$, $DE = x$, $BE = b$ and $EC = c$. Express x in terms of a, b and c. *[CBSE SQP 2023-24]*



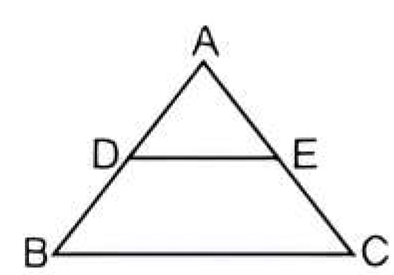
a. $\frac{ac}{b}$

b. $\frac{ac}{b+c}$

c. $\frac{ab}{c}$

d. $\frac{ab}{b+c}$

Q 13. In the given figure, if DE || BC, AD = 3 cm, BD = 4 cm and BC = 14 cm, then DE equals: *[CBSE SQP 2021 Term-I]*



a. 7 cm b. 6 cm c. 4 cm d. 3 cm

Q 14. In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = \frac{1}{2}DE$. Then, the two triangles are: *[CBSE 2021 Term-I]*

a. congruent but not similar
b. similar but not congruent
c. neither congruent nor similar
d. congruent as well as similar

Q 15. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF = 7.5 cm. Then, which of the following is true? *[NCERT EXEMPLAR]*

a. DE = 12 cm, $\angle F = 50°$
b. DE = 12 cm, $\angle F = 100°$
c. EF = 12 cm, $\angle D = 100°$
d. EF = 12 cm, $\angle D = 30°$

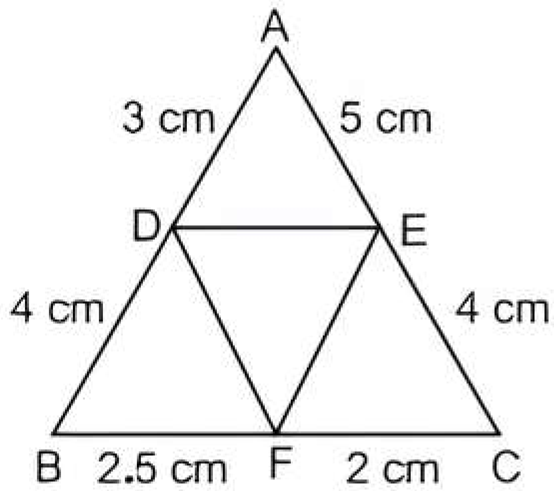
Q 16. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then the ratio of their corresponding sides is: *[CBSE 2015]*

a. 25 : 49
b. 5 : 7
c. 7 : 5
d. 49 : 25

Q 17. ABCD is a trapezium with AD || BC and AD = 4 cm. If the diagonals AC and BD intersect each other at O such that AO/OC = DO/OB = 1/2, then BC = *[CBSE SQP 2022-23]*

a. 6 cm b. 7 cm c. 8 cm d. 9 cm

Q 18. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then:



a. $DE \parallel BC$ b. $DF \parallel AC$
c. $EF \parallel AB$ d. None of these

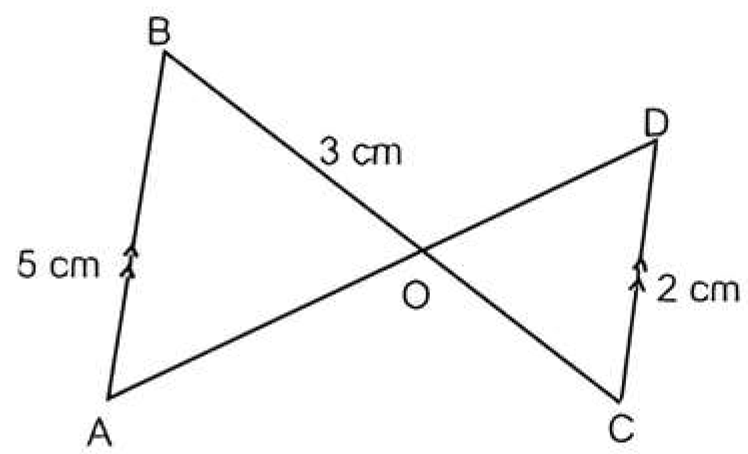
Q 19. It is given that, $\triangle ABC \sim \triangle EDF$, such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm, then the sum of the remaining sides of the triangles is:

a. 23.05 cm b. 16.8 cm
c. 6.25 cm d. 24 cm

Q 20. The sides of two similar triangles are in the ratio 4 : 7. The ratio of their perimeters is: *[CBSE 2023]*

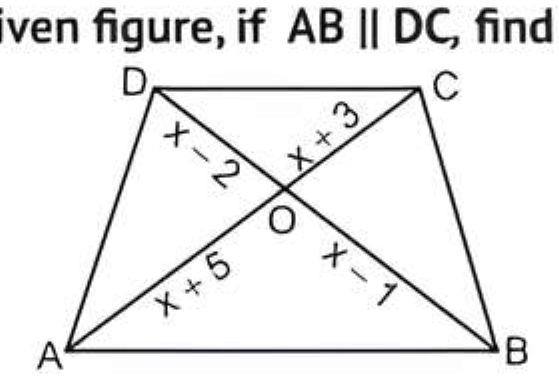
a. 4 : 7 b. 12 : 21
c. 16 : 49 d. 7 : 4

Q 21. In the given figure, $AB \parallel CD$. If $AB = 5$ cm, $CD = 2$ cm and $OB = 3$ cm, then the length of OC is: *[CBSE 2023]*



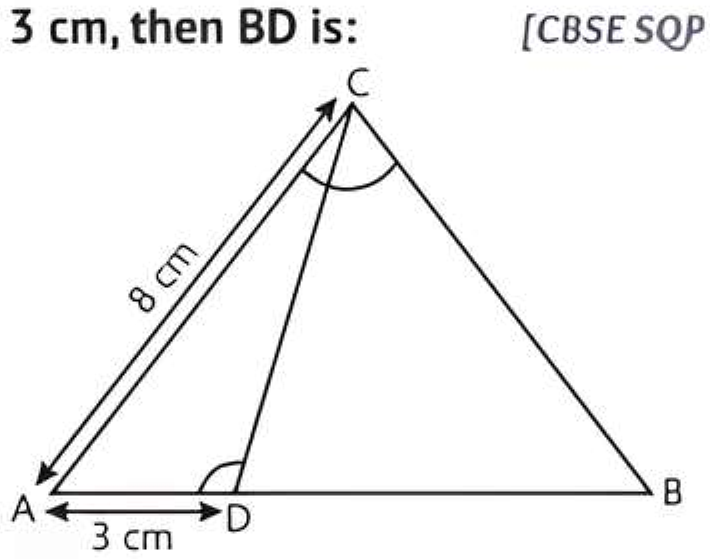
a. $\frac{15}{2}$ cm b. $\frac{10}{3}$ cm
c. $\frac{6}{5}$ cm d. $\frac{3}{5}$ cm

Q 22. In the given figure, if $AB \parallel DC$, find the value of x.



a. 5 b. 7 c. 6 d. 4

Q 23. In the given figure, if $\angle ACB = \angle CDA$, $AC = 8$ cm, $AD = 3$ cm, then BD is: *[CBSE SQP 2021 Term-I]*



a. 22/3 cm b. 26/3 cm
c. 55/3 cm d. 64/3 cm

Q 24. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts the shadow 40 m long on the ground. Then, the height of the tower is:

a. 65 m b. 60 m c. 70 m d. 72 m

Assertion & Reason Type Questions

Directions (Q.Nos. 25-29): *In the following questions given below, there are two statement marked as Assertion (A) and Reason (R). Read the statements and choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 25. **Assertion (A):** All regular polygons of the same number of sides such as equilateral triangles, squares etc, are similar.

Reason (R): Two polygons of the same number of sides are said to be similar, if their corresponding angles are equal and lengths of corresponding sides are proportional.

Q 26. **Assertion (A):** In a $\triangle ABC$, if $DE \parallel BC$ and intersects AB at D and AC at E, then $\frac{AB}{AD} = \frac{AC}{AE}$.

Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then these sides are divided in the same ratio.

Q 27. **Assertion (A):** If the bisector of an angle of a triangle bisects the opposite side, then the triangle is isosceles.

Reason (R): The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Q 28. **Assertion (A):** In a $\triangle ABC$, D and E are points on sides AB and AC respectively such that $BD = CE$. If $\angle B = \angle C$, then DE is not parallel to BC.

Reason (R): If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Q 29. **Assertion (A):** In $\triangle ABC$, $DE \parallel BC$, such that $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x + 4)$ cm and $EC = 3x$ cm then x is equal to 5.

Reason (R): If a line is drawn parallel to one side of triangle to intersect the other two sides at a distinct point, then the other two sides are divided in the same ratio.

Fill in the Blanks Type Questions

Q 30. Two triangles are similar, if their corresponding sides are .... *[NCERT EXERCISE; CBSE 2020]*

Q 31. In ΔABC and ΔDEF, if $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 2DE$. Then, the two triangles are ....

Q 32. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then $\frac{DE}{QR} = \frac{DF}{PQ} =$ ....

Q 33. The line segment joining the mid-points of any two sides of a triangle is .... to the third side.

Q 34. All congruent triangles are similar but the similar triangles need not to be ....

True/False Type Questions

Q 35. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. *[NCERT EXEMPLAR]*

Q 36. If AD and PM are medians of ΔABC and ΔPQR respectively, where $\Delta ABC \sim \Delta PQR$, then $\frac{AB}{PQ} = \frac{AD}{PM}$. *[NCERT EXERCISE]*

Q 37. In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then two triangles are similar.

Solutions

1. (c) Two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same ratio *i.e.* proportional.

2. (c) Given $\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \angle A = \angle P, \angle B = \angle Q$ and $\angle C = \angle R$

$\therefore \quad \angle A = \angle P = 32^\circ$ and $\angle R = \angle C = 65^\circ$

Now, in ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad 32^\circ + \angle B + 65^\circ = 180^\circ$$

$$\Rightarrow \quad \angle B = 180^\circ - 97^\circ = 83^\circ$$

3. (b) Given, $\Delta ABC \sim \Delta QPR$

$$\therefore \quad \frac{AB}{QP} = \frac{BC}{PR} = \frac{CA}{RQ}$$

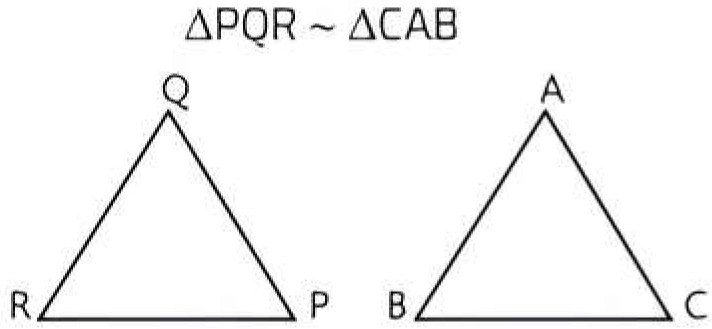
$$\Rightarrow \quad \frac{BC}{PR} = \frac{CA}{RQ} \quad \Rightarrow \quad \frac{5}{x} = \frac{6}{3}$$

$$\Rightarrow \quad x = \frac{5 \times 3}{6} = \frac{5}{2} = 2.5 \text{ cm}$$

4. (a) Given, in two ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

which shows that sides of one triangle are proportional to the side of the other triangle, then their corresponding angles are also equal, so by sss similarity, triangles are similar.

i.e., $\quad \Delta PQR \sim \Delta CAB$



5. (d) AAA, AA , SSS and SAS all are similarity criterion of triangles while RHS is one of the congruency rule of triangles.

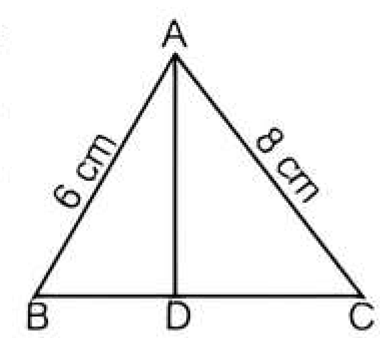
6. (c) In ΔABC and ΔPQR, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2}$

and $\quad \angle B = \angle Q = 50^\circ$

$\therefore \quad \Delta ABC \sim \Delta PQR$

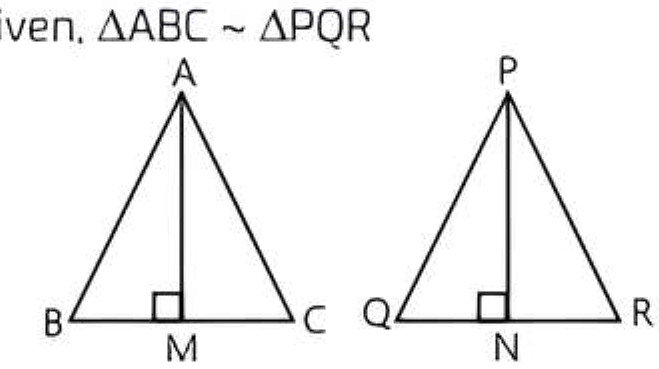
By SAS (Side-Angle-Side) similarity criterion.

7. (a) We know that, the bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.



$\therefore$ BD : DC = AB : AC = 6 : 8 = 3 : 4

8. (d) Given, $\Delta ABC \sim \Delta PQR$



TiP

If two triangles are similar, then their corresponding sides and altitudes are in same proportion.

AM and PN are altitudes of ΔABC and ΔPQR respectively.

Also, $\quad \frac{AB^2}{PQ^2} = \frac{4}{9} \quad \Rightarrow \quad \frac{AB}{PQ} = \frac{2}{3} \quad$ ...(1)

As we know that,

Ratio of altitudes = Ratio of sides for similar triangles.

So, $\quad \frac{AM}{PN} = \frac{AB}{PQ} = \frac{2}{3}$

9. (a) In ΔABC, PQ || AC

$$\therefore \quad \frac{AP}{BP} = \frac{CQ}{BQ} \quad \text{(By Thales theorem)}$$

$$\Rightarrow \quad \frac{2.4}{4} = \frac{CQ}{5}$$

$$\Rightarrow \quad CQ = \frac{2.4 \times 5}{4} = 3 \text{ cm}$$

$\therefore$ Length of BC = BQ + CQ = 5 + 3 = 8 cm

10. (b) In Δ ABC, BC || DE

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE} \quad \text{(By Thales theorem)}$$

$$\Rightarrow \quad \frac{AD}{BD}+1=\frac{AE}{CE}+1$$

$$\Rightarrow \quad \frac{AD+BD}{BD}=\frac{AE+CE}{CE}$$

$$\Rightarrow \quad \frac{AB}{AB-AD}=\frac{AE+CE}{CE}$$

$$\Rightarrow \quad \frac{7}{7-3}=\frac{AE+3}{3}$$

$$\Rightarrow \quad \frac{7}{4}=\frac{AE}{3}+1$$

$$\Rightarrow \quad \frac{AE}{3}=\frac{7}{4}-1=\frac{3}{4}$$

$$\Rightarrow \quad AE=\frac{9}{4}=2.25 \text{ cm}$$

11. (b) Given that,

In ΔABC, $PQ \parallel BC$

$PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm

$$\therefore \quad \frac{AP}{AB}=\frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{AP}{AP+PB}=\frac{AQ}{AC} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{4}{4+6}=\frac{8}{AC} \quad \Rightarrow \quad \frac{4}{10}=\frac{8}{AC}$$

$$\Rightarrow \quad AC=\frac{8\times 10}{4}=20 \text{ cm}$$

12. (b) Given $AB = a$, $DE = x$

$BE = b$ and $EC = c$

In ΔABC, $DE \parallel AB$

By basic proportionality theorem,

$$\Rightarrow \quad \frac{DE}{ABC}=\frac{EC}{BC}$$

$$\Rightarrow \quad \frac{DE}{AB}=\frac{EC}{BE+EC}$$

$$\Rightarrow \quad \frac{x}{a}=\frac{c}{b+c} \quad \Rightarrow \quad x=\frac{ac}{b+c}$$

13. (b) Given, $AD = 3$ cm, $BD = 4$ cm, $BC = 14$ cm and $DE \parallel BC$

$$\therefore \quad \Delta ADE \sim \Delta ABC \qquad \text{(By AA similarity)}$$

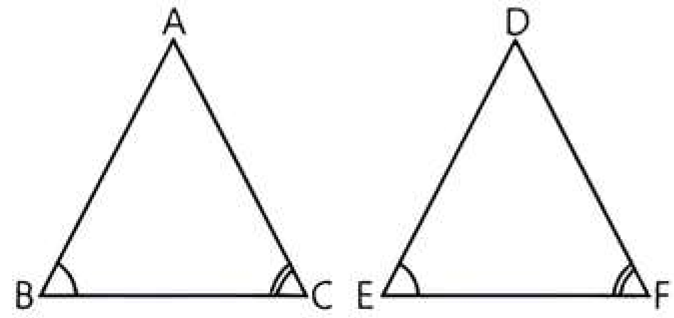
$$\Rightarrow \quad \frac{AD}{AB}=\frac{DE}{BC} \quad \Rightarrow \quad \frac{AD}{AD+BD}=\frac{DE}{BC}$$

$$\Rightarrow \quad \frac{3}{3+4}=\frac{DE}{14} \quad \Rightarrow \quad DE=\frac{3\times 14}{7}=6 \text{ cm}$$

14. (b) In ΔABC and ΔDEF,

$$\angle B = \angle E, \angle F = \angle C \text{ and } AB = \frac{1}{2}DE$$

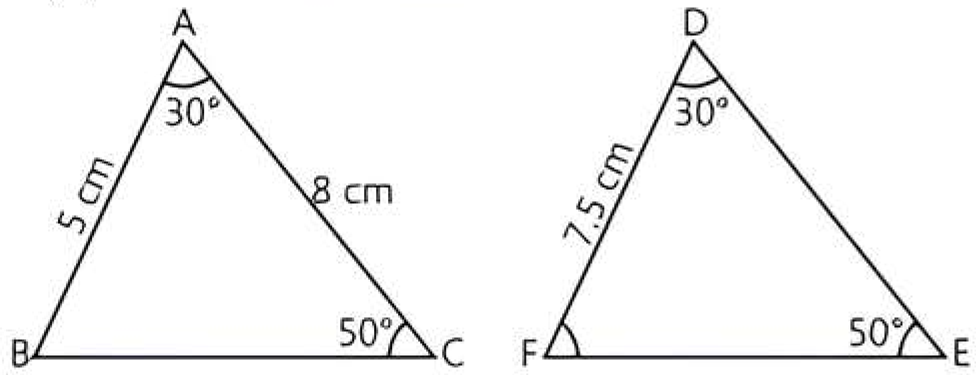
We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.



Since, $AB \neq DE$

Therefore, ΔABC and ΔDEF are not congruent.

15. (b) Given, $\Delta ABC \sim \Delta DFE$



Then, $\angle A = \angle D = 30°$,

$\angle C = \angle E = 50°$

$\therefore \quad \angle B = \angle F = 180° - (30° + 50°) = 100°$

Also, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm

$$\frac{AB}{DF}=\frac{AC}{DE} \quad \Rightarrow \quad \frac{5}{7.5}=\frac{8}{DE}$$

$$\Rightarrow \quad DE=\frac{8\times 7.5}{5}=12 \text{ cm}$$

Hence, $DE = 12$ cm, $\angle F = 100°$

16. (b) Let ABC and PQR be two similar triangles with medians AD and PS respectively.

$$\text{Then,} \quad \frac{AD}{PS}=\frac{5}{7} \qquad \text{(Given)}$$

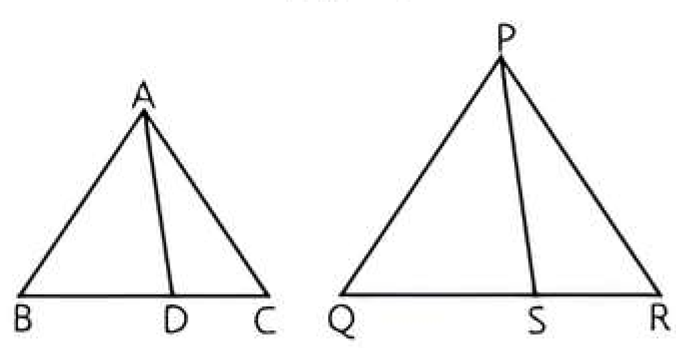
TR!CK

The ratios of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$$\because \quad \Delta ABC \sim \Delta PQR$$

$$\frac{AB}{PQ}=\frac{AD}{PS}$$

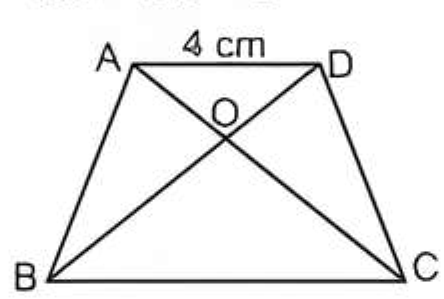
$$\Rightarrow \quad \frac{AB}{PQ}=\frac{5}{7}$$



Hence, the ratio of corresponding sides is 5 : 7.

17. (c) Given, $AD = 4$ cm

$$\text{and} \quad \frac{AO}{OC}=\frac{DO}{OB}=\frac{1}{2} \qquad \text{...(1)}$$



In ΔAOD and ΔCOB

$\angle ADO = \angle CBO$

(∵ AD || BC, so alternate angles are equal)

$\angle AOD = \angle COB$ (Vertically opposite angles)

and $\angle OAD = \angle OCB$

(Alternate angles are equal)

∴ $\Delta AOD \sim \Delta COB$ (By AAA similarity rule)

$$\Rightarrow \quad \frac{AO}{OC} = \frac{OD}{OB} = \frac{AD}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{4}{BC} \quad \text{[From eq. (1)]}$$

$$\Rightarrow \quad BC = 8 \text{ cm}$$

18. (c) Given, AD = 3 cm, AE = 5 cm, BD = 4 cm, CE = 4 cm, CF = 2 cm, BF = 2.5 cm

Here $\frac{CF}{FB} = \frac{2}{2.5}$

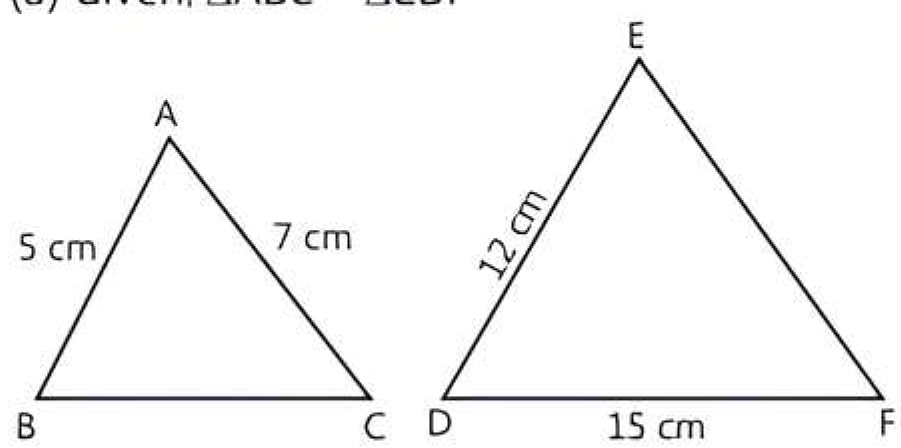
$= \frac{20}{25} = \frac{4}{5}$

and $\frac{CE}{AE} = \frac{4}{5}$

∴ $\frac{CF}{FB} = \frac{CE}{AE}$

⇒ EF || AB (By converse of Thales theorem)

19. (a) Given, $\Delta ABC \sim \Delta EDF$



Since, $\Delta ABC \sim \Delta EDF$

$$\therefore \quad \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5} = 16.8 \text{ cm}$$

On taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$= EF + BC = 16.8 + 6.25 = 23.05$ cm

20. (a) Given, ratio of the sides of two similar triangles

= 4 : 7

TR!CK

If $\Delta PQR \sim \Delta ABC$

$$\therefore \quad \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \frac{PQ + QR + PR}{AB + BC + AC}$$

$$\Rightarrow \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta ABC}$$

∴ Ratio of their perimeters = Ratio of the sides of two similar triangles = 4 : 7.

21. (c) In the given figure, AB ||CD.

In Δ COD and ΔBOA,

$\angle OCD = \angle OBA$ [Alternate interior angles]

$\angle ODC = \angle OAB$ [Alternate interior angles]

and $\angle COD = \angle BOA$ [Vertically opposite angles]

∵ $\Delta COD \sim \Delta BOA$

$$\therefore \quad \frac{CO}{BO} = \frac{OD}{OA} = \frac{CD}{BA}$$

Given, AB = 5 cm , CD = 2 cm and OB = 3 cm

$$\Rightarrow \quad \frac{CO}{BO} = \frac{CD}{BA} \quad \Rightarrow \frac{CO}{3} = \frac{2}{5}$$

$$\Rightarrow CO = \frac{6}{5} \text{cm}$$

22. (b) Given, AB || DC

∴ $\angle ODC = \angle OBA$ (Alternate interior angles)

and $\angle OCD = \angle OAB$ (Alternate interior angles)

∴ $\Delta DOC \sim \Delta BOA$ (By AA similarity criterion)

Thus, $\frac{OD}{OB} = \frac{OC}{OA}$

$$\Rightarrow \quad \frac{x-2}{x-1} = \frac{x+3}{x+5}$$

$$\Rightarrow \quad (x-2)(x+5) = (x+3)(x-1)$$

$$\Rightarrow \quad x^2 + 3x - 10 = x^2 + 2x - 3$$

$$\Rightarrow \quad x = 7$$

23. (c) Given, AC = 8 cm, AD = 3 cm

In ΔACD and ΔABC,

$\angle CDA = \angle ACB$ (Given)

$\angle CAD = \angle CAB$ (Common)

∴ $\Delta ACD \sim \Delta ABC$ (By AA similarity)

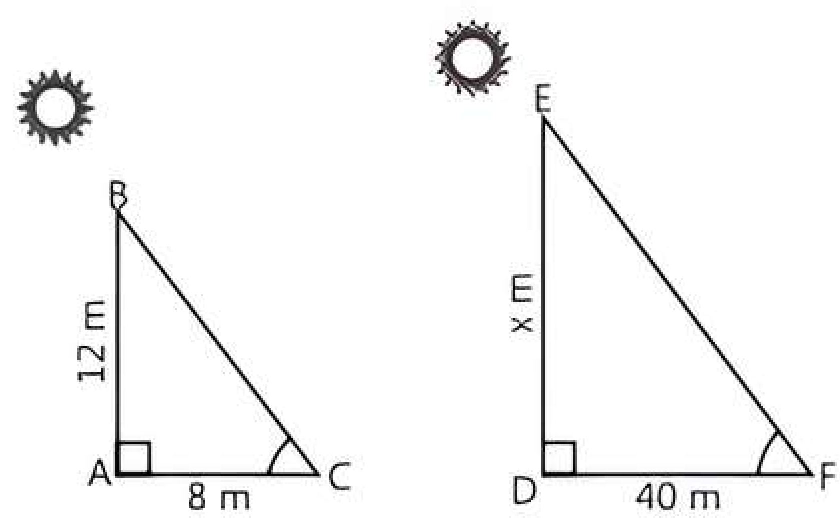
$$\Rightarrow \quad \frac{AC}{AB} = \frac{AD}{AC} \quad \text{(By CPCT)}$$

$$\Rightarrow \quad \frac{8}{AB} = \frac{3}{8}$$

$$\Rightarrow \quad AB = \frac{64}{3} \text{cm}$$

So, $BD = AB - AD = \frac{64}{3} - 3 = \frac{55}{3}$ cm

24. (b) Let AB be the vertical stick and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Join BC and EF. Let DE = x m



We have, AB = 12 m, AC = 8 m and DF = 40 m

In ΔABC and ΔDEF, we have

$\angle A = \angle D = 90°$

$\angle C = \angle F$ (Angular elevation of the sun)

∴ $\Delta ABC \sim \Delta DEF$ (By AA similarity criterion)

$$\Rightarrow \quad \frac{AB}{DE}=\frac{AC}{DF} \Rightarrow \frac{12}{x}=\frac{8}{40}$$

$$\Rightarrow \quad \frac{12}{x}=\frac{1}{5} \Rightarrow x=60\,\text{m}$$

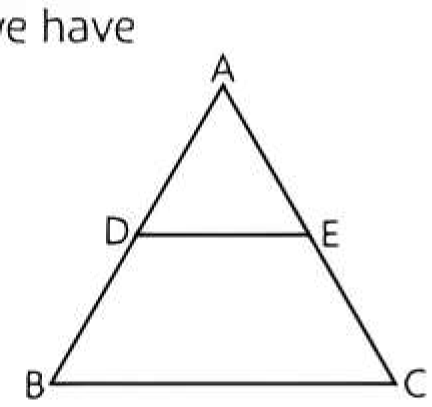
25. (a) **Assertion (A) :** Two polygons of the same number of sides are similar, if their corresponding angles are equal and corresponding sides are proportional.

∵ In equilateral triangles or squares, each angle is equal and sides are also proportional, therefore all regular polygons are similar.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

26. (a) **Assertion (A):** In ΔABC, $DE \parallel BC$, by using Thale's theorem, we have



$$\frac{AD}{DB}=\frac{AE}{EC} \Rightarrow \frac{DB}{AD}=\frac{EC}{AE}$$

$$\Rightarrow \quad 1+\frac{DB}{AD}=1+\frac{EC}{AE}$$

$$\Rightarrow \quad \frac{AD+DB}{AD}=\frac{AE+EC}{AE}$$

$$\Rightarrow \quad \frac{AB}{AD}=\frac{AC}{AE}$$

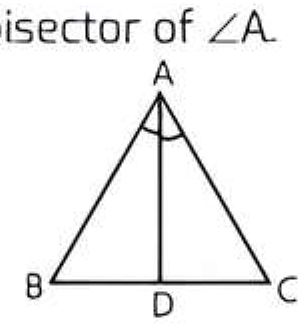
So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

27. (a) **Assertion (A):** In ΔABC, AD is the bisector of $\angle A$.

$$\therefore \quad \frac{AB}{AC}=\frac{BD}{DC}$$



$$\Rightarrow \quad \frac{AB}{AC}=1$$

(∵ D is the mid-point of BC, ∴ $BD = DC$)

$$\Rightarrow \quad AB = AC$$

Hence, ΔABC is an isosceles.

So, Assertion (A) is true.

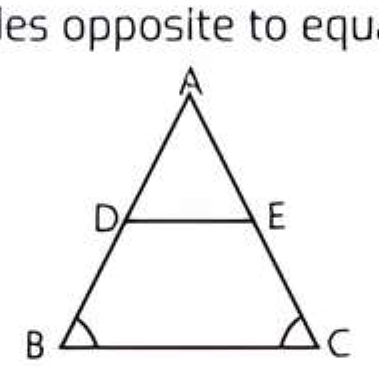
Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

28. (d) **Assertion (A):** In ΔABC, we have $\angle B = \angle C$

$$\Rightarrow \quad AC = AB$$

(∵ Sides opposite to equal angles are equal)



$$\Rightarrow \quad AE+EC = AD+DB$$

$$\Rightarrow \quad AE+CE = AD+DB$$

$$\Rightarrow \quad AE+CE = AD+CE \qquad [BD = CE \text{ (Given)}]$$

$$\Rightarrow \quad AE = AD$$

Thus, we have

$$AD = AE$$

and

$$BD = CE$$

$$\therefore \quad \frac{AD}{BD}=\frac{AE}{CE} \quad \Rightarrow \frac{AD}{DB}=\frac{AE}{EC}$$

$$\Rightarrow \quad DE \parallel BC$$

So, Assertion (A) is false.

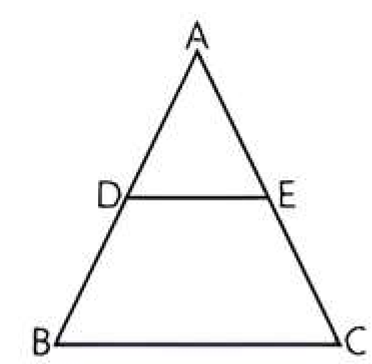
Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

29. (d) **Assertion (A):** We have,

$$\frac{AD}{DB}=\frac{AE}{EC} \qquad [\because DE \parallel BC]$$

$$\Rightarrow \quad \frac{7x-4}{3x+4}=\frac{5x-2}{3x}$$



$$\Rightarrow \quad 21x^2-12x=15x^2+20x-6x-8$$

$$\Rightarrow \quad 6x^2-26x+8=0$$

$$\Rightarrow \quad 3x^2-13x+4=0$$

TR!CK

Product of extreme terms $= 3 \times 4 = 12$

$\because \quad 12 = 6 \times 2 = 3 \times 4 = 12 \times 1$

Here, we will take 12 and 1 as a factors of 12. So, middle term

$$-13 = -12 - 1$$

$$\Rightarrow \quad 3x^2-12x-x+4=0$$

$$\Rightarrow \quad 3x(x-4)-1(x-4)=0$$

$$\Rightarrow \quad (x-4)(3x-1)=0$$

$$\Rightarrow \quad x=4, \frac{1}{3}$$

So, Assertion (A) is false.

Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

30. Proportional

31. In ΔABC and ΔDEF,

$$\angle B = \angle E \qquad \text{(Given)}$$

$$\angle C = \angle F \qquad \text{(Given)}$$

$$\therefore \quad \Delta ABC \sim \Delta DEF \qquad \text{(By AA similarity)}$$

32. In ΔDEF and ΔQRP,

$$\angle D = \angle Q \qquad \text{(Given)}$$

$$\angle E = \angle R \qquad \text{(Given)}$$

$$\therefore \quad \Delta DEF \sim \Delta QRP \qquad \text{(By AA similarity)}$$

$$\Rightarrow \quad \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{RP}$$

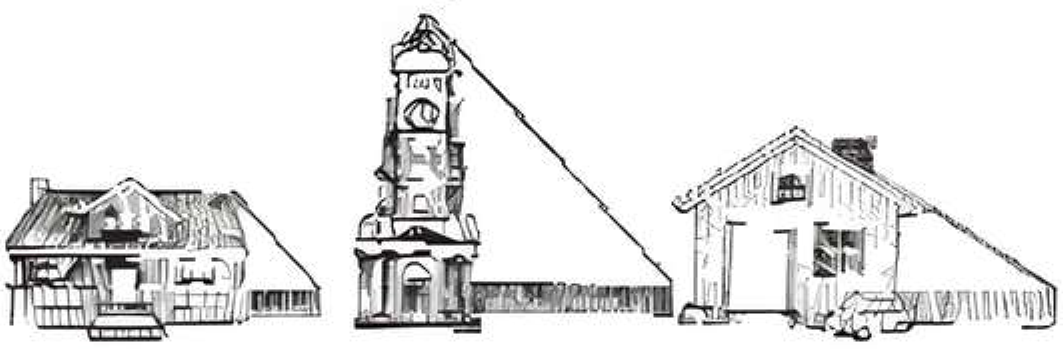
33. Parallel **34.** Congruent
35. True **36.** True
37. True

Case Study Based Questions

Case Study 1

Digvijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Digvijay's house is 20 m when Digvijay's house casts a shadow 10 m long on the ground.

At the same time, the tower casts a shadow 50 m long on the ground and the house of Anshul casts 20 m shadow on the ground.



Based on the above information, solve the following questions:

Q 1. The height of the tower is:

a. 10 m b. 20 m c. 50 m d. 100 m

Q 2. When Digvijay's house casts a shadow of 18 cm, the length of the shadow of the tower is:

a. 18 m b. 20 m c. 90 m d. 100 m

Q 3. The height of Anshul's house is:

a. 20 m b. 40 m c. 50 m d. 100 m

Q 4. When the tower casts a shadow of 40 m, same time the length of the shadow of Anshul's house is:

a. 16 m b. 40 m
c. 100 m d. None of these

Q 5. Which of the following similarity criterion does not exist?

a. AA b. SAS c. SSS d. RHS

Solutions

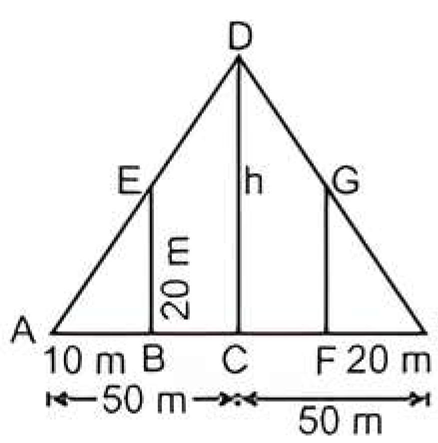
1. Let CD = h m be the height of the tower.
Let BE = 20 m be the height of Digvijay house and GF be the height of Anshul's house.

$$\Delta ACD \sim \Delta ABE$$

$$\frac{AC}{AB} = \frac{CD}{EB}$$

$$\Rightarrow \quad \frac{50}{10} = \frac{h}{20}$$

$$\Rightarrow \quad h = 100 \text{ m}$$



So, option (d) is correct.

2. Given AB = 18 m, let AC = x
In similar ΔABE and ΔACD

$$\frac{AB}{AC} = \frac{BE}{CD} \Rightarrow \frac{18}{x} = \frac{20}{100}$$

$$\Rightarrow \quad x = \frac{18 \times 100}{20} = 18 \times 5 = 90 \text{ m}$$

So, option (c) is correct.

3. Let height of Anshul's house be GF = h_1
Since, $\Delta HFG \sim \Delta HCD$

$$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD} \Rightarrow \frac{20}{50} = \frac{h_1}{100}$$

$$h_1 = \frac{20 \times 100}{50} = 40 \text{ m}$$

So, option (b) is correct.

4. Given, HC = 40 cm
Let length of the shadow of Anshul's house be HF = l m.
Since, $\Delta HFG \sim \Delta HCD$

$$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD}$$

$$\Rightarrow \quad \frac{l}{40} = \frac{40}{100}$$

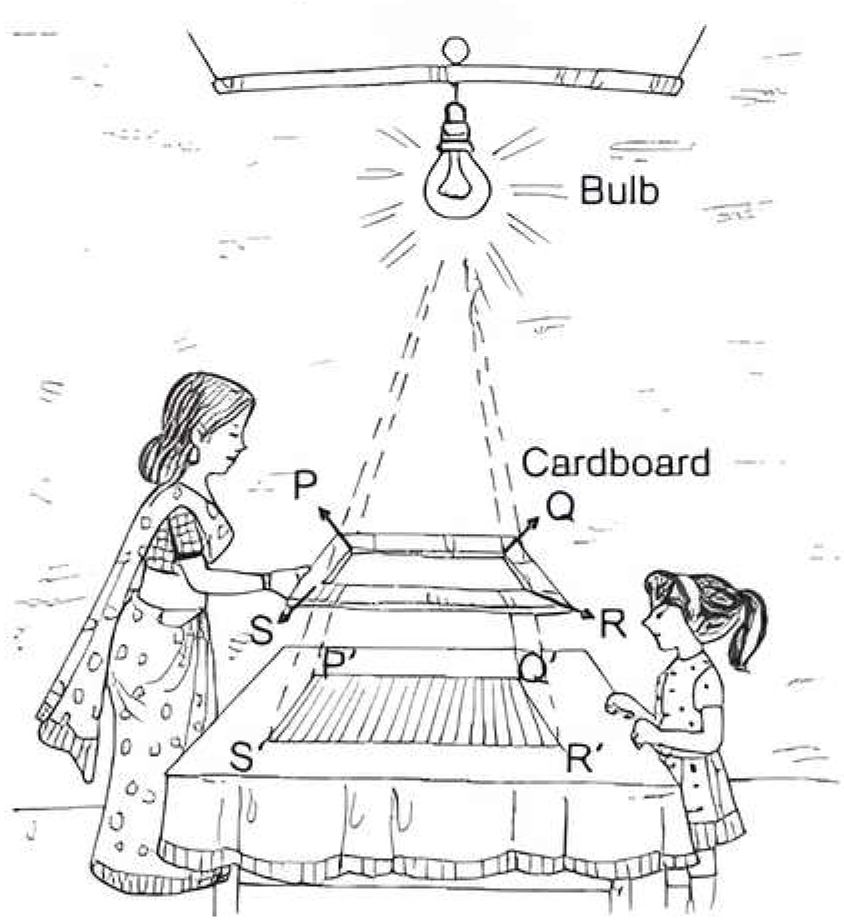
$$\Rightarrow \quad l = \frac{40 \times 40}{100} = 16 \text{ m}$$

So, option (a) is correct.

5. RHS similarity
Criterion does not exist.
So, option (d) is correct.

Case Study 2

Gaurav placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P′Q′R′S′. Quadrilateral P′Q′R′S′ is an enlargement of the quadrilateral PQRS with scale factor 1 : 3. Given that PQ = 2.5 cm, QR = 3.5 cm. RS = 3.4 cm and PS = 3.1 cm; $\angle P = 115^\circ$, $\angle Q = 95^\circ$, $\angle R = 65^\circ$ and $\angle S = 85^\circ$.



Based on the given information, solve the following questions:

Q 1. The length of R′S′ is:

a. 3.4 cm b. 10.2 cm c. 6.8 cm d. 9.5 cm

Q 2. The ratio of sides P′Q′ and Q′R′ is:

a. 5 : 7 b. 7 : 5 c. 7 : 2 d. 2 : 7

Q 3. The measurement of $\angle Q'$ is:

a. 115° b. 95° c. 65° d. 85°

Q 4. The sum of the lengths Q′R′ and P′S′ is:

a. 12.3 cm b. 6.7 cm

c. 19.8 cm d. 9 cm

Q 5. The sum of angles of quadrilateral P′Q′R′S′ is:

a. 180° b. 270° c. 300° d. 360°

Solutions

1. Given, scale factor is 1 : 3.

$\because$ $R'S' = 3RS$

$\therefore$ $R'S' = 3 \times 3.4 = 10.2$ cm

So, option (b) is correct.

2. Since, $P'Q' = 3\,PQ = 3 \times 2.5 = 7.5$ cm

and $Q'R' = 3\,QR = 3 \times 3.5 = 10.5$ cm

$$\therefore \quad \frac{P'Q'}{Q'R'} = \frac{7.5}{10.5} = \frac{5}{7} \text{ or } 5:7$$

So, option (a) is correct.

3. Quadrilateral P′Q′R′S′ is similar to PQRS

$\therefore$ $\angle Q' = \angle Q = 95°$

So, option (b) is correct.

4. $Q'R' = 3\,QR = 3 \times 3.5 = 10.5$ cm

and $P'S' = 3\,PS = 3 \times 3.1 = 9.3$ cm

$\therefore$ $Q'R' + P'S' = 10.5 + 9.3 = 19.8$ cm

So, option (c) is correct.

5. Since, $PQRS \sim P'Q'R'S'$

$\therefore$ $\angle P' = \angle P = 115°$

$\angle Q' = \angle Q = 95°$

$\angle R' = \angle R = 65°$

and $\angle S' = \angle S = 85°$

$\therefore$ $\angle P' + \angle Q' + \angle R' + \angle S' = 115° + 95° + 65° + 85°$

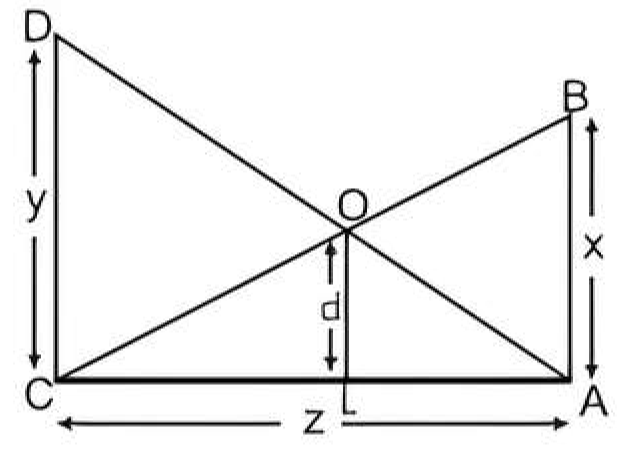
$= 360°$

i.e., the sum of angles of quadrilateral P′Q′R′S′ is 360°.

So, option (d) is correct.

Case Study 3

Anika is studying in class X. She observe two poles DC and BA. The heights of these poles are *x* m and *y* m respectively as shown in figure:



These poles are *z* m apart and O is the point of intersection of the lines joining the top of each pole to the foot of opposite pole and the distance between point O and L is *d*. Few questions came to his mind while observing the poles.

Based on the above information, solve the following questions:

Q 1. Which similarity criteria is applicable in ΔCAB and ΔCLO?

Q 2. If $x = y$, prove that BC : DA = 1 : 1.

Q 3. If CL = *a*, then find *a* in terms of *x*, *y* and *d*.

OR

If AL = *b*, then find *b* in terms of *x*, *y* and *d*.

Solutions

1. In ΔCAB and ΔCLO, we have

$\angle CAB = \angle CLO = 90°$

$\angle C = \angle C$ (common)

$\therefore$ By AA similarity criterion,

$\Delta CAB \sim \Delta CLO$

2. In ΔDCA and ΔBAC,

$DC = BA$ [$\because x = y$ (Given)]

$\angle DCA = \angle BAC$ [Each 90°]

$CA = AC$ [Common]

By SAS similarity criterian,

$\Delta DCA \sim \Delta BAC$

$$\frac{DA}{BC} = \frac{DC}{BA} = \frac{y}{x}$$

$$\Rightarrow \quad \frac{BC}{DA} = \frac{x}{y} = \frac{x}{x} = \frac{1}{1}$$

$\therefore$ BC : DA = 1 : 1 **proved.**

3. $\because$ $\Delta CAB \sim \Delta CLO$

TiP

Corresponding sides of similar triangles are proportional.

$$\therefore \quad \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{z}{a} = \frac{x}{d} \Rightarrow a = \frac{zd}{x}$$

OR

In ΔALO and ΔACD ,

We have

$\angle ALO = \angle ACD = 90°$

$\angle A = \angle A$ (common)

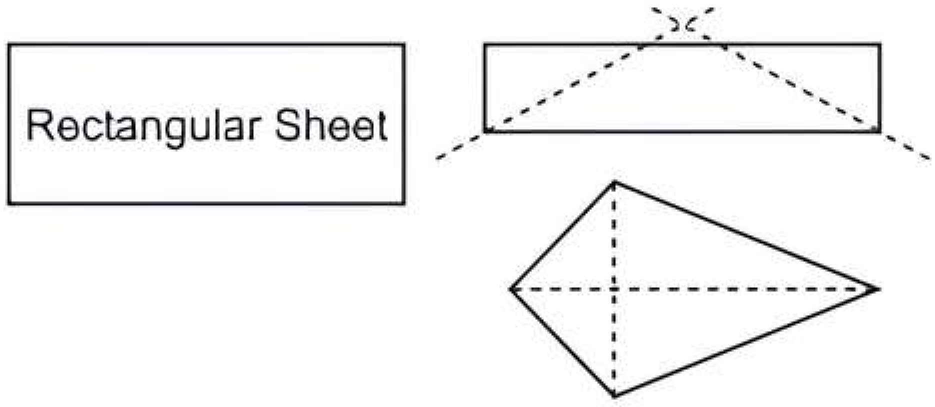
$\therefore$ By AA similarity criterion,

$\Delta ALO \sim \Delta ACD$

$$\therefore \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{b}{z} = \frac{d}{y} \Rightarrow b = \frac{zd}{y}$$

Case Study 4

Before Basant Panchami, Samarth is trying to make kites at home. So, he take a rectangular sheet and fold it horizontally, then vertically and fold it transversally. After cutting transversally, he gets a kite shaped figure as shown below:



Based on the above information, solve the following questions:

Q 1. What is the angle between diagonals of a rectangle?

Q 2. Prove that two triangles divided by a diagonal in rectangle are similar as well as congruent.

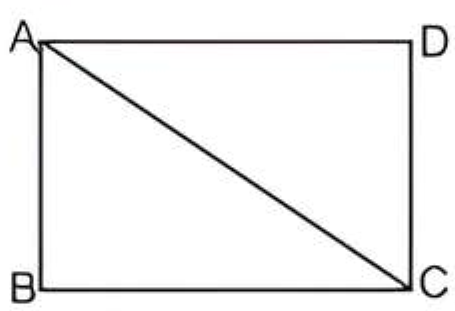
Q 3. Prove that the longest diagonal of a kite bisect a pair of opposite angle.

OR

By which similarity criterion the triangles formed by longest diagonal in a kite are similar?

Solutions

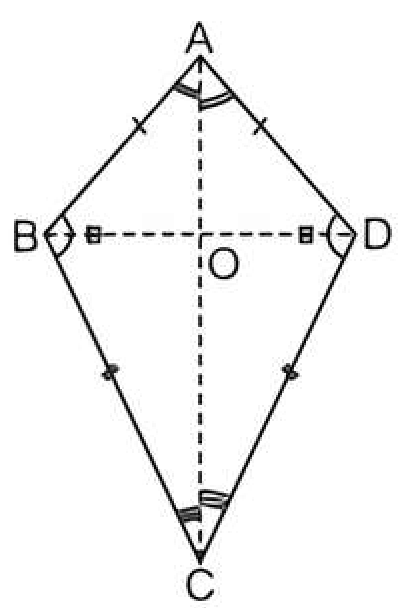
1. Diagonals of a rectangle can bisect each other at any angle.
2. In $\triangle ABC$ and $\triangle CDA$



$AB = CD$
$\angle B = \angle D$
$BC = DA$
$\triangle ABC \cong \triangle CDA$
(By SAS)

When two triangles are congruent, then they are similar also.

3. In $\triangle AOB$ and $\triangle AOD$,



$AB = AD$
$OA = OA$ (common)
$BO = DO$
(diagonal AC bisect the other diagonal BD)
$\therefore$ $\triangle AOB \sim \triangle AOD$
(by SSS similarity)
$\Rightarrow$ $\angle BAO = \angle DAO$ ...(1)

In $\triangle BOC$ and $\triangle DOC$,
$BC = DC$
$OC = OC$ (common)
$BO = OD$
[Diagonal AC bisect the other diagonal BD]
$\therefore$ $\triangle BOC \sim \triangle DOC$ [by SSS similarity]
$\Rightarrow$ $\angle BCO = \angle DCO$ ...(2)

From (1) and (2), it is clear that, the longest diagonal of a kite bisect a pair of opposite angle.

OR

In $\triangle ABC$ and $\triangle ADC$,
$AB = AD$
$BC = DC$
$AC = AC$ (common)
$\therefore$ $\triangle ABC \sim \triangle ADC$
(by SSS criterion)

In $\triangle ABC$ and $\triangle ADC$,
$AB = AD$
$\angle ABC = \angle ADC$
$BC = DC$
$\therefore$ $\triangle ABC \sim \triangle ADC$
(by SAS criterion)

In $\triangle ABC$ and $\triangle ADC$,
$\angle B = \angle D$
$\angle BAC = \angle DAC$
($\because$ $\angle BAO = \angle BAC$, $\angle DAO = \angle DAC$, proved above)
$\angle BCA = \angle DCA$
($\because$ $\angle BCO = \angle BCA$, $\angle DCO = \angle DCA$, proved above)
$\therefore$ $\triangle ABC \sim \triangle ADC$ (by AAA similarity)

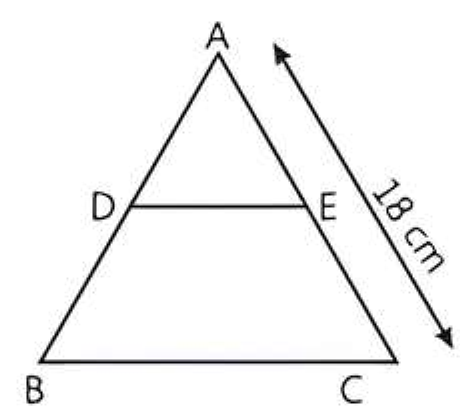
So, required similarity criterions are SSS, SAS and AAA.

Very Short Answer Type Questions

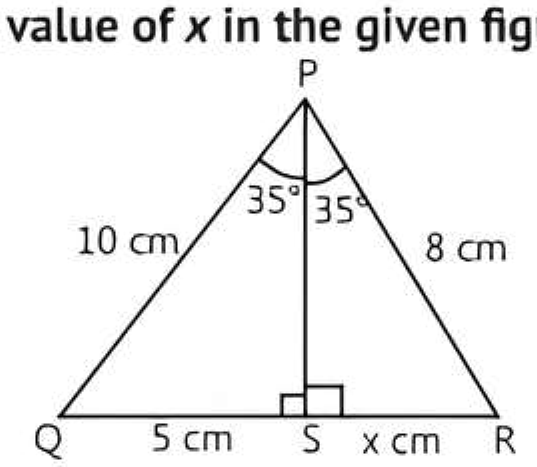
Q 1. In $\triangle PQR$, S and T are points on the sides PQ and PR respectively, such that ST || QR. If PS = 4 cm, PQ = 9 cm and PR = 4.5 cm, then find PT.
[CBSE 2015, 16, 17]

Q 2. In two triangles ABC and DEF, if $\angle A = \angle E$ and $\angle B = \angle F$. Then, prove that $\frac{AB}{AC} = \frac{EF}{ED}$.

Q 3. In the given figure, DE is parallel to BC. If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, then find AE.

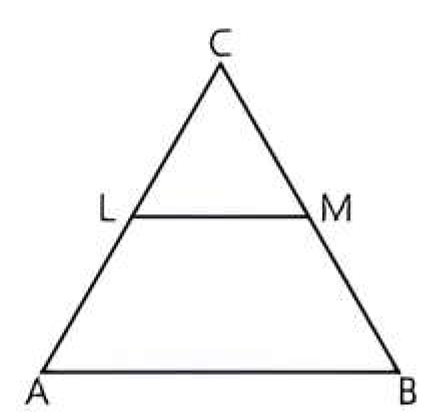


Q 4. Find the value of x in the given figure.



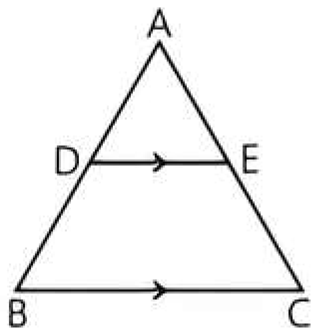
Q 5. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then find the ratio of their corresponding sides. ***[CBSE 2015]***

Q 6. In the given figure, LM || AB. If AL = $x - 3$, AC = $2x$, BM = $x - 2$ and BC = $2x + 3$, find the value of x.

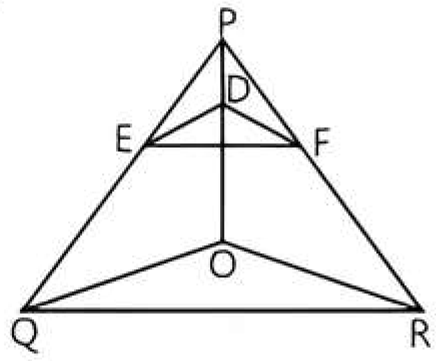


Short Answer Type-I Questions

Q 1. In the given figure, a $\triangle ABC$, $DE \parallel BC$, so that $AD = (4x - 3)$ cm, $AE = (8x - 7)$ cm, $BD = (3x - 1)$ cm and $CE = (5x - 3)$ cm. Find the value of x. [CBSE 2015]



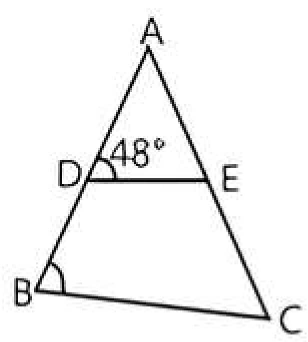
Q 2. In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$. [NCERT EXERCISE]



Q 3. X and Y are points on the sides AB and AC respectively of a triangle ABC, such that $\frac{AX}{AB} = \frac{1}{4}$, $AY = 2$ cm and $YC = 6$ cm. Find whether $XY \parallel BC$ or not. [CBSE 2015]

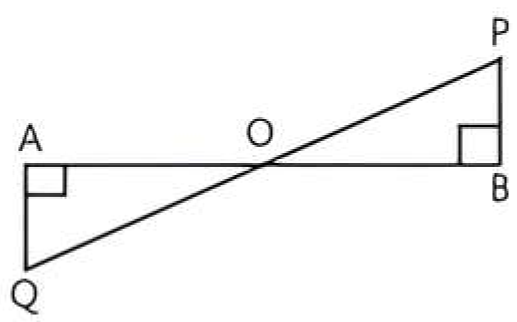
Q 4. In figure, if $AD = 6$cm, $DB = 9$cm, $AE = 8$cm and $EC = 12$cm and $\angle ADE = 48°$. Find $\angle ABC$.

[CBSE SQP 2023-24]

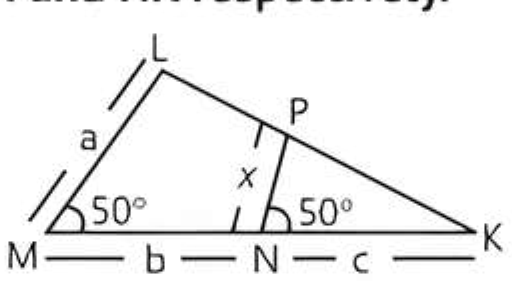


Q 5. In the given figure, $QA \perp AB$ and $PB \perp AB$. If $AO = 20$ cm, $BO = 12$ cm, $PB = 18$ cm, find AQ.

[CBSE 2017]

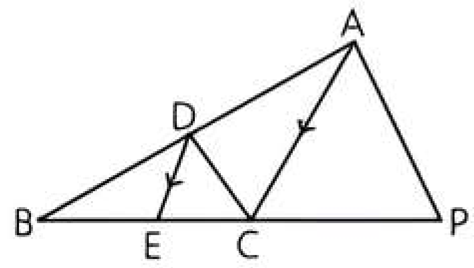


Q 6. In the given figure, $\angle M = \angle N = 50°$. Express x in terms of a, b and c where a, b and c are the lengths of LM, MN and NK respectively.

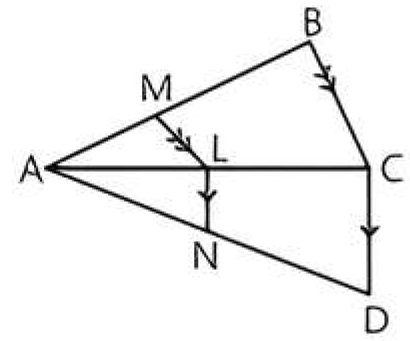


Q 7. A vertical stick which is 15 cm long casts a 12 cm long shadow on ground. At the same time, a vertical tower casts a 50 m long shadow on the ground. Find the height of the tower. [CBSE 2016]

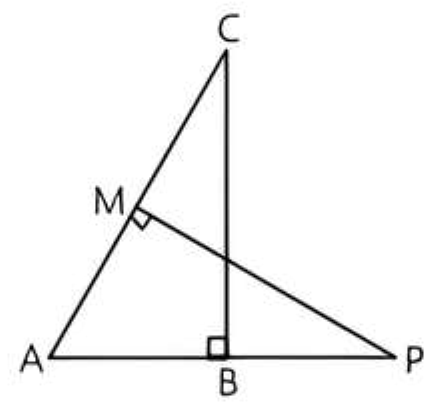
Q 8. In the given figure, $DE \parallel AC$ and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that $DC \parallel AP$.



Q 9. In the given figure, $LM \parallel CB$ and $LN \parallel CD$. Prove that $\frac{AM}{AN} = \frac{AB}{AD}$. [CBSE 2023, NCERT EXERCISE]

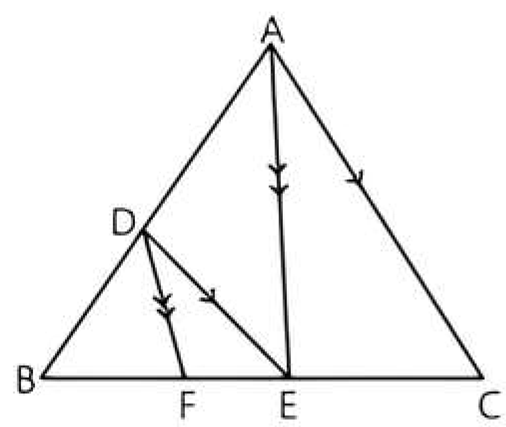


Q 10. In the given figure, ABC and AMP are two right triangles, right angled at B and M, respectively. Prove that $\triangle ABC \sim \triangle AMP$. [CBSE 2023]



Q 11. In the given figure, $DE \parallel AC$ and $DF \parallel AE$.

[CBSE 2023, NCERT EXERCISE]

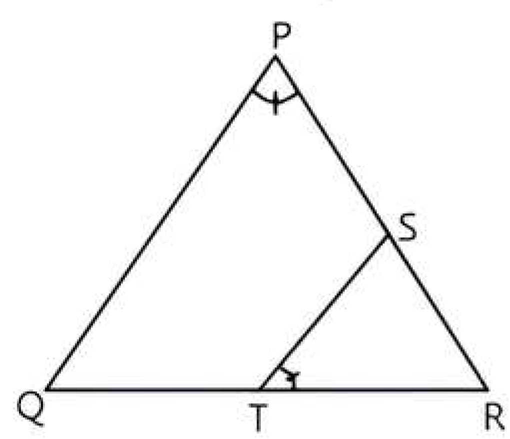


Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.

Q 12. ABCD is a trapezium such that $BC \parallel AD$ and $AB = 4$cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2}$, then find CD.

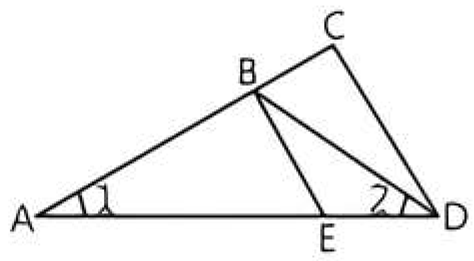
Q 13. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.

[CBSE 2023, NCERT EXERCISE]



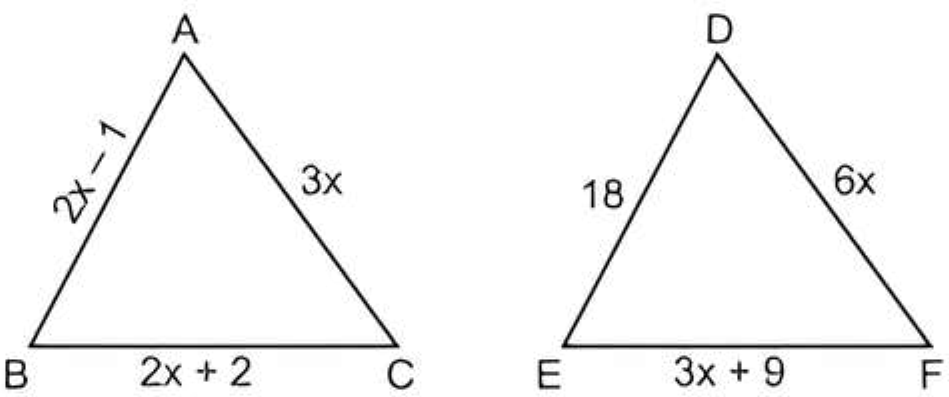
Q 14. The diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$. Show that $\frac{AB}{BC} = \frac{AD}{CD}$.

Q 15. In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that $\Delta BAE \sim \Delta CAD$. *[CBSE SQP 2022-23]*

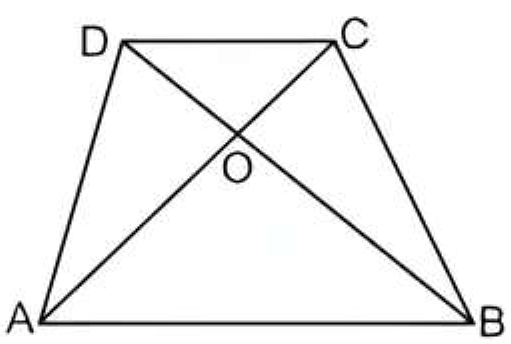


Short Answer Type-II Questions

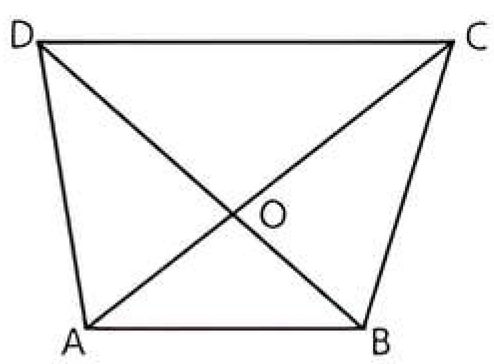
Q 1. In the given figure, if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. *[NCERT EXEMPLAR; CBSE 2020]*



Q 2. Diagonals AC and BD of trapezium ABCD with AB || DC intersect each other at point O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$. *[CBSE 2023, NCERT EXERCISE]*

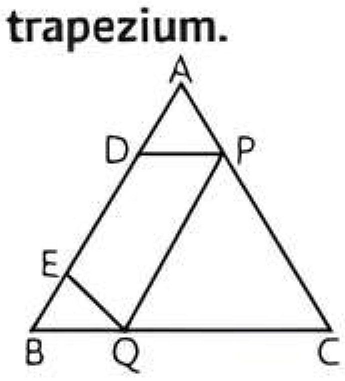


Q 3. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{OD}$.



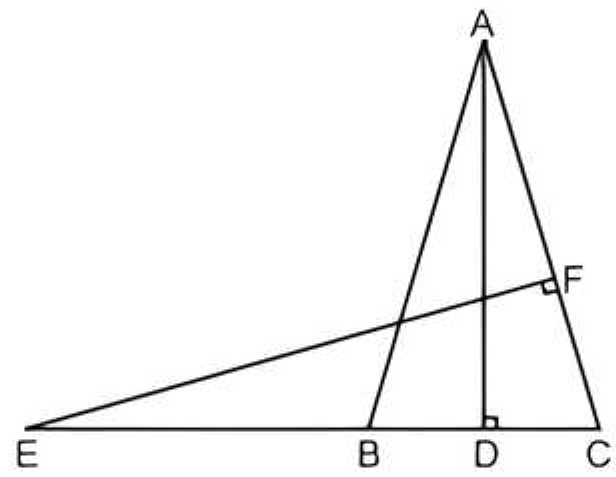
Show that quadrilateral ABCD is a trapezium. *[CBSE 2023, NCERT EXERCISE]*

Q 4. In the given figure, D and E are two points lying on side AB, such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.

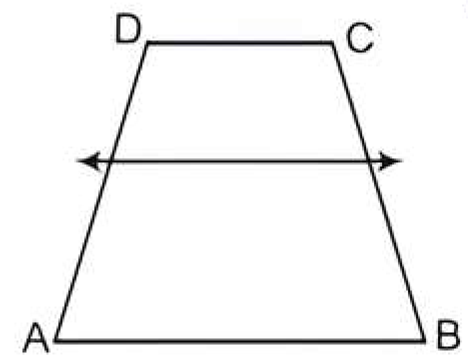


Q 5. If AD and PM are medians of triangles ABC and PQR, respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. *[NCERT EXERCISE; CBSE 2017]*

Q 6. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with AB = AC. If $AD \perp BC$ and $EF \perp AC$, then prove that $\Delta ABD \sim \Delta ECF$. *[NCERT EXERCISE, CBSE 2023]*

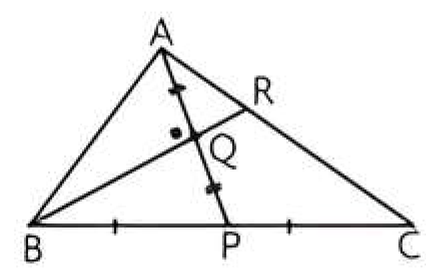


Q 7. In the given figure, if ABCD is a trapezium in which AB || CD || EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$. *[CBSE SQP 2023-24]*



Long Answer Type Questions

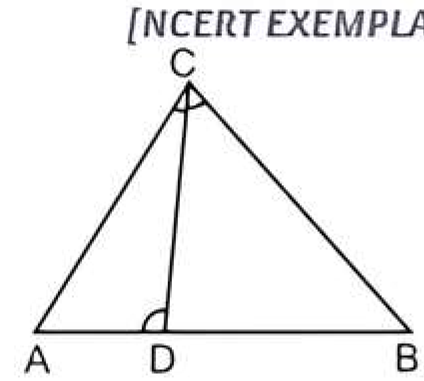
Q 1. In the given figure of Δ ABC, P is the mid-point of BC and Q is the mid-point of AP. If extended BQ meets AC in R, prove that $AR = \frac{1}{3}CA$. *[CBSE 2016]*



Q 2. ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that:

(i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Q 3. In the given figure, $\angle ADC = \angle BCA$; prove that $\Delta ACB \sim \Delta ADC$. Hence find BD if AC = 8 cm and AD = 3 cm. *[NCERT EXEMPLAR, CBSE SQP 2023-24]*



Q 4. Prove that if a lines is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In ΔPQR, S and T are points on PQ and PR respectively. $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle. *[CBSE SQP 2023-24]*

OR

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. *[CBSE 2023]*

Solutions

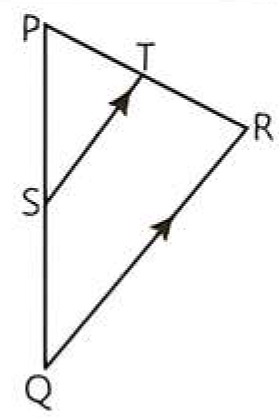
Very Short Answer Type Questions

1. Given, PS = 4 cm, PQ = 9 cm and PR = 4.5 cm

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Since, ST || QR, then by BPT, we have



$$\frac{PS}{PQ}=\frac{PT}{PR}$$

$$\Rightarrow \quad \frac{4}{9}=\frac{PT}{4.5}$$

or $\quad PT=\frac{4\times4.5}{9}=2\,\text{cm}$

Hence, PT = 2 cm.

2. In ΔABC and ΔEFD,

$$\angle A=\angle E \text{ and } \angle B=\angle F \quad \text{(Given)}$$

$\therefore \quad \Delta ABC \sim \Delta EFD$ (By AA similarity criterion)

$\therefore \quad \frac{AB}{EF}=\frac{AC}{ED}\Rightarrow\frac{AB}{AC}=\frac{EF}{ED}$ **Hence proved.**

3. In ΔABC, DE || BC.

$\therefore \quad \frac{AD}{DB}=\frac{AE}{EC}$ (By Thales theorem)

$$\Rightarrow \quad \frac{2}{3}=\frac{AE}{EC} \quad \Rightarrow \quad \frac{3}{2}=\frac{EC}{AE}$$

Adding 1 on both the sides, we get

$$\frac{3}{2}+1=\frac{EC}{AE}+1 \quad \Rightarrow \quad \frac{3+2}{2}=\frac{EC+AE}{AE}$$

$$\Rightarrow \quad \frac{5}{2}=\frac{AC}{AE} \quad \Rightarrow \quad \frac{5}{2}=\frac{18}{AE}$$

$$\Rightarrow \quad 5AE=36$$

$$\Rightarrow \quad AE=\frac{36}{5}\Rightarrow AE=7.2\,\text{cm}$$

4. In ΔPSQ and ΔPSR,

$\angle QSP=\angle RSP=90^\circ$

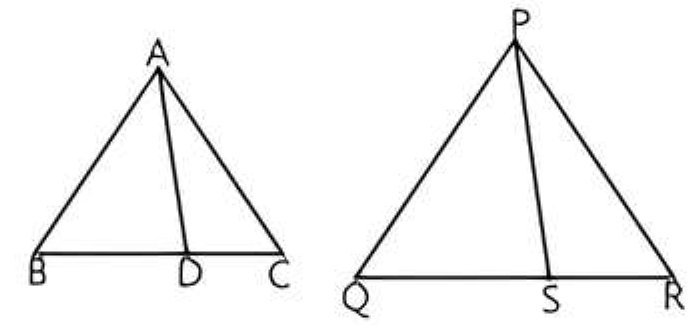
and $\angle QPS=\angle RPS=35^\circ$

$\therefore \quad \Delta PSQ \sim \Delta PSR$ (By AA similarity)

$$\frac{PQ}{PR}=\frac{SQ}{SR}$$

$$\Rightarrow \quad \frac{10}{8}=\frac{5}{x}\Rightarrow x=4\,\text{cm}$$

5. Let ABC and PQR are two similar triangles with medians AD and PS respectively.



Then, $\quad \frac{AD}{PS}=\frac{5}{7}$ (Given)

TR!CK

The ratio of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$\Rightarrow \quad \frac{AB}{PQ}=\frac{AD}{PS}=\frac{5}{7}$ $(\because \Delta ABC \sim \Delta PQR)$

Hence, the ratio of corresponding sides is 5 : 7.

6.

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In ΔABC, LM || AB

$\therefore \quad \frac{AL}{LC}=\frac{BM}{MC}$ (By Thales theorem)

$$\Rightarrow \quad \frac{AL}{AC-AL}=\frac{BM}{BC-BM}$$

$$\Rightarrow \quad \frac{x-3}{2x-(x-3)}=\frac{x-2}{(2x+3)-(x-2)}\Rightarrow\frac{x-3}{x+3}=\frac{x-2}{x+5}$$

$$\Rightarrow \quad (x-3)(x+5)=(x-2)(x+3)$$

$$\Rightarrow \quad x^2+2x-15=x^2+x-6$$

$$\Rightarrow \quad x=9$$

Short Answer Type-I Questions

1. In Δ ABC, DE || BC, so by BPT,

$$\frac{AD}{BD}=\frac{AE}{CE} \quad \Rightarrow \quad \frac{4x-3}{3x-1}=\frac{8x-7}{5x-3}$$

$$\Rightarrow \quad (4x-3)(5x-3)=(8x-7)(3x-1)$$

$$\Rightarrow \quad 20x^2-12x-15x+9=24x^2-8x-21x+7$$

$$\Rightarrow \quad 24x^2-20x^2-29x+27x+7-9=0$$

$$\Rightarrow \quad 4x^2-2x-2=0$$

$$\Rightarrow \quad 2(2x^2-x-1)=0$$

TR!CK

$\because \quad 2=2\times1$

$\therefore$ Here, we have taken 2 and 1 as a factors of 2.
So, middle term, $-1=1-2$.

$$\Rightarrow \quad 2x^2-2x+x-1=0$$

$$\Rightarrow \quad 2x(x-1)+1(x-1)=0$$

$$\Rightarrow \quad (x-1)(2x+1)=0$$

$\Rightarrow \quad x - 1 = 0$ and $2x + 1 = 0$

$\Rightarrow \quad x = 1$ and $x = -\frac{1}{2}$

When $x = -\frac{1}{2}$, then AD, BD, AE and CE all are negative.

$\therefore \quad x \neq -\frac{1}{2}$

Hence, the value of x is 1.

COMMON ERR(!)R

Sometimes students take both value of x as a answer but it is wrong. Students should cross check the value of x.

2.

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In Δ POQ, DE || OQ

$\therefore \quad \frac{PE}{EQ} = \frac{PD}{DO}$ (By BPT) ...(1)

In Δ POR, DF || OR

$\therefore \quad \frac{PF}{FR} = \frac{PD}{DO}$ (By BPT) ...(2)

From eqs. (1) and (2), we get

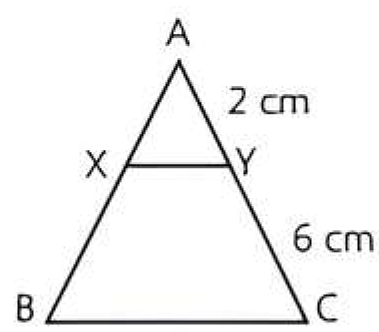
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\therefore$ EF || QR (Converse of BPT) **Hence proved.**

3. Given, $\frac{AX}{AB} = \frac{1}{4}$



Let AX = k, AB = $4k$

$\therefore$ XB = AB – AX = $4k - k = 3k$

Now, $\frac{AX}{XB} = \frac{k}{3k} = \frac{1}{3}$

and $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\therefore \quad \frac{AX}{XB} = \frac{AY}{YC}$

Hence, XY || BC (By converse of BPT)

4. Given, AD = 6cm,
DB = 9cm, AE = 8cm,
EC = 12cm and $\angle ADE = 48°$

Here, $\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$

and $\frac{AE}{EC} = \frac{8}{12}$

$\because \quad \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ DE || BC (By Converse of BPT)

In ΔABC, DE||BC

$\therefore \angle ABC = \angle ADE$ (Corresponding angles)

$= 48°$

5. In ΔOAQ and ΔOBP,

$\angle OAQ = \angle OBP$ (Each 90°)

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

$\therefore \quad \Delta OAQ \sim \Delta OBP$ (By AA similarity)

$\Rightarrow \quad \frac{AO}{BO} = \frac{AQ}{PB}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{20}{12} = \frac{AQ}{18}$

$\Rightarrow \quad AQ = \frac{18 \times 20}{12} = 30$ cm

6. Given, LM = a, PN = x, MN = b and NK = c

In Δ PNK and Δ LMK,

$\angle PNK = \angle LMK$ (Each 50°)

$\angle PKN = \angle LKM$ (Common angle)

$\therefore \quad \Delta PNK \sim \Delta LMK$ (By AA similarity)

So, $\frac{NK}{MK} = \frac{PN}{LM}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{NK}{MN + NK} = \frac{PN}{LM}$

$\Rightarrow \quad \frac{c}{b+c} = \frac{x}{a}$ or $x = \frac{ac}{b+c}$

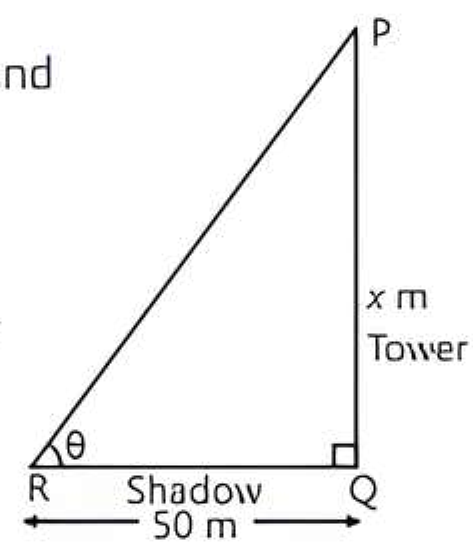
7. Let AB be the vertical stick and BC be its shadow.

Given, AB = 15 cm = 0.15 m

and BC = 12 cm = 0.12 m

Let PQ be the vertical tower and QR be its shadow.

In Δ ABC and Δ PQR,



TiP

Two triangles are similar, if their corresponding sides are in proportional.

$\angle ABC = \angle PQR$ (Each 90°)

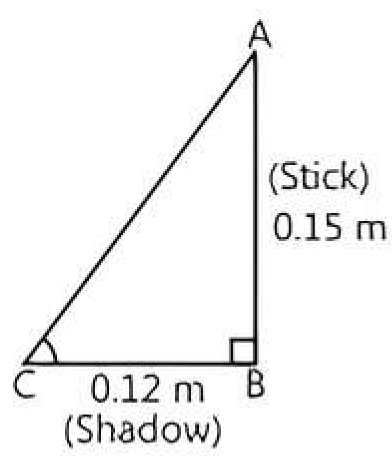
$\angle ACB = \angle PRQ$

(Angular elevation of the sun at the same time)

$\therefore \Delta ABC \sim \Delta PQR$ (By AA similarity)

So, $\frac{AB}{PQ} = \frac{BC}{QR}$

(Corresponding sides are proportional)



$\Rightarrow \quad \frac{0.15}{x} = \frac{0.12}{50}$

$\Rightarrow \quad x = \frac{0.15 \times 50}{0.12}$

or $\quad x = 62.5$ m

Hence, the height of the tower is 62.5 m.

8. In ΔABC

Given that, $DE \parallel AC$

$\therefore \quad \frac{BE}{EC} = \frac{BD}{DA}$ (By BPT) ... (1)

But $\frac{BE}{EC} = \frac{BC}{CP}$ (given) ... (2)

from eqs. (1) and (2), we get $\frac{BD}{DA} = \frac{BC}{CP}$

$\Rightarrow DC \parallel AP$. (By converse of BPT) **Hence proved.**

9. Given: Line segment $LM \parallel CB$ and $LN \parallel CD$.

To Prove: $\frac{AM}{AN} = \frac{AB}{AD}$

Proof: In ΔABC, M is a point on side AB and L is a point on side AC from which line segment $LM \parallel CB$.

$\therefore \quad \frac{AM}{MB} = \frac{AL}{LC}$ [From BPT]

$\Rightarrow \quad \frac{MB}{AM} = \frac{LC}{AL}$ (Taking reciprocals)

$\therefore \quad 1 + \frac{MB}{AM} = 1 + \frac{LC}{AL}$ (Adding 1 on both sides)

$\Rightarrow \quad \frac{AM + MB}{AM} = \frac{AL + LC}{AL}$

$\therefore \quad \frac{AB}{AM} = \frac{AC}{AL}$ [$\because AB = AM + MB, AC = AL + LC$] ...(1)

Similarly, In ΔACD, $LN \parallel CD$

$\therefore \quad \frac{AN}{ND} = \frac{AL}{LC}$ [From BPT]

$\Rightarrow \quad \frac{ND}{AN} = \frac{LC}{AL}$ (Taking reciprocals)

$\Rightarrow \quad 1 + \frac{ND}{AN} = 1 + \frac{LC}{AL}$

$\Rightarrow \quad \frac{AN + ND}{AN} = \frac{AL + LC}{AL}$

$\Rightarrow \quad \frac{AD}{AN} = \frac{AC}{AL}$ [$\because AD = AN + ND, AC = AL + LC$] ...(2)

From eqs (1) and (2), we get

$\frac{AB}{AM} = \frac{AD}{AN}$

$\Rightarrow \quad \frac{AM}{AB} = \frac{AN}{AD}$ (Taking reciprocals)

$\Rightarrow \quad \frac{AM}{AN} = \frac{AB}{AD}$ **Hence proved.**

10. Given: ABC and AMP are two right triangles, right angled at B and M, respectively.

Prove that: $\Delta ABC \sim \Delta AMP$

Proof: In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ [Each $90°$]

$\angle CAB = \angle PAM$ [Common angle]

$\angle ACB = \angle APM$ [Same itself]

$\therefore \quad \Delta ABC \sim \Delta AMP$ [By AAA similarity criterion]

Hence proved

11. Given: In ΔABC, D is a point on side AB and two points E and F on the side BC. Line segments DF, DE and AE are drawn. $DE \parallel AC$ and $DF \parallel AE$.

Prove that : $\frac{BF}{FE} = \frac{BE}{EC}$

Proof : $\because$ In ΔABE, $DF \parallel AE$

$\therefore \quad \frac{BF}{FE} = \frac{BD}{DA}$ ..... (1) (By BPT)

and In ΔABC, $DE \parallel AC$

$\therefore \quad \frac{BE}{EC} = \frac{BD}{DA}$ ...(2) (By BPT)

From eqs. (1) and (2), we get

$\frac{BF}{FE} = \frac{BE}{EC}$ **Hence proved.**

12. In

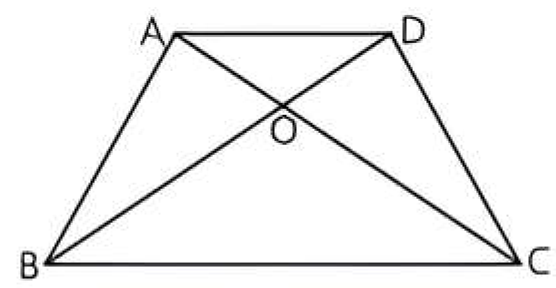
TIP

When two non-parallel rays intersect at a point the angles formed between these rays at point of intersection in opposite directions are called vertically opposite angle.

In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2}$ (Given)



$\therefore \quad \Delta AOB \sim \Delta COD$ (By SAS similarity criterion)

$\therefore \quad \frac{AB}{CD} = \frac{AO}{OC} \Rightarrow \frac{4}{CD} = \frac{1}{2} \Rightarrow CD = 8$ cm

13. Given : In the given figure,

$\angle P = \angle RTS$

To Prove : $\Delta RPQ \sim \Delta RTS$

Proof : In ΔRPQ and ΔRTS

$\angle P = \angle RTS$ (Given)

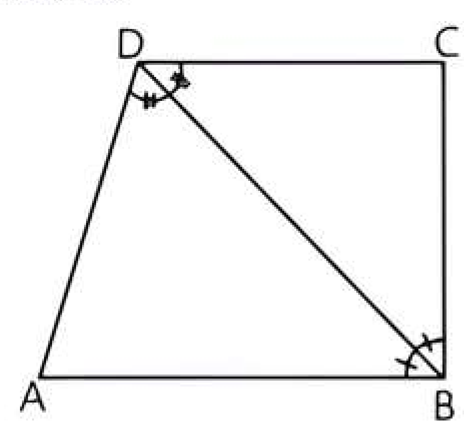
$\angle R = \angle SRT$ (common angle)

$\angle Q = \angle RST$ (some itself)

$\therefore RPQ \sim \Delta RTS$ (By AAA similarity criterion)

Hence proved.

14. Given: Diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$.



To Prove : $\frac{AB}{BC} = \frac{AD}{CD}$

Proof: In ΔABD and ΔCBD

$\angle ABD = \angle CBD$ ($\therefore$ BD bisects $\angle B$)

$BD = BD$ (Common side)

$\angle ADB = \angle CDB$ (∴ BD bisects $\angle D$)

Thus, $\Delta ABD \sim \Delta CBD$ (By ASA similarity criterion)

$$\therefore \quad \frac{AB}{CB} = \frac{AD}{CD} \text{ or } \frac{AB}{BC} = \frac{AD}{CD}$$ **Hence proved.**

15. In ΔABD,

$$\angle 1 = \angle 2 \quad \text{(Given)}$$

$$\therefore \quad BD = AB \quad \text{(Sides opposite to equal angles are equal)} \quad ...(1)$$

Given,

$$\frac{AD}{AE} = \frac{AC}{BD}$$

Using eq. (1),

$$\frac{AD}{AE} = \frac{AC}{AB} \quad ...(2)$$

In ΔBAE and ΔCAD,

$$\frac{AC}{AB} = \frac{AD}{AE} \quad \text{(From eq. (2))}$$

$$\angle A = \angle A \quad \text{(Common)}$$

$$\Delta BAE \sim \Delta CAD$$

(By SAS similarity criterion)

Hence proved.

Short Answer Type-II Questions

1. Since, $\Delta ABC \sim \Delta DEF$, so ratio of their corresponding sides is equal.

So, $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{1}{2}$$

Taking first and third parts, we get

$$\frac{2x-1}{18} = \frac{1}{2}$$

$$\Rightarrow \quad 2x = 9 + 1$$

$$\Rightarrow \quad x = \frac{9+1}{2} = \frac{10}{2} = 5$$

∴ Sides of ΔABC,

$AB = (2x - 1) = 2 \times 5 - 1 = 10 - 1 = 9$ cm,

$BC = 2x + 2 = 2 \times 5 + 2 = 10 + 2 = 12$ cm

and $AC = 3x = 3 \times 5 = 15$ cm

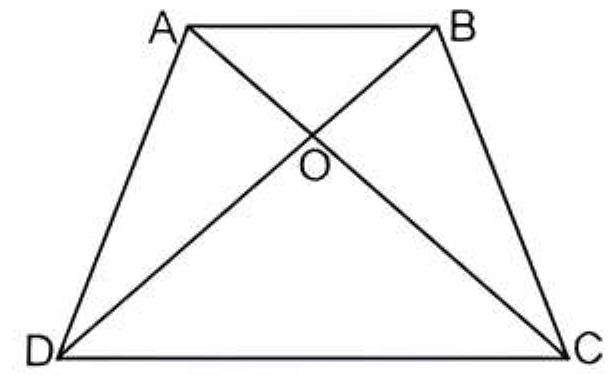
∴ Sides of ΔDEF,

$DE = 18$ cm,

$EF = 3 \times 5 + 9 = 15 + 9 = 24$ cm

and $DF = 6x = 6 \times 5 = 30$ cm

2. **Given:** ABCD is a trapezium in which AB||CD and its diagonals AC and BD intersect at point O.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof : ∵ AB||CD and AC is a transversal

∴ $\angle OAB = \angle OCD$ (alternate angles)

and $\angle AOB = \angle COD$ (vertically opposite angles)

Now, In ΔAOB and ΔOCD,

$\angle AOB = \angle COD$ and $\angle OAB = \angle OCD$

From AA similarity,

$$\Delta AOB \sim \Delta OCD$$

$$\frac{OA}{OC} = \frac{OB}{OD} \quad \text{(From the proportionality of side)}$$

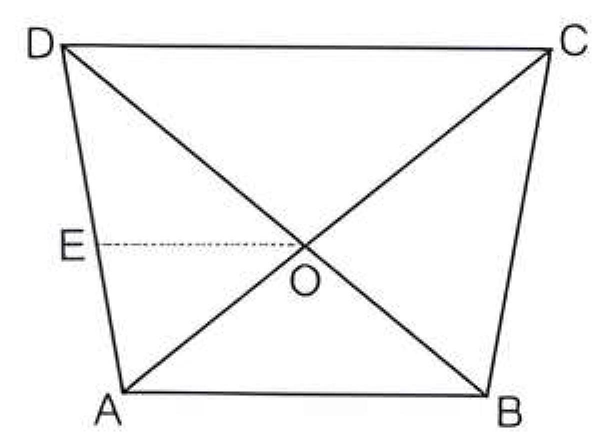
Hence proved.

3. **Given :** In a quadrilateral ABCD, $\frac{AO}{BO} = \frac{CO}{DO}$

To Prove : ABCD is a trapezium.

Construction : Let us draw a quadrilateral ABCD.

Draw a line OE || AB.



Proof : In Δ ABD, OE || AB

> **TiP**
>
> *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.*

By using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{BO}{OD} \quad ...(1)$$

But, it is given that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD} \quad ...(2)$$

From eqs. (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{OC}$$

> **TR!CK**
>
> *If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.*

$\Rightarrow$ EO || DC (By the converse of basic proportionality theorem)

$\Rightarrow$ AB || OE || DC

$\Rightarrow$ AB || DC

$\therefore$ ABCD is a trapezium. **Hence proved.**

4. Given : AD = BE, DP || BC and EQ || AC

To Prove : PQ || AB

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Proof : In Δ ABC, by BPT we have

$$\frac{AD}{DB}=\frac{AP}{PC} \quad (\because DP \parallel BC) \ldots(1)$$

Again, in Δ ABC, by BPT we have

$$\frac{BE}{EA}=\frac{BQ}{QC} \quad (\because EQ \parallel AC)$$

or $$\frac{AD}{DB}=\frac{BQ}{QC} \quad \ldots(2)$$

($\because$ AD = BE and EA = ED + DA = ED + BE = DB)

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

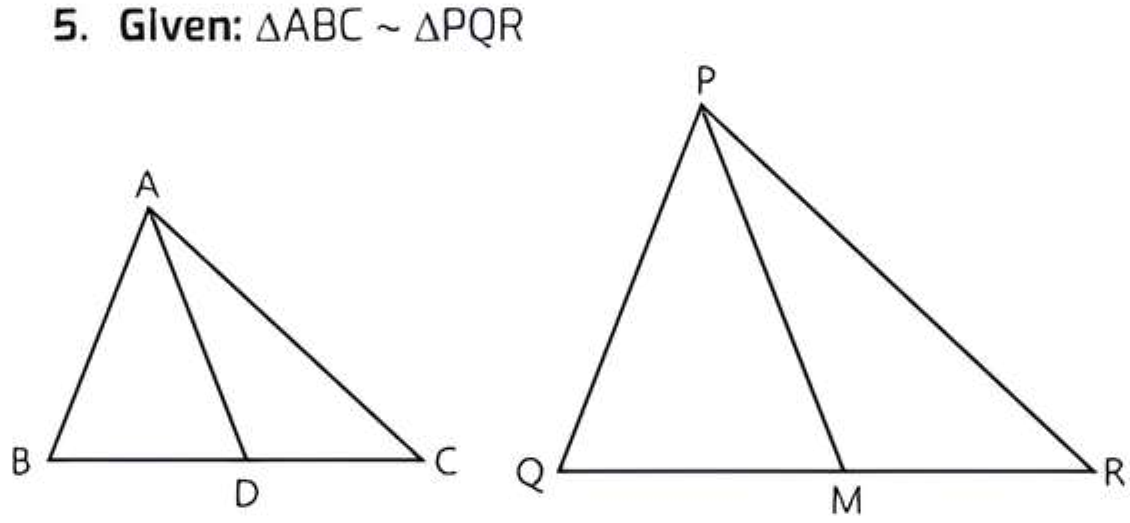
From eqs. (1) and (2), we get

$$\frac{AP}{PC}=\frac{BQ}{QC}$$

In ΔABC, P and Q divide the sides CA and CB respectively in the same ratio.

$\therefore$ PQ || AB **Hence proved.**

5. Given: $\Delta ABC \sim \Delta PQR$



To Prove : $\frac{AB}{PQ}=\frac{AD}{PM}$

Proof : Since, $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR} \quad \ldots(1)$$

(Corresponding sides are proportional)

Also, $\angle A=\angle P, \angle B=\angle Q, \angle C=\angle R$ ...(2)

(Corresponding angles are equal)

Since, AD and PM are medians, they will divide their opposite sides.

$$\therefore BD=\frac{BC}{2} \text{ and } QM=\frac{QR}{2} \quad \ldots(3)$$

From eqs. (1) and (3), we get

$$\frac{AB}{PQ}=\frac{BD}{QM} \quad \ldots(4)$$

Now, in Δ ABD and Δ PQM,

$\angle B=\angle Q$ [Using eq. (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using eq. (4)]

$\Delta ABD \sim \Delta PQM$ (By SAS similarity)

$$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$$

(Corresponding sides are proportional)

Hence proved.

6. It is given that ABC is an isosceles triangle.

$\therefore$ AB = AC

$\angle ABD = \angle ECF$

[$\therefore$ Angles opposite to equal sides are equal]

In ΔABD and ΔECF

$\angle ADB = \angle EFC$ (Each 90°)

$\angle ABD = \angle ECF$ (Proved above)

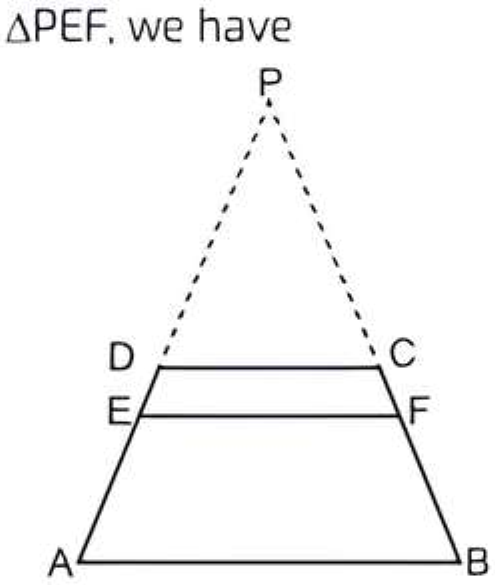
$\therefore$ $\Delta ABD \sim \Delta ECF$ (By AA similarity) **Proved.**

7. Given: In the given figure, AB || CD || EF.

To Prove: $\frac{AE}{ED}=\frac{BF}{FC}$.

Construction: Produce AD and BC to meet at P (say).

Proof: In ΔPEF, we have



DC || EF

$$\therefore \frac{PD}{DE}=\frac{PC}{CF} \quad \text{(By Thales theorem)}$$

$$\Rightarrow \frac{PD}{DE}+1=\frac{PC}{CF}+1 \quad \text{(Adding 1 on both sides)}$$

$$\Rightarrow \frac{PD+DE}{DE}=\frac{PC+CF}{CF}$$

$$\Rightarrow \frac{PE}{DE}=\frac{PF}{CF} \quad (\because PE = PD + DE, PF = PC + CF) \ldots(1)$$

In ΔPAB, we have

EF || AB

$$\therefore \quad \frac{PE}{EA} = \frac{PF}{FB} \quad \text{(By BPT) ...(2)}$$

On dividing eq. (1) by eq. (2), we get

$$\frac{\frac{PE}{DE}}{\frac{PE}{EA}} = \frac{\frac{PF}{CF}}{\frac{PF}{FB}}$$

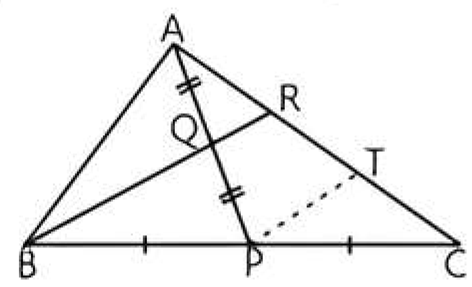
$$\Rightarrow \quad \frac{EA}{DE} = \frac{FB}{CF}$$

$$\text{or} \quad \frac{AE}{ED} = \frac{BF}{FC}$$

Hence proved.

Long Answer Type Questions

1. **Given:** In ΔABC, P is the mid-point of BC and Q is the mid-point of AP.



To Prove: $RA = \frac{1}{3}CA$

Construction: Draw PT || BR.

Proof: In Δ CBR, PT || BR

$$\frac{CT}{TR} = \frac{CP}{PB} \quad \text{(By BPT)}$$

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\Rightarrow \quad \frac{CT}{TR} = 1 \quad (\because \text{P is mid-point of BC } i.e., PB = CP)$$

$$\Rightarrow \quad CT = TR \quad \text{...(1)}$$

In Δ APT, QR || PT

$$\frac{AQ}{QP} = \frac{AR}{RT} \quad \text{(By BPT)}$$

$$\Rightarrow \quad 1 = \frac{AR}{RT} \quad (\because \text{Q is mid-point of AP } i.e., AQ = QP)$$

$$\Rightarrow \quad AR = RT \quad \text{...(2)}$$

From eqs. (1) and (2), we get

$$AR = RT = CT$$

$$\therefore \quad AR = \frac{1}{3}AC$$

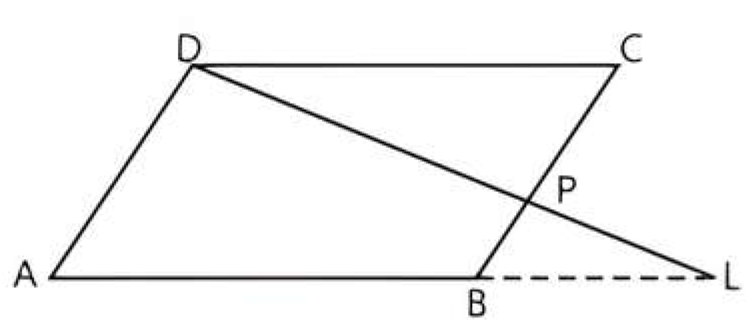
Hence proved.

2. **Given:** A parallelogram ABCD in which P is a point on side BC such that DP produced meets AB produced at L.

To Prove: (i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Proof: (i) In ΔALD, we have

BP || AD



$$\therefore \quad \frac{LB}{BA} = \frac{LP}{PD}$$

$$\Rightarrow \quad \frac{BL}{AB} = \frac{PL}{DP}$$

$$\Rightarrow \quad \frac{BL}{DC} = \frac{PL}{DP} \quad (\because AB = DC)$$

$$\Rightarrow \quad \frac{DP}{PL} = \frac{DC}{BL} \quad \text{(Taking reciprocals of both sides)}$$

Hence proved.

(ii) From part (i), we have

$$\frac{DP}{PL} = \frac{DC}{BL}$$

$$\Rightarrow \quad \frac{PL}{DP} = \frac{BL}{DC} \quad \text{(Taking reciprocals of both sides)}$$

$$\Rightarrow \quad \frac{PL}{DP} = \frac{BL}{AB} \quad (\because DC = AB)$$

$$\Rightarrow \quad \frac{PL}{DP} + 1 = \frac{BL}{AB} + 1$$

$$\Rightarrow \quad \frac{DP + PL}{DP} = \frac{BL + AB}{AB}$$

$$\Rightarrow \quad \frac{DL}{DP} = \frac{AL}{AB}$$

$$\Rightarrow \quad \frac{DL}{DP} = \frac{AL}{DC} \quad (\because AB = DC)$$

Hence proved.

3. Given $\angle BCA = \angle ADC$

In ΔACB and ΔADC,

$$\angle BCA = \angle ADC \quad \text{(Given)}$$

$$\angle CAB = \angle DAC \quad \text{(Common angle)}$$

$$\therefore \quad \Delta ACB \sim \Delta ADC \quad \text{(By AA similarity)}$$

Hence proved.

Also given , AC = 8 cm and AD = 3 cm.

We know that,

Sides of similar triangle are in same proportion.

$$\therefore \quad \frac{AC}{AD} = \frac{AB}{AC} \Rightarrow AC^2 = AB \times AD$$

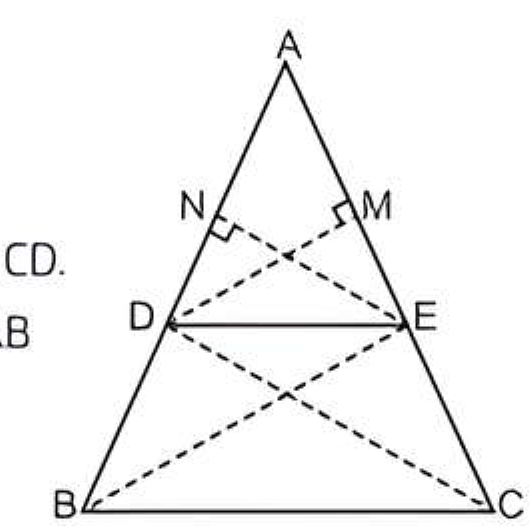
$$AB = \frac{AC^2}{AD} = \frac{(8)^2}{3} = \frac{64}{3}$$

So,

$$BD = AB - AD$$

$$= \frac{64}{3} - 3 = \frac{64 - 9}{3} = \frac{55}{3}$$

4. **Given:** In ΔABC, DE || BC



To Prove: $\frac{AD}{BD} = \frac{AE}{EC}$

Construction: Join BE and CD.

Draw DM $\perp$ AC and EN $\perp$ AB

Proof: Here,

$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EN$$

$$(\because \text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\text{and} \quad ar(\Delta BDE) = \frac{1}{2} \times DB \times EN$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{...(1)}$$

Also, $\quad ar(\Delta ADE) = \frac{1}{2} \times AE \times DM$

and $\quad ar(\Delta DEC) = \frac{1}{2} \times EC \times DM$

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$$

Since, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

$$\therefore \quad ar(\Delta BDE) = ar(\Delta CED)$$

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad ...(2)$$

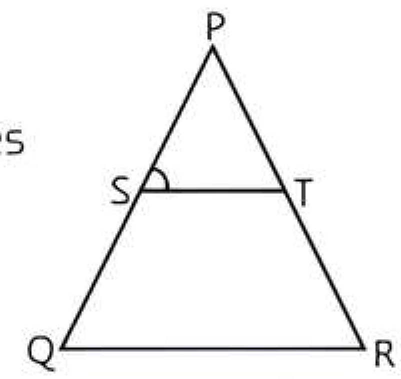
From eqs. (1) and (2),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

Given: In triangle PQR, S and T are points on PQ and PR respectively.

$\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$



To Prove: PQR is an isosceles triangle.

Proof: Since, $\frac{PS}{SQ} = \frac{PT}{TR}$

$\Rightarrow ST \parallel QR$ [By converse of BPT]

$\Rightarrow \angle PST = \angle PQR$ [Corresponding angles]

$\Rightarrow \angle PRQ = \angle PQR$ [$\because \angle PST = \angle PRQ$ {Given}]

$\Rightarrow PQ = PR$ [$\therefore$ Sides opposite to equals angles are also equal]

$\Rightarrow \Delta PQR$ is an isosceles triangle. **Hence proved.**

Chapter Test

Multiple Choice Questions

Q 1. If in ΔABC, AB = 6 cm and DE || BC such that $AE = \frac{1}{4}AC$, then the length of AD is:

a. 2 cm b. 12 cm c. 1.5 cm d. 4 cm

Q 2. In ΔPQR and ΔMNS, $\frac{PQ}{NS} = \frac{QR}{MS} = \frac{PR}{MN}$, then symbolically we write as:

a. $\Delta QRP \sim \Delta SMN$ b. $\Delta PQR \sim \Delta SMP$

c. $\Delta PQR \sim \Delta MNS$ d. None of these

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): ABC is a triangle in which AB = AC and D is a point on AC such that $BC^2 = AC \times CD$. Then $\Delta ABC \sim \Delta BDC$ by SAS similarity criterion.

Reason (R): If two angles of one triangle are respectively equal to the two angles of another triangle, then the two triangles are similar. This is known as SAS similarity criterion.

Q 4. Assertion (A): In a ΔABC, D and E are points on sides AB and AC respectively, such that BD = CE. If $\angle B = \angle C$, then DE is not parallel to BC.

Reason (R): If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Fill in the Blanks

Q 5. Two polygons of the same number of sides are similar, if their corresponding angles are .... and their corresponding sides are, in the .... ratio.

Q 6. All equilateral triangles are .... (similar/not similar).

True/False

Q 7. Two figures having the same shapes is said to be similar figures.

Q 8. In two triangles, if one pair of the corresponding sides are proportional and the included angles are also equal, then two triangles are not similar.

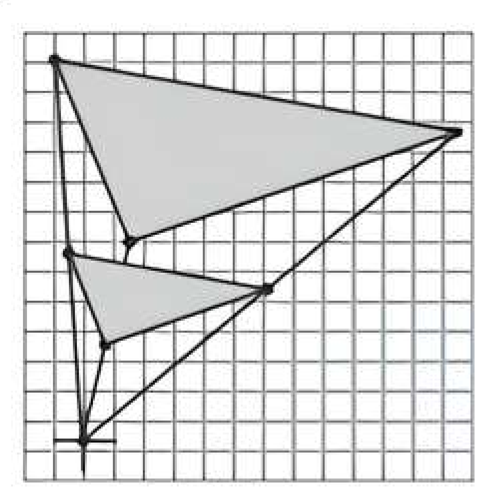
Case Study Based Question

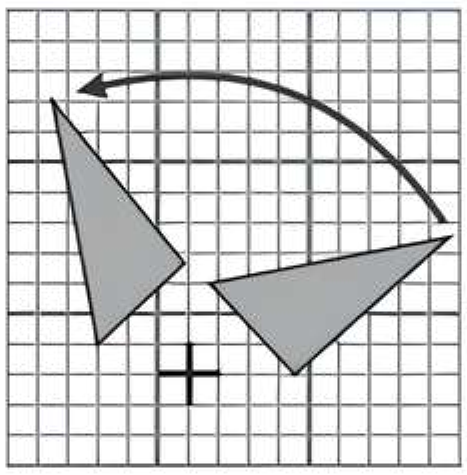
Q 9. Scale Factor: A scale drawing of an object is of the same shape as the object but of a different dimension.

The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.

Similar Figures: The ratio of two corresponding sides in similar figures is called the scale factor.

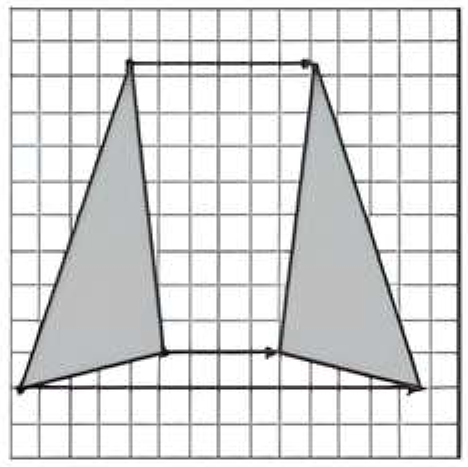
If one shape can become another using resizing then the shapes are similar.

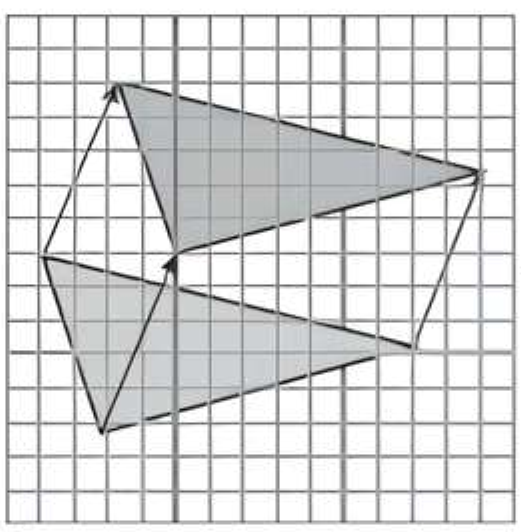




Rotation or Turn

Reflection or Flip





Translation or Slide

Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn.
Based on the above information, solve the following questions:

(i) A model of an aeroplane is made to a scale of 1 : 400. Find the length (in cm) of the model, if the length of the aeroplane is 40 m.

(ii) Find the length (in m) of the aeroplane if length of its model is 16 cm.

(iii) A $\triangle ABC$ has been enlarged by scale factor $m = 2.5$ to the $\triangle A'B'C'$. Find the length of A'B', if AB is 6 cm.

OR

Find the length of C'A', if CA = 4 cm.

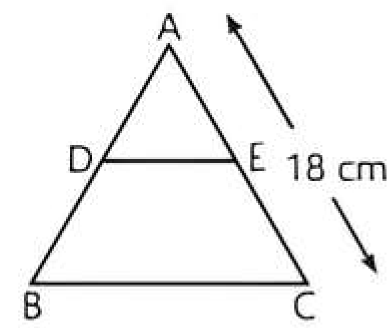
Very Short Answer Type Questions

Q 10. If the corresponding altitudes of two similar triangles are in the ratio 3 : 5, then find the ratio of their corresponding sides.

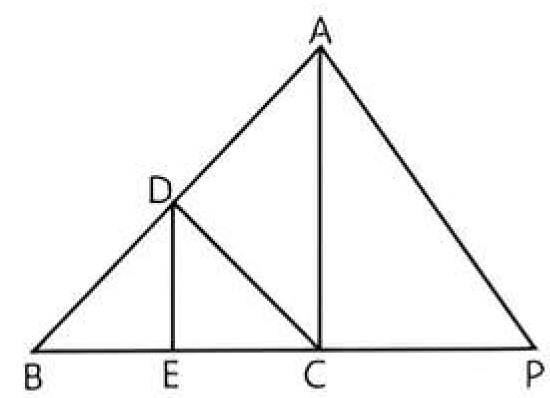
Q 11. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$ and $\angle B = \angle D$, then these triangles will be similar by which criteria?

Short Answer Type-I Questions

Q 12. In the given figure, DE || BC. If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE.

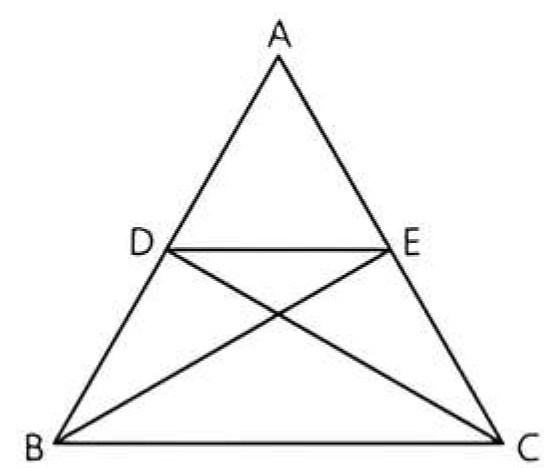


Q 13. In the given figure, DE || AC and DC || AP. Prove that $\frac{BC}{CP} = \frac{BE}{EC}$.

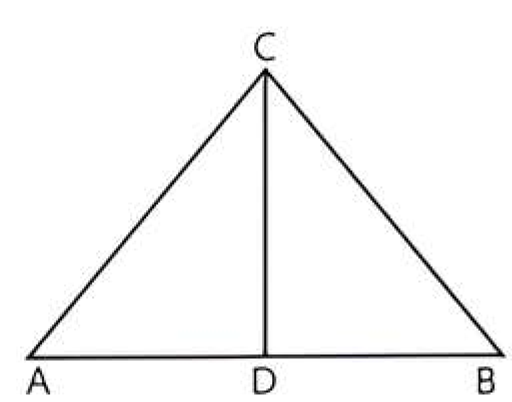


Short Answer Type-II Questions

Q 14. In the given figure, if $\triangle ABE \cong \triangle ACD$, prove that $\triangle ADE \sim \triangle ABC$.



Q 15. In the given figure, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.



Long Answer Type Question

Q 16. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that

$$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}.$$