θ	0°	30°	45°	60°	90°
Sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\frac{1}{0} = \infty$
Cosec θ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
Cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

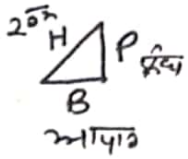
Sin	Cos	Tan
S	C	T
P	B	P
H	H	B

(Pandit Bhadri Prasad Har Har Bole Sona chandi tole)

$\sin\theta = \frac{P}{H}$

$\cos\theta = \frac{B}{H}$

$\tan\theta = \frac{P}{B}$



ਅਧਿਆਇ - 8 | chapter - 8

ਤ੍ਰਿਕੋਣਮਿਤੀ ਬਾਰੇ ਜਾਣ - ਪਛਾਣ | Trigonometry

ਅਭਿਆਸ - 8.1 | Exercise - 8.1

(1) In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm find.

In ΔABC, ਵਿੱਚ, ਜਿਸਦਾ ਕੋਣ B ਸਮਕੋਣ ਹੈ, AB = 24 ਸੈਂ.ਮੀ ਅਤੇ BC = 7 ਸੈਂ.ਮੀ ਹੈ ਤਾਂ ਹੇਠ ਲਿਖਿਆਂ ਦਾ ਮੁੱਲ ਪਤਾ ਕਰੋ।

(i) sin A, cos A

Sol:-

ΔABC ਵਿੱਚ (In ΔABC)

$H^2 = P^2 + B^2$

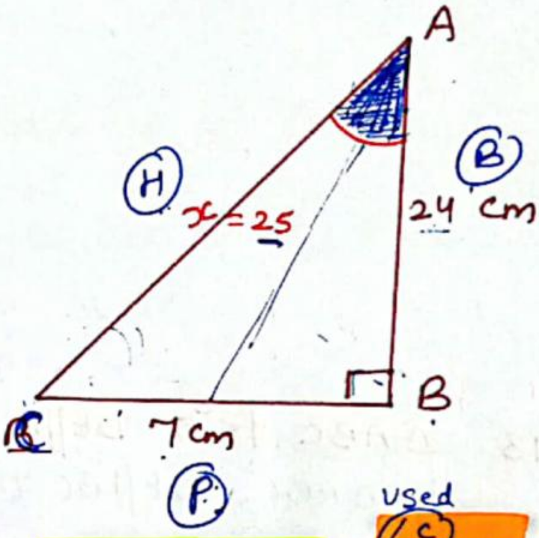
$x^2 = (24)^2 + (7)^2$

$x^2 = 576 + 49$

$x^2 = 625$

$x^2 = 25^2$

$x = 25$ cm



∠A used → P = 7, B = 24, H = 25

∠C used: P = 24, B = 7, H = 25

$\sin A = \frac{P}{H} = \frac{7}{25}$ Ans

$\cos A = \frac{B}{H} = \frac{24}{25}$ Ans

Sin	Cos	Tan
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$

(ii) sin C, cos C

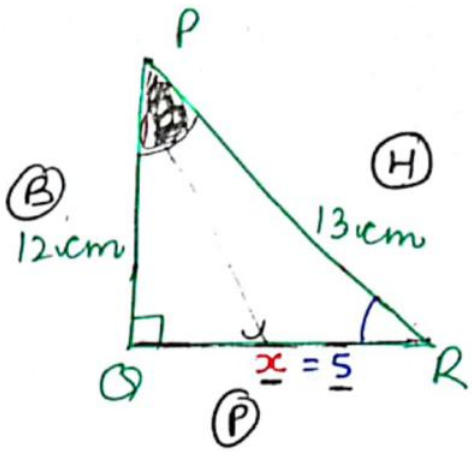
$\sin C = \frac{P}{H} = \frac{24}{25}$

$\cos C = \frac{B}{H} = \frac{7}{25}$ Ans

(2) दिया है, ज्ञात करें:

In fig., find

(1) $\tan P - \cot R$



S C T
P B P/B
H H
cot

Sol:-

ΔPQR में (In ΔPQR)

$H^2 = P^2 + B^2$

$(13)^2 = (12)^2 + x^2$

$169 = 144 + x^2$

$169 - 144 = x^2$

$25 = x^2$

$5^2 = x^2$

$\boxed{x = 5}$

$\tan P - \cot R$

$\Rightarrow \frac{5}{12} - \frac{5}{12} = 0$ Ans

LP used: $P = 5$, $B = 12$, $H = 13$

LR used: $P = 12$, $B = 5$, $H = 13$

$\tan P = \frac{P}{B} = \frac{5}{12}$

$\cot R = \frac{B}{P} = \frac{5}{12}$

(3) जेवढ $\sin A = \frac{3}{4}$, तर $\cos A$ आणि $\tan A$ ची किंमत काढा.

If $\sin A = \frac{3}{4}$, find $\cos A$ and $\tan A$

S P/H
C B H
T P B

Sol:-

$$\sin A = \frac{3}{4} = \frac{P}{H}$$

$$H^2 = P^2 + B^2$$

$$(4)^2 = (3)^2 + B^2$$

$$16 = 9 + B^2$$

$$16 - 9 = B^2$$

$$7 = B^2$$

$$\sqrt{7} = B$$

$$B = \sqrt{7}$$

$\angle A$ used

$P = 3$
$B = \sqrt{7}$
$H = 4$

$$\cos A = \frac{B}{H} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{P}{B} = \frac{3}{\sqrt{7}}$$ Ans

(9) यदि $15\cot A = 8$, $\sin A$ & $\sec A$ ज्ञात करें।

If $15\cot A = 8$, find $\sin A$ & $\sec A$.

S	C	T
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$
cosec	Sec	Cot

Sol:- $15\cot A = 8$

$\cot A = \frac{8}{15} = \frac{B}{P}$

$B = 8,\ P = 15,\ H = 17$

$H^2 = P^2 + B^2$

$H^2 = (15)^2 + (8)^2$

$H^2 = 225 + 64$

$H^2 = 289$

$H^2 = 17^2$

$\boxed{H = 17}$

$\sin A = \frac{P}{H} = \frac{15}{17}$ Ans

$\sec A = \frac{H}{B} = \frac{17}{8}$ Ans

(5) ਜੇਕਰ $\sec\theta = \frac{13}{12}$ ਹੋਵੇ ਤਾਂ ਬਾਕੀ ਸਾਰੇ ਤਿਕੋਣਮਿਤੀ ਅਨੁਪਾਤ ਪਤਾ ਕਰੋ।

Given $\sec\theta = \frac{13}{12}$, calculate other trigonometric ratios.

Sin Cos Tan
P B P
H H B
cosecθ secθ cotθ

Sol:- $\sec\theta = \frac{13}{12} = \frac{H}{B}$

$H^2 = P^2 + B^2$

$(13)^2 = P^2 + (12)^2$

$169 = P^2 + 144$

$169 - 144 = P^2$

$25 = P^2$

$5^2 = P^2$

$P = 5$

H=13, B=12, P=5

$\sin\theta = \frac{P}{H} = \frac{5}{13}$

$\cos\theta = \frac{B}{H} = \frac{12}{13}$

$\tan\theta = \frac{P}{B} = \frac{5}{12}$

$\text{cosec}\theta = \frac{H}{P} = \frac{13}{5}$

$\cot\theta = \frac{B}{P} = \frac{12}{5}$ Ans.

(14)
(7)

यदि $\cot\theta = \frac{7}{8}$. ज्ञात कीजिए

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

(ii)

$\cot\theta = \frac{7}{8} = \frac{B}{P}$

$H^2 = P^2 + B^2$

$H^2 = 8^2 + 7^2$

$H^2 = 64 + 49$

$H^2 = 113$

$H = \sqrt{113}$

$\sin\theta = \frac{P}{H} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{B}{H} = \frac{7}{\sqrt{113}}$

(i)

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} \Rightarrow \frac{\left(\frac{1}{1}+\frac{8}{\sqrt{113}}\right)\left(\frac{1}{1}-\frac{8}{\sqrt{113}}\right)}{\left(\frac{1}{1}+\frac{7}{\sqrt{113}}\right)\left(\frac{1}{1}-\frac{7}{\sqrt{113}}\right)}$

$\Rightarrow \frac{(1)^2-\left(\frac{8}{\sqrt{113}}\right)^2}{(1)^2-\left(\frac{7}{\sqrt{113}}\right)^2}$

$\Rightarrow \frac{1-\frac{64}{113}}{1-\frac{49}{113}} \Rightarrow \frac{\frac{113-64}{113}}{\frac{113-49}{113}}$

$\Rightarrow \frac{49}{\cancel{113}} \times \frac{\cancel{113}}{64}$

$= \frac{49}{64}$ Ans

(ii) $\cot^2\theta$

$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ Ans

(8) यदि / If $3\cot A = 4$ तो दर्शाइए कि / Show that

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

हल करें।

Sin Cos Tan
P B P
H H B
Cosec Sec Cot

Sol:- $3\cot A = 4$

$\cot A = \frac{4}{3}$ $\frac{B}{P}$

$H^2 = P^2 + B^2$

$H^2 = 3^2 + 4^2$

$H^2 = 9 + 16$

$H^2 = 25$

$H^2 = 5^2$

$H = 5$

$\sin A = \frac{P}{H} = \frac{3}{5}$

$\cos A = \frac{B}{H} = \frac{4}{5}$

$\tan A = \frac{P}{B} = \frac{3}{4}$

A.T.Q

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

$$\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\frac{\frac{1}{1}-\frac{9}{16}}{\frac{1}{1}+\frac{9}{16}} = \frac{16}{25} - \frac{9}{25}$$

$$\frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{16-9}{25}$$

$$\frac{7}{16} \times \frac{16}{25} = \frac{7}{25}$$

$$\frac{7}{25} = \frac{7}{25}$$ Ans

(9) ΔABC ਇੱਕ, ਜਿਸਦਾ ਕੋਣ B ਸਮਕੋਣ ਹੈ, ਜੇਕਰ $\tan A = \frac{1}{\sqrt{3}}$ ਤਾਂ ਹੇਠਾਂ ਦਿੱਤਿਆਂ ਦੇ ਮੁੱਲ ਪਤਾ ਕਰੋ।

In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:-

(i) $\sin A \cos C + \cos A \sin C$ (ii) $\cos A \cos C - \sin A \sin C$

S C Tan
P B P
H H B

Sol $\tan A = \frac{1}{\sqrt{3}} = \frac{P}{B}$

$H^2 = P^2 + B^2$

$H^2 = (1)^2 + (\sqrt{3})^2$

$H^2 = 1 + 3$

$H^2 = 2^2$

$H = 2$

∠A ਦੇ ਤਿੰਨ:
$P = 1$
$B = \sqrt{3}$
$H = 2$

∠C ਦੇ ਤਿੰਨ:
$P = \sqrt{3}$
$B = 1$
$H = 2$

$\sin A = \frac{P}{H} = \frac{1}{2}$

$\cos A = \frac{B}{H} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{P}{H} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{B}{H} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{1+3}{4} = \frac{4}{4} = 1$ Ans.

(ii) $\cos A \cos C - \sin A \sin C$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$ Ans.

(ii)

(10) ΔPQR, ਜਿੱਥੇ ਕੋਣ Q ਸਮਕੋਣ ਹੈ, $PR + QR = 25$ ਸੈ.ਮੀ. ਅਤੇ $PQ = 5$ ਸੈ.ਮੀ. ਹੈ। $\sin P$, $\cos P$ ਅਤੇ $\tan P$ ਦਾ ਮੁੱਲ ਪਤਾ ਕਰੋ।

In ΔPQR, right angled at Q, $PR + QR = 25$ cm & $PQ = 5$ cm. Determine the value of $\sin P$, $\cos P$ & $\tan P$.

Sol:-

$PR + QR = 25$

$PR + x = 25$

$PR = 25 - x$

$H^2 = P^2 + B^2$

$(25-x)^2 = (5)^2 + (x)^2$

$(25)^2 + x^2 - 2(25)(x) = 25 + x^2$

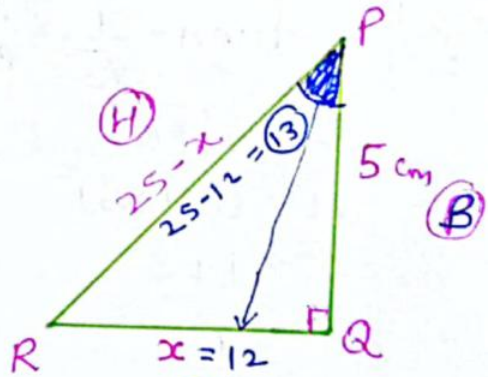
$625 - 50x = 25$

$625 - 25 = 50x$

$600 = 50x$

$12\ \frac{600}{50} = x$

$x = 12$ cm



$\angle P$ ਦੇ ਤੋਂ

P = 12

B = 5

H = 13

S	C	T
P	B	P
H	H	B

$\sin P = \frac{P}{H} = \frac{12}{13}$

$\cos P = \frac{B}{H} = \frac{5}{13}$

$\tan P = \frac{P}{B} = \frac{12}{5}$ Ans

ਧੰਨਵਾਦ ਸਹਿਤ :-
Mr. Vivek Goyel
(Math Master)
GSSS Nangajja
(Jal).

θ	0°	30°	45°	60°	90°
$\sin\theta$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\frac{1}{0} = \infty$
$\operatorname{Cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

अभ्यास → 8.2
Exercise → 8.2

निम्न त्रिकोणमितीय का मान ज्ञात करें: (Find the value of followings :-

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

Sol:- $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$\frac{3}{4} + \frac{1}{4} = \frac{3+1}{4}$

$= \frac{4}{4}$

$= 1$ Ans.

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Sol:- $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2$ Ans

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Sol:-

$$\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{1}}$$

$$\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$\frac{\sqrt{3}}{2(\sqrt{6}+\sqrt{2})} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$\frac{\sqrt{18}-\sqrt{6}}{2\left((\sqrt{6})^2-(\sqrt{2})^2\right)}$$

$$= \frac{\sqrt{2\times3\times3} - \sqrt{3\times2}}{2(6-2)}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$ Ans

(iv) $\frac{5\cos^2 60 + 4\sec^2 30 - \tan^2 45}{\sin^2 30 + \cos^2 30}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + 4\left(\frac{4}{3}\right) - \frac{1}{1}}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - \frac{1}{1}}{\frac{1+3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - \frac{1}{1}}{\frac{4}{4}}$$

$$= \frac{5}{4} + \frac{16}{3} - \frac{1}{1}$$

$$= \frac{15+64}{12} - \frac{1}{1}$$

$$= \frac{79}{12} - \frac{1}{1}$$

$$= \frac{79-12}{12} = \frac{67}{12}$$ Ans

ਪ੍ਰਸ਼ਨ 2 ਸਹੀ ਉੱਤਰ ਚੁਣੋ ਅਤੇ ਉਸਦਾ ਕਾਰਨ ਦਿਓ :-

Choose the correct option & Justify:-

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

$\Rightarrow \frac{2\tan 30}{1+\tan^2 30} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}+\frac{1}{3}} \Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4_2} = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}\times 2} = \frac{\sqrt{3}}{2}$

$= \sin 60^\circ$ Ans

option (A) ✓

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

Sol:-

$\Rightarrow \frac{1-\tan^2 45}{1+\tan^2 45} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

option d ✓

(iii) $\sin 2A = 2\sin A\cos A$, $A =$

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\frac{2\tan 30}{1-\tan^2 30}$

(A) $\cos 60$ (B) $\sin 60$ (C) $\tan 60$ (D) $\sin 30^\circ$

Sol:- $\frac{2\tan 30}{1-\tan^2 30}$

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{1}}$$

$$= \sqrt{3}$$

$$= \tan 60$$

Option C Ans

प्रश्न 3 यदि $\tan(A+B)=\sqrt{3}$ और $\tan(A-B)=\frac{1}{\sqrt{3}}$, $0^\circ<A+B<90^\circ$; $A>B$ तो ज्ञात कीजिए A और B के मान लिखिए।

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$, $0^\circ<A+B<90^\circ$; $A>B$ then find A & B.

Sol: $\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60^\circ$

$A+B=60$ —— (1)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\tan(A-B)=\tan 30$

$A-B=30^\circ$ —— (2)

(1) और (2) को जोड़ने पर (Add (1) & (2))

$A+B+A-B=60+30$

$2A=90$

$A=\frac{90}{2}=45$

$A=45^\circ$

$45+B=60$ ((1) में)

$B=60-45$

$B=15^\circ$ Ans

पूरक कोणों के त्रिकोणमितीय अनुपात
(Trigonometry Ratio of complementary Angles)

1. $\sin(90-\theta) = \cos\theta$
2. $\cos(90-\theta) = \sin\theta$
3. $\tan(90-\theta) = \cot\theta$
4. $\cot(90-\theta) = \tan\theta$
5. $\sec(90-\theta) = \operatorname{cosec}\theta$
6. $\operatorname{cosec}(90-\theta) = \sec\theta$

Ex 8.2 & 8.1

3x3 Reciprocal

Sin	Cos	Tan
P	B	P
H	H	B
Cosec	Sec	Cot

$\sin\theta \longleftrightarrow \cos\theta$

$\tan\theta \longleftrightarrow \cot\theta$

$\sec\theta \longleftrightarrow \operatorname{cosec}\theta$

Ex 8.3 Conversion (परिवर्तन)

* $\cos 20^\circ = \sin(90-20)$
  $= \sin 70^\circ$

* $\tan 40 = \cot(90-40)$
  $= \cot 50$

अभ्यास → 8.3
Exercise → 8.3

प्रश्न 1 तेह स्थितिमां दा मूल्य थशे (Evaluate followings)

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

Sol:- $\frac{\sin 18}{\cos 72}$

$= \frac{\cos(90-18)}{\cos 72}$

$= \frac{\cos 72}{\cos 72}$

$= 1$ Ans.

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

$= \frac{\tan 26^\circ}{\cot 64^\circ}$

$= \frac{\cot(90-26)}{\cot 64}$

$= \frac{\cot 64}{\cot 64}$

$= 1$ Ans

(iii) $\cos 48^\circ - \sin 42^\circ$

$\cos 48^\circ - \sin 42^\circ$

$\sin(90-48) - \sin 42$

$\sin 42 - \sin 42$

$= 0$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

$\Rightarrow \operatorname{cosec} 31 - \sec 59$

$\Rightarrow \sec(90-31) - \sec 59$

$\Rightarrow \sec 59 - \sec 59$

$= 0$ Ans

प्रश्न 2: सिद्ध कि: (Show that)

(i) $\tan 48 \tan 23 \tan 42^\circ \tan 67^\circ = 1$

Sol:- L.H.S $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$\tan 48 \tan 42 \tan 23 \tan 67$

$\cot(90-48) \tan 42 \cot(90-23) \tan 67$

$\cot 42 \tan 42 \cot 67 \tan 67$

$\frac{1}{\tan 42} \times \tan 42 \quad \frac{1}{\tan 67} \times \tan 67$

$= 1 = R.H.S$

$\cot\theta = \frac{1}{\tan\theta}$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

ਹੱਲ:- $\cos 38^\circ \cos 52 - \sin 38^\circ \sin 52^\circ$

$\sin(90-38)\cos 52 - \cos(90-38)\sin 52$

$\sin 52 \cos 52 - \cos 52 \sin 52$

$= 0$ Ans

ਸਿੱਧ ਹੋਇਆ

$a5 - 5a = 0$

ਪ੍ਰਸ਼ਨ -3. ਜੇਕਰ $\tan 2A = \cot(A-18^\circ)$, ਜਿੱਥੇ 2A ਇੱਕ ਸੂਖਮ ਕੋਣ ਹੈ, ਤਾਂ A ਦਾ ਮਾਨ ਪਤਾ ਕਰੋ।

यदि $\tan 2A = \cot(A-18^\circ)$, where 2A is an Acute angle, Find value of A.

ਹੱਲ:- $\tan 2A = \cot(A-18)$

$\cot(90-2A) = \cot(A-18)$

$90-2A = A-18$

$90+18 = A+2A$

$108 = 3A$

$\frac{108}{3} = A$

$36^\circ = A$ Ans

ਪ੍ਰਸ਼ਨ 4:- ਜੇਕਰ $\tan A = \cot B$ ਤਾਂ ਸਿੱਧ ਕਰੋ $A+B = 90^\circ$

यदि $\tan A = \cot B$, Prove that $A+B = 90^\circ$

ਹੱਲ:- $\tan A = \cot B$

$\cot(90-A) = \cot B$

$90-A = B$

$90 = B+A$

$A+B = 90^\circ$ Ans

ਪ੍ਰਸ਼ਨ -5 ਜੇਕਰ $\sec 4A = \operatorname{cosec}(A-20^\circ)$, ਜਿੱਥੇ 4A ਇੱਕ ਸਿਖਰ ਕੋਣ ਹੈ ਤਾਂ A ਦਾ ਮੁੱਲ ਪਤਾ ਕਰੋ।

if $\sec 4A = \operatorname{cosec}(A-20^\circ)$, where 4A is an acute angle, Find value of A.

ਹੱਲ:-

$$\sec 4A = \operatorname{cosec}(A-20)$$

$$\operatorname{cosec}(90-4A) = \operatorname{cosec}(A-20)$$

$$90-4A = A-20$$

$$90+20 = A+4A$$

$$110 = 5A$$

$$\frac{110}{5} = A$$

$$22^\circ = A$$

$$A = 22^\circ \text{ Ans}$$

ਪ੍ਰਸ਼ਨ -6 ਜੇਕਰ A, B ਅਤੇ C ਤ੍ਰਿਭੁਜ ABC ਦੇ ਅੰਦਰੂਨੀ ਕੋਣ ਹਨ ਤਾਂ ਸਿੱਧ ਕਰੋ ਕਿ $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

if A, B and C are interior angles of $\triangle ABC$, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

ਹੱਲ:-

$\angle A + \angle B + \angle C = 180$ (ਤ੍ਰਿਭੁਜ ਦੇ ਅੰਦਰੂਨੀ ਕੋਣਾਂ ਦਾ ਜੋੜ 180 ਹੁੰਦਾ ਹੈ)
(Sum of Interior angle of $\triangle$ is 180°)

$$\angle B + \angle C = 180 - \angle A$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180-A}{2}\right)$$

$$= \sin\left(\frac{180}{2} - \frac{A}{2}\right)$$

$$= \sin\left(90 - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2} \text{ Ans}$$

प्रश्न 7 $\sin 67^\circ + \cos 75^\circ$ को 0° और 45° के बीच के कोणों के त्रिकोणमितीय अनुपातों के पदों में व्यक्त करें।

Express $\sin 67 + \cos 75^\circ$ in term of trigonometric Ratios of angles between 0° and 45°

Sol:- $\sin 67^\circ + \cos 75^\circ$

$\cos(90-67) + \sin(90-75)$

$\cot 23^\circ + \sin 15^\circ$ Ans

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त्रिकोणमिती सर्वसमता
Trigonometric Identities

1. $\cos^2\theta + \sin^2\theta = 1$

2. $\sec^2\theta - \tan^2\theta = 1$

3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

1 $\cos^2\theta + \sin^2\theta = 1$

- $\cos^2\theta = 1 - \sin^2\theta$
- $\sin^2\theta = 1 - \cos^2\theta$

2. $\sec^2\theta - \tan^2\theta = 1$

- $\sec^2\theta = 1 + \tan^2\theta$
- $\tan^2\theta = \sec^2\theta - 1$

3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

- $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
- $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

1 $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

2. $\sec\theta = \frac{1}{\cos\theta}$

3. $\cot\theta = \frac{1}{\tan\theta}$

* $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$

પ્રશ્ન 1 ત્રિકોણમિતી અનુપાતો $\sin A$, $\sec A$ અને $\tan A$ ને $\cot A$ ના પદમાં દર્શાવો.

Express the trigonometric Ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Sol:-

Sin Cos Tan
P B P
H H B
Cosec Sec Cot

ધારો કે (Let) $\cot A = \frac{x}{1} = \frac{B}{P}$

$\angle A$ ને સાપેક્ષ
$P = 1$
$B = x$
$H = \sqrt{1+x^2}$

$H^2 = P^2 + B^2$

$H^2 = (1)^2 + (x)^2$

$H^2 = 1 + x^2$

$H = \sqrt{1+x^2}$

1. $\sin A = \frac{P}{H} = \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+\cot^2 A}}$ Ans

2. $\sec A = \frac{H}{B} = \frac{\sqrt{1+x^2}}{x} = \frac{\sqrt{1+\cot^2 A}}{\cot A}$ Ans

3. $\tan A = \frac{P}{B} = \frac{1}{x} = \frac{1}{\cot A}$ Ans

પ્રશ્ન 2 $\angle A$ ના બાકી બધા ત્રિકોણમિતી અનુપાતો ને $\sec A$ ના પદમાં દર્શાવો.

Write all other trigonometric Ratios of $\angle A$ in terms of $\sec A$

Sol:-

Sin Cos Tan
P B P
H H B
Cosec Sec Cot

ધારો કે (Let) $\sec A = \frac{x}{1} = \frac{H}{B}$

$H = x$
$B = 1$
$P = \sqrt{x^2-1}$

$H^2 = P^2 + B^2$

$x^2 = P^2 + 1^2$

$x^2 = P^2 + 1$

$x^2 - 1 = P^2$

$\sqrt{x^2-1} = P$

1. $\sin A = \frac{P}{H} = \frac{\sqrt{x^2-1}}{x} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$ Ans

2. $\cos A = \frac{B}{H} = \frac{1}{x} = \frac{1}{\sec A}$ Ans

3. $\tan A = \frac{P}{B} = \frac{\sqrt{x^2-1}}{1} = \sqrt{\sec^2 A - 1}$ Ans

4. $\operatorname{cosec} A = \frac{H}{P} = \frac{x}{\sqrt{x^2-1}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$ Ans

5. $\cot A = \frac{B}{P} = \frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{\sec^2 A - 1}}$ Ans

ਪ੍ਰਸ਼ਨ -3 ਹੇਠਾਂ ਪੜਾ ਹਨੇ :- Evaluate

(i) $\frac{\sin^2 63 + \sin^2 27}{\cos^2 17 + \cos^2 73}$

$\Rightarrow \frac{\cos^2(90-63) + \sin^2 27}{\sin^2(90-17) + \cos^2 73}$

$= \frac{\cos^2 27 + \sin^2 27}{\sin^2 73 + \cos^2 73}$

$= \frac{1}{1}$

$= 1$ Ans

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$\Rightarrow \cos(90-25)\cos 65 + \sin(90-25)\sin 65$

$\Rightarrow \cos 65 \cos 65 + \sin 65 \sin 65$

$\Rightarrow \cos^2 65 + \sin^2 65$

$= 1$ Ans

ਪ੍ਰਸ਼ਨ -4 ਸਹੀ ਉੱਤਰ ਚੁਣੋ ਅਤੇ ਆਪਣੇ ਉੱਤਰ ਦੀ ਪੁਸ਼ਟੀ ਕਰੋ।
Choose the correct option. Justify

(i) $9\sec^2 A - 9\tan^2 A$

(A) 1 (B) 9 (C) 8 (D) 0

Sol:- $9\sec^2 A - 9\tan^2 A$

$9(\sec^2 A - \tan^2 A)$

$9(1) = 9$

Option = B ✓

$\cos^2\theta + \sin^2\theta = 1$
$\sec^2\theta - \tan^2\theta = 1$
$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=$

(A) 0 (B) 1 (C) 2 (D) −1

Sol:- $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$$\left(\frac{1}{1}+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$\frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta\sin\theta}=$$

$$\Rightarrow \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta\sin\theta}$$

$$\Rightarrow \frac{1+2\cos\theta\sin\theta-1}{\cos\theta\sin\theta}=\frac{2\cos\theta\sin\theta}{\cos\theta\sin\theta}=2$$

Option = c ✓

$\sec\theta=\frac{1}{\cos\theta}$

$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

$\cot\theta=\frac{\cos\theta}{\sin\theta}$

$(a+b)(a-b)=a^2-b^2$

$(a+b)^2=a^2+b^2+2ab$

$\cos^2\theta+\sin^2\theta=1$

(iii) $(\sec A+\tan A)(1-\sin A)$

(A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$

Sol $(\sec A+\tan A)(1-\sin A)$

$$\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$\frac{(1+\sin A)(1-\sin A)}{\cos A}=\frac{1^2-\sin^2 A}{\cos A}$$

$$=\frac{1-\sin^2 A}{\cos A}$$

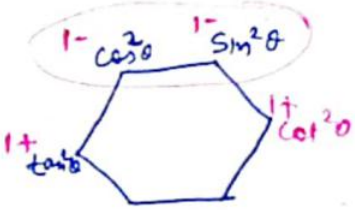
$$=\frac{\cos^2 A}{\cos A}=\cos A$$

Option = d ✓

$\sec\theta=\frac{1}{\cos\theta}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

$(a+b)(a-b)=a^2-b^2$



(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (d) $\tan^2 A$

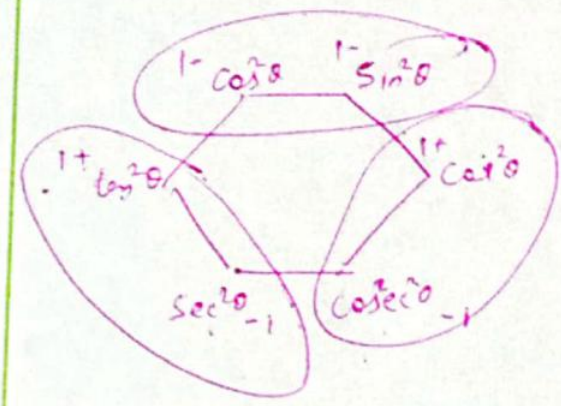
$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

Option = d ✓



$\sec A = \frac{1}{\cos A}$

$\operatorname{cosec} A = \frac{1}{\sin A}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

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ਪ੍ਰਸ਼ਨ -5

ਹੇਠਾਂ ਦਿੱਤੀਆਂ ਸਰਬਸਮਤਾਵਾਂ (ਤਤਸਮਕਾਂ) ਸਿੱਧ ਕਰੋ, ਜਿੱਥੇ ਤੇ ਕੋਣ ਜਿਨ੍ਹਾਂ ਲਈ ਵਿਅੰਜਕ ਪਰਿਭਾਸ਼ਿਤ ਹਨ, ਨਿਊਨ ਕੋਣ ਹਨ:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

ਹੱਲ:

ਖੱਬਾ ਪਾਸਾ (L.H.S) $(\operatorname{cosec}\theta - \cot\theta)^2$

$$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

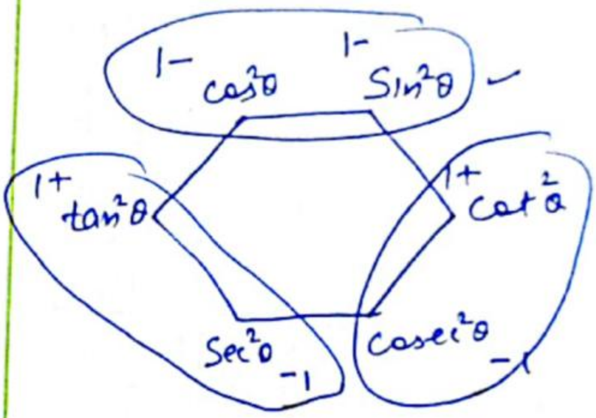
$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \text{ਸੱਜਾ ਪਾਸਾ (R.H.S)}$$

ਫਾਰਮੂਲੇ:

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$



$\sin^2\theta = 1-\cos^2\theta$

$a^2-b^2 = (a+b)(a-b)$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

L.H.S $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$\Rightarrow \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$= \frac{\cos^2 A + (1)^2 + \sin^2 A + 2(1)(\sin A)}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$

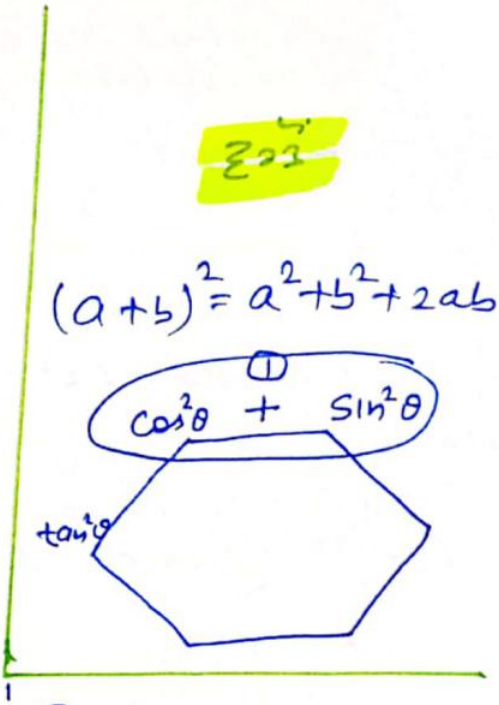
$= \frac{2+2\sin A}{(1+\sin A)\cos A} \Rightarrow \frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$\Rightarrow 2 \times \frac{1}{\cos A}$

$\Rightarrow 2\sec A = R.H.S$

Eg:

$(a+b)^2 = a^2+b^2+2ab$



(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

L.H.S

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \mathrm{cosec}\,\theta\sec\theta + 1.$$

$$= \text{R.H.S}$$

$\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\rightarrow (a-b) = -(b-a)$

$a^3-b^3 = (a-b)(a^2+b^2+ab)$

$\sin^2\theta + \cos^2\theta = 1$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

L.H.S

$$\frac{1+\sec A}{\sec A}$$

$$\Rightarrow \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$\Rightarrow \frac{\cos A+1}{\cancel{\cos A}} \times \frac{\cancel{\cos A}}{1}$$

$$\Rightarrow 1+\cos A \quad \text{—①}$$

R.H.S

$$\frac{\sin^2 A}{1-\cos A}$$

$$\frac{1^2-\cos^2 A}{1-\cos A}$$

$$\frac{(1+\cos A)\cancel{(1-\cos A)}}{\cancel{(1-\cos A)}}$$

$$1+\cos A \quad \text{—②}$$

① ३ ② ३ (from ① & ②)

बचो पाश्चा = बचा पाश्चा

L.H.S = R.H.S Ans

Ex 5.3

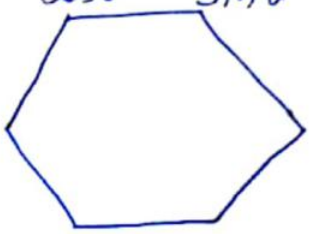
$\sec\theta = \frac{1}{\cos\theta}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$1-\cos^2\theta$ $1-\sin^2\theta$



$a^2-b^2=(a+b)(a-b)$

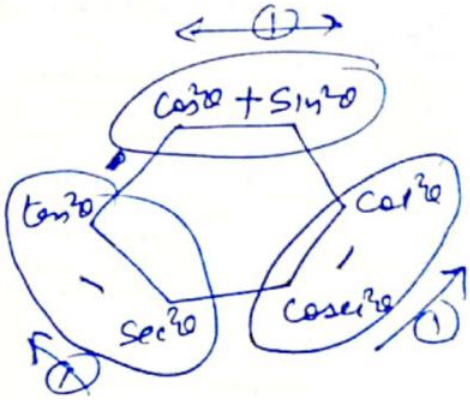
(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$\operatorname{cosec} A = \frac{1}{\sin A}$ ✓

$\sec A = \frac{1}{\cos A}$

$\cot A = \frac{\cos A}{\sin A}$ ✓

$\tan A = \frac{\sin A}{\cos A}$



Sol $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$\Rightarrow \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$

$\Rightarrow \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$\Rightarrow \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$\Rightarrow \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$ $\quad a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A - \operatorname{cosec} A + 1}$

$\Rightarrow \frac{(\cot A + \operatorname{cosec} A)\left[1 - \operatorname{cosec} A + \cot A\right]}{\cot A - \operatorname{cosec} A + 1}$

$= \cot A + \operatorname{cosec} A$

$= R.H.S$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Ex 8.3

L.H.S

$$\sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

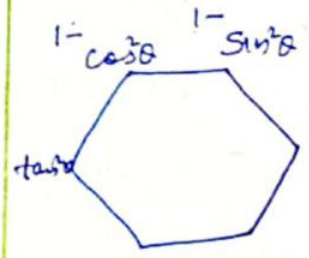
$$\sqrt{\frac{(1+\sin A)(1+\sin A)}{1-\sin^2 A}}$$

$$\sqrt{\frac{(1+\sin A)(1+\sin A)}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \sec A + \tan A =$ R.H.S



$(a+b)(a-b) = a^2 - b^2$

$1 - \sin^2\theta = \cos^2\theta$

$\sec A = \frac{1}{\cos A}$

$\operatorname{cosec} A = \frac{1}{\sin A}$

$\tan A = \frac{\sin A}{\cos A}$ ✓

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Ex 8.3

Sol L.H.S $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$\Rightarrow \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

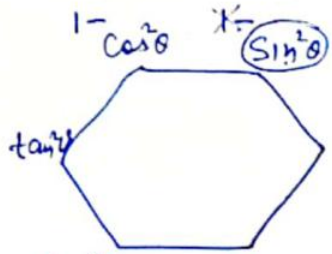
$$\Rightarrow \tan\theta \frac{(1-2\sin^2\theta)}{(2\cos^2\theta-1)}$$

$$\Rightarrow \tan\theta \frac{(1-2(1-\cos^2\theta))}{2\cos^2\theta-1}$$

$$\Rightarrow \tan\theta\left[\frac{1-2+2\cos^2\theta}{2\cos^2\theta-1}\right] \Rightarrow \tan\theta\left[\frac{2\cos^2\theta-1}{2\cos^2\theta-1}\right]$$

$\Rightarrow \tan\theta =$ R.H.S Hence Proved

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



$\sin^2\theta = 1 - \cos^2\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S

$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$\sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$

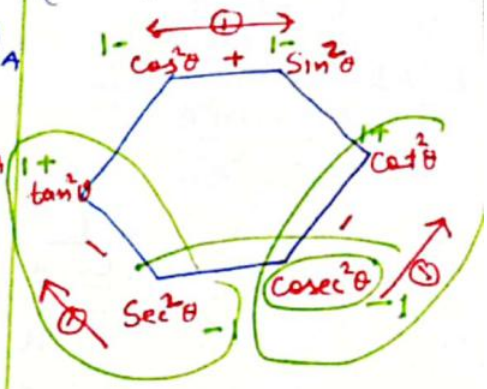
$(\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 \sin A \operatorname{cosec} A + 2 \cos A \sec A$

$1 + 1 + \cot^2 A + 1 + \tan^2 A + \frac{2 \sin A}{\sin A} + \frac{2 \cos A}{\cos A}$

$1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$7 + \cot^2 A + \tan^2 A =$ R.H.S

$(a+b)^2 = a^2 + b^2 + 2ab$



$\operatorname{cosec} A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

L.H.S

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$\left(\frac{1}{\sin A} - \frac{\sin A}{1}\right)\left(\frac{1}{\cos A} - \frac{\cos A}{1}\right)$

$\left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

$= \cos A \sin A$ — (1)

R.H.S

$\frac{1}{\tan A + \cot A}$

$\frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$\frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$

$\sin A \cos A$ — (2)

$\operatorname{cosec} A = \frac{1}{\sin A}$ | $\sec A = \frac{1}{\cos A}$

$\tan A = \frac{\sin A}{\cos A}$

$\cot A = \frac{\cos A}{\sin A}$

(1) और (2) से (from (1) & (2))

बाँया भाग = दाँया भाग

(L.H.S = R.H.S)

(X) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$

L.H.S $= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$

$= \frac{\sin^2 A}{\cos^2 A}$

$= \tan^2 A$

$= R.H.S$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$\frac{(1-\tan A)^2}{\left(\frac{\tan A - 1}{\tan A}\right)^2}$

$(1-\tan A)^2 \times \left(\frac{\tan A}{\tan A - 1}\right)^2$

$\frac{(1-\tan A)^2}{(\tan A - 1)^2} \times \tan^2 A$

$= \tan^2 A = R.H.S$

$\sec A = \frac{1}{\cos A}$

$\operatorname{cosec} A = \frac{1}{\sin A}$

$\tan A = \frac{\sin A}{\cos A}$

$\cot A = \frac{\cos A}{\sin A}$

$\cot\theta = \frac{1}{\tan\theta}$

$\frac{(a-b)^2}{(b-a)^2} = 1$

Ques 30:

$\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Ans

Ques 31:-

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