CBSE Sample Paper -03
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

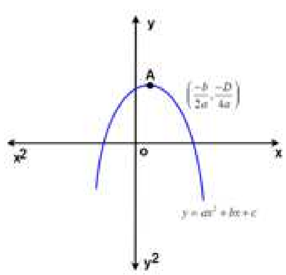
2. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ΔABC is a right triangle.

3. In $\sin A + \sin^2 A = 1$, then show that $\cos^2 A + \cos^4 A = 1$

4. If $A + B = 90^o$ and $\tan A = \frac{3}{4}$, what is cot B?

SECTION – B

5. The graph of $y = ax^2 + bx + c$ is given in the following figure. Identify the signs of *a*, *b* and *c*.

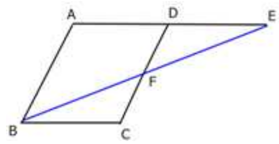


6. Following table shows the weight of the bags of 12 students:

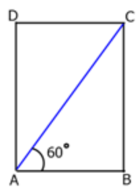
Weight (Kg)	67	70	72	73	75
Number of students	4	3	2	2	1

Find the mean weight.

7. E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. Prove that ΔABE~ΔCFB.



8. In a rectangle ABCD, AB = 20 cm, ∠BAC = 60°. Calculate side BC.



9. There is a circular path around a sports field. Prenu takes 18 minutes to drive 1 round of the field, while Raj takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

10. Find a cubic polynomial with the sum, sum of the products of its zeros taken two at a time, and product of its zeros as 2, –7 and –14, respectively.

SECTION – C

11. If n is an old positive integer, show that $(n^2 - 1)$ divisible by 8.

12. If tan A = n tan B and sin A = m sin B, prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$.

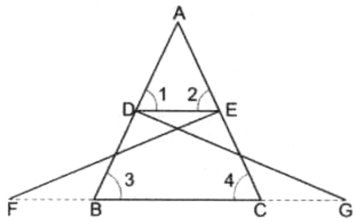
13. If $\tan\theta + \sin\theta = m$ *and* $\tan\theta - \sin\theta = n$, *show that* $(m^2 - n^2) = 4\sqrt{mn}$.

14. Points A and B are 70 km apart on a highway. A car starts from a and another car starts from B at the same time. If they travel in same direction, they meet in 7 hours but if they travel in opposite direction, they meet in one hour. What are their speeds?

15. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, $a$and$a + b$. Find a and b.

16. If the diagonals of a quadrilateral divide each other proportionally, the quadrilateral is a trapezium.

17. In given figure, $\Delta FEC \cong \Delta GDB$ *and* $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$.



18. Find an acute θ,when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$.

19. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0-20	20-0	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the model lifetimes of the components.

20. A 2-digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number.

SECTION – D

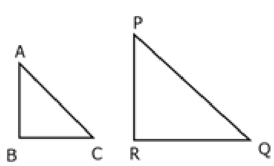
21. Let a, b , c, k be rational number such that k is not a perfect cube. If $a + bk^{\frac{1}{3}} + ck^{\frac{2}{3}} = 0,$ then prove that a=b=c=0.

22. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

23. State and prove Pythagoras theorem.

24. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

25. If ∠B and ∠Q are acute angles such that sinB = sinQ, then prove that ∠B = ∠Q.



26. Solve the following system of equations in x and y

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2$$

$$(a+b)(x+y) = a^2 + b^2$$

27. During the medical checkup of 35 studetns of a class, their weights were recorded as follows:

Weight (kg)	Less than 38	Less than 40	Less than 42	Less than 44	Less than 46	Less than 48	Less than 50	Less than 52
Number of students	0	3	5	9	14	28	32	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

28. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

29. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

30. Let a, b, c and d be positive rationals such that $a+\sqrt{b}=c+\sqrt{d}$, then either $a = c$ and $b = d$ or b and d are squares of rationals.

31. A man hires a taxi to cover a certain distance. The fare is Rs 50 for first kilometre and Rs 25 for subsequent kilometers. Taking total distance covered as x km and total fare as y:

    a. Write a linear equation for this.

    b. The man covers a distance of 10 km and gave Rs 300 to the driver. Driver said "It is not the correct amount" and returned him the balance. Find the correct fare and the amount paid back by the driver.

    c. Which values are depicted by the driver in the question?

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Time allowed: 3 hours	**ANSWERS**	Maximum Marks: 90

SECTION – A

1. $\frac{6}{1250}=\frac{3}{625}=\frac{3}{5^4}=\frac{2^4}{2^4}=\frac{48}{(5\times 2)^4}=\frac{48}{10^4}=0.0048$

This representation will terminate after 4 decimal places.

2. We have AC = BC and $AB^2 = 2AC^2$

Now, $AB^2 = 2AC^2$

$\Rightarrow$ $AB^2 = AC^2 + AC^2$

$\Rightarrow$ $AB^2 = AC^2 + BC^2$ [$\because$ AC = BC (Given)]

$\Rightarrow$ ΔABC is a right triangle right angled at C.

3. $\sin A+\sin^2 A=1 \Rightarrow \sin a=1-\sin^2 a=\cos^4 A=1$

$\therefore$ $\cos^2 a+\cos^4 a=\sin A+\sin^4 A=1.$

4. $\cot B=\cot(90^o-A)$ $(\because A+B=90^o)$

$=\tan A$ $(\because \cot(90^o-\theta)=\tan\theta)$

$=\frac{3}{4}$

SECTION – B

5. We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a< 0$.

We also observe that the vertex of the parabola is in first quadrant.

$\therefore -\frac{b}{2a}>0 \Rightarrow -b<0 \Rightarrow b>0$

Parabola $y = ax^2 + bx + c$ cuts Y-axis at P. On Y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of P are $(0, c)$. As P lies on the positive direction of Y-axis, therefore, $c> 0$.

Hence, $a< 0$, $b> 0$ and $c> 0$.

6.

Calculation of arithmetic mean

Weight (Kg) x_i	Frequency f_i	$f_i x_i$

67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
	$N = \sum f_i = 12$	$\sum f_i x_i = 843$

∴ Mean = $\overline{X} = \frac{\sum f_i x_i}{N} = \frac{843}{12} = 70.25$

7. In triangles ABE and CFB, we have

$\angle AEB = \angle CBF$ [Alternate angles]

$\angle A = \angle C$ [Opposite angles of a parallelogram]

Thus, by AA-criterion of similarity, we have

$\Delta ABE \sim \Delta CFB$

8. In ΔABC, we have

AB = 20, $\angle BAC = 60°$

∴ $\tan\angle BAC = \frac{BC}{AB}$

⇒ $\tan 60° = \frac{BC}{20}$

⇒ $\sqrt{3} = \frac{BC}{20}$ ⇒ $BC = 20\sqrt{3}$ cm

9. Required number of minutes is the LCM of 18 and 12.

We have,

$18 = 2 \times 3^2$ and $12 = 2^2 \times 3$

∴ LCM of 18 and 12 = $2^2 \times 3^2 = 36$

Thus, Prenu and Raj will meet again at the starting point after 36 minutes.

10. If α, β and γ are the zeros of a cubic polynomial $f(x)$, then

$f(x) = k\left\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\right\}$,

where k is any non-zero real number.

Here, $\alpha+\beta+\gamma = 2$, $\alpha\beta+\beta\gamma+\gamma\alpha = -7$ and $\alpha\beta\gamma = -14$

∴ $f(x) = k\left(x^3 - 2x^2 - 7x + 14\right)$, where k is any non-zero real number.

SECTION – C

11. We know that an old positive integer n is of the form (4q+1) or (4q+3) for some integer q.

Case I When n=(4q+1)

In this case

$$n^2 - 1 = (4q+1)^2 - 1 = 16q^2 + 8q = 8q(2q+1)$$

Which is clearly divisible by 8.

Case II When n=(4q+3)

In this case, We have

$$n^2 - 1 = (4q+3)^2 - 1 = 16q^2 + 14q + 8 = 8(2q^2 + 3q + 1)$$

Which is clearly divisible by 8.

12. We have find $\cos^2 A$ in term of m and n. This means that the angle B is to be eliminated from the given relations.

Now, tan A = n tan B

$$\tan B = \frac{1}{n}\tan A = \cot B = \frac{n}{\tan A}$$

and sinA = m sinB

$$\sin B = \frac{1}{m}\sin A = \cos ecB = \frac{m}{\sin A}$$

Substituting the values of cot B and cosec B in $\cos ec^2 B - \cot^2 B = 1$, we get

$$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$m^2 - n^2 \cos^2 A = \sin^2 A$$

$$m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$m^2 - 1 = n^2 \cos^2 - \cos^2 A$$

$$m^2 - 1 = \left(n^2 - 1\right)\cos^2 A$$

$$\frac{m^2 - 1}{n^2 - 1} = \cos^2 A$$

13. We have given $\tan\theta + \sin\theta = m$, and $\tan\theta - \sin\theta = n$, then

L.H.S = $\left(m^2 - n^2\right) = \left(\tan\theta + \sin\theta\right)^2 - \left(\tan\theta - \sin\theta\right)^2$

$\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta = 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}\left(1-\cos^2\theta\right)}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

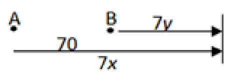
$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{\left(\tan\theta - \sin\theta\right)\left(\tan\theta + \sin\theta\right)}$

$= 4\sqrt{mn}$ = R.H.S

14. Let the speed of faster car at A = x km/hr and the speed of slower car at B = y km.hr.

Case 1: When they travel in same direction



Distance covered by faster car in 7 hours = $7x$ km

Distance covered by slower car in 7 hours = $7y$ km

$\Rightarrow \quad 7x = 7y + 70$

$\Rightarrow \quad 7(x - y) = 70$

$\Rightarrow \quad x - y = 10$ ...(i)

Case 2: When they travel in opposite direction

Distance travelled by faster car in 1 hour = x km

Distance travelled by slower car in 1 hour = y km

$\Rightarrow \quad x + y = 70$ ...(ii)

Adding (i) and (ii), we get

$2x = 80 \quad \Rightarrow \quad x = 40$

Substituting $x = 40$ in (i), we get

$40 - y = 10 \quad \Rightarrow \quad y = 40 - 10 = 30$

$\therefore$ Speeds of cars would be 40 km/hr and 30 km/hr.

15. $x^3 - 3x^2 + x + 1$ is a cubic polynomial.

$\therefore$ Sum of its zeros = $\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-(-3)}{1} = 3$

$\Rightarrow$ $a - b + a + a + b = 3$

$\Rightarrow$ $3a = 3$

$\Rightarrow$ $a = 3$

Also, product of its zeros = $\frac{-(\text{constant term})}{\text{coefficient of } x^3} = \frac{-1}{1} = -1$

$\Rightarrow$ $(a - b) \times a \times (a + b) = -1$

$\Rightarrow$ $a(a^2 - b^2) = -1$

$\Rightarrow$ $1(1 - b^2) = -1$ $[\because a = 1]$

$\Rightarrow$ $1 - b^2 = -1$

$\Rightarrow$ $b^2 = 2$

$\Rightarrow$ $b = \pm\sqrt{2}$

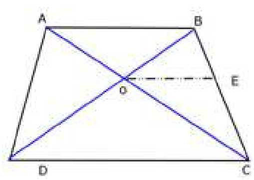
Thus, $a = 1$ and $b = \pm\sqrt{2}$.

16. Given: A quadrilateral ABCD whose diagonals AC and BC intersect at O such that

$\frac{AO}{OC} = \frac{BO}{OD}$

To prove: ABCD is a trapezium.

Construction: Through O, draw OE || AB.



$\therefore$ By Basic Proportionality Theorem, we have

$\frac{AO}{OC} = \frac{BE}{EC}$

But, $\frac{AO}{OC} = \frac{BO}{OD}$ [Given]

$\therefore$ $\frac{BE}{EC} = \frac{BO}{OD}$

Now, in ΔBCD, we have $\frac{BE}{EC} = \frac{BO}{OD}$

$\therefore$ By Basic Proportionality Theorem, we have

OE || DC

Now, OE || AB [By construction]

and, OE || DC

$\therefore$ AB || DC

Thus, ABCD is a trapezium.

17. Since, $\Delta FEC \cong \Delta GDB$

$\Rightarrow EC = BD$ ...(i)

It is given that

$\angle 1 = \angle 2$

$\Rightarrow AE = AD$ [Sides opposite to equal angles are equal] ...(ii)

From (i) and (ii)

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow DE \| BC$ [By the converse of basic proportionality theorem]

$\Rightarrow \angle 1 = \angle 2 \text{ and } \angle 2 = \angle 4$ [Corresponding]

Thus, in $\Delta's$ ADE and ABC, we have

$\angle A = \angle A$ [Common]

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ [Proved above]

So, by AAA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$

18. We have,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \Rightarrow \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

[Dividing numerator & denominator of the LHS by $\cos\theta$]

$$\Rightarrow \frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

On comparing we get,

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^o \Rightarrow \theta = 60^o$

19. Here, the maximum class frequency is 61 and the class corresponding to this frequency is 60-80. So, the model class is 60-80.

Here, $l=60, h=20,\ f_1=52,\ f_0=38$

$$\therefore\ Model = l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$$

$$=60+\frac{31-52}{2\times 61-52-38}\times 20=60+\frac{9}{122-90}\times 20$$

$$=60+\frac{9}{32}\times 20=60+\frac{45}{8}$$

$$=60+5.625=65.625$$

Hence, model lifetime of the components is 65.625 hours.

20. Let the tens and units digits of the required number be x and y, respectively. Then $xy = 14$.

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore$ $(10x + y) + 45 = (10y + x)$

$\Rightarrow$ $9(y - x) = 45$

$\Rightarrow$ $y - x = 45$ ...(i)

Now, $(y+x)^2-(y-x)^2 = 4xy$

$\Rightarrow$ $(y+x)=\sqrt{(y-x)^2+4xy}$

$= \sqrt{25+4\times 14}=\sqrt{81}$

$\Rightarrow$ $y + x = 9$ ...(ii)

On adding (i) and (ii), we get

$2y = 14 \quad \Rightarrow \quad y = 7$

Putting $y = 7$ in (ii), we get

$7 + x = 9 \quad \Rightarrow \quad x = 9 - 7 = 2$

$\therefore$ $x = 2$ and $y = 9$

SECTION – D

21. Given, $a+bk^{\frac{1}{3}}+ck^{\frac{2}{3}}=0,$ ... (i)

Multiplying both sides by $k^{\frac{1}{3}}$, we have

$bk^{\frac{1}{3}}+ck^{\frac{2}{3}}+ck=0,$ …(ii)

Multiplying (i) by b and (ii) by c and then subtracting , we have

$(ab + b^2k^{1/2} + bck^{2/3}) - ack^{1/3} + bck^{2/3} + c^2k) = 0$

$\Rightarrow (b^2 - ac)k^{1/3} + ab - c^2k = 0$

$\Rightarrow b^2 - ac = 0$ *and* $ab - c^2k = 0$ [Since $k^{1/3}$ *is* irrational]

$\Rightarrow$ $b^2 - ac = 0$ *and* $ab = c^2k$

$\Rightarrow b^2 = ac$ *and* $a^2b^2 = c^4k^2$

$\Rightarrow a^2(ac) = c^4k^2$ [By putting $b^2 = ac$ in $a^2b^2 = c^4k^2$]

$\Rightarrow a^3c - k^2c^4 = 0 \Rightarrow (a^3 - k^2c^3)c = 0$

$\Rightarrow a^3 - k^2c^3 = 0$, *or* $c = o$

Now, $a^3 - k^2c^3 = 0$

$$\Rightarrow k^2 = \frac{a^3}{c^3} \Rightarrow (k^2)^{1/3} = \left(\frac{a^3}{c^3}\right) \Rightarrow k^{2/3} = \frac{a}{c}$$

This is impossible as $k^{2/3}$ is irrational and $\frac{a}{c}$ is rational.

$\therefore \quad a^3 - k^2c^3 \neq 0$

Hence, c=0

Substituting c =0 in $b^2 - ac = 0$, we get b = 0

Substituting b = 0 and c = 0 in $a + bk^{1/3} + ck^{2/3} = 0$, we get a = 0

Hence, c = b = c = 0.

22. Let $p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

by division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow \quad g(x) \times q(x) = p(x) - r(x)$

$\Rightarrow \quad g(x)(x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4)$

$= x^3 - 3x^2 + x + 2 + 2x - 4$

$= x^3 - 3x^2 + 3x - 2$

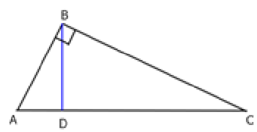
$\Rightarrow \quad g(x)$ is a factor of $x^3 - 3x^2 + 3x - 2$ other than $(x - 2)$.

Dividing $x^3 - 3x^2 + 3x - 2$ by $(x - 2)$, we obtain $g(x)$ as follows:

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore \quad g(x) = x^2 - x + 1$

23. Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: A right-angled triangle ABC in which $\angle B = 90°$.

To prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction: From B, draw $BD \perp AC$.

Proof: In triangles ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad AB^2 = AD \times AC$ ...(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \quad \frac{DC}{BC} = \frac{BC}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad BC^2 = AC \times DC$ ...(ii)

Adding equations (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow \quad AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow \quad AB^2 + BC^2 = AC \times AC$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

Or, $\quad AC^2 = AB^2 + BC^2$

24. Let the number of students be x and the number of rows be y.

Then, number of students in each row = $\frac{x}{y}$

When one student is extra in each row, there are 2 rows less, i.e., when each row has $\left(\frac{x}{y}+1\right)$ students, the number of rows is $(y-2)$.

$\therefore \quad$ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}+1\right)(y-2)$

$\Rightarrow \quad x = x-\frac{2x}{y}+y-2$

$\Rightarrow \quad -\frac{2x}{y}+y-2=0$ ...(i)

If one student is less in each row, then there are 3 rows more, i.e., when each row has $\left(\frac{x}{y}-1\right)$ students, the number of rows is $(y+3)$.

$\therefore \quad$ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}-1\right)(y+3)$

$\Rightarrow \quad x = x+\frac{3x}{y}-y-3$

$\Rightarrow \quad \frac{3x}{y}-y-3=0$ ...(ii)

Putting $\frac{x}{y} = u$ in (i) and (ii), we get

$-2u + y - 2 = 0$ ...(iii)

and, $\quad 3u - y - 3 = 0$ ...(iv)

Adding (iii) and (iv), we get

$u - 5 = 0 \quad \Rightarrow \quad u = 5$

Putting $u = 5$ in (iii), we get y = 12

Now, $u = 5 \quad \Rightarrow \quad \frac{x}{y} = 5 \quad \Rightarrow \quad \frac{x}{12} = 5 \quad \Rightarrow \quad x = 60$

Thus, the number of students in the class is 60.

25. Consider two right triangles ABC and PQR such that sinB = sinQ.

We have,

$\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

$\therefore \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ...(i)

$\Rightarrow \quad AC = kPR$ and $AB = kPQ$ ...(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(iii)

From (i) and (iii), we have

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \Delta ACB \sim \Delta PRQ$

$\Rightarrow \quad \angle B = \angle Q$

26. The given system of equations may be written as

$(a-b)x + (a+b)y - (a^2 - 2ab - b^2) = 0$

$(a+b)x+(a+b)y-(a^2+b^2)=0$

By cross-multiplication, we have

$$\frac{x}{(a+b)\times(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)}=\frac{-y}{(a-b)\times-(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)}=\frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}=\frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}=\frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{(a+b)\{(a^2+b^2)+(a^2-2ab-b^2)\}}=\frac{-y}{(a+b)(a^2-2ab-b^2)-(a-b)(a^2+b^2)}=\frac{1}{(a+b)(a-b-a-b)}$$

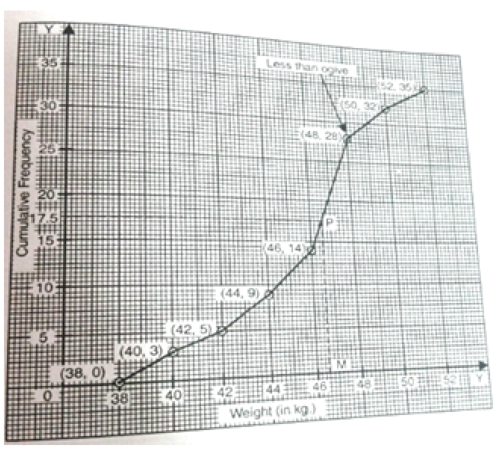
$$\Rightarrow \quad \frac{x}{(a+b)(-2ab-2b^2)}=\frac{-y}{a^3-a^2b-3ab^2-b^3-a^3-ab^2+a^2b+b^3}=\frac{1}{-(a+b)2b}$$

$$\Rightarrow \quad \frac{x}{-2b(a+b)^2}=\frac{-y}{-4ab^2}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad x=\frac{-2b(a+b)^2}{-2b(a+b)}=a+b \text{ and } y=\frac{4ab^2}{-2b(a+b)}=\frac{-2ab}{a+b}$$

Hence, the solution of the given system of equations is $x = a + b$, $y= \frac{-2ab}{a+b}$.

27. To draw the required ogive, we plot the points (38, 0), (40, 3), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) and join them by a freehand curve.



To obtain the value of the median, we locate the point $\frac{n}{2}=\frac{35}{2}$ = 17.5 on the y-axis. From this point, we draw a line parallel to the x-axis, meeting the ogive at the point P. From P, we draw a perpendicular PM on the x-axis. The x-coordinate of the point where this perpendicular meets the x-axis, i.e., M gives the value of the median.

$\therefore$ The required value of the median is 46.5 kg.

Verification:

Weight (kg)	Number of students (f_i)	Cumulative frequency (cf)
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Here, $n = 35$, $\therefore$ $\frac{n}{2} = 17.5$

Median class is 46-48

$\therefore$ $l = 46, f = 14, cf = 14, h = 2$

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$

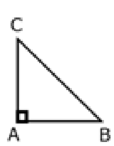
$= 46 + \frac{3.5}{14} \times 2$

= 46.5 kg

The value of the median in both the cases is same, i.e., 46.5 kg.

Hence verified.

28. Draw a right triangle ABC in which $\angle ABC = \theta$



Since, $\cot\theta = \frac{AB}{AC} = \frac{7}{8}$

$\therefore$ Let AB = 7 units and AC = 8 units

$\therefore$ $BC = \sqrt{AB^2 + AC^2}$ (By Pythagoras Theorem)

$= \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$

$= \sqrt{113}$ units

$\therefore \quad \sin\theta = \frac{AC}{BC} = \frac{8}{\sqrt{113}}$ and $\cos\theta = \frac{AB}{BC} = \frac{7}{\sqrt{113}}$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{113-64}{113-49} = \frac{49}{64}$$

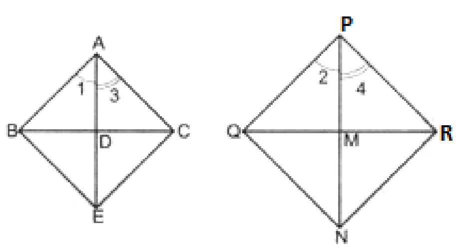
29. Given: In ΔABC *and* ΔPQR; AD and PM are their medians respectively such that

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ ... (i)

To Prove: $\Delta ABC \sim \Delta PQR$

Construction: Produce AD to E such that AD = DE and produce PM to N such that PM = MN . Join BE, CE, QN, RN.

Proof: Quadrilateral ABEC and PQNR are $\|^{gm}$ because their diagonals bisects each other at D and M respectively.



$\Rightarrow BE = AC$ *and* $QN = PR$

$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ $[From(i)]$

$i.e., \frac{AB}{PQ} = \frac{BE}{QN}$ ...(ii)

From $(i)\,\dfrac{AB}{PQ}=\dfrac{AD}{PM}=\dfrac{2AD}{APM}=\dfrac{AE}{PN}$

$\Rightarrow \dfrac{AB}{PQ}=\dfrac{AE}{PN}$ ...(iii)

From (ii) and (iii)

$\dfrac{AB}{PQ}=\dfrac{BE}{QN}=\dfrac{AE}{PN}$

$\Rightarrow \Delta ABE \sim \Delta PQN$ (SSS similarity criterion)

$\Rightarrow \angle 1=\angle 2$ ...(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \;\; \Rightarrow \angle 3=\angle 4$ ...(v)

Adding (iv) and (v), we get

$\angle 1+\angle 3=\angle 2+\angle 4 \;\Rightarrow\; \angle A=\angle P$

And $\dfrac{AB}{PQ}=\dfrac{AC}{PR}$ (Given)

$\therefore\; \Delta ABC \sim \Delta PQR$ (By SAS criterion of similarity)

30. If $a = c$, then

$a+\sqrt{b}=c+\sqrt{d} \quad \Rightarrow \quad \sqrt{b}=\sqrt{d} \quad \Rightarrow \quad b = d$

So, let $a \neq c$. Then, there exists a positive rational number x such that $a = c + x$.

Now, $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \quad c+x+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \quad x+\sqrt{b}=\sqrt{d}$ ...(i)

$\Rightarrow \quad \left(x+\sqrt{b}\right)^2=\left(\sqrt{d}\right)^2$

$\Rightarrow \quad x^2+2\sqrt{b}x+b=d$

$\Rightarrow \quad d-x^2-b=2x\sqrt{b}$

$\Rightarrow \quad \sqrt{b}=\dfrac{d-x^2-b}{2x}$

$\Rightarrow \quad \sqrt{b}$ is rational. $\left[\because d, x \text{ and } b \text{ are rationals}, \therefore \dfrac{d-x^2-b}{2x} \text{ is rational}\right]$

From (i), we have

$\sqrt{d} = x + \sqrt{b}$

$\Rightarrow$ $\sqrt{d}$ is rational

$\Rightarrow$ d is the square of a rational number.

Thus, either $a = c$ and $b = d$ or b and d are the squares of rationals.

31. (a) According to the given condition:

$y = 50 + 25(x - 1)$

$= 50 + 25x - 25$

$\Rightarrow$ $y = 25x + 25$

(b) Correct fare = 25 × 10 + 25

= 250 + 25

= Rs 275

Amount paid back by the driver = 300 – 275 = Rs 25

(c) The values depicted by the driver in the question are honesty and truthfulness.