CHAPTER

14

Lines and Angles

AN INTRODUCTION TO GEOMETRY

Euclid's Geometry

The word 'geometry' comes from the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure'. Geometry appears to have originated from the need for measuring land. This branch of mathematics was studied in various forms in every ancient civilisation, be it in Egypt, Babylonia, China, India, Greece, the Incas, etc. The people of these civilisations faced several practical problems which required the development of geometry in various ways.

Euclid's Five Postulates :

Postulate 1 : A straight line may be drawn from any one point to any other point.

Postulate 2 : A terminated line can be produced indefinitely.

Postulate 3 : A circle can be drawn with any centre and any radius.

Postulate 4 : All right angles are equal to one another.

Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

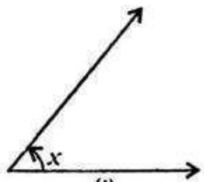
LINE AND ANGLES

Basic Terms and Definitions

A line with two end points is called a line-segment and a part of a line with one end point is called a ray. Note that the line segment AB is denoted by $\overline{AB}$, and its length is denoted by AB. The ray AB is denoted by $\overrightarrow{AB}$, and a line is denoted by $\overleftrightarrow{AB}$. However, we will not use these symbols, and will denote the line segment AB, ray AB, length AB and line AB by the same symbol, AB. The meaning of all these will be clear from the context.

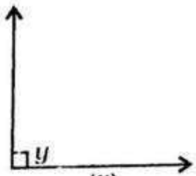
If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points.

Recall that an angle is formed when two rays originate from the same end point. The rays making an angle are called the arms of the angle and the end point is called the vertex of the angle. You have studied different types of angles, such as acute angle, right angle, obtuse angle, straight angle and reflex angle.



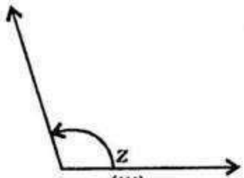
(i)

Acute angle : $0° < x < 90°$



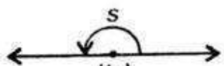
(ii)

Right angle : $y = 90°$



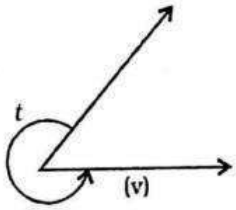
(iii)

Obtuse angle : $90° < z < 180°$



(iv)

Straight angle : $s = 180°$



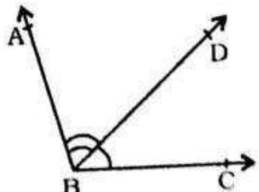
(v)

Reflex angle : $180° < t < 360°$

An acute angle measures between 0° and 90°, whereas a **right angle** is exactly equal to 90°. An angle greater than 90° but less than 180° is called an **obtuse angle**.

Also, recall that a **straight angle** is equal to 180°. An angle which is greater than 180° but less than 360° is called a **reflex angle**. Further, two angles whose sum is 90° are called **complementary angles**, and two angles whose sum is 180° are called **supplementary angles**.

Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm. In the given figure, ∠ABD and ∠DBC are adjacent angles. Ray BD is their common arm and point B is their common vertex. Ray BA and ray BC are non common arms. Moreover, when two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms. So, we can write

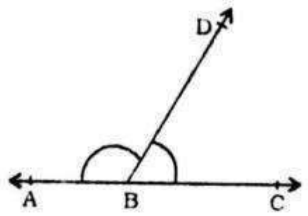


Adjacent Angles

$\angle ABC = \angle ABD + \angle DBC.$

Note that $\angle ABC$ and $\angle ABD$ are not adjacent angles as their non-common arms BD and BC lie on the same side of the common arm BA.

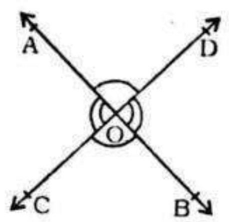
If the non-common arms BA and BC form a line then it will look like the figure given below. In this case, $\angle ABD$ and $\angle DBC$ are called **linear pair of angles**.



Linear Pair of Angles

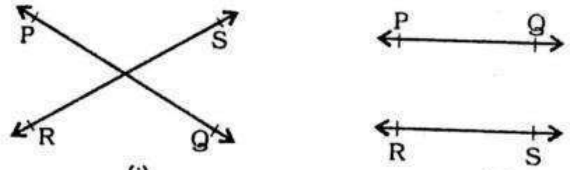
When two lines, say AB and CD, intersect each other, say at the point O there are two pairs of vertically opposite angles.

One pair is $\angle AOD$ and $\angle BOC$.



Intersecting Lines and Non-intersecting Lines

Draw two different lines PQ and RS on a paper. You will see that you can draw them in two different ways as shown in the figures given below :



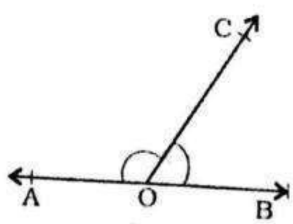
(i) Intersecting lines

(ii) Non-intersecting (parallel) lines

Recall the notion of a line, that it extends indefinitely in both directions. Lines PQ and RS in Figure (i) are intersecting lines and in Figure (ii) are parallel lines. Note that the lengths of the common perpendiculars at different points on these parallel lines is the same. This equal length is called the distance between two parallel lines.

Pairs of Angles

Draw a figure in which a ray stands on a line as shown in the Figure given below. Name the line as AB and the ray as OC. What are the angles formed at the point O? They are $\angle AOC$, $\angle BOC$ and $\angle AOB$.



$\angle AOC + \angle BOC = \angle AOB$ ...... (1)

$\angle AOB = 180°$. ...... (2)

From (1) and (2), $\angle AOC + \angle BOC = 180°$

From the above discussion, we can state the following Axioms :

Axiom 1 : If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.

Recall that when the sum of two adjacent angles is 180°, then they are called a linear pair of angles.

In Axiom 1, it is given that 'a ray stands on a line'. From this 'given', we have concluded that 'the sum of two adjacent angles so formed is 180°'.

(A) If the sum of two adjacent angles is 180°, then a ray stands on a line (that is, the non-common arms form a line).

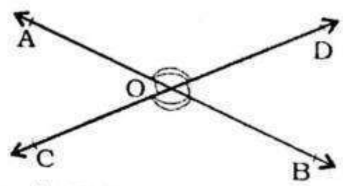
Axiom 2 : If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

For obvious reasons, the two axioms above together is called the **Linear Pair Axiom**.

Let us now examine the case when two lines intersect each other.

Theorem 1. If two lines intersect each other, then the vertically opposite angles are equal.

Proof : In the statement above, it is given that 'two lines intersect each other'. So, let AB and CD be two lines intersecting at O as shown in the figure given below. They lead to two pairs of vertically opposite angles, namely,



(i) $\angle AOC$ and $\angle BOD$ (ii) $\angle AOD$ and $\angle BOC$.

We need to prove that $\angle AOC = \angle BOD$ and $\angle AOD = \angle BOC$.

Now, ray OA stands on line CD.

Therefore,

$\angle AOC + \angle AOD = 180°$ (Linear pair axiom) (1)

Can we write $\angle AOD + \angle BOD = 180°$ (2)

From (1) and (2), we can write.

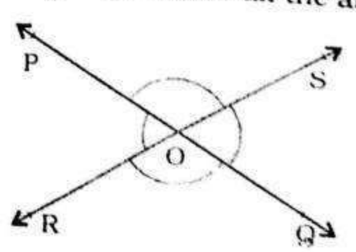
$\angle AOC + \angle AOD = \angle AOD + \angle BOD$

This implies that $\angle AOC = \angle BOD$

Similarly, it can be proved that $\angle AOD = \angle BOC$

Example 1 : In the given figure, lines PQ and RS intersect each other at point O. If

$\angle POR : \angle ROQ = 5 : 7$, find all the angles.



Solution : $\angle POR + \angle ROQ = 180°$
(Linear pair of angles)
But $\angle POR : \angle ROQ = 5 : 7$ (Given)

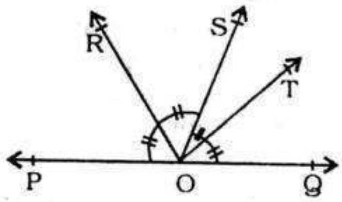
Therefore, $\angle POR = \frac{5}{12} \times 180° = 75°$

Similarly, $\angle ROQ = \frac{7}{12} \times 180° = 105°$

Now, $\angle POS = \angle ROQ = 105$ (Vertically opposite angles) and

$\angle SOQ = \angle POR = 75°$ (Vertically opposite angles)

Example 2 : In the given figure, ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$, respectively. If $\angle POS = x$, find $\angle ROT$.



Solution : Ray OS stands on the line POQ.
Therefore, $\angle POS + \angle SOQ = 180°$
But, $\angle POS = x$
Therefore, $x + \angle SOQ = 180°$
So, $\angle SOQ = 180° - x$
Now, ray OR bisects $\angle POS$, therefore,

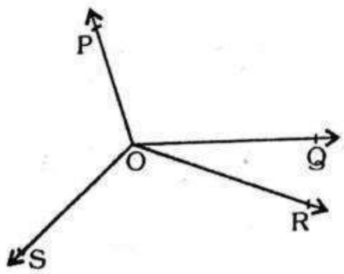
$\angle ROS = \frac{1}{2} \times \angle POS = \frac{1}{2} \times x = \frac{x}{2}$

Similarly,

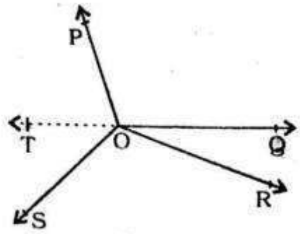
$\angle SOT = \frac{1}{2} \times \angle SOQ = \frac{1}{2} \times (180° - x) = 90° - \frac{x}{2}$

Now, $\angle ROT = \angle ROS + \angle SOT = \frac{x}{2} + 90° - \frac{x}{2} = 90°$

Example 3 : In the given figure, OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$.



Solution : In the given figure, you need to produce any of the rays OP, OQ, OR or OS backwards to a point. Let us produce ray OQ backwards to a point T so that TOQ is a line.



Now, ray OP stands on line TOQ.
Therefore, $\angle TOP + \angle POQ = 180°$ (1)
(Linear pair axiom)
Similarly, ray OS stands on line TOQ.
Therefore, $\angle TOS + \angle SOQ = 180°$ (2)
But $\angle SOQ = \angle SOR + \angle QOR$
So, (2) becomes
$\angle TOS + \angle SOR + \angle QOR = 180°$ (3)
Now, adding (1) and (3), you get
$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360°$ (4)
But $\angle TOP + \angle TOS = \angle POS$
Therefore, (4) becomes
$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$

Parallel Lines and a Transversal

A line which intersects two or more lines at distinct points is called a transversal. Line l intersects lines m and n at points P and Q respectively. Therefore, line l is a transversal for lines m and n. Observe that four angles are formed at each of the points P and Q.

Let us name these angles as $\angle 1, \angle 2, \ldots, \angle 8$ as shown in the figure given above.

$\angle 1, \angle 2, \angle 7$ and $\angle 8$ are called **exterior angles**, while $\angle 3, \angle 4, \angle 5$ and $\angle 6$ are called interior angles.

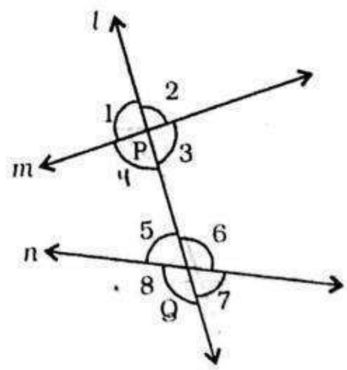
(a) Corresponding angles :
(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$
(iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$

(b) Alternate interior angles :
(i) $\angle 4$ and $\angle 6$ (ii) $\angle 3$ and $\angle 5$

(c) Alternate exterior angles:
(i) $\angle 1$ and $\angle 7$ (ii) $\angle 2$ and $\angle 8$

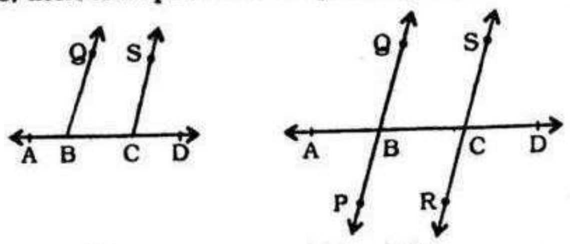
(d) Interior angles on the same side of the transversal:
(i) $\angle 4$ and $\angle 5$ (ii) $\angle 3$ and $\angle 6$

Interior angles on the same side of the transversal are also referred to as **consecutive interior** angles or **allied angles** or **co-interior** angles. Further, many a times, we simply use the words alternate angles for alternate interior angles.



Now, let us find out the relation between the angles in these pairs when line m is parallel to line n.

Axiom 3 : If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

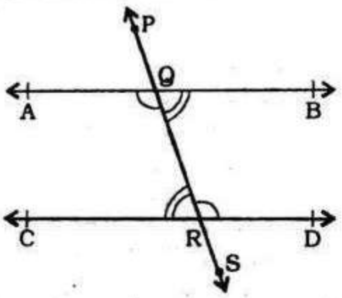


(i) (ii)

Axiom 3 is also referred to as the **corresponding angles axiom**. Now, let us discuss the converse of this axiom which is as follows:

If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel.

Axiom 4 : If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.



In the figure given above, transveral PS intersects parallel lines AB and CD at points Q and R respectively.

Is $\angle BQR = \angle QRC$ and $\angle AQR = \angle QRD$?

You know that $\angle PQA = \angle QRC$ (1)

(Corresponding angles axiom)

$\angle PQA = \angle BQR$ (2)

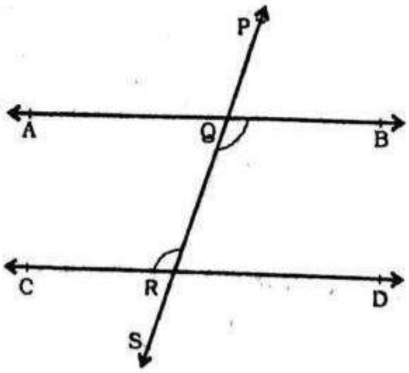
So, from (1) and (2), you may conclude that

$\angle BQR = \angle QRC$.

Similarly, $\angle AQR = \angle QRD$.

This result can be stated as a theorem given below:

Theorem 2. If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.



Now, using the converse of the corresponding angles axiom, can we show the two lines parallel if a pair of alternate interior angles is equal? In the figure given above, the transversal PS intersects lines AB and CD at points Q and R respectively such that $\angle BQR = \angle QRC$.

Is AB || CD?

$\angle BQR = \angle PQA$ (1)

But, $\angle BQR = \angle QRC$ (Given) (2)

So, from (1) and (2), you may conclude that

$\angle PQA = \angle QRC$

But they are corresponding angles.

So, AB || CD (Converse of corresponding angles axiom)

This result can be stated as a theorem given below:

Theorem 3. If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

In a similar way, you can obtain the following two theorems related to interior angles on the same side of the transversal.

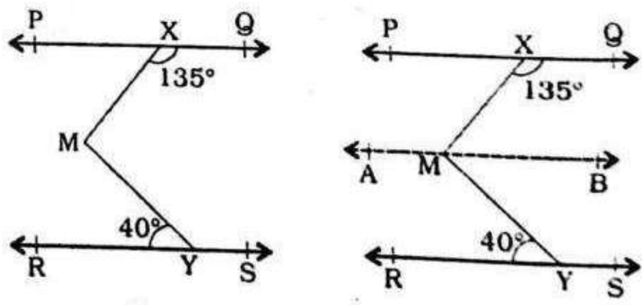
Theorem 4. If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Theorem 5. If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

Theorem 6. Lines which are parallel to the same line are parallel to each other.

Note : The property above can be extended to more than two lines also.

Example 4 : In the figure (i), if PQ || RS, $\angle MXQ = 135°$ and $\angle MYR = 40°$, find $\angle XMY$.



(i) (ii)

Solution : Here, we need to draw a line AB parallel to line PQ, through point M as shown in the figure (ii). Now, AB || PQ and PQ || RS.

Therefore, AB || RS (WHY?)

Now, $\angle QXM + \angle XMB = 180°$

(AB || PQ, Interior angles on the same side of the transversal XM)

But $\angle QXM = 135°$

So, $135° + \angle XMB = 180°$

Therefore, $\angle XMB = 45°$ (1)

Now, $\angle BMY = \angle MYR$ (AB || RS, Alternate angles)

Therefore, $\angle BMY = 40°$ (2)

Adding (1) and (2), you get

$\angle XMB + \angle BMY = 45° + 40°$

That is, $\angle XMY = 85°$

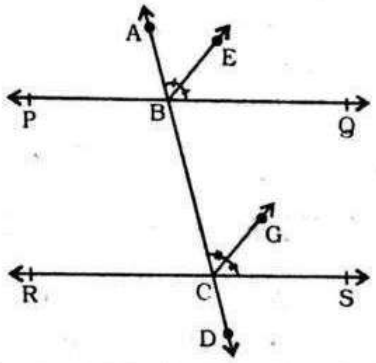
Example 5 : If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Solution : In the figure, a transversal AD intersects two lines PQ and RS at points B and C respectively. Ray BE is the bisector of $\angle ABQ$ and ray CG is the bisector of $\angle BCS$; and BE || CG.

We are to prove that PQ || RS.

It is given that ray BE is the bisector of $\angle ABQ$.

Therefore, $\angle ABE = \frac{1}{2}\angle ABQ$ (1)



Similarly, ray CG is the bisector of $\angle BCS$.

Therefore, $\angle BCG = \frac{1}{2}\angle BCS$ (2)

But BE || CG and AD is the transversal.

Therefore, $\angle ABE = \angle BCG$

(Corresponding angles axiom) (3)

Substituting (1) and (2) in (3), you get

$\frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS$

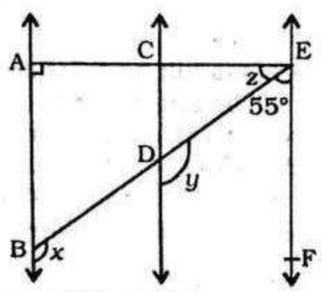
That is, $\angle ABQ = \angle BCS$

But, they are the corresponding angles formed by transversal AD with PQ and RS; and are equal.

Therefore, PQ || RS

(Converse of corresponding angles axiom)

Example 6 : In the figure, AB || CD and CD || EF. Also EA $\perp$AB. If $\angle BEF = 55°$, find the values of x, y and z.



Solution : $y + 55° = 180°$

(Interior angles on the same side of the of the transversal ED)

Therefore, $y = 180° - 55° = 125°$

Again $x = y$

(AB || CD, Corresponding angles axiom)

Therefore $x = 125°$

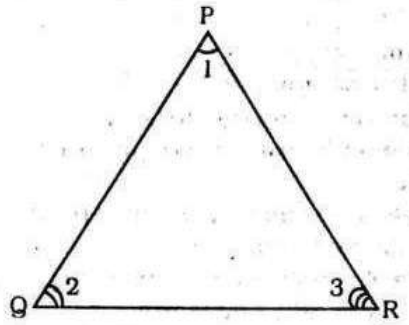
Now, since AB || CD and CD || EF, therefore, AB || EF.

So, $\angle EAB + \angle FEA = 180°$ (Interior angles on the same side of the transversal EA)

Therefore, $90° + z + 55° = 180°$

Which gives $z = 35°$

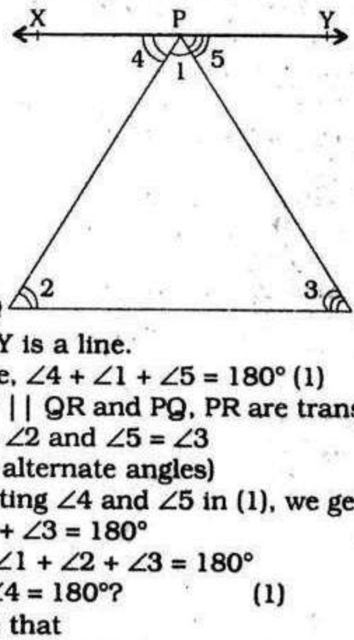
Angle Sum Property of a Triangle



Theorem 7. The sum of the angles of a triangle is 180°.

Proof : Let us see what is given in the statement above, that is, the hypothesis and what we need to prove. We are given a triangle PQR and $\angle 1$, $\angle 2$ and $\angle 3$ are the angles of PQR.

We need to prove that $\angle 1 + \angle 2 + \angle 3 = 180°$. Let us draw a line XPY parallel to QR through the opposite vertex P, as shown in the figure, so that we can use the properties related to parallel lines.



Now, XPY is a line.

Therefore, $\angle 4 + \angle 1 + \angle 5 = 180°$ (1)

But XPY || QR and PQ, PR are transversals.

So, $\angle 4 = \angle 2$ and $\angle 5 = \angle 3$

(Pairs of alternate angles)

Substituting $\angle 4$ and $\angle 5$ in (1), we get

$\angle 2 + \angle 1 + \angle 3 = 180°$

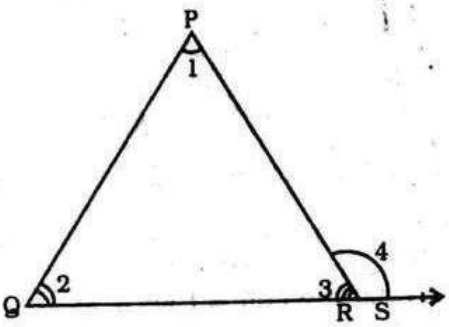
That is, $\angle 1 + \angle 2 + \angle 3 = 180°$

Is $\angle 3 + \angle 4 = 180°$? (1)

Also, see that

$\angle 1 + \angle 2 + \angle 3 = 180°$ (2)

From (1) and (2), you can see that $\angle 4 = \angle 1 + \angle 2$.

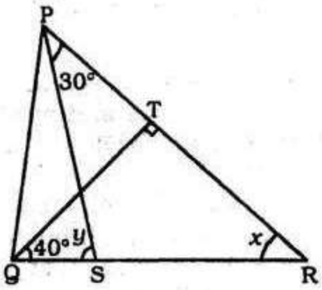


This result can be stated in the form of a theorem as given below:

Theorem 8. If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

It is obvious from the above theorem that an exterior angle of a triangle is greater than either of its interior apposite angles.

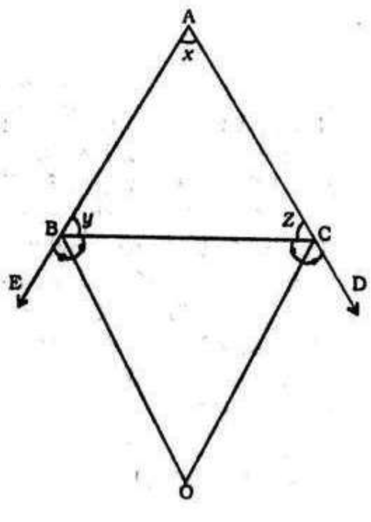
Example 7 : In the given figure, if QT ∠PR, ∠TQR = 40° and ∠SPR = 30°, find x and y.



Solution : In Δ TQR, 90° + 40° + x = 180°
(Angle sum property of a triangle)
Therefore, $x = 50°$
Now, $y = \angle SPR + x$ (Theorem 8)
Therefore, $y = 30° + 50° = 80°$

Example 8 : In the figure given below, the sides AB and AC of ABC are produced to points E and D respectively. If bisectors BO and CO of ∠CBE and ∠BCD respectively meet at point O, then prove that

$\angle BOC = 90° - \frac{1}{2}\angle BAC$



Solution : Ray BO is the bisector of ∠CBE.

Therefore, $\angle CBO = \frac{1}{2}\angle CBE = \frac{1}{2}(180° - y)$

$= 90° - \frac{y}{2}$ (1)

Similarly, ray CO is the bisector of ∠BCD.

Therefore, $\angle BCO = \frac{1}{2}\angle BCD$

$= \frac{1}{2}(180° - z) = 90° - \frac{z}{2}$ (2)

In ΔBOC, ∠BOC + ∠BCO + ∠CBO = 180° (3)

Substituting (1) and (2) in (3), you get

$\angle BOC = 90° - \frac{z}{2} + 90° - \frac{y}{2} = 180°$

So, $\angle BOC = \frac{1}{2}(y + z)$ (4)

But, $x + y + z = 180°$ (Angle sum property of a triangle)

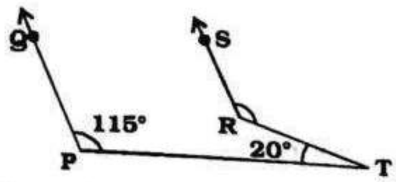
Therefore, $y + z = 180° - x$

Therefore, (4) becomes

$\angle BOC = \frac{1}{2}(180° - x) = 90° - \frac{x}{2} = 90° - \frac{1}{2}\angle BAC$

SOLVED OBJECTIVE QUESTIONS

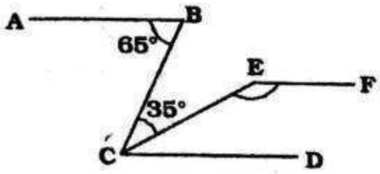
1. If P and Q are points on the opposite sides of a straight line AB. If O is a point on AB such that $\angle AOP = \angle BOQ$, then when one of the following is correct?
   (1) $\angle AOQ < \angle BOP$
   (2) $\angle AOQ > \angle BOP$
   (3) $\angle AOP = 180° - \angle AOQ$
   (4) $\angle AOP = 90° - \angle AOQ$
2. In the given figure. If $PQ \parallel RS$ $\angle QPT = 115°$ and $\angle PTR = 20°$, then $\angle SRT$ is equal to :



   (1) 155° (2) 150°
   (3) 135° (4) 95°
3. Two parallel lines are cut by a transversal then which of the following are true?
   I. Pair of alternate interior angles are congruent.
   II. Pair of corresponding angles are congruent.
   III. Pair of interior angles on the same side of the transversal are supplementary.
   (1) I, II, III are true (2) I, III are true
   (3) I, II are true (4) II, III are true
4. AB and CD are two parallel lines. PQ cuts AB and CD at E and F respectively. EL is the bisector of $\angle FEB$. If $\angle LEB = 35$; then $\angle CFQ$ will be :

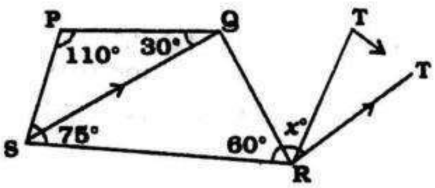
(1) 130° (2) 85°
(3) 70° (4) 95°

5. AB and CD are two parallel lines. The points B and C are joined such that $\angle ABC = 65°$. A line CE is drawn making angle of 35° with the line CB. EF is drawn parallel to AB. as show in figure then $\angle CEF$ is equal to :



(1) 160° (2) 155°
(3) 150° (4) 145°

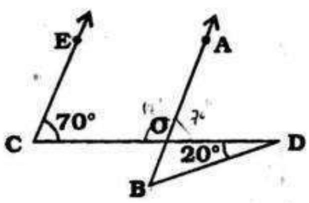
6. In the figure below, RT is drawn parallel to the line SQ. The value of x is:



(1) 85° (2) 45°
(3) 120° (4) 75°

7. AB is a straight line and O is a point on AB. If one draws a line OC not coinciding with OA or OB, then the $\angle AOC$ and $\angle BOC$ are:
(1) Equal
(2) Complementary
(3) Supplementary
(4) Together equal to 130°

8. In the given figure if EC ‖ AB, $\angle ECD = 70°$, $\angle BDO = 20°$, then $\angle OBD$ is equal to :

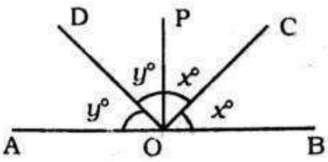


(1) 70° (2) 60°
(3) 50° (4) 20°

9. Two parallel lines AB and CD are intersected by a transversal line EF at M and N respectively. The lines MP and NP are the bisectors of the interior angles BMN and DNM on the same side of the transversal. Then $\angle MPN$ is equal to :
(1) 90° (2) 45°
(3) 135° (4) 60°

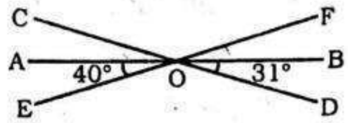
10. AB and CD are parallel straight lines of lengths 5 cm and 4 cm respectively. AD and BC intersect at a point O such that AO = 10cm, then OD equals :
(1) 7 cm (2) 8 cm
(3) 5 cm (4) 6 cm

11. In the following figure $\angle BOP = 2x°$, $\angle AOP = 2y°$, OC and OD are angle bisectors of $\angle BOP$ and $\angle AOP$ respectively. Find the value of $\angle COD$:



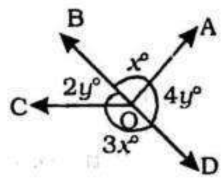
(1) 75° (2) 90°
(3) 100° (4) 120°

12. In the following figure find the value of $\angle BOC$:



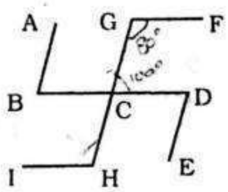
(1) 101° (2) 149°
(3) 71° (4) 140°

13. Find y, if $x° = 36°$, as per the given diagram :



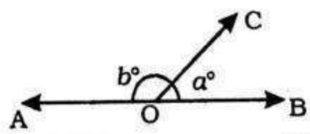
(1) 36° (2) 16°
(3) 12° (4) 42°

14. In the given diagram AB ‖ GH ‖ DE and GF ‖ BD ‖ HI, $\angle FGC = 80°$. Find the value of $\angle CHI$:



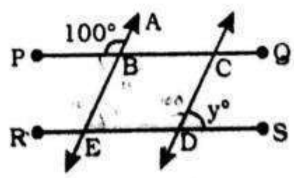
(1) 80° (2) 120°
(3) 100° (4) 160°

15. In the given figure, $\angle a$ is greater than one sixth of right angle, then :



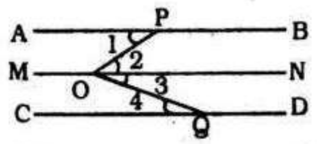
(1) $b > 165°$ (2) $b < 165°$
(3) $b \leq 165°$ (4) $b \geq 165°$

16. In the adjoining figure AE || CD and BC || ED, then find Y :



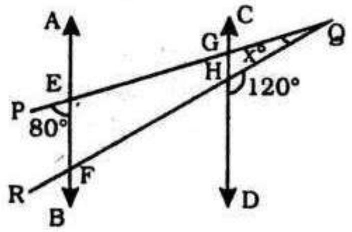
(1) 60° (2) 80°
(3) 90° (4) 75°

17. In the adjoining figure ∠APO=42° and ∠CQO = 38°. Find the value of ∠POQ :



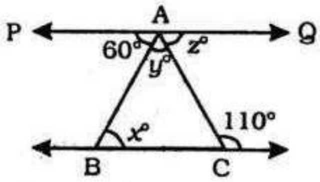
(1) 68° (2) 72°
(3) 80° (4) 126°

18. In the adjoining figure AB||CD and PQ, QR intersect AB and CD both at E, F and G, H respectively. Given that m∠PEB = 80°, m∠QHD=120° and m∠PQR = X°, find the value of X :



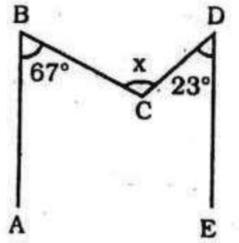
(1) 40° (2) 20°
(3) 100° (4) 30°

19. In the following figure, find the value of y :



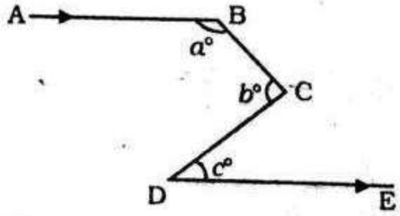
(1) 70° (2) 60°
(3) 50° (4) 80°

20. In the adjoining figure AB||DE, ∠ABC = 67° and ∠EDC = 23°. Find ∠BCD :



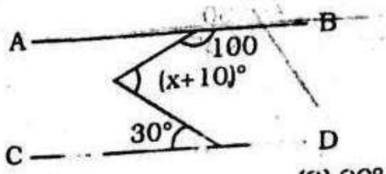
(1) 90° (2) 44°
(3) 46° (4) None of these

21. In the given figure AB||DE. Find $a° + b° - c°$:



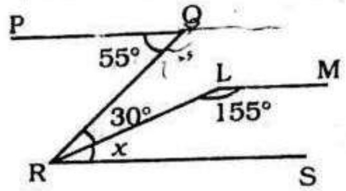
(1) 160° (2) 120°
(3) 180° (4) 210°

22. AB ∥ CD, shown in the figure. Find the value of x:



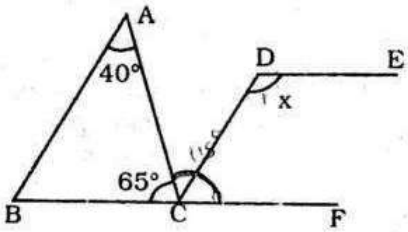
(1) 100° (2) 90°
(3) 110° (4) 140°

23. In the figure PQ||LM||RS. Find the value of ∠LRS :



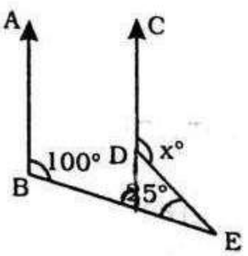
(1) 30° (2) 25°
(3) 35° (4) 40°

24. In the figure AB||DC and DE||BF. Find the value of x :



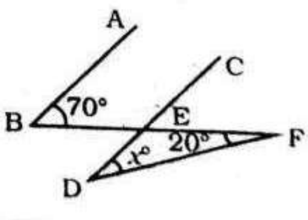
(1) 140° (2) 155°
(3) 105° (4) 115°

25. In the figure AB||CD, ∠ABE = 100. Find m∠CDE :



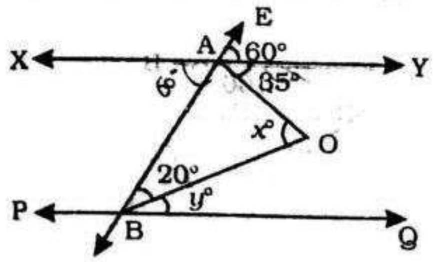
(1) 125° (2) 55°
(3) 65° (4) 75°

26. In the figure AB||CD, find $x°$ (i.e., ∠CDF) :



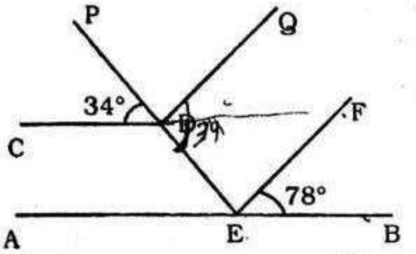
(1) 50° (2) 90°
(3) 30° (4) 70°

27. In the given figure XY | | PQ, find the value of x :



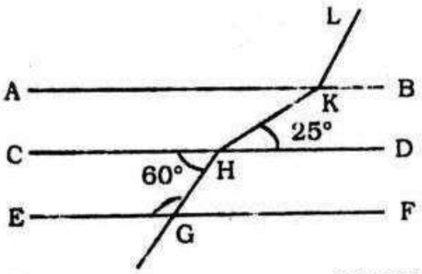
(1) 70° (2) 40°
(3) 75° (4) 15°

28. In the given figure AB | | CD and EF | | DQ. Find the value of ∠DEF :



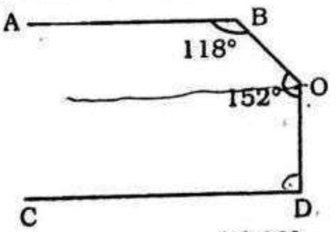
(1) 68° (2) 78°
(3) 34° (4) 39°

29. In the given figure AB | | CD | | EF and GH | | KL. Find m∠HKL :



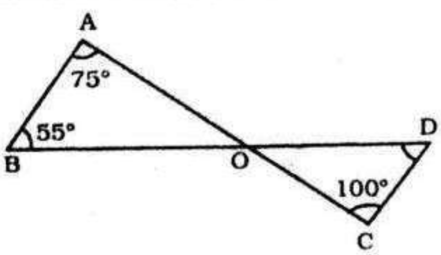
(1) 85° (2) 145°
(3) 120° (4) 95°

30. AB | | CD, ∠ABO = 118°, BOD = 152°, find ∠ODC :



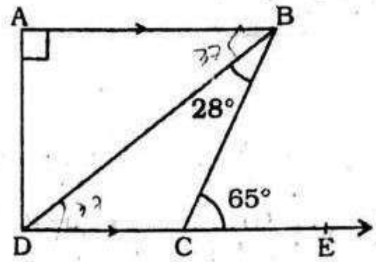
(1) 70° (2) 80°
(3) 90° (4) 34°

31. In the given figure, $\angle OAB = 75°$, $\angle OBA = 55°$ and $\angle OCD = 100°$. Then ∠ODC = ?



(1) 20° (2) 25°
(3) 30° (4) 35°

32. In the given figure, $AB \parallel DC$, ∠BAD = 90°, $\angle CBD = 28°$ and $\angle BCE = 65°$. Then $\angle ABD$ = ?

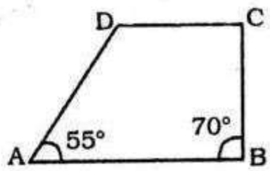


(1) 32° (2) 37°
(3) 43° (4) 53°

33. ABC is an equilateral triangle. If a, b and c denotes the lengths of perpendiculars from A, B and C respectively on the opposite sides then :

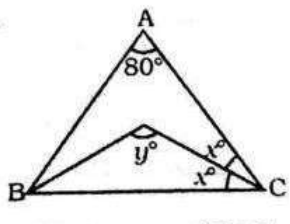
(1) $a \neq b \neq c$ (2) $a = b = c$
(3) $a = b = 2c$ (4) $a - b = c$

34. In the adjoining figure, ABCD is a trapezium in which AB | | DC. If ∠ A = 55° and ∠ B = 70° then ∠ C and ∠ D are respectively :



(1) 140°, 125° (2) 100°, 135°
(3) 110°, 125° (4) 105°, 130°

35. In the given figure, ∠A = 80°, ∠B = 60°, $\angle C = 2x$ and $\angle BDC = y°$. BD and CD bisect angles B and C respectively. The values of x and y respectively are :



(1) 15° and 70° (2) 10° and 160°
(3) 20° and 130° (4) 20° and 125°

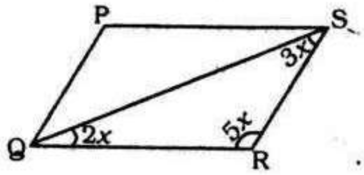
36. The measures of the angles of a triangle are in the ratio 2 : 7 : 11. Measures of angles are

(1) 16°, 56°, 88° (2) 18°, 63°, 99°
(3) 20°, 70°, 90° (4) 25°, 175°, 105°

37. The angles of a hexagon are $x°$, $(x-5)°$, $(x-5)°$, $(2x-5)°$, $(2x-5)$ and $(2x-20)°$ then the value of x is :

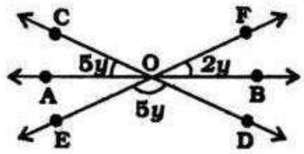
(1) 60° (2) 80°
(3) 90° (4) 45°

38. In the adjoining figure, the $\angle QPS$ is equal to :



(1) 85° (2) 75°
(3) 100° (4) 90°

39. In figure determine the value of y :

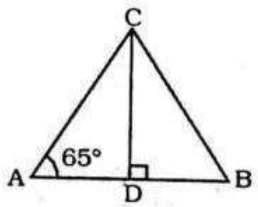


(1) 25° (2) 35°
(3) 15° (4) 40°

40. In each of the following, the measures of the three angles are given. In which case the angles can possibly be those of a triangle?

(1) 59°, 72°, 61° (2) 45°, 61°, 73°
(3) 30°, 125°, 20° (4) 63°, 37°, 80°

41. In ΔABC, $\angle C = 90°$ and $CD \perp AB$, also $\angle A = 65°$ then $\angle CBA$ is equal to :

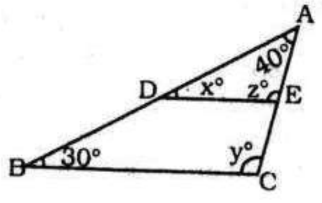


(1) 25° (2) 35°
(3) 65° (4) 40°

42. The angles of a triangle are as 2 : 3 : 4. The angles of triangle are respectively :

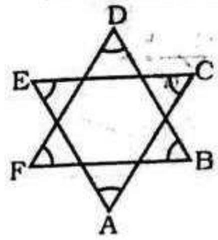
(1) 30°,60°,90° (2) 40°,60°,80°
(3) 60°,40°,80° (4) 20°,60°,80°

43. In the adjoining figure, D and E are points on sides AB and AC of ΔABC such that DE || BC. If $\angle B = 30°$ and $\angle A = 40°$, then x, y and z are respectively :



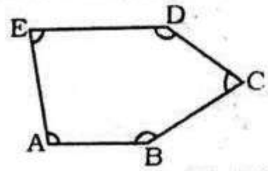
(1) 30°,110°,110° (2) 30°,105°,105°
(3) 30°, 85°,85° (4) 30°,95°,95°

44. Here in the adjoining figure :
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F =$



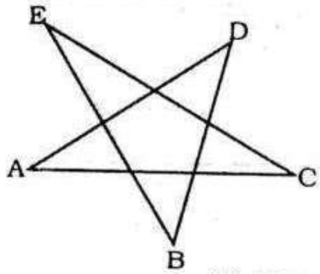
(1) 360° (2) 720°
(3) 180° (4) 300°

45. In the adjoining figure :
$\angle ABC + \angle BCD + \angle CDE + \angle DEA + \angle EAB = ?$



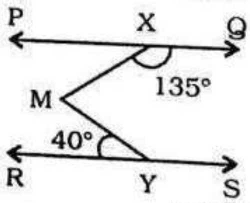
(1) 360° (2) 540°
(3) 720° (4) None of these

46. In the adjoining figure :
$\angle A + \angle B + \angle C + \angle D + \angle E =$



(1) 900° (2) 720°
(3) 180° (4) 540°

47. In the following figure, if PQ || RS, $\angle MXQ = 135°$ and $\angle MYR = 40°$. Find $\angle XMY$.

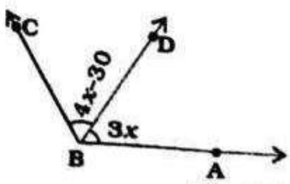


(1) 85° (2) 75°
(3) 65° (4) 80°

48. In a ΔABC, if $\angle A = 120°$ and AB = AC, then $\angle B$ is equal to

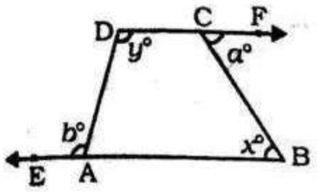
(1) 35° (2) 60°
(3) 30° (4) 80°

49. In figure, the value of x which would make ABC a line :



(1) 40° (2) 35°
(3) 45° (4) 30°

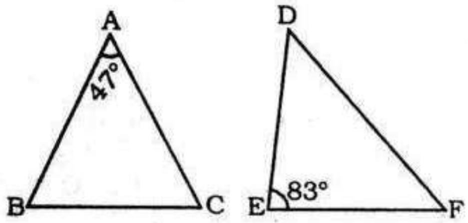
50. The sides BA and DC of quadrilateral ABCD are produced as shown in figure. Then which of the following statements is correct?



(1) $2x° + y° = a° + b°$ (2) $x° + \frac{1}{2}y° = \frac{a° + b°}{2}$

(3) $x° + y° = a° + b°$ (4) $x° + a° = y° + b°$

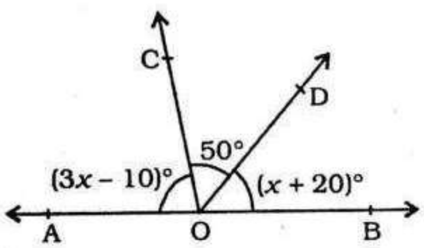
51. If ABC and DEF are similar triangles in which $\angle A = 47°$ and $\angle E = 83°$, then $\angle C$ is :



(1) 50° (2) 70°

(3) 60° (4) 80°

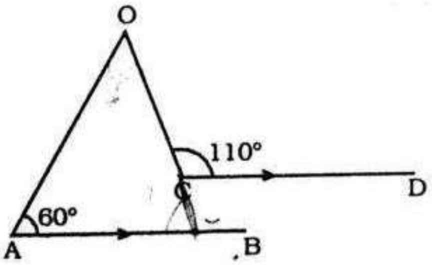
52. In the given figure, AOB is a straight line. If $\angle AOC = (3x - 10)°$, $\angle COD = 50°$ and $\angle BOD = (x + 20)°$, then $\angle AOC = ?$



(1) 40° (2) 60°

(3) 80° (4) 50°

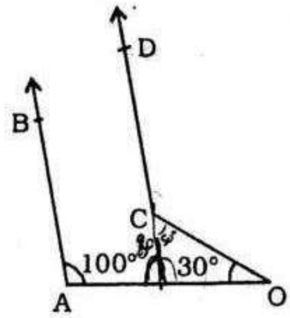
53. In the given figure, $AB \parallel CD$. If $\angle BAO = 60°$ and $\angle OCD = 110°$, then $\angle AOC = ?$



(1) 70° (2) 60°

(3) 50° (4) 40°

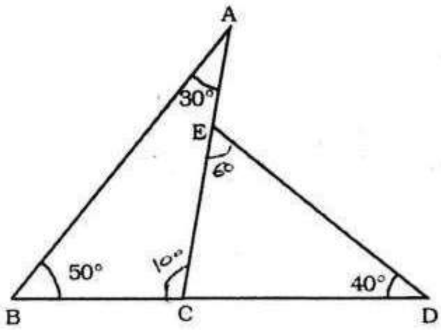
54. In the given figure, $AB \parallel CD$. If $\angle AOC = 30°$ and $\angle OAB = 100°$, then $\angle OCD = ?$



(1) 130° (2) 150°

(3) 80° (4) 100°

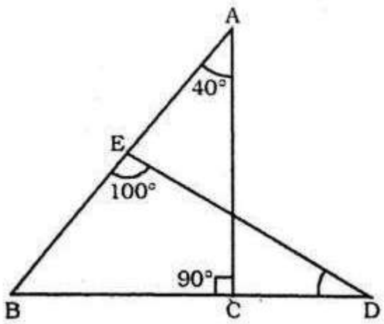
55. In the given figure, $\angle BAC = 30°$, $\angle ABC = 50°$ and $\angle CDE = 40°$. Then $\angle AED = ?$



(1) 120° (2) 100°

(3) 80° (4) 110°

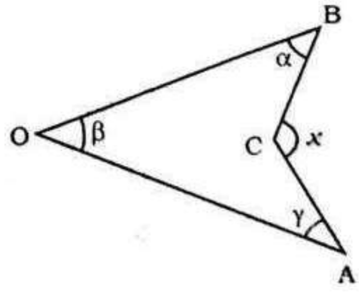
56. In the given figure, $\angle BAC = 40°$, $\angle ACB = 90°$ and $\angle BED = 100°$. Then $\angle BDE = ?$



(1) 50° (2) 30°

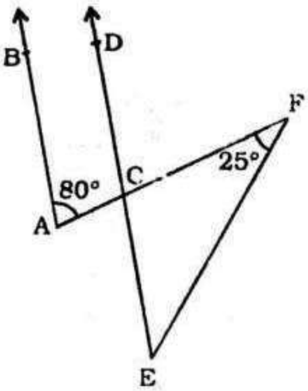
(3) 40° (4) 25°

57. In the given figure, $x = ?$



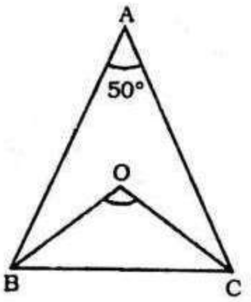
(1) $\alpha + \beta - \gamma$ (2) $\alpha - \beta + \gamma$
(3) $\alpha + \beta + \gamma$ (4) $\alpha + \gamma - \beta$

58. In the given figure, $AB \parallel CD$. If $\angle CAB = 80°$ and $\angle EFC = 25°$, then $\angle CEF$ = ?



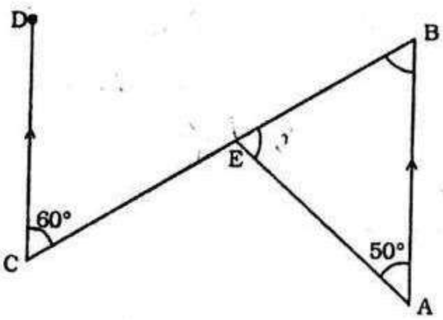
(1) 65° (2) 55°
(3) 45° (4) 75°

59. In the given figure, BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively. If $\angle A = 50°$, then $\angle BOC$ = ?



(1) 130° (2) 100°
(3) 115° (4) 120°

60. In the given figure, $AB \parallel CD$. If $\angle EAB = 50°$ and $\angle ECD = 60°$, then $\angle AEB$ = ?



(1) 50° (2) 60°
(3) 70° (4) 55°

61. It is given that $\Delta ABC \cong \Delta FDE$ in which AB = 5cm, $\angle B = 40°$, $\angle A = 80°$ and FD = 5cm. Then, which of the following is true?
(1) $\angle D = 60°$ (2) $\angle E = 60°$
(3) $\angle F = 60°$ (4) $\angle D = 80°$

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

62. In an obtuse-angled triangle ABC, ∠A is the obtuse angle and O is the orthocentre. If ∠BOC = 54°, then ∠BAC is
(1) 108° (2) 126°
(3) 136° (4) 116°

[SSC Graduate Level Tier-I Exam, 2012]

63. If I is the In-centre of ΔABC and ∠A = 60°, then the value of ∠BIC is
(1) 100° (2) 120°
(3) 150° (4) 110°

[SSC Graduate Level Tier-I Exam, 2012]

64. If a straight line L makes an angle θ (θ > 90°) with the positive direction of x-axis, then the acute angle made by a straight line L_1, perpendicular to L, with the y-axis is

(1) $\frac{\pi}{2} + \theta$ (2) $\frac{\pi}{2} - \theta$
(3) $\pi + \theta$ (4) $\pi - \theta$

[SSC Graduate Level Tier-I Exam, 2012]

65. A,O,B are three points on a line segment and C is a point not lying on AOB. If ∠AOC = 40° and OX, OY are the internal and external bisectors of ∠AOC respectively, then ∠BOY is
(1) 70° (2) 80°
(3) 72° (4) 68°

[SSC Graduate Level Tier-I Exam, 2012]

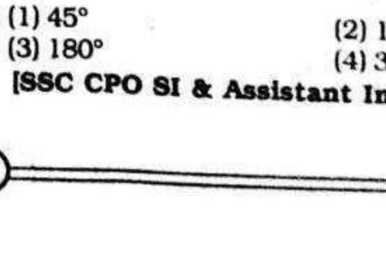
66. The side BC of Δ ABC is produced to D. If ∠ACD = 108° and $\angle B = \frac{1}{2} \angle A$ then ∠A is
(1) 36° (2) 72°
(3) 108° (4) 59°

[SSC Graduate Level Tier-I Exam, 2012]

67. If in any triagle ABC, the base BC is produced in both ways, the sum of the exterior angles at B and C is
(1) $\pi - A$ (2) $\pi + A$
(3) $\frac{\pi}{2} + A$ (4) $\pi - \frac{A}{2}$

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

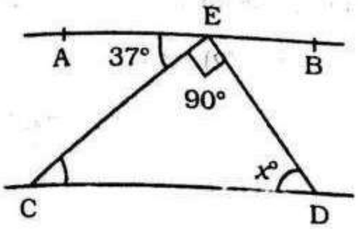
68. In the figure below, lines k and l are parallel. The value of $a° + b°$ is



(1) 45° (2) 100°
(3) 180° (4) 360°

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

69. In the figure below, if AB ∥ CD and CE ⊥ ED, then the value of x is



(1) 53
(2) 63
(3) 37
(4) 45

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

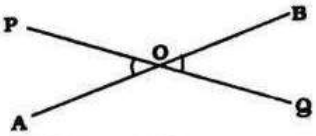
ANSWERS

1. (3)	**2.** (3)	**3.** (1)	**4.** (3)	**5.** (3)
6. (1)	**7.** (3)	**8.** (3)	**9.** (1)	**10.** (2)
11. (2)	**12.** (2)	**13.** (1)	**14.** (1)	**15.** (2)
16. (2)	**17.** (3)	**18.** (2)	**19.** (3)	**20.** (1)
21. (3)	**22.** (1)	**23.** (2)	**24.** (3)	**25.** (1)
26. (1)	**27.** (3)	**28.** (1)	**29.** (2)	**30.** (3)
31. (3)	**32.** (2)	**33.** (2)	**34.** (3)	**35.** (3)
36. (2)	**37.** (2)	**38.** (1)	**39.** (3)	**40.** (4)
41. (1)	**42.** (2)	**43.** (1)	**44.** (1)	**45.** (2)
46. (3)	**47.** (1)	**48.** (3)	**49.** (4)	**50.** (3)
51. (1)	**52.** (3)	**53.** (3)	**54.** (1)	**55.** (1)
56. (2)	**57.** (3)	**58.** (2)	**59.** (3)	**60.** (3)
61. (2)	**62.** (2)	**63.** (2)	**64.** (4)	**65.** (1)
66. (2)	**67.** (2)	**68.** (1)	**69.** (1)	

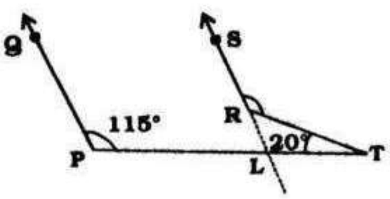
EXPLANATIONS

1. (3) Here, as given
$\angle AOP = \angle BOQ$
POQ is a straight line.
So $\angle AOP + \angle AOQ = 180°$
$\angle AOP = 180° - \angle AOQ$

2. (3) Here $\angle QPL + \angle PLR = 180°$



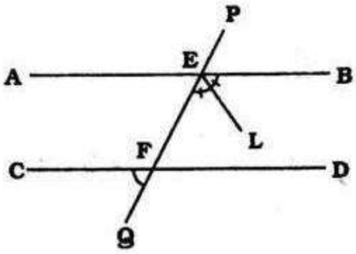
$\angle PLR = 180° - \angle QPL$
$= 180° - 115° = 65°$
$\angle RLP = \angle LRT + \angle RTL$



$\angle LRT = \angle RLP - \angle RTL = 65° - 20° = 45°$
$\angle SRT + \angle LRT = 180°$
$\angle SRT = 180° - \angle LRT = 180° - 45° = 135°$
$\angle SRT = 135°$

3. (1) All the three statements are true regarding the given condition.

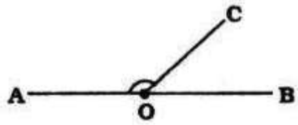
4. (3) $\angle LEB = 35°$
$\angle FEB = 2 \times \angle LEB = 70°$
$\angle CFQ = \angle FEB$ (alternate angles)
$\angle CFQ = 70°$



5. (3) $\angle ABC = \angle BCD$ as $AB \parallel CD$
$\angle BCD = 65°$
$\angle ECD = 65° - \angle BCE$
$= 65° - 35° = 30°$
$\angle CEF + \angle ECD = 180°$
$\angle CEF = 180° - 30° = 150°$

6. (1) Here $\angle PSQ = 180° - (110° + 30°)$
$\angle PSQ = 40°$
$\angle QSR = 75° - 40° = 35°$
$\angle QSR + \angle SRT = 180°$ [$\because$ SQ ∥ RT]
$35° + 60° + x = 180°$
$x = 180° - 95°$
$x = 85°$

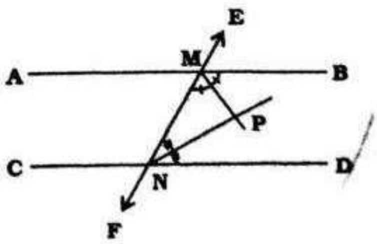
7. (3) As AOB are collinear so



$\angle AOC + \angle BOC = 180°$
Hence are supplementary.

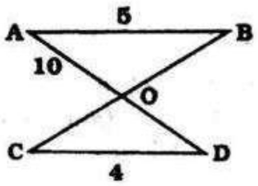
8. (3) $\angle AOD = \angle ECO \Rightarrow \angle AOD = 70°$
So $\angle BOD = 110°$
Hence in Δ BOD
$\angle OBD + \angle BOD + \angle ODB = 180°$
$\angle OBD = 180° - (110° + 20°)$
$\angle OBD = 50°$

9. (1) As $\angle BMN + \angle DNM = 180°$
$\angle PMN + \angle PNM = 90°$



$\angle MPN = 180° - (\angle PMN + \angle PNM)$
$= 180° - (90°)$
$\Rightarrow \angle MPN = 90°$

10. (2) Here as $AB \parallel CD$



So $\frac{AB}{CD} = \frac{AO}{OD}$

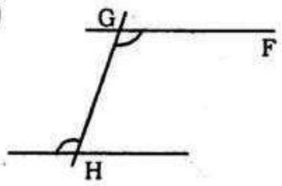
$\frac{5}{4} = \frac{10}{OD} \Rightarrow \text{OD} = \frac{4 \times 10}{5} = 8 \text{ cm}$

11. (2) $2x + 2y = 180$
$\Rightarrow x + y = 90°$
$\therefore \angle COD = 90°$

12. (2) $\angle DOE = 180 - (40 + 31) = 109°$
and $\angle BOF = 40°$ ($\because \angle AOE = \angle BOF$)
$\therefore \angle BOC = \angle BOF + \angle FOC = 40 + 109$
($\because \angle FOC = \angle DOE$)
$= 149°$

13. (1) $2y + 3x = 180$
$\Rightarrow y = 36°$ ($\because x = 36°$) or $x + 4y = 180°$
$\Rightarrow y = 36°$ ($\because x = 36°$)

14. (1)



(Pair of alternate angles)

15. (2) $a > \frac{90°}{6} \Rightarrow a > 15°$
$\because a + b = 180°$
$\therefore b < 165°$ ($\because a > 15°$)

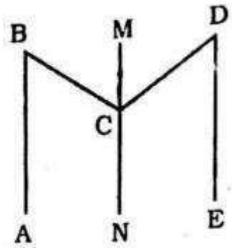
16. (2) $\angle ABP = \angle CDR = 100°$
$y = 80°$ ($\because \angle CDR + \angle CDS = 180°$

17. (3) $\angle APO = 42°$ and $\angle CQO = 38°$
$\angle POQ = \angle PON + \angle NOQ$
$= \angle APO + \angle OQC = 42° + 38° = 80°$

18. (2) $\angle PGH = 80° \Rightarrow \angle QGH = 100°$
$\angle QHD = 120° \Rightarrow \angle CHQ = 60°$
$\therefore \angle x + 100 + 60 = 180°$
$\Rightarrow x = 20°$

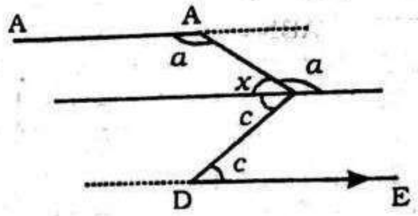
19. (3) $\angle z = 180° - 110° = 70°$
($\because \angle QAC = \angle BCA$) and $60° + y° + z° = 180°$
$\therefore y = 180° - (70° + 60°)$
$y = 50°$

20. (1) $\angle BCD = \angle BCM + \angle DCM$
$= \angle ABC + \angle EDC = 67° + 23° = 90°$



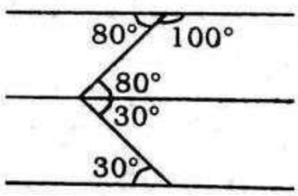
(Draw a line parallel to AB or DE through C.)

21. (3) $a + x = 180°$
$c + x = b \Rightarrow x = b - c$



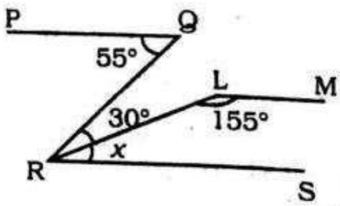
$\therefore a + (b - c) = 180°$
$\Rightarrow a + b - c = 180°$

22. (1) $(a + b - c) = 180°$
$\Rightarrow 100° + (x + 10°) - 30° = 180°$
$\Rightarrow x = 100°$
Alternatively :



$\therefore x + 10° = 80° + 30°$
$\Rightarrow x = 100°$

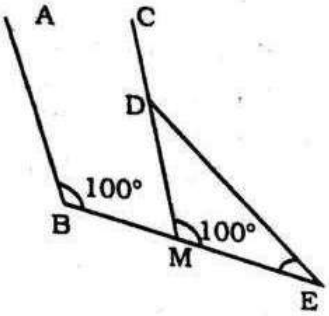
23. (2)



$\angle PQR = \angle QRS$
$55° = \angle QRS$
$55° = \angle QRL + \angle LRS$
$55° = 30° + \angle LRS$
$\Rightarrow \angle LRS = 25°$

24. (3) $\angle ABC = 180° - (40° + 65°)$
$\angle ABC = 75°$
and $\angle DCF = \angle ABC = 75°$
$\therefore \angle EDC = 180° - \angle DCF = 105°$

25. (1) Extend CD to M, then
$\angle DME = \angle ABE = 100°$
$\angle MED = 25°$



$\therefore \angle MDE = 180° - (100° + 25°) = 55°$
$\therefore \angle CDE = 180° - 55° = 125°$

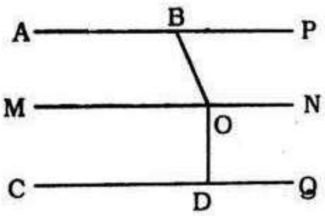
26. (1) $\angle CEF = \angle ABF = 70°$
$\therefore \angle FED = 180° - 70° = 110°$
$\angle EDF + \angle DFE + \angle FED = 180°$
$\therefore \angle FDE = 180° - (110° + 20°) = 50°$

27. (3) $\angle BAO = 180° - (60° + 35°) = 85°$
$\angle AOB = 180° - (85° + 20°) = 75°$

28. (1) $\angle AEP = \angle CDP = 34°$
$\angle AEP + \angle BEF + \angle DEF = 180°$
$34° + 78° + \angle DEF = 180°$
$\Rightarrow \angle DEF = 68°$

29. (2) $\angle AKH = \angle KHD = 25°$
$\angle EGH = \angle AKL = 180° - 60° = 120°$
$\therefore \angle HKL = 120° + 25° = 145°$

30. (3)



$\angle ABO = 118°$
$\therefore MOB = 180° - 118° = 62°$
$\therefore \angle MOD = 152° - 62° = 90°$
$\therefore \angle ODC = 180° - 90° = 90°$

31. (3) In ΔOAB, we have :
$\angle OAB + \angle OBA + \angle AOB = 180°$
$\therefore 55° + 75° + \angle AOB = 180°$
$\Rightarrow \angle AOB = 50°$.
$\therefore \angle COD = \angle AOB = 50°$ (vert. opp. $\angle$s)
In ΔOCD, we have:
$\angle COD + \angle OCD + \angle ODC = 180°$
$50° + 100° + x = 180°$
$\Rightarrow x = 30°$.

32. (2) $\angle BCD + \angle BCE = 180°$
$\Rightarrow \angle BCD + 65° = 180°$
$\Rightarrow \angle BCD = (180° - 65°) = 115°$
In ΔBCD, we have:
$\angle CBD + \angle BCD + \angle BDC = 180°$
$\Rightarrow 28° + 115° + \angle BDC = 180°$
$\Rightarrow \angle BDC = (180° - 143°) = 37°$
Since $AB \parallel DC$, we have:
$\angle ABD = \angle BDC = 37°$ [Alt. Int. $\angle$s]
$\therefore \angle ABD = 37°$.

33. (2) Lengths of perpendiculars from vertices to the opposite sides are equal. So, $a = b = c$.

34. (3) As AB || DC
so $\angle A + \angle D = 180°$
$\Rightarrow \angle D = 180° - 55° = 125°$
Also, $\angle B + \angle C = 180°$
$\Rightarrow \angle C = 180° - \angle B = 180° - 70° = 110°$
So, $\angle C = 110°$ and $\angle D = 125°$

35. (3) $\angle A + \angle B + \angle C = 180°$.
$\angle C = 180° - (\angle A + \angle B)$
$= 180 - (80 + 60)$
$\angle C = 40°$ [$\because \angle C = 2x$]
$\Rightarrow 2x = 40 \Rightarrow x = 20°$

$$y = 180° - \left(\frac{1}{2}\angle B + x\right) \quad [\because \Delta BDC]$$

$$= 180° - \left(\frac{1}{2} \times 60° + 20°\right)$$

$= 180° - (30° + 20°) = 130°$
So $x = 20°$ and $y = 130°$

36. (2) Let the measures of three angles of triangle are 2x, 7x and 11x respectively.
$\therefore 2x + 7x + 11x = 180°$
$\Rightarrow 20x = 180°$

$$\Rightarrow x = \frac{180}{20} = 9°$$

$\therefore$ First angle $= 2x = 2 \times 9 = 18°$
Second angle $= 7x = 7 \times 9 = 63°$
Third angle $= 11x = 11 \times 9 = 99°$

37. (2) Sum of interior angles of a hexagon = 720°
$\therefore x° + (x-5)° + (x-5)° + (2x-5)° + (2x-5)° + (2x + 20)° = 720°$
$9x = 720°$

$$x = \frac{720°}{9} = 80°$$

38. (4) As QSR is a triangle so,
$2x + 3x + 5x = 180°$
$\Rightarrow 10x = 180° \Leftrightarrow x = 18°$
$\therefore \angle P = \angle R = 5x$
$\angle P = 5 \times 18° = 90°$

39. (3) Here as OA, OB are opposite rays.

$\angle AOC + \angle COF + \angle FOB = 180°$

$5y + 5y + 2y = 180°$

$12y = 180° \Rightarrow y = 15°$

40. (4) Sum of three angles of a triangle is 180°. If sum of three angles is greater or less than 180° then the triangle is not possible.

41. (1) In ΔBCD, $\angle BCD = 65°$

and $\angle BDC = 90°$

$\angle CBD = 180° - (\angle BCD + \angle CDB)$

$= 180° - (65° + 90°)$

$= 180° - 155° = 25°$

42. (2) Let angles of triangle be $2x$, $3x$, and $4x$, Then

$2x + 3x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow x = 20°$

So, angles are : $2x = 40°$

$3x = 60°$

$4x = 80°$

43. (1) In Δ ABC

$\angle A + \angle B + y = 180°$

$\Rightarrow y = 180° - (40 + 30)°$

$\Rightarrow y = 110°$

$\angle ADE = \angle ABC$

{pair of corresponding angles}

$\Rightarrow x = 30°$

Similarly, $y = z = 110°$

44. (1) Here in Δ AEC,

$\angle A + \angle E + \angle C = 180°$ .....(i)

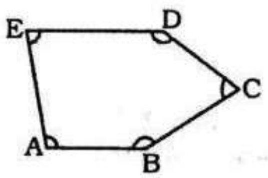
In Δ BFD, $\angle B + \angle F + \angle D = 180°$ ....(ii)

Adding (i) and (ii) we get,

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

45. (2) Here

$\angle ABC + \angle BCD + \angle CDE + \angle DEA + \angle EAB$

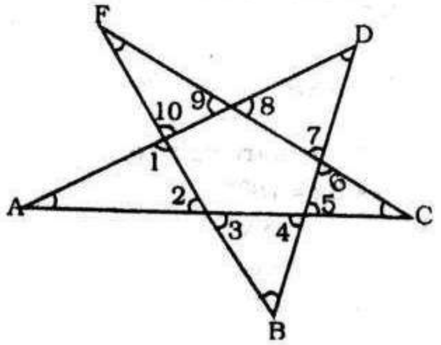


= 3 [sum of angles of triangle]

$= 3 \times 180° = 540°$

46. (3) Here

$\angle A + \angle 1 + \angle 2 = \angle B + \angle 3 + \angle 4$



$= \angle C + \angle 6 + \angle 5 = \angle D + \angle 7 + \angle 8$

$= \angle E + \angle 9 + \angle 10 = 180°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle E + \angle 1$

$+ \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7$

$+ \angle 8 + \angle 9 + \angle 10 = 180° \times 5 = 900°$

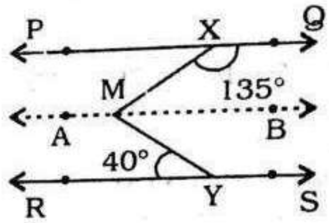
Also $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 +$

$\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 720°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle E$

$= 900° - 720° = 180°$

47. (1)



Through point M draw a line AB parallel to PQ.

$\therefore$ AB || RS

Now, $\angle QXM + \angle XMB = 180°$

$\Rightarrow \angle XMB = 180° - 135° = 45°$

Now, AB || RS and $\angle$ BMY and $\angle$ MYR are alternate angles.

$\therefore \angle BMY = \angle MYR$

$\Rightarrow \angle BMY = 40°$

$\therefore \angle XMY = \angle XMB + \angle BMY$

$= 45° + 40° = 85°$

48. (3) $\angle A = 120°$ and AB = AC

$\Rightarrow \angle B = \angle C$

But, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 120° + \angle B + \angle B = 180°$

$\Rightarrow 2\angle B = 60°$

$\Rightarrow \angle B = 30°$

49. (4) Here if ABC is a straight line

then $\angle ABD + \angle DBC = 180°$

$4x - 30° + 3x = 180°$

$7x = 180° + 30°$

$7x = 210° \Rightarrow x = 30°$

50. (3) As $\angle A + b° = 180°$

$\Rightarrow \angle A = 180° - b$

Also $\angle C + a° = 180°$ (linear pair)

$\Rightarrow \angle C = 180° - a°$

But $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow (180° - b°) + x + (180° - a°) + y° = 360°$

$\Rightarrow x° + y° = a° + b°$

51. (1) Since $\Delta ABC \sim \Delta DEF$

$\therefore \angle B = \angle E = 83°$

Hence, in ΔABC, $\angle C = 180° - (\angle A + \angle B)$

$\Rightarrow \angle C = 180° - (47 + 83)° = 50°$.

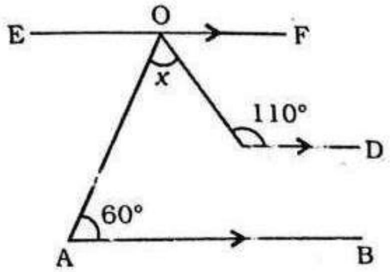
52. (3) Since AOB is a straight line, we have:

$3x - 10 + 50 + x + 20 = 180$

$\Rightarrow 4x = 120 \Rightarrow x = 30$.

$\therefore \angle AOC = (3 \times 30 - 10)° = 80°$.

53. (3)



Let $\angle AOC = x°$.

Draw $EOF \parallel AB \parallel CD$.

Now, $EO \parallel AB$ and OA is a transversal.

$\therefore \angle EOA = \angle OAB = 60°$ [Alt. Int.∠s]

Again, $OF \parallel CD$ and OC is the transversal.

$\therefore \angle COF + \angle OCD = 180°$

$\Rightarrow \angle COF = 110° = 180°$

$\Rightarrow \angle COF = 70°$.

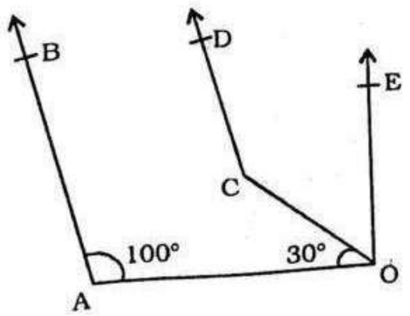
Now, EOF is a straight line.

$\therefore \angle EOA + \angle AOC + \angle COF = 180°$

$\Rightarrow 60 + x + 70 = 180$

$\Rightarrow x = (180 - 130) = 50°$.

54. (1)



Let $\angle OCD = x°$. Draw $OE \parallel AB \parallel CD$.

Now, $AB \parallel OE$ and OA is the transversal.

$\therefore \angle OAB + \angle AOE = 180°$

$\Rightarrow \angle OAB + \angle AOC + \angle COE = 180°$

$\therefore 100° + 30° + \angle COE = 180°$

$\Rightarrow \angle COE = 50°$.

Again, $CD \parallel OE$ and OC is the transversal.

$\therefore \angle COD + \angle EOC = 180°$

$\Rightarrow x + 50 = 180 \Rightarrow x = 130°$.

55. (1) In ΔABC, $\angle BAC + \angle ABC + \angle ACB = 180°$

$\therefore 30° + 50° + \angle ACB = 180°$

$\Rightarrow \angle ACB = 100°$

$\therefore \angle ECD = (180° - 100°) = 80°$.

$\angle AED = \angle ECD + \angle EDC = (80° + 40°) = 120°$,

$\therefore \angle AED = x° = 120°$.

56. (2) In ΔABC,

$40° + 90° + \angle B = 180° \Rightarrow \angle B = 50°$

In ΔEBD,

$\angle B + \angle BED + \angle BDE = 180°$

$\therefore 50° + 100° + x° = 180° \Rightarrow x = 30°$.

57. (3) Join OC and produce it to D.

Let $\angle BOC = m°$, $\angle AOC = n°$,

$\angle BCD = p°$ and $\angle ACD = q°$.

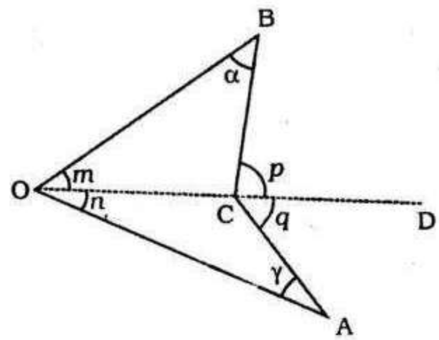
Then, $m + n = \beta$ and $p + q = x$.

In ΔBOC, side OC has been produced to D.

$\therefore p = \alpha + m$ ...(i)

In ΔAOC, side OC has been produced to D.

$q = n + \gamma$ ... (ii)



Adding (i) and (ii), we get:

$p + q = \alpha + \gamma + (m + n)$

$\Rightarrow x = \alpha + \gamma + \beta$

Hence, $x = \alpha + \beta + \gamma$.

58. (2) Let $\angle CEF = x°$.

Now, $AB \parallel CD$ and AF is a transversal.

$\therefore \angle DCF = \angle CAB = 80°$ (corresponding ∠s)

In ΔCEF, side EC has been produced to D

$\therefore x + 25 = 80 \Rightarrow x = (80 - 25) = 55°$.

59. (3) $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 50° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 130°$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 65°$

In ΔOBC,

$\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180°$

$\Rightarrow 65° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 115°$.

60. (3) Let $\angle AEB = x^\circ$.

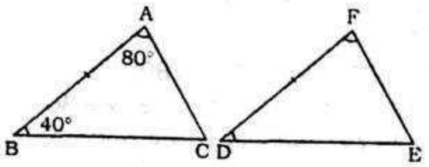
Now, $AB \parallel CD$ and BC is the transversal.

$\therefore \angle ABE = \angle BCD = 60^\circ$ [Alternate Interior $\angle$s]

In ΔABE,

we have : $50^\circ + 60^\circ + x = 180^\circ \Rightarrow x = 70^\circ$.

61. (2) Given : $\Delta ABC \cong \Delta FDE$.



$AB = FD = 5\text{cm}$,

$\angle B = 40^\circ$ and $\angle A = 80^\circ$.

$\therefore \angle C = 180^\circ - (80^\circ + 40^\circ) = 60^\circ$

So, we must have

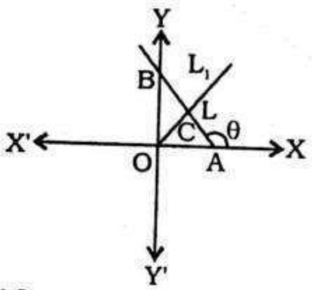
$\angle E = \angle C = 60^\circ$

Hence, $\angle E = 60^\circ$

62. (2) $\angle BAC = 180^\circ - \angle BOC = 180^\circ - 54^\circ = 126^\circ$

63. (2) $\angle BIC = 90^\circ + \frac{A}{2} = 90^\circ + 30^\circ = 120^\circ$

64. (4)

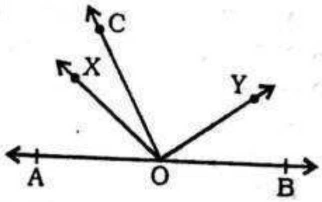


$\angle CAO = \pi - \theta$

$\angle COA = \pi - \frac{\pi}{2} - \pi + \theta = \theta - \frac{\pi}{2}$

$\therefore \angle BOC = \frac{\pi}{2} - \left(\theta - \frac{\pi}{2}\right) = \pi - \theta$

65. (1)



OY is the bisector of $\angle AOC$.

$\therefore \angle AOC = 2\angle COX$

OX is the bisector of $\angle BOC$.

$\therefore \angle BOC = 2\angle COY$

$\therefore \angle AOC + \angle BOC = 2\angle COY + 2\angle COX = 180^\circ$

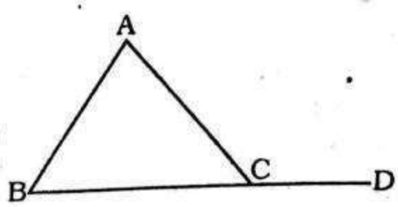
$\Rightarrow 2(\angle COX + \angle YOC) = 180^\circ$

$\Rightarrow \angle XOY = 90^\circ$

$\therefore \angle AOX + \angle XOY + \angle BOY = 180^\circ$

$\therefore \angle BOY = 180^\circ - 90^\circ - 20^\circ = 70^\circ$

66. (2)



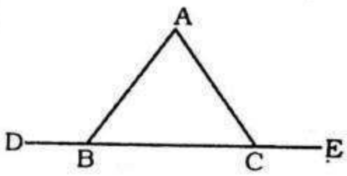
$\angle ACD = \angle ABC + \angle BAC$

$\Rightarrow 108^\circ = \frac{\angle A}{2} + \angle A$

$\Rightarrow \frac{3\angle A}{2} = 108^\circ$

$\Rightarrow \angle A = \frac{108 \times 2}{3} = 72^\circ$

67. (2)



$\angle ABD = \pi - B$

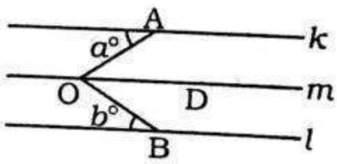
$\angle ACE = \pi - C$

$\angle ABD + \angle ACE = 2\pi - (B + C)$

$= 2\pi - (\pi - A)$

$= \pi + A$

68. (1)



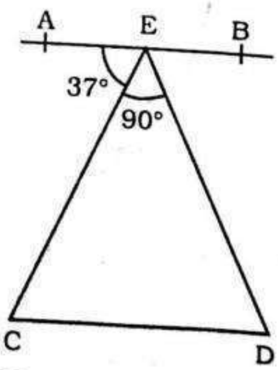
$k \parallel l \parallel m$

$\angle BOA = 45^\circ$

$\Rightarrow \angle AOD = a^\circ$ and $\angle DOB = b^\circ$

$\therefore a^\circ + b^\circ = \angle AOB = 45^\circ$

69. (1)



$\angle AEC + \angle CAD + \angle DEB = 180^\circ$

$\Rightarrow 37^\circ + 90^\circ + \angle DEB = 180^\circ$

$\Rightarrow \angle DEB = 180^\circ - 127^\circ = 53^\circ$

$EB \parallel CD$

$\therefore \angle BED = \angle EDC = 53^\circ$