

महत्वपूर्ण सूत्र

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ जहाँ $n \neq -1$
- $\int \frac{1}{x} dx = \log_e x + c$
- $\int e^x dx = e^x + c$
- $\int \sin x dx = -\cos x + c$
- $\int \cos x dx = \sin x + c$
- $\int \sec^2 x dx = \tan x + c$
- $\int \operatorname{cosec}^2 x dx = -\cot x + c$
- $\int \sec x \cdot \tan x dx = \sec x + c$
- $\int \operatorname{cosec} x \cdot \cot x dx = -\operatorname{cosec} x + c$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$
- $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
- $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + c$
- $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$
- $\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c$
- $\int a^x dx = \frac{a^x}{\log_e a} + c$
- $\int \tan x dx = \log(\sec x) + c$
- $\int \cot x dx = \log \sin x + c$
- $\int \sec x dx = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) = \log(\sec x + \tan x)$
- $\int \operatorname{cosec} x dx = \log \left| \tan \frac{x}{2} \right| = -\log(\operatorname{cosec} x + \cot x)$

महत्वपूर्ण नियम

- $\int dx = \int x^0 \cdot dx = \frac{x^{0+1}}{0+1} = x$
- $\int k \cdot f(x) dx = k \cdot \int f(x) dx$ जहाँ k कोई constant है।
जैसे - $\int 5x^4 dx = 5 \int x^4 dx = 5 \cdot \frac{x^5}{5} = x^5 + c$
- $\int (u \pm v \pm w \dots) dx = \int u dx \pm \int v dx \pm \int w dx \pm \dots$
जहाँ $u, v, w \dots$ इत्यादि सभी x के फलन हैं।
जैसे - $\int (4x^3 - 3e^x + 5) dx$
 $= 4 \int x^3 dx - 3 \int e^x dx + 5 \int dx$
 $= 4 \cdot \frac{x^4}{4} - 3e^x + 5x + c = x^4 - 3e^x + 5x + c$
- $\int uv dx = u \int v dx - \int \left[v dx \cdot \frac{du}{dx} \right] dx$
- $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ जहाँ $a < c < b$

समाकलन पर आधारित प्रश्न

1. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = ?$

Speedy Solution :-

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = \int \left(x + 2 + \frac{1}{x} \right) dx = \frac{x^2}{2} + 2x + \log_e x + c$$

2. $\int \frac{1}{1+\cos x} dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \frac{1}{1+\cos x} dx &= \int \frac{1}{2\cos^2 \frac{x}{2}} dx = \frac{1}{2} \int \sec^2 \frac{x}{2} dx \\ &= \frac{1}{2} \times \frac{\tan \frac{x}{2}}{\frac{1}{2}} + c = \tan \frac{x}{2} + c \end{aligned}$$

3. $\int \cos^{-1} \left(\frac{1-\tan^2 x}{1+\tan^2 x} \right) dx = ?$

Speedy Solution :-

$$\begin{aligned} & \int \cos^{-1} \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right) dx \\ &= \int \cos^{-1}(\cos 2x) dx \left[\because \frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x \right] \\ &= 2 \int x dx = \frac{2x^2}{2} + c = x^2 + c \end{aligned}$$

4. $\int \cot^{-1} \left(\frac{\sin 2x}{1 - \cos 2x} \right) dx = ?$

Speedy Solution :-

$$\begin{aligned} & \int \cot^{-1} \left(\frac{\sin 2x}{1 - \cos 2x} \right) dx \\ &= \int \cot^{-1} \left(\frac{2 \sin x \cos x}{2 \sin^2 x} \right) dx \\ &= \int \cot^{-1}(\cot x) dx = \int x dx = \frac{x^2}{2} + c \end{aligned}$$

5. $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \frac{1 - \cos 2x}{1 + \cos 2x} dx &= \int \frac{2 \sin^2 x}{2 \cos^2 x} dx = \int \tan^2 x dx \\ &= \int (\sec^2 x - 1) dx = \tan x - x + c \end{aligned}$$

6. $\int \frac{dx}{1 + \sin x} = ?$

Speedy Solution :-

$$\begin{aligned} \int \frac{dx}{1 + \sin x} &= \int \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \\ &= \int \frac{1 - \sin x}{1 - \sin^2 x} dx = \int \frac{1 - \sin x}{\cos^2 x} dx = \int \tan x - \sec x + c \end{aligned}$$

7. $\int \cot^2 x dx = ?$

Speedy Solution :-

$$\int \cot^2 x dx = \int (\operatorname{cosec}^2 x - 1) dx = -\cot x - x + c$$

8. $\int \sin^2 x dx = ?$

Speedy Solution :-

$$\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + c$$

9. $\int \frac{1}{1 - \cos 2x} dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \frac{1}{1 - \cos 2x} dx &= \int \frac{1}{2 \sin^2 x} dx \\ &= \frac{1}{2} \int \operatorname{cosec}^2 x dx = -\frac{1}{2} \cot x + c \end{aligned}$$

10. $\int \sqrt{1 - \cos 2x} dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \sqrt{1 - \cos 2x} dx &= \int \sqrt{2 \sin^2 x} dx \\ &= \sqrt{2} \int \sin x dx = -\sqrt{2} \cos x + c \end{aligned}$$

11. $\int \frac{x^2 - 1}{x^4 - 1} dx = ?$

Speedy Solution :-

$$\int \frac{x^2 - 1}{x^4 - 1} dx = \int \frac{x^2 - 1}{(x^2 + 1)(x^2 - 1)} dx = \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c$$

12. $\int \frac{x}{x+1} dx = ?$

Speedy Solution :-

$$\int \frac{x}{x+1} dx = \int \frac{x+1-1}{x+1} dx = \int \left(1 - \frac{1}{x+1} \right) dx = x - \log(x+1) + c$$

13. $\int x^{-7} dx = ?$

Speedy Solution :-

$$\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} = \frac{-x^{-6}}{6} + c$$

14. $\int \frac{1}{1 - \cos x} dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \frac{1}{1 - \cos x} dx &= \int \frac{1}{2 \sin^2 \frac{x}{2}} dx = \frac{1}{2} \int \operatorname{cosec}^2 \frac{x}{2} dx \\ &= \frac{1}{2} \times \frac{-\cot \frac{x}{2}}{\frac{1}{2}} = -\cot \frac{x}{2} + c \end{aligned}$$

15. $\int \tan^{-1} \left(\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right) dx = ?$

Speedy Solution :-

$$\begin{aligned} \int \tan^{-1} \left(\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right) dx &= \int \tan^{-1} \left(\sqrt{\frac{2 \sin^2 x}{2 \cos^2 x}} \right) dx = \int \tan^{-1}(\tan x) dx \\ &= \int x \cdot dx = \frac{x^2}{2} + c \end{aligned}$$

PREVIOUS YEAR'S RRB'S QUESTIONS

1. $\int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx$ का मान ज्ञात करें -
 (A) 5 (B) 8 (C) 6 (D) 1

(RRB महेंद्रगढ़ Diesel Driver, 2001)

Speedy Solution : (B)

$$\begin{aligned} \int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx &= \int_0^{2\pi} \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2 \sin \frac{x}{4} \cdot \cos \frac{x}{4}} dx \\ &= \int_0^{2\pi} \sqrt{\left(\sin \frac{x}{4} + \cos \frac{x}{4}\right)^2} dx = \left(\frac{1}{4}\right) \left[-\cos \frac{x}{4} + \sin \frac{x}{4}\right]_0^{2\pi} \\ &= 4 \left[\left(-\cos \frac{\pi}{2} + \sin \frac{\pi}{2}\right) - \left(-\cos 0 + \sin 0\right) \right] \\ &= 4[(0+1) - (-1+0)] = 8 \end{aligned}$$

2. $\int \frac{dx}{1 + \sin x}$ का मान निकालें -
 (A) $\tan x - \sec x + c$ (B) $\sin x - \sec x + c$
 (C) $\sec x + c$ (D) $-\tan x + c$

(RRB कोलकता Apprentice, Supervisor, 2001)

Speedy Solution : (A)

$$\begin{aligned} \int \frac{1}{1 + \sin x} dx &= \int \frac{1(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx \\ &= \int \frac{1 - \sin x}{1 - \sin^2 x} dx = \int \frac{1 - \sin x}{\cos^2 x} dx \\ &= \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx \\ &= \int \sec^2 x dx - \int \tan x \cdot \sec x dx = \tan x - \sec x + c \end{aligned}$$

3. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ का मान क्या होगा ?

- (A) 2π (B) π (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

(RRB कोलकता Apprentice, Supervisor, 2001)

Speedy Solution : (D)

माना कि $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots (i)$

तब, $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

या, $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots (ii)$

समीकरण (i) और (ii) को जोड़ने पर

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\frac{\pi}{2}} 1 dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} \end{aligned}$$

$$\therefore I = \frac{\pi}{2}$$

4. $\int \tan^2 x dx$ का मान निकालें ?

- (A) $\tan x - x + c$ (B) $\tan x - \sec x + c$
 (C) $\sec x + c$ (D) $\sec x$

(RRB भुवनेश्वर Tech., 2001)

Speedy Solution : (A)

$$\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \int \sec^2 x dx - \int 1 dx = \tan x - x + c$$

5. $\int \frac{dx}{1 - \sin x} = ?$

- (A) $\tan x + \sec x + c$ (B) $\sin x + \sec x + c$
 (C) $\sin x + \tan x$ (D) $\tan x + c$

(RRB मालदा A.S.M., 2004)

Speedy Solution : (A)

$$\int \frac{dx}{1 - \sin x} = \int \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)} dx = \int \frac{1 + \sin x}{1 - \sin^2 x} dx$$

$$= \int \frac{1 + \sin x}{\cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) dx$$

$$= \int \sec^2 x dx + \int \sec x \cdot \tan x dx = \tan x + \sec x + c$$

6. $\int \frac{d\theta}{1 + \cos \theta} = ?$

- (A) $\sin \frac{\theta}{2} + c$ (B) $\tan \frac{\theta}{2} + c$ (C) $\sin \theta + c$ (D) $\tan \theta + c$

(RRB राँची A.S.M., 2002)

Speedy Solution : (B)

$$\int \frac{d\theta}{1 + \cos \theta} = \int \frac{d\theta}{2 \cos^2 \frac{\theta}{2}} = \frac{1}{2} \int \sec^2 \frac{\theta}{2} d\theta$$

$$= \frac{1}{2} \cdot 2 \tan \frac{\theta}{2} + c = \tan \frac{\theta}{2} + c$$

7. $\int \frac{e^x}{2 + e^x} dx = ?$

- (A) $\log(2 + e^x) + c$ (B) $\log e^x$
(C) $\log e^x + c$ (D) $\sin e^x + c$

(RRB गोरखपुर E.S.M. - II, 2003)

Speedy Solution : (A)

माना कि $2 + e^x = y$

$$\therefore e^x dx = dy$$

$$\therefore \int \frac{dy}{y} = \log y + c = \log(2 + e^x) + c$$

8. $\int \sqrt{1 - \cos 2x} dx = ?$

- (A) $\sin x + c$ (B) $\cos x + c$
(C) $-\sqrt{2} \sin x + c$ (D) $-\sqrt{2} \cos x + c$

(RRB चेन्नई J.E., 2002)

Speedy Solution : (D)

$$\int \sqrt{1 - \cos 2x} dx = \int \sqrt{2 \sin^2 x} dx = \sqrt{2} \int \sin x dx = -\sqrt{2} \cos x + c$$

9. $\int \frac{x^2 - 1}{x^4 - 1} dx = ?$

- (A) $\sec^{-1} x + c$ (B) $\cos^{-1} x + c$
(C) $\tan^{-1} x + c$ (D) $\sin^{-1} x + c$

(RRB कोलकाता Asst. Driver, 2003)

Speedy Solution : (C)

$$\int \frac{x^2 - 1}{x^4 - 1} dx = \int \frac{x^2 - 1}{(x^2 + 1)(x^2 - 1)} dx = \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c$$

10. $\int \frac{1}{\sin^2 x \cdot \cos^2 x} dx = ?$

- (A) $-2 \cot 2x + c$ (B) $\cot x + c$
(C) $\tan x + \cot x + c$ (D) $\tan x - \cot x + c$

(RRB गोरखपुर Goods Guard, 2003)

Speedy Solution : (A)

$$\int \frac{1}{\sin^2 x \cdot \cos^2 x} dx = 4 \int \frac{1}{4 \sin^2 x \cdot \cos^2 x} dx$$

$$= 4 \int \frac{1}{(2 \sin x \cdot \cos x)^2} dx = 4 \int \frac{1}{\sin^2 2x} dx$$

$$= 4 \int \operatorname{cosec}^2 2x dx = 4 \times -\frac{\cot 2x}{2} + c$$

$$= -2 \cot 2x + c$$

11. $\int \tan^2 x dx = ?$

- (A) $-\cot x - x + c$ (B) $\tan x - x + c$
(C) $\tan x - \frac{x^2}{2} + c$ (D) $\cot x + x + c$

(RRB मुम्बई Loco Pilot, 2004)

Speedy Solution : (B)

$$\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + c$$

12. $\int \sin^2 \frac{x}{2} dx = ?$

- (A) $x - \sin x + c$ (B) $\frac{x}{2} - \sin x + c$
(C) $\frac{x}{2} - \frac{\sin x}{2} + c$ (D) $\frac{x}{2} - \cos x + c$

(RRB सिकन्दराबाद T.A., 2004)

Speedy Solution : (C)

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2} (x - \sin x) + c = \frac{x}{2} - \frac{\sin x}{2} + c$$

13. $\int (3x + 5)^7 dx = ?$

- (A) $\frac{(3x + 5)^8}{8} + c$ (B) $\frac{(3x + 5)^6}{6} + c$
(C) $\frac{(3x + 5)^8}{24} + c$ (D) $18(3x + 5)^6$

(RRB कोलकाता Diesel Driver, 2004)

Speedy Solution : (C)

$$\int (3x + 5)^7 dx = \frac{(3x + 5)^{7+1}}{(7+1) \times 3} + c = \frac{(3x + 5)^8}{24} + c$$