MIND MAP : LEARNING MADE SIMPLE CHAPTER - 13

Limits and Derivatives

Derivative of a function at 'a'

The derivative of a function f at a is defined by

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Eg: Find derivative of $f(x)=\frac{1}{x}$.

Sol: We have $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]=\frac{-1}{x^2}$$

Derivative of polynomial function

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + -- + a_1x + a_0$ be a polynomial function, where a_i s are all real numbers and $a_n \neq 0$. Then the derivative function is given by

$$\frac{d\,f(x)}{dx} = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + -- + 2a_2x + a_1$$

Algebra of derivatives of functions

For functions u and v the following holds:

- $(u \pm v)' = u' \pm v'$
- $(uv)' = u'v + v'u$
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ provided all are defined and $v \neq 0$

Here, $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$

Some Standard Derivatives

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$

Some Standard Limits

- $\lim_{x\to a}\frac{x^n - a^n}{x-a} = na^{n-1}$
- $\lim_{x\to 0}\frac{\sin x}{x} = 1$
- $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

Algebra of Limits

For functions f and g the following holds:

- $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
- $\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$
- $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, $\therefore \lim_{x\to a} g(x) \neq 0$

Limit

- We say $\lim_{x\to a^-} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the left of a. This value is called the left hand limit of f at 'a'
- We say $\lim_{x\to a^+} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the right of a. This value is called the right hand limit of f 'at a'.
- If the right and left hand limits coincide, we call that common value as the limit of $f(x)$ at $x=a$ and denoted it by $\lim_{x\to a} f(x)$.

Eg: Find limit of function $f(x)=(x-1)^2$ at $x=1$.

Sol: For $f(x)=(x-1)^2$

Left hand limit (LHL) (at x=1)= $\lim_{x\to 1^-}(x-1)^2 = 0$

and Right hand limit (RHL) (at x=1)= $\lim_{x\to 1^+}(x-1)^2 = 0$

$\therefore$ LHL = RHL

$\lim_{x\to 1}(x-1)^2 = 0$