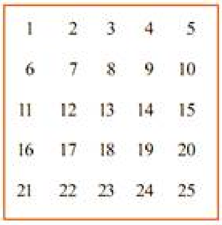
4. Identities

Questions Pg-68

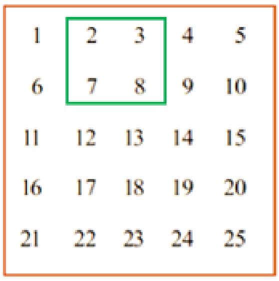
1 A. Question

Write number like this:



As in the calendar; mark four numbers in a square and find the difference of diagonal products. Is it the same for all squares of four numbers?

Answer



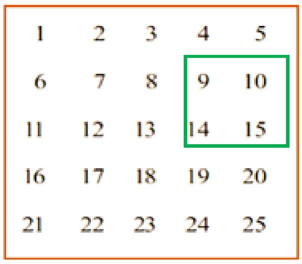
Let us consider this green square.

Diagonal products are:

2×8 = 16

7×3 = 21

Difference = 21-16 = 5

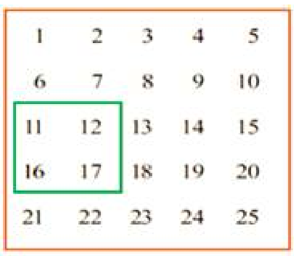


Now let us consider this green square.

Diagonal products: 9× 15 = 135

14× 10 = 140

Difference = 140 – 135 = 5



Now consider this green square.

Diagonal products: 11× 17 = 187

16× 12 = 192

Difference = 192 – 187 = 5

So, we observe that in all the above cases the difference of diagonal products is coming out to be same and is equal to 5.

1 A. Question

Write number like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

As in the calendar; mark four numbers in a square and find the difference of diagonal products. Is it the same for all squares of four numbers?

Answer

1	2	3	4	5
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Let us consider this green square.

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Now consider this green square.

Diagonal products: 11× 17 = 187

16× 12 = 192

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So, we observe that in all the above cases the difference of diagonal products is coming out to be same and is equal to 5.

1 B. Question

Write number like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Explain why this is so, using algebra.

Answer

Let the first number in the square be x, the others can be filled in as below,

x	x+1
x+5	x+6

Diagonal products: x (x+6) = $x^2 + 6x$

(x+5)(x+1) = x(x+1) + 5(x+1) = $(x^2 + x) + (5x + 5) = x^2 + 6x + 5$

Clearly, the difference between the two diagonal products is always 5.

1 B. Question

Write number like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
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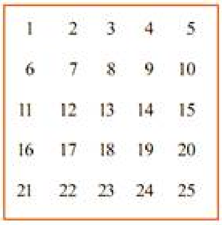
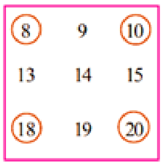
Clearly, the difference between the two diagonal products is always 5.

1 C. Question

Write number like this:

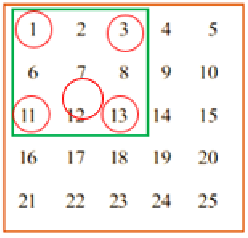


Instead of a square of four numbers, take a square of nine numbers and mark only the numbers at the four corners.



What is the difference of diagonal products? Explain using algebra.

Answer

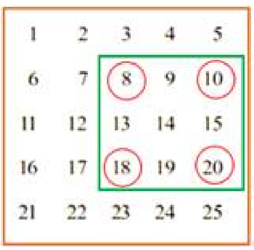


Consider the green square and consider the four corner elements (red circles)

Diagonal products: 1× 13 = 13

3× 11 = 33

Difference between diagonal products = 33 - 13 = 20



Now, consider this green square and consider the corner elements (red circles)

Diagonal products: 8× 20 = 160

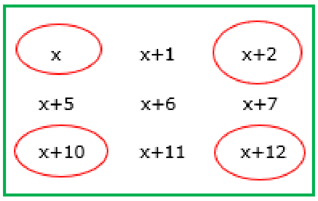
18× 10 = 180

Difference between diagonal products = 180 – 160 = 20

So, we observe that the difference between diagonal products in the above cases is also coming out to be same and is equal to 20.

Explanation using Algebra,

Let the first number in the square be x, the others can be filled in as below,



Diagonal products: x (x+12) = $x^2 + 12x$

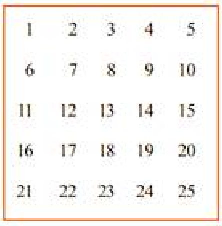
(x+10)(x+2) = x(x+2) + 10(x+2) = (x^2 + 2x) + (10x+ 20)

= $x^2 + 12x + 20$

Clearly, we can observe that the difference between diagonal products is always equal to 20.

1 C. Question

Write number like this:



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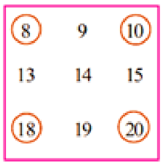
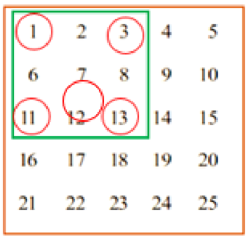


What is the difference of diagonal products? Explain using algebra.

Answer



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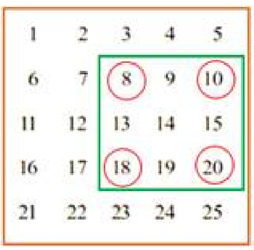

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Diagonal products: 8× 20 = 160

18× 10 = 180

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So, we observe that the difference between diagonal products in the above cases is also coming out to be same and is equal to 20.

Explanation using Algebra,

Let the first number in the square be x, the others can be filled in as below,

x	x+1	x+2
x+5	x+6	x+7
x+10	x+11	x+12

Diagonal products: x (x+12) = x^2 + 12x

(x+10)(x+2) = x(x+2) + 10(x+2) = (x^2 + 2x) + (10x+ 20)

= x^2 + 12x + 20

Clearly, we can observe that the difference between diagonal products is always equal to 20.

2 A. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
9	12	15
12	16	20

What is the difference of diagonal sums?

Answer

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

In this part, we are considering this red square of a multiplication table.

Diagonal sums: 6+20 = 26

10+12 = 22

Difference between diagonal sums = 26 – 22 = 4

2 A. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
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What is the difference of diagonal sums?

Answer

MULTIPLICATION TABLE

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3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
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In this part, we are considering this red square of a multiplication table.

Diagonal sums: 6+20 = 26

10+12 = 22

Difference between diagonal sums = 26 – 22 = 4

2 B. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
9	12	15
12	16	20

Explain using algebra, why this difference is the same for all such squares.

Answer

we can write a row of multiplication table as

x 2x 3x 4x 5x 6x 7x 8x 9x

Next row will be:

(x+1) 2(x+1) 3(x+1) 4(x+1) 5(x+1) 6(x+1) 7(x+1) 8(x+1) 9(x+1)

If we take a general number from the first row as yx then the next number in this row will be (y+1)x.

So, we can obtain a general square of 9 terms from this multiplication table as:

yx	(y+1)x	(y+2)x
y(x+1)	(y+1)(x+1)	(y+2)(x+1)
y(x+2)	(y+1)(x+2)	(y+2)(x+2)

Diagonal sums: (yx)+(y+2)(x+2) = yx+y(x+2)+2(x+2) = yx + yx + 2y +2x +4

= 2yx +2y +2x +4

Y(x+2)+ (y+2)x = (yx +2y)+(yx + 2x) = 2yx +2y +2x

Clearly, the difference between diagonal sums is equal to 4.

2 B. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
9	12	15
12	16	20

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we can write a row of multiplication table as

x 2x 3x 4x 5x 6x 7x 8x 9x

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= 2yx +2y +2x +4

Y(x+2)+ (y+2)x = (yx +2y)+(yx + 2x) = 2yx +2y +2x

Clearly, the difference between diagonal sums is equal to 4.

2 C. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
9	12	15
12	16	20

What if we take a square of sixteen numbers?

Answer

Now, if we take a square of 16 numbers, then the general terms will be:

yx	(y+1)x	(y+2)x	(y+3)x
y(x+1)	(y+1)(x+1)	(y+2)(x+1)	(y+3)(x+1)
y(x+2)	(y+1)(x+2)	(y+2)(x+2)	(y+3)(x+2)
y(x+3)	(y+1)(x+3)	(y+2)(x+3)	(y+3)(x+3)

Diagonal sums: yx + (y+3)(x+3) = yx + y(x+3) + 3(x+3) = yx +yx +3y +3x +9 = 2yx +3y+3x+9

Y(x+3) + (y+3)x = yx +3y + yx +3x = 2yx + 3y + 3x

Clearly, the difference between the diagonal sums is equal to 9.

2 C. Question

In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

6	8	10
9	12	15
12	16	20

What if we take a square of sixteen numbers?

Answer

Now, if we take a square of 16 numbers, then the general terms will be:

yx	(y+1)x	(y+2)x	(y+3)x
y(x+1)	(y+1)(x+1)	(y+2)(x+1)	(y+3)(x+1)
y(x+2)	(y+1)(x+2)	(y+2)(x+2)	(y+3)(x+2)
y(x+3)	(y+1)(x+3)	(y+2)(x+3)	(y+3)(x+3)

Diagonal sums: yx + (y+3)(x+3) = yx + y(x+3) + 3(x+3) = yx +yx +3y +3x +9 = 2yx +3y+3x+9

Y(x+3) + (y+3)x = yx +3y + yx +3x = 2yx + 3y + 3x

Clearly, the difference between the diagonal sums is equal to 9.

3. Question

Look at these:

1 × 4 = (2 × 3) – 2

2 × 5 = (3 × 4) – 2

3 × 6 = (4 × 5) – 2

4 × 7 = (5 × 6) – 2

i) Write the next two lines in this pattern.

ii) If we take four consecutive natural numbers, what is the relation between the products of the first and the last, and the product of the middle two?

iii) Write this as a general principle in algebra and explain it.

Answer

i) 5× 8 = (6× 7) – 2

6× 9 = (7× 8) – 2

ii) The above pattern is based on this statement only.

Let the four consecutive numbers be 1,2,3,4.

Product of first and last = (1× 4) = 4

Product of middle two terms = (2× 3) = 6

and 4 = 6 - 2

Let the four consecutive numbers be 4,5,6,7.

Product of first and last = (4× 7) = 28

Product of middle two terms = (5× 6) = 30

And 28 = 30 - 2

iii) General principle,

Let the four consecutive natural numbers be x , (x+1) , (x+2) , (x+3).

Product of first and last = x(x+3) = $x^2 + 3x$

Product of middle two terms = (x+1)(x+2) = x(x+2) + 1(x+2)

= $x^2 + 2x + x + 2$

= $x^2 + 3x + 2$

Clearly, we can see that the difference between the product of middle two terms and the product of first and last term is equal to 2.

3. Question

Look at these:

1 × 4 = (2 × 3) – 2

2 × 5 = (3 × 4) – 2

3 × 6 = (4 × 5) – 2

4 × 7 = (5 × 6) – 2

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Product of first and last = (4× 7) = 28

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iii) General principle,

Let the four consecutive natural numbers be x , (x+1) , (x+2) , (x+3).

Product of first and last = x(x+3) = $x^2 + 3x$

Product of middle two terms = (x+1)(x+2) = x(x+2) + 1(x+2)

= $x^2 + 2x + x + 2$

= $x^2 + 3x + 2$

Clearly, we can see that the difference between the product of middle two terms and the product of first and last term is equal to 2.

4. Question

Shown below is a method to the product 46× 28.

$4 \times 2 = 8$	8×100	800
$(4 \times 8) + (6 \times 2) = 44$	44×10	440
6×8		48
46×28		1288

i) Check this method for some other two-digit numbers.

ii) Explain why this is correct, using algebra. (Recall that any two-digit number can be written 10m + n, as seen in the section, Two-digit numbers of the lesson, Numbers and Algebra, in the class 7 textbook).

Answer

i)

Let us calculate 37× 12 using this method.

$3 \times 1 = 3$	3×100	300
$(3 \times 2)+(7 \times 1) = 13$	13×10	130
7×2		14
37×12		444

Let us now calculate 56× 89

$5 \times 8 = 40$	40×100	4000
$(5 \times 9)+(6 \times 8) = 93$	93×10	930
6×9		54
56×89		4984

ii) We know that we can express any 2-digit number as 10m+n , where m is the digit at tens place and n is the digit at ones place.

So, let (10m+n) and (10x+y) be the two numbers to be multiplied.

$(10m+n)(10x+y) = 10m(10x+y)+n(10x+y) = 100\,mx + 10my + 10nx + ny$

$= (100 \times mx) + ((m \times y)+(n \times x)) \times 10 + (n \times y)$

Now, carefully observe the above method, you will see that we were finding these terms only and then addition of these terms gives us the required product.

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(10m+n)(10x+y) = 10m(10x+y)+n(10x+y) = 100 mx +10my +10nx + ny

= (100× mx) + ((m×y)+(n×x))× 10 +(n×y)

Now, carefully observe the above method, you will see that we were finding these terms only and then addition of these terms gives us the required product.

1 A. Question

Write number like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
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21	22	23	24	25

As in the calendar; mark four numbers in a square and find the difference of diagonal products. Is it the same for all squares of four numbers?

Answer

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Let us consider this green square.

Diagonal products are:

2×8 = 16

7×3 = 21

Difference = 21-16 = 5

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Now let us consider this green square.

Diagonal products: 9× 15 = 135

14× 10 = 140

Difference = 140 – 135 = 5

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Now consider this green square.

Diagonal products: 11× 17 = 187

16× 12 = 192

Difference = 192 – 187 = 5

So, we observe that in all the above cases the difference of diagonal products is coming out to be same and is equal to 5.

Questions Pg-74

1. Question

Is there a general method to compute the squares of numbers like $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2} \ldots ?$ Explain it using algebra.

Answer

we can write $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, \ldots \ldots$ generally as $x\frac{1}{2}$.

So, to find the square we have to calculate $\left(x\frac{1}{2}\right)^2$

using, $(x+y)^2 = x^2 + 2xy + y^2$

$$\left(x\frac{1}{2}\right)^2 = \left(x + \frac{1}{2}\right)^2$$

$$= (x)^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= x^2 + x + \frac{1}{4}$$

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Answer

we can write $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, \ldots\ldots$ generally as $x\frac{1}{2}$.

So, to find the square we have to calculate $\left(x\frac{1}{2}\right)^2$

using, $(x+y)^2 = x^2 + 2xy + y^2$

$$\left(x\frac{1}{2}\right)^2 = \left(x + \frac{1}{2}\right)^2$$

$$= (x)^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= x^2 + x + \frac{1}{4}$$

2 A. Question

Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

Check this for some more two-digit numbers.

Answer

$4^2 = 16$	16×100	1600
$2 \times (4 \times 5) = 40$	40×10	400
5^2		25
45^2		2025

Now, let's calculate 72^2

$7^2 = 49$	49×100	4900
$2 \times (7 \times 2) = 28$	28×10	280
2^2		4
72^2		5184

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5^2		25
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Now, let's calculate 72^2

$7^2 = 49$	49×100	4900
$2\times (7\times2) = 28$	28×10	280
2^2		4
72^2		5184

2 B. Question

Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

Explain why this is correct, using algebra.

Answer

Now lets try to understand this method using algebra.

We know that we can express any 2 digit number as 10m+n , where m is the digit at tens place and n is the digit at ones place.

Now we have to find its square, i.e. $(10m + n)^2$

$(10m + n)^2 = (10m)^2 + 2\times 10m\times n + n^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$=(m^2 \times 100) + (2mn\times 10)+ (n^2)$

Now, carefully observe the above method, you will see that we were finding these terms only and then addition of these terms gives us the required square.

2 B. Question

Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

Explain why this is correct, using algebra.

Answer

Now lets try to understand this method using algebra.

We know that we can express any 2 digit number as 10m+n , where m is the digit at tens place and n is the digit at ones place.

Now we have to find its square, i.e. $(10m + n)^2$

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2 C. Question

Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

Find an easy method to compute squares of number ending in 5.

Answer

Method to find the square of number ending with 5.

Suppose we have to find 45^2.

Let the number at tens place be x.

So, we first have to find x(x+1).

For example, here we have x=4.

So, x(x+1) = 4(4+1) = 4×5 = 20

Now, we just need to write this 20 before 25, i.e. 2025 and it is the required square.

Let us now find the square of 85.

So, 8(8+1) = 8×9 = 72

∴ $85^2 = 7225$

2 C. Question

Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

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Answer

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For example, here we have x=4.

So, x(x+1) = 4(4+1) = 4×5 = 20

Now, we just need to write this 20 before 25, i.e. 2025 and it is the required square.

Let us now find the square of 85.

So, 8(8+1) = 8×9 = 72

∴ $85^2 = 7225$

3. Question

Look at this pattern

$1^2 + (4 \times 2) = 3^2$

$2^2 + (4 \times 3) = 4^2$

$3^2 + (4 \times 4) = 5^2$

i) Write the next two lines.

ii) Explain the general principle using algebra.

Answer

i) $4^2 + (4\times5) = 6^2$

$5^2 + (4\times6) = 7^2$

ii) General principle,

By observing the pattern we can write a general line as:

$x^2 + (4\times (x+1)) = (x+2)^2$

Now, let us prove this using algebra.

$x^2 + (4\times (x+1)) = x^2 + 4x + 4$

$(x+2)^2 = x^2 + 4x + 4$ ……using , $(x+y)^2 = x^2 + 2xy + y^2$

Cleary, both these terms are equal.

Hence, proved.

3. Question

Look at this pattern

$1^2 + (4 \times 2) = 3^2$

$2^2 + (4 \times 3) = 4^2$

$3^2 + (4 \times 4) = 5^2$

i) Write the next two lines.

ii) Explain the general principle using algebra.

Answer

i) $4^2 + (4\times5) = 6^2$

$5^2 + (4\times6) = 7^2$

ii) General principle,

By observing the pattern we can write a general line as:

$x^2 + (4\times (x+1)) = (x+2)^2$

Now, let us prove this using algebra.

$x^2 + (4\times (x+1)) = x^2 + 4x + 4$

$(x+2)^2 = x^2 + 4x + 4$ ……using , $(x+y)^2 = x^2 + 2xy + y^2$

Cleary, both these terms are equal.

Hence, proved.

4. Question

Explain using algebra, the fact that the square of any natural number which is not a multiple of 3, leaves remainder 1 on division by 3.

Answer

We know that, We can express a number which is not a multiple of 3 as (3m + 1) or (3m +2).

$(3m+1)^2 = (3m)^2 + 2\times3m\times1 + 1^2 = 9m^2 + 6m + 1 = 3 (3m^2 + 2m) + 1$ ……….(1)

…………..using, $(x+y)^2 = x^2 + 2xy + y^2$

$(3m+2)^2 = (3m)^2 + 2\times3m\times2 + 2^2 = 9m^2 + 12m + 4 = 9m^2 + 12m + 3 +1$

$= 3(3m^2 + 4m + 1) + 1$ ....(2)

.............using, $(x+y)^2 = x^2 + 2xy + y^2$

Now, observe (1) and (2) carefully, clearly on division by 3 both these terms will leave remainder 1.

4. Question

Explain using algebra, the fact that the square of any natural number which is not a multiple of 3, leaves remainder 1 on division by 3.

Answer

We know that, We can express a number which is not a multiple of 3 as (3m + 1) or (3m +2).

$(3m+1)^2 = (3m)^2 + 2\times3m\times1 + 1^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1$ ....(1)

.............using, $(x+y)^2 = x^2 + 2xy + y^2$

$(3m+2)^2 = (3m)^2 + 2\times3m\times2 + 2^2 = 9m^2 + 12m + 4 = 9m^2 + 12m + 3 + 1$

$= 3(3m^2 + 4m + 1) + 1$ ....(2)

.............using, $(x+y)^2 = x^2 + 2xy + y^2$

Now, observe (1) and (2) carefully, clearly on division by 3 both these terms will leave remainder 1.

5. Question

Prove that for any natural number ending in 3, its square ends in 9. What about numbers ending in 5? And numbers ending in 4?

Answer

We know that we can express any 2 digit number as 10m+n , where m is the digit at tens place and n is the digit at ones place.

∴ the number ending with 3 can be expressed as 10m+3.

$(10m + 3)^2 = (10m)^2 + 2\times10m\times3 + 3^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 10\times 6m + 9$

Clearly, we can observe that the square ends in 9.

The number ending with 5 can be expressed as 10m+5.

$(10m + 5)^2 = (10m)^2 + 2\times10m\times5 + 5^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 100m + (25)$

$= 100(m^2 + m) + (10\times2 + 5)$

Clearly, we can observe that the square ends in 5.

The number ending with 4 can be expressed as 10m+4.

$(10m + 4)^2 = (10m)^2 + 2\times10m\times4 + 4^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 10\times 8m + (16)$

$= 100m^2 + 10\times 8m + (10 + 6)$

$= 100m^2 + 10\times(8m+1) + 6$

Clearly, we can observe that the square ends in 6.

5. Question

Prove that for any natural number ending in 3, its square ends in 9. What about numbers ending in 5? And numbers ending in 4?

Answer

We know that we can express any 2 digit number as 10m+n , where m is the digit at tens place and n is the digit at ones place.

∴ the number ending with 3 can be expressed as 10m+3.

$(10m + 3)^2 = (10m)^2 + 2\times10m\times3 + 3^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 10\times 6m + 9$

Clearly, we can observe that the square ends in 9.

The number ending with 5 can be expressed as 10m+5.

$(10m + 5)^2 = (10m)^2 + 2\times10m\times5 + 5^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 100m + (25)$

$= 100 (m^2 + m) + (10\times2 + 5)$

Clearly, we can observe that the square ends in 5.

The number ending with 4 can be expressed as 10m+4.

$(10m + 4)^2 = (10m)^2 + 2\times10m\times4 + 4^2$ ....using, $(x+y)^2 = x^2 + 2xy + y^2$

$= 100m^2 + 10\times 8m + (16)$

$= 100m^2 + 10\times 8m + (10 + 6)$

$= 100 m^2 + 10\times (8m+1) + 6$

Clearly, we can observe that the square ends in 6.

Questions Pg-79

1 A. Question

Compute the squares of these numbers.

49

Answer

$49^2 = (50-1)^2$

Using identity $(x-y)^2 = x^2 + y^2 - 2xy$

So $49^2 = (50-1)^2$

$= 50^2 + 1^2 - 2\times 50\times 1$

= 2500 + 1- 100

= 2401

1 A. Question

Compute the squares of these numbers.

49

Answer

$49^2 = (50-1)^2$

Using identity $(x-y)^2 = x^2 + y^2 - 2xy$

So $49^2 = (50-1)^2$

$= 50^2 + 1^2 - 2 \times 50 \times 1$

= 2500 + 1- 100

= 2401

1 B. Question

Compute the squares of these numbers.

98

Answer

98 can be written as (100-2)

$98^2 = (100 - 2)^2$

Using identity $(x - y)^2 = x^2 + y^2 - 2xy$

So $98^2 = (100 - 2)^2$

$= 100^2 + 2^2 - 2 \times 100 \times 2$

= 10000+4 - 400

= 9604

1 B. Question

Compute the squares of these numbers.

98

Answer

98 can be written as (100-2)

$98^2 = (100 - 2)^2$

Using identity $(x - y)^2 = x^2 + y^2 - 2xy$

So $98^2 = (100 - 2)^2$

$= 100^2 + 2^2 - 2 \times 100 \times 2$

= 10000+4 - 400

= 9604

1 C. Question

Compute the squares of these numbers.

$7\frac{3}{4}$

Answer

$(7\frac{3}{4})^2$ can be written as $(7 + \frac{3}{4})^2$

Using identity $(x + y)^2 = x^2 + y^2 + 2xy$

so $(7\frac{3}{4})^2 = (7 + \frac{3}{4})^2$

$= (7)^2 + (\frac{3}{4})^2 + 2 \times 7 \times \frac{3}{4}$

$= 49 + \frac{9}{16} + \frac{21}{2}$

$= 49 + (\frac{9+168}{16})$

$= 49 + \frac{177}{16}$

$= 49\frac{177}{16}$

1 C. Question

Compute the squares of these numbers.

$7\frac{3}{4}$

Answer

$(7\frac{3}{4})^2$ can be written as $(7 + \frac{3}{4})^2$

Using identity $(x + y)^2 = x^2 + y^2 + 2xy$

so $(7\frac{3}{4})^2 = (7 + \frac{3}{4})^2$

$= (7)^2 + (\frac{3}{4})^2 + 2 \times 7 \times \frac{3}{4}$

$= 49 + \frac{9}{16} + \frac{21}{2}$

$= 49 + (\frac{9+168}{16})$

$= 49 + \frac{177}{16}$

$= 49\frac{177}{16}$

1 D. Question

Compute the squares of these numbers.

9.25

Answer

9.25^2 can be written as $(9 + 0.25)^2$

Using identity $(x + y)^2 = x^2 + y^2 + 2xy$

$(9 + 0.25)^2 = (9)^2 + (0.25)^2 + 2 \times 9 \times 0.25$

$= 81 + 0.0625 + 4.5$

$= 85.5625$

1 D. Question

Compute the squares of these numbers.

9.25

Answer

9.25^2 can be written as $(9 + 0.25)^2$

Using identity $(x + y)^2 = x^2 + y^2 + 2xy$

$(9 + 0.25)^2 = (9)^2 + (0.25)^2 + 2\times9\times0.25$

$= 81 + 0.0625 + 4.5$

$= 85.5625$

2. Question

Look at this pattern:

$$\left(\frac{1}{2}\right)^2 + \left(1\frac{1}{2}\right)^2 = 2\frac{1}{2} \quad 2 = 2\times1^2$$

$$\left(1\frac{1}{2}\right)^2 + \left(2\frac{1}{2}\right)^2 = 8\frac{1}{2} \quad 8 = 2\times2^2$$

$$\left(2\frac{1}{2}\right)^2 + \left(3\frac{1}{2}\right)^2 = 18\frac{1}{2} \quad 18 = 2\times3^2$$

Explain the general principal using algebra.

Answer

we can write it as $(1-\frac{1}{2})^2 + (1+\frac{1}{2})^2$

Using identity $(x-y)^2 = x^2 + y^2 - 2xy$

and $(x+y)^2 = x^2 + y^2 + 2xy$

$= 1+\frac{1^2}{2} - 1 + 1+\frac{1^2}{2} + 1$

$= 2 + \frac{1}{2}$

$= 2\frac{1}{2}$

Similarly, we can do it for other numbers.

2. Question

Look at this pattern:

$$\left(\frac{1}{2}\right)^2 + \left(1\frac{1}{2}\right)^2 = 2\frac{1}{2} \quad 2 = 2\times1^2$$

$$\left(1\frac{1}{2}\right)^2 + \left(2\frac{1}{2}\right)^2 = 8\frac{1}{2} \quad 8 = 2\times2^2$$

$$\left(2\frac{1}{2}\right)^2 + \left(3\frac{1}{2}\right)^2 = 18\frac{1}{2} \quad 18 = 2\times3^2$$

Explain the general principal using algebra.

Answer

we can write it as $(1-\frac{1}{2})^2 + (1+\frac{1}{2})^2$

Using identity $(x-y)^2 = x^2 + y^2 - 2xy$

and $(x+y)^2 = x^2 + y^2 + 2xy$

$= 1+\frac{1^2}{2} - 1 + 1+\frac{1^2}{2} + 1$

$= 2 + \frac{1}{2}$

$= 2\frac{1}{2}$

Similarly, we can do it for other numbers.

3 A. Question

Some natural numbers can be written as a difference of two perfect squares in two ways. For example.

$24 = 7^2 - 5^2 = 5^2 - 1^2$

$32 = 9^2 - 7^2 = 6^2 - 2^2$

$40 = 11^2 - 9^2 = 7^2 - 3^2$

Explain using algebra, the method of writing all multiples of 8, starting with 24 as the difference of two perfect squares in two ways.

Answer

Let's use algebra. starting with x,y, the square of the difference is is $(x-y)^2 = x^2 + y^2 - 2xy$

the square of the sum is

$(x+y)^2 = x^2 + y^2 + 2xy$

what if we subtract the square of the difference from the square of the sum.

$(x+y)^2 - (x-y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$

$(x+y)^2 - (x-y)^2 = 4xy$

writing this in reverse

$4xy = (x+y)^2 - (x-y)^2$

for example, $24 = 4 \times 6 \times 1$

= here x=6 and y=1

$= (x+y)^2 - (x-y)^2$

$= (6+1)^2 - (6-1)^2$

$= 7^2 - 5^2$

similarly 24 can also be written as

$24 = 4 \times 3 \times 2$

= here x=3 and y=2

$= (x+y)^2 - (x-y)^2$

$= (3+2)^2 - (3-2)^2$

$= 5^2 - 1^2$

so $24 = 7^2 - 5^2 = 5^2 - 1^2$

other multiple of 8,

$32 = 9^2 - 7^2 = 6^2 - 2^2$

$32 = 4 \times 8 \times 1$

= here x=8 and y=1

$= (x + y)^2 - (x - y)^2$

$= (8 + 1)^2 - (8 - 1)^2$

$= 9^2 – 7^2$

similarly 32 can also be written as

$32 = 4 \times 4 \times 2$

= here x=4 and y=2

$= (x + y)^2 - (x - y)^2$

$= (4 + 2)^2 - (4 - 2)^2$

$32 = 9^2 – 7^2 = 6^2 – 2^2$

3 A. Question

Some natural numbers can be written as a difference of two perfect squares in two ways. For example.

$24 = 7^2 – 5^2 = 5^2 – 1^2$

$32 = 9^2 – 7^2 = 6^2 – 2^2$

$40 = 11^2 – 9^2 = 7^2 – 3^2$

Explain using algebra, the method of writing all multiples of 8, starting with 24 as the difference of two perfect squares in two ways.

Answer

Let's use algebra. starting with x,y, the square of the difference is is $(x - y)^2 = x^2 + y^2 - 2xy$

the square of the sum is

$(x + y)^2 = x^2 + y^2 + 2xy$

what if we subtract the square of the difference from the square of the sum.

$(x + y)^2 - (x - y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$

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$= (x + y)^2 - (x - y)^2$

$= (6 + 1)^2 - (6 - 1)^2$

$= 7^2 – 5^2$

similarly 24 can also be written as

$24 = 4 \times 3 \times 2$

= here x=3 and y=2

$= (x + y)^2 - (x - y)^2$

$= (3 + 2)^2 - (3 - 2)^2$

$= 5^2 - 1^2$

so $24 = 7^2 - 5^2 = 5^2 - 1^2$

other multiple of 8,

$32 = 9^2 - 7^2 = 6^2 - 2^2$

$32 = 4\times8\times1$

= here x=8 and y=1

$= (x+y)^2 - (x-y)^2$

$= (8+1)^2 - (8-1)^2$

$= 9^2 - 7^2$

similarly 32 can also be written as

$32 = 4\times4\times2$

= here x=4 and y=2

$= (x+y)^2 - (x-y)^2$

$= (4+2)^2 - (4-2)^2$

$32 = 9^2 - 7^2 = 6^2 - 2^2$

3 B. Question

Some natural numbers can be written as a difference of two perfect squares in two ways. For example.

$24 = 7^2 - 5^2 = 5^2 - 1^2$

$32 = 9^2 - 7^2 = 6^2 - 2^2$

$40 = 11^2 - 9^2 = 7^2 - 3^2$

In how many different ways can we write multiples of 16, starting with 48 as the difference of two perfect squares?

Answer

Let's use algebra. starting with x,y, the square of the difference is $(x-y)^2 = x^2 + y^2 - 2xy$

the square of the sum is

$(x+y)^2 = x^2 + y^2 + 2xy$

what if we subtract the square of the difference from the square of the sum.

$(x+y)^2 - (x-y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$

$(x+y)^2 - (x-y)^2 = 4xy$

writing this in reverse

$4xy = (x+y)^2 - (x-y)^2$

multiples of 16 are 16,32,48,64,80.....starting with 48

$48 = 4\times12\times1$

= here x=12 and y=1

$= (x+y)^2 - (x-y)^2$

= $(12+1)^2$- $(12-1)^2$

= $(13)^2$- $(11)^2$

similarly 48 can also be written as

48 = 4×4×3

= here x=4 and y=3

= $(x+y)^2$- $(x-y)^2$

= $(4+3)^2$- $(4-3)^2$

= $(7)^2$- $(1)^2$

48 can also be written as

48 = 4×6×2

= here x=6 and y=2

= $(x+y)^2$- $(x-y)^2$

= $(6+2)^2$- $(6-2)^2$

= $(8)^2$- $(4)^2$

Similarly for 64 = 4×16×1 = $(17)^2$- $(15)^2$

64 = 4×8×2 = $(10)^2$- $(6)^2$

64 = 4×4×4 = $(8)^2$- $(0)^2$

Similarly we can do it for other multiples of 16.

3 B. Question

Some natural numbers can be written as a difference of two perfect squares in two ways. For example.

24 = $7^2 - 5^2 = 5^2 - 1^2$

32 = $9^2 - 7^2 = 6^2 - 2^2$

40 = $11^2 - 9^2 = 7^2 - 3^2$

In how many different ways can we write multiples of 16, starting with 48 as the difference of two perfect squares?

Answer

Let’s use algebra. starting with x,y, the square of the difference is $(x-y)^2 = x^2 + y^2 - 2xy$

the square of the sum is

$(x+y)^2 = x^2 + y^2 + 2xy$

what if we subtract the square of the difference from the square of the sum.

$(x+y)^2 - (x-y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$

$(x+y)^2 - (x-y)^2 = 4xy$

writing this in reverse

$4xy = (x+y)^2 - (x-y)^2$

multiples of 16 are 16,32,48,64,80…..starting with 48

$48 = 4\times12\times1$

= here x=12 and y=1

$= (x+y)^2 - (x-y)^2$

$= (12+1)^2 - (12-1)^2$

$= (13)^2 - (11)^2$

similarly 48 can also be written as

$48 = 4\times4\times3$

= here x=4 and y=3

$= (x+y)^2 - (x-y)^2$

$= (4+3)^2 - (4-3)^2$

$= (7)^2 - (1)^2$

48 can also be written as

$48 = 4\times6\times2$

= here x=6 and y=2

$= (x+y)^2 - (x-y)^2$

$= (6+2)^2 - (6-2)^2$

$= (8)^2 - (4)^2$

Similarly for $64 = 4\times16\times1 = (17)^2 - (15)^2$

$64 = 4\times8\times2 = (10)^2 - (6)^2$

$64 = 4\times4\times4 = (8)^2 - (0)^2$

Similarly we can do it for other multiples of 16.

Questions Pg-83

1 A. Question

Compute the following in head:

$68^2 - 32^2$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$68^2 - 32^2 = (68+32)(68-32)$

= (100) (36)

= 3600

1 A. Question

Compute the following in head:

$68^2 - 32^2$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$68^2 - 32^2 = (68+32)(68-32)$

$= (100)(36)$

$= 3600$

1 B. Question

Compute the following in head:

$$\left(3\frac{1}{2}\right)^2 - \left(2\frac{1}{2}\right)^2$$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$(\frac{7}{2})^2 - (\frac{5}{2})^2 = (\frac{7}{2} + \frac{5}{2})(\frac{7}{2} - \frac{5}{2})$

$= (\frac{12}{2})(\frac{2}{2})$

$= 6 \times 1$

$= 6$

1 B. Question

Compute the following in head:

$$\left(3\frac{1}{2}\right)^2 - \left(2\frac{1}{2}\right)^2$$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$(\frac{7}{2})^2 - (\frac{5}{2})^2 = (\frac{7}{2} + \frac{5}{2})(\frac{7}{2} - \frac{5}{2})$

$= (\frac{12}{2})(\frac{2}{2})$

$= 6 \times 1$

$= 6$

1 C. Question

Compute the following in head:

$(3.6)^2 - (1.4)^2$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$(3.6)^2 - (1.4)^2 = (3.6+1.4)(3.6 - 1.4)$

$= (5)(2.2)$

$= 11$

1 C. Question

Compute the following in head:

$(3.6)^2 – (1.4)^2$

Answer

using identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$(3.6)^2 – (1.4)^2 = (3.6+1.4)\ (3.6 - 1.4)$

= (5) (2.2)

= 11

1 D. Question

Compute the following in head:

201×199

Answer

it can be written as $(200+1)\times(200 - 1)$

using identity $(x+y) \times (x-y) = (x)^2 - (y)^2$

$(200+1)\times(200 - 1) = (200)^2 - (1)^2$

= 40000-1

= 39999

1 D. Question

Compute the following in head:

201×199

Answer

it can be written as $(200+1)\times(200 - 1)$

using identity $(x+y) \times (x-y) = (x)^2 - (y)^2$

$(200+1)\times(200 - 1) = (200)^2 - (1)^2$

= 40000-1

= 39999

1 E. Question

Compute the following in head:

$$2\frac{1}{3} \times 1\frac{2}{3}$$

Answer

$(2 + \frac{1}{3}) \times (1 + + \frac{2}{3})$

using identity (x+y) (u+v) = xu+xv+yu+yv

$=(2\times 1) + (2 \times \frac{2}{3}) + (\frac{1}{3} \times 1) + (\frac{1}{3} \times \frac{2}{3})$

$= 2+\frac{4}{3}+\frac{1}{3}+\frac{2}{9}$

$= 2+\frac{5}{3}+\frac{2}{9}$

$= \frac{18+15+2}{9}$

$= \frac{35}{9}$

1 E. Question

Compute the following in head:

$2\frac{1}{3} \times 1\frac{2}{3}$

Answer

$(2 + \frac{1}{3}) \times (1 + + \frac{2}{3})$

using identity (x+y) (u+v) = xu+xv+yu+yv

$=(2\times 1) + (2 \times \frac{2}{3}) + (\frac{1}{3} \times 1) + (\frac{1}{3} \times \frac{2}{3})$

$= 2+\frac{4}{3}+\frac{1}{3}+\frac{2}{9}$

$= 2+\frac{5}{3}+\frac{2}{9}$

$= \frac{18+15+2}{9}$

$= \frac{35}{9}$

1 F. Question

Compute the following in head:

10.7×9.3

Answer

it can be written as (10+0.7) (10-0.7)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.7) (10-0.7) = $(10)^2 - (0.7)^2$

= 100 - 0.49

= 99.51

1 F. Question

Compute the following in head:

10.7×9.3

Answer

it can be written as (10+0.7) (10-0.7)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.7) (10-0.7) = $(10)^2 - (0.7)^2$

= 100 - 0.49

= 99.51

2. Question

Look at this pattern:

$$\left(1\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=2$$

$$\left(2\frac{1}{2}\right)^2-\left(1\frac{1}{2}\right)^2=4$$

$$\left(3\frac{1}{2}\right)^2-\left(2\frac{1}{2}\right)^2=6$$

Explain the general principle using algebra.

Answer

we can write it as $\left(1+\frac{1}{2}\right)^2$ - $\left(1-\frac{1}{2}\right)^2$

Using identity $(x-y)^2$= x^2+ y^2- 2xy

and $(x+y)^2$= x^2+ y^2+ 2xy

$= 1+\frac{1}{2}^{2}+1-(1+\frac{1}{2}^{2}-1)$

$= 1+\frac{1}{2}^{2}+1-1-\frac{1}{2}^{2}+1$

= 2

Similarly it can be done for other numbers.

2. Question

Look at this pattern:

$$\left(1\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=2$$

$$\left(2\frac{1}{2}\right)^2-\left(1\frac{1}{2}\right)^2=4$$

$$\left(3\frac{1}{2}\right)^2-\left(2\frac{1}{2}\right)^2=6$$

Explain the general principle using algebra.

Answer

we can write it as $\left(1+\frac{1}{2}\right)^2$ - $\left(1-\frac{1}{2}\right)^2$

Using identity $(x-y)^2$= x^2+ y^2- 2xy

and $(x+y)^2$= x^2+ y^2+ 2xy

$= 1+\frac{1}{2}^{2}+1-(1+\frac{1}{2}^{2}-1)$

$= 1+\frac{1}{2}^{2}+1-1-\frac{1}{2}^{2}+1$

= 2

Similarly it can be done for other numbers.

3 A. Question

Find out the larger product of each pair below, without actual multiplication.

25 × 75, 26 × 74

Answer

Using identity $(x+y) \times (x-y) = (x)^2$- $(y)^2$

25 × 75 = (50-25)(50+25) = $(50)^2$- $(25)^2$

26 × 74 = (50- 24)(50+24)= $(50)^2$- $(24)^2$

clearly 26 × 74 is the larger product

since in this we are subtracting $(24)^2$ from $(50)^2$whereas in 25 × 75 we are subtracting $(25)^2$from $(50)^2$.

example- (10-5) and (10-4)

(10-4) is larger since we are subtracting 4 from 10 whereas we are subtracting 5 from 10 in (10-5).

3 A. Question

Find out the larger product of each pair below, without actual multiplication.

25 × 75, 26 × 74

Answer

Using identity $(x+y) \times (x-y) = (x)^2$- $(y)^2$

25 × 75 = (50-25)(50+25) = $(50)^2$- $(25)^2$

26 × 74 = (50- 24)(50+24)= $(50)^2$- $(24)^2$

clearly 26 × 74 is the larger product

since in this we are subtracting $(24)^2$ from $(50)^2$whereas in 25 × 75 we are subtracting $(25)^2$from $(50)^2$.

example- (10-5) and (10-4)

(10-4) is larger since we are subtracting 4 from 10 whereas we are subtracting 5 from 10 in (10-5).

3 B. Question

Find out the larger product of each pair below, without actual multiplication.

76 × 24, 74 × 26

Answer

Using identity $(x+y) \times (x-y) = (x)^2$- $(y)^2$

76 × 24 = (50+26)(50-26) = $(50)^2$- $(26)^2$

74 × 26 = (50+24)(50-24) = $(50)^2$- $(24)^2$

clearly 74 × 26 is the larger product

since in this we are subtracting $(24)^2$ from $(50)^2$whereas in 76 × 24 we are subtracting $(26)^2$from $(50)^2$. since we are subtracting smaller number from same number.

3 B. Question

Find out the larger product of each pair below, without actual multiplication.

76 × 24, 74 × 26

Answer

Using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

76 × 24 = (50+26)(50-26) = $(50)^2 - (26)^2$

74 × 26 = (50+24)(50-24) = $(50)^2 - (24)^2$

clearly 74 × 26 is the larger product

since in this we are subtracting $(24)^2$ from $(50)^2$ whereas in 76 × 24 we are subtracting $(26)^2$ from $(50)^2$. since we are subtracting smaller number from same number.

3 C. Question

Find out the larger product of each pair below, without actual multiplication.

10.6 × 9.4, 10.4 × 9.6

Answer

Using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

10.6 × 9.4 = (10+0.6)(10-0.6)= $(10)^2 - (0.6)^2$

10.4 × 9.6 = (10+0.4)(10-0.4)= $(10)^2 - (0.4)^2$

clearly 10.4 × 9.6 is the larger product

since in this we are subtracting $(0.4)^2$ from $(10)^2$ whereas in 10.6×9.4 we are subtracting $(0.6)^2$ from $(10)^2$

3 C. Question

Find out the larger product of each pair below, without actual multiplication.

10.6 × 9.4, 10.4 × 9.6

Answer

Using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

10.6 × 9.4 = (10+0.6)(10-0.6)= $(10)^2 - (0.6)^2$

10.4 × 9.6 = (10+0.4)(10-0.4)= $(10)^2 - (0.4)^2$

clearly 10.4 × 9.6 is the larger product

since in this we are subtracting $(0.4)^2$ from $(10)^2$ whereas in 10.6×9.4 we are subtracting $(0.6)^2$ from $(10)^2$

4 A. Question

Compute the following differences:

(125 × 75) – (126 × 74)

Answer

(125 × 75) can be written as (100+25)(100-25)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(100+25)(100-25) = $(100)^2 - (25)^2$

= 10000 - 625

= 9375

now (126 × 74) can be written as (100+26)(100-26)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(100+26)(100-26) = $(100)^2 - (26)^2$

= 10000 - 676

= 9324

now putting values in question

(125 × 75) – (126 × 74) = 9375-9324

= 51

4 A. Question

Compute the following differences:

(125 × 75) – (126 × 74)

Answer

(125 × 75) can be written as (100+25)(100-25)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(100+25)(100-25) = $(100)^2 - (25)^2$

= 10000 - 625

= 9375

now (126 × 74) can be written as (100+26)(100-26)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(100+26)(100-26) = $(100)^2 - (26)^2$

= 10000 - 676

= 9324

now putting values in question

(125 × 75) – (126 × 74) = 9375-9324

= 51

4 B. Question

Compute the following differences:

(124 × 76) – (126 × 74)

Answer

(124 × 76) can be written as (100+24)(100-24)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(100+24)(100-24) = $(100)^2 - (24)^2$

= 10000 - 576

= 9424

now (126 × 74) can be written as (100+26)(100-26)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(100+26)(100-26) = $(100)^2 - (26)^2$

= 10000 - 676

= 9324

now putting values in question

(124 × 76) – (126 × 74) = 9424-9324

= 100

4 B. Question

Compute the following differences:

(124 × 76) – (126 × 74)

Answer

(124 × 76) can be written as (100+24)(100-24)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(100+24)(100-24) = $(100)^2 - (24)^2$

= 10000 - 576

= 9424

now (126 × 74) can be written as (100+26)(100-26)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(100+26)(100-26) = $(100)^2 - (26)^2$

= 10000 - 676

= 9324

now putting values in question

(124 × 76) – (126 × 74) = 9424-9324

= 100

4 C. Question

Compute the following differences:

(224 × 176) – (226 × 174)

Answer

(224 × 176) can be written as (200+24)(200-24)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(200+24)(200-24) = $(200)^2 - (24)^2$

= 40000 - 576

= 39424

now (226 × 174) can be written as (200+26)(200-26)

using identity $(x+y)\times(x-y) = (x)^2 - (y)^2$

(200+26)(200-26) = $(200)^2 - (26)^2$

= 40000 - 676

= 39324

now putting values in question

(224 × 176) – (226 × 174) = 39424 - 39324

= 100

4 C. Question

Compute the following differences:

(224 × 176) – (226 × 174)

Answer

(224 × 176) can be written as (200+24)(200-24)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(200+24)(200-24) = $(200)^2 - (24)^2$

= 40000 - 576

= 39424

now (226 × 174) can be written as (200+26)(200-26)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(200+26)(200-26) = $(200)^2 - (26)^2$

= 40000 - 676

= 39324

now putting values in question

(224 × 176) – (226 × 174) = 39424 - 39324

= 100

4 D. Question

Compute the following differences:

(10.3 × 9.7) – (10.7 × 9.3)

Answer

(10.3 × 9.7) can be written as (10+0.3)(10-0.3)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.3)(10-0.3) = $(10)^2 - (0.3)^2$

= 100 - 0.09

= 99.91

now (10.7 × 9.3) can be written as (10+0.7)(10-0.7)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.7)(10-0.7) = $(10)^2 - (0.7)^2$

= 100 - 0.49

= 99.51

now putting values in question

(10.3 × 9.7) – (10.7 × 9.3) = 99.91 - 99.51

= 0.40

4 D. Question

Compute the following differences:

(10.3 × 9.7) – (10.7 × 9.3)

Answer

(10.3 × 9.7) can be written as (10+0.3)(10-0.3)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.3)(10-0.3) = $(10)^2 - (0.3)^2$

= 100 - 0.09

= 99.91

now (10.7 × 9.3) can be written as (10+0.7)(10-0.7)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(10+0.7)(10-0.7) = $(10)^2 - (0.7)^2$

= 100 - 0.49

= 99.51

now putting values in question

(10.3 × 9.7) – (10.7 × 9.3) = 99.91 - 99.51

= 0.40

4 E. Question

Compute the following differences:

(11.3 × 10.7) – (11.7 × 10.3)

Answer

(11.3 × 10.7) can be written as (11+0.3)(11-0.3)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(11+0.3)(11-0.3) = $(11)^2 - (0.3)^2$

= 121 - 0.09

= 120.91

now (11.7 × 10.3) can be written as (11+0.7)(11-0.7)

using identity $(x + y) \times (x - y) = (x)^2 - (y)^2$

(11+0.7)(11-0.7) = $(11)^2 - (0.7)^2$

= 121 - 0.49

= 120.51

now putting values in question

(11.3 × 10.7) – (11.7 × 10.3) = 120.91- 120.51

= 0.40

4 E. Question

Compute the following differences:

(11.3 × 10.7) – (11.7 × 10.3)

Answer

(11.3 × 10.7) can be written as (11+0.3)(11-0.3)

using identity $(x+y) \times (x-y) = (x)^2 - (y)^2$

(11+0.3)(11-0.3) = $(11)^2 - (0.3)^2$

= 121 - 0.09

= 120.91

now (11.7 × 10.3) can be written as (11+0.7)(11-0.7)

using identity $(x+y) \times (x-y) = (x)^2 - (y)^2$

(11+0.7)(11-0.7) = $(11)^2 - (0.7)^2$

= 121 - 0.49

= 120.51

now putting values in question

(11.3 × 10.7) – (11.7 × 10.3) = 120.91- 120.51

= 0.40

Questions Pg-84

1 A. Question

Mark four numbers forming a square in a calendar:

4	5
11	12

Add the squares of the diagonal pair and find the difference of these sums:

$4^2 + 12^2 = 160$

$11^2 + 5^2 = 146$

160 – 146 = 14

Do this for other four numbers.

Answer

Let the numbers be 6,7,13 and 14

6	7
13	14

the squares of the diagonal are

6^2 and 14^2

7^2 and 13^2

Add the squares of the diagonal pair

$6^2 + 14^2$

= 36+196

= 232

$7^2 + 13^2$

= 49+ 169

= 218

find the difference of these sums:

232 - 218

=14

1 A. Question

Mark four numbers forming a square in a calendar:

4	5
11	12

Add the squares of the diagonal pair and find the difference of these sums:

$4^2 + 12^2 = 160$

$11^2 + 5^2 = 146$

160 – 146 = 14

Do this for other four numbers.

Answer

Let the numbers be 6,7,13 and 14

6	7
13	14

the squares of the diagonal are

6^2 and 14^2

7^2 and 13^2

Add the squares of the diagonal pair

$6^2 + 14^2$

= 36+196

= 232

$7^2 + 13^2$

= 49+ 169

= 218

find the difference of these sums:

232 - 218

=14

1 B. Question

Mark four numbers forming a square in a calendar:

4	5
11	12

Add the squares of the diagonal pair and find the difference of these sums:

$4^2 + 12^2 = 160$

$11^2 + 5^2 = 146$

160 – 146 = 14

Explain using algebra, why the difference is 14 always.

Answer

Let’s use algebra to see this.

Taking the first number in the square as x, the others can be filled as below

x	x+1
x+7	x+8

the squares of diagonal are

x^2 and $(x+8)^2$

$(x+7)^2$ and $(x+1)^2$

Add the squares of the diagonal pair

$x^2 + (x+8)^2$

$= x^2 + (x^2+8^2+2\times x\times 8)$ [Using identity $(x+y)^2 = x^2 + y^2 + 2xy$]

$= 2x^2 + 64+16x$

Add the squares of the other diagonal pair

$= (x+1)^2+(x+7)^2$

$= (x^2 +1 +2\times x\times 1) + (x^2+7^2+2\times x\times 7)$

$= (x^2 +1 +2x) + (x^2 +49 +14x)$

$= 2x^2 +16x+ 50$

find the difference of these sums:

$= (2x^2 + 64+16x) - (2x^2 +16x+ 50)$

$= 2x^2 + 64+16x - 2x^2 -16x - 50$

= 64-50

= 14

Hence the difference is 14, we can take any number as x; which means this holds in any part of the calendar.

1 B. Question

Mark four numbers forming a square in a calendar:

4	5
11	12

Add the squares of the diagonal pair and find the difference of these sums:

$4^2 + 12^2 = 160$

$11^2 + 5^2 = 146$

160 – 146 = 14

Explain using algebra, why the difference is 14 always.

Answer

Let's use algebra to see this.

Taking the first number in the square as x, the others can be filled as below

x	x+1
x+7	x+8

the squares of diagonal are

x^2 and $(x+8)^2$

$(x+7)^2$ and $(x+1)^2$

Add the squares of the diagonal pair

$x^2 + (x+8)^2$

$= x^2 + (x^2+8^2+2\times x\times 8)$ [Using identity $(x+y)^2 = x^2+ y^2+ 2xy$]

$= 2x^2 + 64+16x$

Add the squares of the other diagonal pair

$= (x+1)^2+(x+7)^2$

$= (x^2 +1 +2\times x\times 1) + (x^2+7^2+2\times x\times 7)$

$= (x^2 +1 +2x) + (x^2 +49 +14x)$

$= 2x^2 +16x+ 50$

find the difference of these sums:

$= (2x^2 + 64+16x)- (2x^2 +16x+ 50)$

$= 2x^2 + 64+16x- 2x^2 -16x- 50$

= 64-50

= 14

Hence the difference is 14, we can take any number as x; which means this holds in any part of the calendar.

2 A. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Add the squares of diagonal pairs and find the difference of the sums.

$3^2 + 19^2 = 370$

$17^2 + 5^2 = 314$

370 – 314 = 56

Do this for other such nine numbers.

Answer

let the other numbers be

6	7	8
13	14	15
20	21	22

The squares of diagonal are

6^2 and 22^2

20^2 and 8^2

Add the squares of the diagonal pair

$6^2 + 22^2$

= 36+ 484

= 520

$20^2 + 8^2$

= 400+64

= 464

find the difference of these sums:

= 520 - 464

= 56

2 A. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Add the squares of diagonal pairs and find the difference of the sums.

$3^2 + 19^2 = 370$

$17^2 + 5^2 = 314$

370 – 314 = 56

Do this for other such nine numbers.

Answer

let the other numbers be

6	7	8
13	14	15
20	21	22

The squares of diagonal are

6^2 and 22^2

20^2 and 8^2

Add the squares of the diagonal pair

$6^2 + 22^2$

= 36+ 484

= 520

$20^2 + 8^2$

= 400+64

= 464

find the difference of these sums:

= 520 - 464

= 56

2 B. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Add the squares of diagonal pairs and find the difference of the sums.

$3^2 + 19^2 = 370$

$17^2 + 5^2 = 314$

370 – 314 = 56

Explain using algebra, why the difference is always 56. (it is convenient to take the number at the centre of the squares as x – see the section, Another trick of the lesson, Unchanging Relation of the Class 7 textbook)

Answer

Let's use algebra to see this.

Taking the first number in the square as x, the others can be filled as below

x	x+1	x+2
x+7	x+8	x+9
x+14	x+15	x+16

The squares of diagonal are

x^2 and $(x+16)^2$

$(x+14)^2$ and $(x+2)^2$

Add the squares of the diagonal pair

$= x^2 + (x+16)^2$

$= x^2 + (x^2+256+ 2\times x \times 16$ [Using identity $(x + y)^2 = x^2 + y^2 + 2xy$]

$=2 x^2 +256+ 32x$

Add the squares of the other diagonal pair

$= (x+14)^2+(x+2)^2$

$=(x^2 +14^2 +2\times x \times 14) + (x^2+2^2+2\times x \times 2)$

$= (x^2 +196 +28x) + (x^2 +4 +4x)$

$= 2x^2 +32x+ 200$

find the difference of these sums:

$(2 x^2 +256+ 32x) - (2x^2 +32x+ 200)$

$= 2 x^2 +256+ 32x - 2x^2 - 32x- 200$

$= 256-200$

$= 56$

Hence the difference is 56, we can take any number as x; which means this hold in any part of the calendar.

Yes we can take x in the center but this will complicate the calculations.

since then the numbers will be

x-8	x-7	x-6
x-1	x	x+1
x+6	x+7	x+8

2 B. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Add the squares of diagonal pairs and find the difference of the sums.

$3^2 + 19^2 = 370$

$17^2 + 5^2 = 314$

370 – 314 = 56

Explain using algebra, why the difference is always 56. (it is convenient to take the number at the centre of the squares as x – see the section, Another trick of the lesson, Unchanging Relation of the Class 7 textbook)

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x+7	x+8	x+9
x+14	x+15	x+16

The squares of diagonal are

x^2 and $(x+16)^2$

$(x+14)^2$ and $(x+2)^2$

Add the squares of the diagonal pair

$= x^2 + (x+16)^2$

$= x^2 +(x^2+256+ 2\times x \times 16$ [Using identity $(x+y)^2 = x^2 + y^2 + 2xy$]

$=2x^2 +256+ 32x$

Add the squares of the other diagonal pair

$= (x+14)^2+(x+2)^2$

$=(x^2 +14^2 +2\times x \times 14) + (x^2+2^2+2\times x \times 2)$

$= (x^2 +196 +28x) + (x^2 +4 +4x)$

$= 2x^2 +32x+ 200$

find the difference of these sums:

$(2x^2 +256+ 32x) - (2x^2 +32x+ 200)$

$= 2x^2 +256+ 32x - 2x^2 - 32x- 200$

$= 256-200$

$= 56$

Hence the difference is 56, we can take any number as x; which means this hold in any part of the calendar.

Yes we can take x in the center but this will complicate the calculations.

since then the numbers will be

x-8	x-7	x-6
x-1	x	x+1
x+6	x+7	x+8

3 A. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Multiply the diagonal pairs and find the difference of these products.

$3 \times 19 = 57$

$17 \times 5 = 85$

$85 – 57 = 28$

Do this for other such squares.

Answer

let the other numbers be

6	7	8
13	14	15
20	21	22

Multiply the diagonal pairs

6x22 = 132

20x8 = 160

Difference of these products

160-132

= 28

3 A. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Multiply the diagonal pairs and find the difference of these products.

3 × 19 = 57

17 × 5 = 85

85 – 57 = 28

Do this for other such squares.

Answer

let the other numbers be

6	7	8
13	14	15
20	21	22

Multiply the diagonal pairs

6x22 = 132

20x8 = 160

Difference of these products

160-132

= 28

3 B. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Multiply the diagonal pairs and find the difference of these products.

3 × 19 = 57

17 × 5 = 85

85 – 57 = 28

Explain using algebra, why the difference is always 28 (It is convenient to take the number at the centre as x).

Answer

Let’s use algebra to see this.

Taking the first number in the square as x, the others can be filled as below

x	x+1	x+2
x+7	x+8	x+9
x+14	x+15	x+16

Multiply the diagonal pairs

$(x)(x+16)$

$= x^2+16x$

Other diagonal product

$(x+14)(x+2)$ [using identity $(x+y)(u+v)= xu+xv+yu+yv$]

$= x^2+2x+14x+28$

$= x^2+16x+28$

Difference of these products

$=(x^2+16x+28) - (x^2+16x)$

$=x^2+16x+28 - x^2-16x$

$= 28$

Hence the difference is 28, we can take any number as x; which means this hold in any part of the calendar.

We can take x at the center, but this will complicate our calculations.

3 B. Question

Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

3	4	5
10	11	12
17	18	19

Multiply the diagonal pairs and find the difference of these products.

3 × 19 = 57

17 × 5 = 85

85 – 57 = 28

Explain using algebra, why the difference is always 28 (It is convenient to take the number at the centre as x).

Answer

Let's use algebra to see this.

Taking the first number in the square as x, the others can be filled as below

x	x+1	x+2
x+7	x+8	x+9
x+14	x+15	x+16

Multiply the diagonal pairs

(x)(x+16)

= x^2+16x

Other diagonal product

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Difference of these products

=(x^2+16x+28) - (x^2+16x)

=x^2+16x+28 - x^2-16x

= 28

Hence the difference is 28, we can take any number as x; which means this hold in any part of the calendar.

We can take x at the center, but this will complicate our calculations.

Questions Pg-72

1 A. Question

Now compute the squares of these numbers in head:

52

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$(52)^2 = (50 + 2)^2$

$=(50)^2 + 2\times50\times2 + (2)$

= 2500 + 200 + 4

= 2704

1 A. Question

Now compute the squares of these numbers in head:

52

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$(52)^2 = (50 + 2)^2$

$=(50)^2 + 2 \times 50 \times 2 + (2)$

$= 2500 + 200 + 4$

$= 2704$

1 B. Question

Now compute the squares of these numbers in head:

105

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$(105)^2 = (100 + 5)^2$

$=(100)^2 + 2 \times 100 \times 5 + (5)^2$

$= 10000 + 1000 + 25$

$= 11025$

1 B. Question

Now compute the squares of these numbers in head:

105

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$(105)^2 = (100 + 5)^2$

$=(100)^2 + 2 \times 100 \times 5 + (5)^2$

$= 10000 + 1000 + 25$

$= 11025$

1 C. Question

Now compute the squares of these numbers in head:

$20\frac{1}{2}$

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$\left(20\frac{1}{2}\right)^2 = \left(20 + \frac{1}{2}\right)^2$

$= (20)^2 + 2 \times 20 \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$= 400 + 20 + \frac{1}{4}$

$= 420\frac{1}{4}$

1 C. Question

Now compute the squares of these numbers in head:

$20\frac{1}{2}$

Answer

Using $(x+y)^2 = x^2 + xy + yx + y^2$

$$\left(20\frac{1}{2}\right)^2 = \left(20 + \frac{1}{2}\right)^2$$

$$= (20)^2 + 2 \times 20 \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= 400 + 20 + \frac{1}{4}$$

$$= 420\frac{1}{4}$$

1 D. Question

Now compute the squares of these numbers in head:

10.2

Answer

$(10.2)^2 = (10 + 0.2)^2$

$=(10)^2 + 2\times10\times0.2 + (0.2)^2$

= 100 + 4 + 0.04

= 104.04

1 D. Question

Now compute the squares of these numbers in head:

10.2

Answer

$(10.2)^2 = (10 + 0.2)^2$

$=(10)^2 + 2\times10\times0.2 + (0.2)^2$

= 100 + 4 + 0.04

= 104.04