





a)  $(6 + 17i)$

b)  $(-6 + 17i)$

c)  $(6 - 15i)$

d)  $(6 - 17i)$

18.  ${}^{36}C_{34} = ?$  [1]

a) 610

b) 630

c) 1224

d) 612

19. **Assertion (A):** The expansion of  $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 + \dots + n_{c_n}x^n$ . [1]

**Reason (R):** If  $x = -1$ , then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. [1]

**Reason (R):** The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

**Section B**

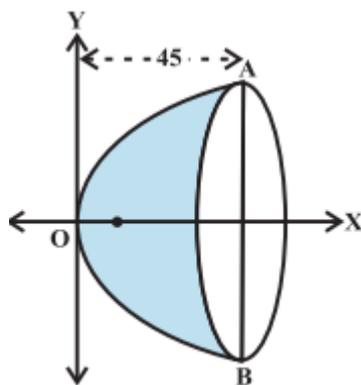
21. Write the domain of the real function  $f(x) = \frac{1}{\sqrt{|x|-x}}$  [2]

OR

Find the domain and range of the real function  $f(x) = \sqrt{9-x^2}$ .

22. Differentiate  $\frac{x}{\sin x}$  respect to  $x$ . [2]

23. The focus of a parabolic mirror as shown in is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, find the distance AB [2]



OR

Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas  $4x^2 + y = 0$ .

24. Two sets A and B are, such that  $n(A \cup B) = 21$ ,  $n(A) = 10$ ,  $n(B) = 15$ , find  $n(A \cap B)$  and  $n(A - B)$ . [2]

25. Find the angles between the pairs of straight lines  $x - 4y = 3$  and  $6x - y = 11$ . [2]

**Section C**

26. Find the domain and range of the function  $f(x) = \frac{x^2-9}{x-3}$  [3]

27. To receive Grade A, in a mathematics course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Ragini's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Ragini must obtain in fifth examination to get Grade A in the course. [3]

28. Find the point on the y-axis which is equidistant from the points A(3, 1, 2) and B(5, 5, 2). [3]

OR

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

29. Using binomial theorem, expand:  $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$  [3]

OR

Find  $(a + b)^4 - (a - b)^4$ . Hence, evaluate  $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$

30. Express the complex number  $\left(-2 - \frac{1}{3}i\right)^3$  in the form of a + ib. [3]

OR

Find the square root of  $3 - 4i$

31. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical  $C_1$ , 50 to chemical  $C_2$  and 30 to both the chemicals  $C_1$  and  $C_2$ . Find the number of individuals exposed to (i) chemical  $C_1$  but not chemical  $C_2$  (ii) Chemical  $C_2$  but not chemical  $C_1$  (iii) Chemical  $C_2$  or chemical  $C_1$ . [3]

#### Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: [5]

- one is red and two are white
- two are blue and one is red
- one is red.

33. Evaluate:  $\lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$ . [5]

OR

Differentiate  $x^2 \sin x$  from first principle.

34. In an increasing GP, the sum of the first and last terms is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in this GP? [5]

35. Prove that:  $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$ . [5]

OR

Prove that  $\cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} = \frac{1}{16}$

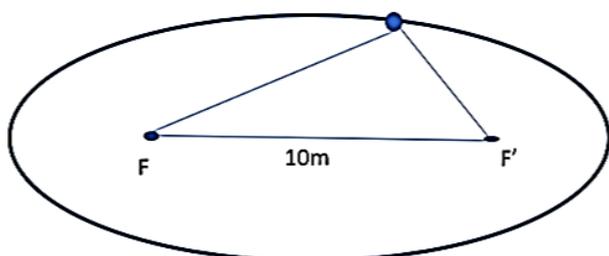
#### Section E

36. Read the following text carefully and answer the questions that follow: [4]

A farmer wishes to install 2 handpumps in his field for watering.



The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



- i. Name the curve traced by farmer and hence find the foci of curve. (1)
- ii. Find the equation of curve traced by farmer. (1)
- iii. Find the length of major axis, minor axis and eccentricity of curve along which farmer moves. (2)

**OR**

- iv. Find the length of latus rectum. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

Consider the data.

Class	Frequency
0-10	6
10-20	7
20-30	15
30-40	16
40-50	4
50-60	2

- i. Find the mean deviation about median. (1)
- ii. Find the Median. (1)
- iii. Write the formula to calculate the Mean deviation about median? (2)

**OR**

Write the formula to calculate median? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

The purpose of the student council is to give students an opportunity to develop leadership by organizing and carrying out school activities and service projects. Create an environment where every student can voice out their concern or need. Raju, Ravi Joseph, Sangeeta, Priya, Meena and Aman are members of student's council. There is a photo session in a school these 7 students are to be seated in a row for photo session.



- i. Find the total number of arrangements so that Raju and Ravi are at extreme positions? (1)
- ii. Find the number of arrangements so that Joseph is sitting in the middle. (1)
- iii. Find the number of arrangements so that three girls are together. (2)

**OR**

Find the number of arrangements so that Aman and Ravi are not together? (2)

# Solution

## Section A

1. (a) 2

**Explanation:** Given  $\sin \theta + \operatorname{cosec} \theta = 2$

Squaring on both sides, we get

$$\sin^2 \theta + \operatorname{csc}^2 \theta + 2 \sin \theta \operatorname{csc} \theta = 4 \quad [ \because \sin \theta \operatorname{cosec} \theta = 1 ]$$

$$\Rightarrow \sin^2 \theta + \operatorname{cosec}^2 \theta = 4 - 2 = 2$$

2. (a)  $(-1, 0) \cup (1, 2) \cup (2, \infty)$

**Explanation:** For  $f(x)$  to be real, we must have

$$4 - x^2 \neq 0 \text{ and } x^3 - x > 0$$

$$\Rightarrow x^2 \neq 4 \text{ and } x(x^2 - 1) > 0$$

$$\Rightarrow x \neq 2, -2 \text{ and } x(x-1)(x+1) > 0$$

$$\Rightarrow x \neq 2, -2 \text{ and } -1 < x < 0, 1 < x < \infty$$

$$\therefore \text{Domain} = (-1, 0) \cup (1, 2) \cup (2, \infty)$$

3. (a)  $\frac{1}{221}$

**Explanation:** Total number of ways drawing 2 cards successively without replacement

$$= {}^{52}C_1 \times {}^{51}C_1 \text{ and number of ways 2 aces without replacement} = {}^4C_1 \times {}^3C_1$$

$$\therefore \text{Required probability} = \frac{{}^4C_1 \times {}^3C_1}{{}^{52}C_1 \times {}^{51}C_1} = \frac{4 \times 3}{52 \times 51}$$

$$= \frac{1}{13 \times 17} = \frac{1}{221}$$

4. (a)  $\frac{1}{2}$

**Explanation:** Given,  $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \rightarrow 0} \frac{x \left[ \frac{\tan 2x}{x} - 1 \right]}{x \left[ 3 - \frac{\sin x}{x} \right]}$

$$\lim_{x \rightarrow 0} \frac{\frac{\tan 2x}{2x} \times 2 - 1}{3 - \frac{\sin x}{x}} = \frac{1 \cdot 2 - 1}{3 - 1} = \frac{2 - 1}{2} = \frac{1}{2}$$

5.

(b) (4, 7)

**Explanation:** Let A (4, 78) and B (-2, 6) be the given vertex. Let C(h, k) be the third vertex.

The centroid of  $\triangle ABC$  is  $\left( \frac{4-2+h}{3}, \frac{8+6+k}{3} \right)$

It is given that the centroid of triangle ABC is (2, 7) as obtained from above formula,

$$\therefore \frac{4-2+h}{3} = 2, \frac{8+6+k}{3} = 7$$

$$\Rightarrow h = 4, k = 7$$

Thus, the third vertex is (4, 7)

6.

(d)  $C - D = E$

**Explanation:**  $C - D = \{a, b, c\} - \{c, d\} = \{a, b\}$

But  $E = \{d\}$

Hence  $C - D \neq E$

7.

(d) 1

**Explanation:** 1

$$\text{Let } z = \frac{1+i}{1-i}$$

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow z = \frac{1+i^2+2i}{1-i^2}$$

$$\Rightarrow z = \frac{2i}{2}$$

$$\Rightarrow z = i$$

$$\Rightarrow z^4 = i^4$$

Since  $i^2 = -1$ , we have:

$$\Rightarrow z^4 = i^2 \times i^2$$

$$\Rightarrow z^4 = 1$$

8.

(d) {2, 4, 6}

**Explanation:** We have,  $x + 2y = 8$

$$y = \frac{8-x}{2}$$

since, x and y are Natural numbers, So x must be an even number.

$$\text{if } x = 2, y = 3;$$

$$\text{if } x = 4, y = 2;$$

$$\text{if } x = 6, y = 1.$$

So, relation  $R = \{(2, 3), (4, 2), (6, 1)\}$

Hence, the domain of R is {2, 4, 6}.

9.

(b)  $\left(\frac{-2}{3}, \infty\right)$

**Explanation:**  $(x + 3) + 4 > -2x + 5$

$$\Rightarrow x + 7 > -2x + 5$$

$$\Rightarrow x + 7 + 2x > -2x + 5 + 2x$$

$$\Rightarrow 3x + 7 > 5$$

$$\Rightarrow 3x + 7 - 7 > 5 - 7$$

$$\Rightarrow 3x > -2$$

$$\Rightarrow x > \frac{-2}{3}$$

$$\Rightarrow x \in \left(\frac{-2}{3}, \infty\right)$$

10.

(b)  $\left(\frac{9\pi}{32}\right)^c$

**Explanation:**  $50^\circ 37' 30'' = 50^\circ + \left(37 \frac{30}{60}\right)' = 50^\circ + \left(\frac{75}{2}\right)' = 50^\circ + \left(\frac{75}{2 \times 60}\right)^\circ = \left(50 \frac{5}{8}\right)^\circ = \left(\frac{405}{8}\right)^\circ$

$$180^\circ = \pi^c \Rightarrow 1^\circ = \left(\frac{\pi}{180}\right)^c \Rightarrow \left(\frac{405}{8}\right)^\circ = \left(\frac{\pi}{180} \times \frac{405}{8}\right)^c = \left(\frac{9\pi}{32}\right)^c$$

11.

(c)  $2^n$

**Explanation:** The total no of subsets =  $2^n$

12.

(b) 3069

**Explanation:** This is a GP in which  $a = 3$ ,  $r = \frac{6}{3} = 2$  and  $l = 1536$

$$\therefore \text{required sum} = \frac{(lr-a)}{(r-1)} = \frac{(1536 \times 2 - 3)}{(2-1)} = (3072 - 3) = 3069.$$

13.

(c)  $(n + 2) \cdot 2^{n-1}$

**Explanation:** Here,  $C_0 + 2C_1 + 3C_2 + \dots + (n + 1)C_n$

$$= (C_0 + C_1 + C_2 + \dots + C_n) + (C_1 + 2C_2 + 3C_3 + \dots + nC_n)$$

$$= 2^n + n \cdot 2^{n-1} = (n + 2) \cdot 2^{n-1}$$

14.

(a)  $-x > -5$

**Explanation:** Given  $x < 5$

Multiplying both sides of the above inequality by -1, we get

$$-x > -5$$

[The sign of the inequality is to be reversed if both sides of an inequality are multiplied by the same negative real number]

15.

(d) A

**Explanation:** Common between set A and  $(A \cup B)$  is set A itself

16.

(d)  $\tan x$

**Explanation:**  $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \frac{2 \sin^2 x + \sin x}{2 \sin x \cos x + \cos x} = \frac{\sin x(2 \sin x + 1)}{\cos x(2 \sin x + 1)} = \tan x$

17. (a)  $(6 + 17i)$

**Explanation:**  $(2 - 3i)(-3 + 4i) = (-6 + 8i + 9i - 12i^2) = (-6 + 17i + 12) = (6 + 17i)$

18.

(b) 630

**Explanation:**  ${}^{36}C_{34} = {}^{36}C_{(36-34)} = {}^{36}C_2 = \frac{36 \times 35}{2} = 630$ .

19.

(b) Both A and R are true but R is not the correct explanation of A.

**Explanation: Assertion:**

$$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 + \dots + n_{c_n}x^n$$

**Reason:**

$$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + \dots + n_{c_n}(1)^{n-n}(-1)^n$$

$$= n_{c_0} - n_{c_1} + n_{c_2} - n_{c_3} + \dots + (-1)^n n_{c_n}$$

Each term will cancel each other

$$\therefore (1 + (-1))^n = 0$$

Reason is also the but not the correct explanation of Assertion.

20.

(c) A is true but R is false.

**Explanation: Assertion** Mean of the given series

$$\begin{aligned} \bar{x} &= \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n} \\ &= \frac{4+7+8+9+10+12+13+17}{8} = 10 \end{aligned}$$

xi	$ xi - \bar{x} $
4	$ 4 - 10  = 6$
7	$ 7 - 10  = 3$
8	$ 8 - 10  = 2$
9	$ 9 - 10  = 1$
10	$ 10 - 10  = 0$
12	$ 12 - 10  = 2$
13	$ 13 - 10  = 3$
17	$ 17 - 10  = 7$
$\sum x_i = 80$	$\sum  x_i - \bar{x}  = 24$

$\therefore$  Mean deviation about mean

$$= \frac{\sum |x_i - \bar{x}|}{n} = \frac{24}{8} = 3$$

**Reason** Mean of the given series

$$\begin{aligned} \bar{x} &= \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n} \\ &= \frac{38+70+48+40+42+55}{10} = 50 \end{aligned}$$

$\therefore$  Mean deviation about mean

$$\begin{aligned} &= \frac{\sum |x_i - \bar{x}|}{n} \\ &= \frac{84}{10} = 8.4 \end{aligned}$$

Hence, Assertion is true and Reason is false.

Section B

21. Case I : When  $x > 0$ . Then, we have,

$$|x| = x$$

$$\Rightarrow \frac{1}{\sqrt{|x|-x}} = \frac{1}{\sqrt{x-x}} = \frac{1}{0} = \infty$$

Case II : When  $x < 0$

$$|x| = -x$$

$$\Rightarrow \frac{1}{\sqrt{|x|-x}} = \frac{1}{\sqrt{-x-x}} = \frac{1}{\sqrt{-2x}} \text{ (exists because when } x < 0, -2x > 0)$$

$\Rightarrow f(x)$  is defined when  $x < 0$

Therefore, domain =  $(-\infty, 0)$

OR

It is clear that,  $f(x) = \sqrt{9 - x^2}$  is not defined when  $(9 - x^2) < 0$ , i.e.

When  $x^2 > 9$  i.e when  $x > 3$  or  $x < -3$

$$\text{dom}(f) = |x \in R : -3 \leq x \leq 3|$$

$$\text{Also, } y = \sqrt{9 - x^2} \Rightarrow y^2 = (9 - x^2)$$

$$\Rightarrow x = \sqrt{9 - y^2}$$

clearly,  $x$  is not defined when  $(9 - y^2) < 0$

$$\text{but } (9 - y^2) < 0 \Rightarrow y^2 > 9$$

$$\Rightarrow y > 3 \text{ or } y < -3$$

$$\text{range}(f) = \{y \in R : -3 \leq y \leq 3\} .$$

22. Let  $f(x) = \frac{x}{\sin x}$

$$\therefore f(x) = \frac{d}{dx} \left( \frac{x}{\sin x} \right)$$

$$= \frac{\sin x \frac{d}{dx} x - x \frac{d}{dx} \sin x}{(\sin x)^2}$$

$$= \frac{\sin x \cdot 1 - x \cdot \cos x}{\sin^2 x}$$

$$= \text{cosec } x - x \cot x \text{ cosec } x$$

$$= (1 - x \cot x) \cdot \text{cosec } x$$

23. Since the distance from the focus to the vertex is 5 cm. We have,  $a = 5$ . If the origin is taken at the vertex and the axis of the mirror lies along the positive x-axis, the equation of the parabolic section is

$$y^2 = 4(5)x = 20x \implies \text{required equation of parabola } y^2 = 20x$$

Note that  $x = 45$ . Thus

$$y^2 = 900$$

$$\text{Therefore } y = \pm 30$$

$$\text{Hence } AB = 2y = 2 \times 30 = 60 \text{ cm}$$

OR

We are given that:

$$4x^2 + y = 0$$

$$\Rightarrow \frac{-y}{4} = x^2$$

Comparing the given equation with  $x^2 = -4ay$

$$4a = \frac{1}{4} \implies a = \frac{1}{16}$$

$$\therefore \text{Vertex} = (0, 0)$$

$$\text{Focus} = (0, -a) = \left(0, \frac{-1}{16}\right)$$

Equation of the directrix:

$$y = a$$

$$\text{i.e. } y = \frac{1}{16}$$

$$\text{Axis} = x = 0$$

$$\text{Therefore, length of the latus rectum} = 4a = \frac{1}{4}$$

24. Using identity,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\begin{aligned}
21 &= 10 + 15 - n(A \cap B) \\
\therefore n(A \cap B) &= (10 + 15) - 21 \\
&= 25 - 21 = 4 \\
\therefore n(A - B) &= n(A \cap B') \\
&= n(A) - n(A \cap B) \\
&= 10 - 4 \\
&= 6
\end{aligned}$$

25. Given that equations of the lines are,

$$x - 4y = 3 \dots (i)$$

$$6x - y = 11 \dots (ii)$$

Let  $m_1$  and  $m_2$  be the slopes of these lines.

$$\text{Here, } m_1 = \frac{1}{4}, m_2 = 6$$

Let  $\theta$  be the angle between the lines.

Then,

$$\begin{aligned}
\tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\
&= \left| \frac{\frac{1}{4} - 6}{1 + \frac{3}{2}} \right| \\
&= \frac{23}{10}
\end{aligned}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{23}{10} \right)$$

Therefore, the acute angle between the lines is  $\tan^{-1} \left( \frac{23}{10} \right)$

### Section C

26. Here  $f(x) = \frac{x^2 - 9}{x - 3}$

$f(x)$  assume real values for all real values of  $x$  except for  $x - 3 = 0$  i.e.  $x = 3$

Thus domain of  $f(x) = \mathbb{R} - \{3\}$

Let  $f(x) = y$

$$\therefore y = \frac{x^2 - 9}{x - 3} = \frac{(x+3)(x-3)}{(x-3)}$$

$$\Rightarrow y = x + 3$$

$y$  takes all real values except 6 as domain  $= \mathbb{R} - \{3\}$

Thus range of  $f(x) = \mathbb{R} - \{6\}$ .

27. Let the marks obtained by Ragini in fifth examination be  $x$ .

$$\text{Then average of five examinations} = \frac{87+92+94+95+x}{5}$$

$$\text{Now } \frac{87+92+94+95+x}{5} \geq 90 \Rightarrow \frac{368+x}{5} \geq 90$$

Multiplying both sides by 5, we have

$$368 + x \geq 450$$

$$\Rightarrow x \geq 450 - 368$$

$$\Rightarrow x \geq 82$$

Thus the minimum marks needed to be obtained by Ragini = 82.

28. Consider,  $C(0, y, 0)$  point which lies on the  $y$ -axis and is equidistant from points  $A(3, 1, 2)$  and  $B(5, 5, 2)$ .

$$\therefore AC = BC$$

$$\sqrt{(0-3)^2 + (y-1)^2 + (0-2)^2} = \sqrt{(0-5)^2 + (y-5)^2 + (0-2)^2}$$

Squaring both sides,

$$\Rightarrow (0-3)^2 + (y-1)^2 + (0-2)^2 = (0-5)^2 + (y-5)^2 + (0-2)^2$$

$$\Rightarrow 9 + y^2 - 2y + 1 + 4 = 25 + y^2 - 10y + 25 + 4$$

$$\Rightarrow 8y = 40$$

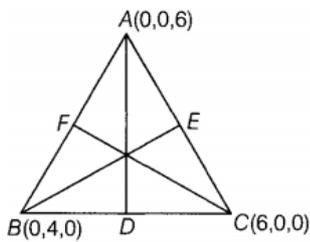
$$\Rightarrow y = 5$$

The coordinate of  $C$  is  $(0, 5, 0)$ .

OR

$ABC$  is a triangle with vertices  $A(0, 0, 6)$ ,  $(0, 4, 0)$  and  $C(6, 0, 0)$ .

Let points  $D$ ,  $E$  and  $F$  are the mid-points of  $BC$ ,  $AC$  and  $AB$ , respectively. So,  $AD$ ,  $BE$  and  $CF$  will be the medians of the triangle.



Coordinates of point  $D = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$

[ $\therefore$  coordinates of mid-point  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ ]

Coordinates of point  $E = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = (3, 0, 3)$

and coordinates of point  $F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$

Now, length of median

$AD =$  Distance between point A and D

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2}$$

[ $\therefore$  distance  $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$ ]

$$= \sqrt{9+4+36}$$

$$= \sqrt{49} = 7$$

Similarly,  $BE = \sqrt{(0-3)^2 + (4-0)^2 + (0-3)^2}$

$$= \sqrt{9+16+9} = \sqrt{34}$$

and  $CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2}$

$$= \sqrt{36+4+9} = \sqrt{49} = 7$$

Hence, length of the medians are 7,  $\sqrt{34}$  and 7.

29. To find: Expansion of  $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$  by means of binomial theorem

Formula used:  ${}^n C_r = \frac{n!}{(n-r)!(r)!}$

$$(a+b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + {}^n C_n b^n$$

Now here We have,  $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$

$$\begin{aligned} &= \left[6C_0 \left(\frac{2x}{3}\right)^{6-0}\right] + \left[6C_1 \left(\frac{2x}{3}\right)^{6-1} \left(-\frac{3}{2x}\right)^1\right] + \left[6C_2 \left(\frac{2x}{3}\right)^{6-2} \left(-\frac{3}{2x}\right)^2\right] \\ &+ \left[6C_3 \left(\frac{2x}{3}\right)^{6-3} \left(-\frac{3}{2x}\right)^3\right] + \left[6C_4 \left(\frac{2x}{3}\right)^{6-4} \left(-\frac{3}{2x}\right)^4\right] \\ &+ \left[6C_5 \left(\frac{2x}{3}\right)^{6-5} \left(-\frac{3}{2x}\right)^5\right] + \left[6C_6 \left(-\frac{3}{2x}\right)^6\right] \\ &= \left[\frac{6!}{0!(6-0)!} \left(\frac{2x}{3}\right)^6\right] - \left[\frac{6!}{1!(6-1)!} \left(\frac{2x}{3}\right)^5 \left(\frac{3}{2x}\right)\right] + \left[\frac{6!}{2!(6-2)!} \left(\frac{2x}{3}\right)^4 \left(\frac{9}{4x^2}\right)\right] - \left[\frac{6!}{3!(6-3)!} \left(\frac{2x}{3}\right)^3 \left(\frac{27}{8x^3}\right)\right] \\ &+ \left[\frac{6!}{4!(6-4)!} \left(\frac{2x}{3}\right)^2 \left(\frac{81}{16x^4}\right)\right] - \left[\frac{6!}{5!(6-5)!} \left(\frac{2x}{3}\right)^1 \left(\frac{243}{32x^5}\right)\right] + \left[\frac{6!}{6!(6-6)!} \left(\frac{729}{64x^6}\right)\right] \\ &= \left[1 \left(\frac{64x^6}{729}\right)\right] - \left[6 \left(\frac{32x^5}{243}\right) \left(\frac{3}{2x}\right)\right] + \left[15 \left(\frac{16x^4}{81}\right) \left(\frac{9}{4x^2}\right)\right] - \left[20 \left(\frac{8x^3}{27}\right) \left(\frac{27}{8x^3}\right)\right] \\ &+ \left[15 \left(\frac{4x^2}{9}\right) \left(\frac{81}{16x^4}\right)\right] - \left[6 \left(\frac{2x}{3}\right) \left(\frac{243}{32x^5}\right)\right] + \left[1 \left(\frac{729}{64x^6}\right)\right] \\ &= \frac{64}{729} x^6 - \frac{32}{27} x^4 + \frac{20}{3} x^2 - 20 + \frac{135}{4} \frac{1}{x^2} - \frac{243}{8} \frac{1}{x^4} + \frac{729}{64} \frac{1}{x^6} \end{aligned}$$

OR

$$(a+b)^4 = [{}^4 C_0 a^4 + {}^4 C_1 a^3 b + {}^4 C_2 a^2 b^2 + {}^4 C_3 a b^3 + {}^4 C_4 b^4]$$

$$\text{and } (a-b)^4 = [{}^4 C_0 a^4 - {}^4 C_1 a^3 b + {}^4 C_2 a^2 b^2 - {}^4 C_3 a b^3 + {}^4 C_4 b^4]$$

$$\therefore (a+b)^4 - (a-b)^4 = 2 [{}^4 C_1 a^3 b + {}^4 C_3 a b^3]$$

$$= 2 [4a^3 b + 4ab^3] = 8ab [a^2 + b^2]$$

$$\therefore (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8 \cdot \sqrt{3} \cdot \sqrt{2} [(\sqrt{3})^2 + (\sqrt{2})^2]$$

$$= 8 \cdot \sqrt{3} \cdot \sqrt{2} [3 + 2] = 40 \cdot \sqrt{3} \cdot \sqrt{2} = 40\sqrt{6}$$

30.  $\left(-2 - \frac{1}{3}i\right)^3 = -\left(2 + \frac{1}{3}i\right)^3$

$$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times (2)^2 \times \frac{1}{3}i + 3 \times 2 \times \left(\frac{1}{3}i\right)^2\right]$$

$$\begin{aligned}
&= - \left[ 8 + \frac{1}{27}i^3 + 4i + \frac{2}{3}i^2 \right] = - \left[ 8 - \frac{1}{27}i + 4i - \frac{2}{3} \right] \left[ \begin{array}{l} \because i^3 = -i \\ i^2 = -1 \end{array} \right] \\
&= \left[ \left( 8 - \frac{2}{3} \right) + \left( 4 - \frac{1}{27} \right) i \right] \\
&= - \left[ \frac{22}{3} + \frac{107}{27}i \right] = \frac{-22}{3} - \frac{107}{27}i
\end{aligned}$$

OR

$$\text{Let } x + yi = \sqrt{3 - 4i}$$

Squaring both sides, we get

$$x^2 - y^2 + 2xyi = 3 - 4i$$

Equating the real and imaginary parts

$$x^2 - y^2 = 3 \dots (i)$$

$$\text{and } 2xy = -4 \Rightarrow xy = -2$$

Now from the identity, we know

$$(x^2 + y^2) = (x^2 - y^2)^2 + 4x^2y^2$$

$$(x^2 + y^2)^2 = (3)^2 + 4(-2)^2$$

$$= 9 + 16 = 25$$

$$\therefore x^2 + y^2 = 5 \dots (ii) \text{ [Neglecting (-) sign as } x^2 + y^2 > 0]$$

Solving (i) and (ii) we get

$$x^2 = 4 \text{ and } y^2 = 1$$

$$x = \pm 2 \text{ and } y = \pm 1$$

Since the sign of  $xy$  is negative

$$\therefore \text{if, } x = 2, y = -1$$

$$\text{and if } x = -2, y = 1$$

$$\therefore \sqrt{-5 + 12i} = \pm(2 - i)$$

31. Let  $S$  denote the universal set consisting of individuals suffering from the skin disorder,  $A$  denote the set of individuals exposed to chemical  $C_1$  and  $B$  denote the set of individuals exposed to chemical  $C_2$ .

Now,

$$n(S) = 200$$

$$n(A) = 120$$

$$n(B) = 50$$

$$\text{and } n(A \cap B) = 30$$

i. Chemical  $C_1$  but not chemical  $C_2$

Number of individuals exposed to chemical  $C_1$  but not chemical  $C_2$  is

$$= n(A \cap B')$$

$$= n(A) - n(A \cap B)$$

$$= 120 - 30 = 90$$

ii. Number of individuals exposed to chemical  $C_2$  but not chemical  $C_1$

$$= n(A' \cap B)$$

$$= n(B) - n(A \cap B)$$

$$= 50 - 30 = 20$$

iii. Number of individuals exposed to chemical  $C_1$  or chemical  $C_2$

$$= n(A \cup B)$$

$$= n(A) + n(B) - n(A \cap B)$$

$$= 120 + 50 - 30$$

$$= 140$$

#### Section D

32. Bag contains:

6 -Red balls

4 -White balls

8 -Blue balls



$$= 1 + \frac{4}{11}$$

$$= \frac{15}{11}$$

OR

We have to find derivative of  $f(x) = x^2 \sin x$

Derivative of a function  $f(x)$  is given by  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  {where  $h$  is a very small positive number}

$\therefore$  Derivative of  $f(x) = x^2 \sin x$  is given as  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

Using  $(a+b)^2 = a^2 + 2ab + b^2$ , we get

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{h^2 \sin(x+h) + x^2 \sin(x+h) + 2hx \sin(x+h) - x^2 \sin x}{h}$$

Using the algebra of limits, we have

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{h^2 \sin(x+h)}{h} + \lim_{h \rightarrow 0} \frac{x^2 \sin(x+h) - x^2 \sin x}{h} + \lim_{h \rightarrow 0} \frac{2hx \sin(x+h)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} h \sin(x+h) + \lim_{h \rightarrow 0} \frac{x^2(\sin(x+h) - \sin x)}{h} + \lim_{h \rightarrow 0} 2x \sin(x+h)$$

$$\Rightarrow f'(x) = 0 \times \sin(x+0) + 2x \sin(x+0) + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

Using the algebra of limits we have

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

We can't evaluate the limits at this stage only as on putting value it will take  $\frac{0}{0}$  form. So, we need to do little modifications.

$$\text{Use: } \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{\cos\left(x + \frac{h}{2}\right) \sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

Using the algebra of limits:

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \cos\left(x + \frac{h}{2}\right)$$

$$\text{By using the formula we get } - \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \cos\left(x + \frac{h}{2}\right)$$

substitute the value of  $h$  to evaluate the limit:

$$\text{Therefore, } f'(x) = 2x \sin x + x^2 \cos(x+0) = 2x \sin x + x^2 \cos x$$

Hence,

$$\text{Derivative of } f(x) = (x^2 \sin x) \text{ is } (2x \sin x + x^2 \cos x)$$

34. Let the given GP contain  $n$  terms. Let  $a$  be the first term and  $r$  be the common ratio of this GP.

Since the given GP is increasing, we have  $r > 1$

$$\text{Now, } T_1 + T_n = 66 \Rightarrow a + ar^{(n-1)} = 66 \dots(i)$$

$$\text{And, } T_2 \times T_{n-1} = 128 \Rightarrow ar \times ar^{(n-2)} = 128$$

$$\Rightarrow a^2 r^{(n-1)} = 128 \Rightarrow ar^{(n-1)} = \frac{128}{a} \dots(ii)$$

Using (ii) and (i), we get

$$a + \frac{128}{a} = 66 \Rightarrow a^2 - 66a + 128 = 0$$

$$\Rightarrow a^2 - 2a - 64a + 128 = 0$$

$$\Rightarrow a(a-2) - 64(a-2) = 0$$

$$\Rightarrow (a-2)(a-64) = 0$$

$$\Rightarrow a = 2 \text{ or } a = 64$$

Putting  $a = 2$  in (ii), we get

$$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{4} = 32 \dots(iii)$$

Putting  $a = 64$  in (ii), we get

$$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{64 \times 64} = \frac{1}{32}, \text{ which is rejected, since } r > 1.$$

Thus,  $a = 2$  and  $r^{(n-1)} = 32$

$$\text{Now, } S_n = 126 \Rightarrow \frac{a(r^n - 1)}{(r - 1)} = 126$$

$$\Rightarrow 2 \left( \frac{r^n - 1}{r - 1} \right) = 126 \Rightarrow \frac{r^n - 1}{r - 1} = 63$$

$$\Rightarrow \frac{r^{(n-1)} \times r - 1}{r - 1} = 63 \Rightarrow \frac{32r - 1}{r - 1} = 63$$

$$\Rightarrow 32r - 1 = 63r - 63 \Rightarrow 31r = 62 \Rightarrow r = 2$$

$$\therefore r^{(n-1)} = 32 = 2^5 \Rightarrow n - 1 = 5 \Rightarrow n = 6$$

Hence, there are 6 terms in the given GP

35. We have to prove that  $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$ .

$$\text{LHS} = \sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$$

By regrouping the LHS and multiplying and dividing by 4 we get,

$$= \frac{1}{4} (2 \sin 66^\circ \sin 6^\circ) (2 \sin 78^\circ \sin 42^\circ)$$

$$\text{But } 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

Then the above equation becomes,

$$= \frac{1}{4} (\cos(66^\circ - 6^\circ) - \cos(66^\circ + 6^\circ)) (\cos(78^\circ - 42^\circ) - \cos(78^\circ + 42^\circ))$$

$$= \frac{1}{4} (\cos(60^\circ) - \cos(72^\circ)) (\cos(36^\circ) - \cos(120^\circ))$$

$$= \frac{1}{4} (\cos(60^\circ) - \cos(90^\circ - 18^\circ)) (\cos(36^\circ) - \cos(180^\circ - 120^\circ))$$

$$\text{But } \cos(90^\circ - \theta) = \sin \theta \text{ and } \cos(180^\circ - \theta) = -\cos(\theta).$$

Then the above equation becomes,

$$= \frac{1}{4} (\cos(60^\circ) - \cos(18^\circ)) (\cos(36^\circ) + \cos(60^\circ))$$

$$\text{Now, } \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

$$\sin(18^\circ) = \frac{\sqrt{5}-1}{4}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Substituting the corresponding values, we get

$$= \frac{1}{4} \left( \frac{1}{2} - \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left( \frac{2 - \sqrt{5} + 1}{4} \right) \left( \frac{\sqrt{5} + 1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left( \frac{3 - \sqrt{5}}{4} \right) \left( \frac{3 + \sqrt{5}}{4} \right)$$

$$= \frac{1}{4} \left( \frac{3^2 - (\sqrt{5})^2}{4 \times 4} \right)$$

$$= \frac{1}{4} \left( \frac{9 - 5}{16} \right)$$

$$= \frac{1}{16}$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

OR

$$\begin{aligned} \text{LHS} &= \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} \\ &= \cos \frac{2\pi}{15} \cos 2 \left( \frac{2\pi}{15} \right) \cos 4 \left( \frac{2\pi}{15} \right) \cos 8 \left( \frac{2\pi}{15} \right) \end{aligned}$$

$$\text{Put } \frac{2\pi}{15} = \alpha$$

$$\Rightarrow \text{LHS} = \cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha$$

$$= \frac{2 \sin \alpha [\cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha]}{2 \sin \alpha} \text{ [multiplying numerator and denominator by } 2 \sin \alpha \text{]}$$

$$= \frac{2 \sin \alpha \cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha}{2 \sin \alpha}$$

$$= \frac{2(\sin 2\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha)}{2(2 \sin \alpha)} \text{ [}\therefore 2 \sin \alpha \cos \alpha = \sin 2\alpha \text{ and multiplying numerator and denominator by } 2 \text{]}$$

$$= \frac{(2 \sin 2\alpha \cdot \cos 2\alpha) \cdot \cos 4\alpha \cdot \cos 8\alpha}{4 \sin \alpha}$$

$$= \frac{2(\sin 4\alpha \cdot \cos 4\alpha) \cos 8\alpha}{2(4 \sin \alpha)} \text{ [}\therefore 2 \sin \alpha \cos \alpha = \sin 2\alpha \text{ and multiplying numerator and denominator by } 2 \text{]}$$

$$= \frac{2(\sin 8\alpha \cdot \cos 8\alpha)}{2(8 \sin \alpha)}$$

$$= \frac{\sin 16\alpha}{16 \sin \alpha} = \frac{\sin(15\alpha + \alpha)}{16 \sin \alpha}$$

Now,  $15\alpha = 2\pi$ ,

$$= \frac{\sin(2\pi + \alpha)}{16 \sin \alpha} = \frac{\sin \alpha}{16 \sin \alpha} = \frac{1}{16} = \text{RHS}$$

$\therefore \text{LHS} = \text{RHS}$

Hence proved.

### Section E

36. i. The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.  
Two positions of hand pumps are foci Distance between two foci =  $2c = 10$  Hence  $c = 5$  Here foci lie on x axis & coordinates of foci =  $(\pm c, 0)$

Hence coordinates of foci =  $(\pm 5, 0)$

ii.  $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci =  $2a$

Sum of distances between the farmer and each hand pump is =  $26 = 2a$

$$\Rightarrow 2a = 26 \Rightarrow a = 13 \text{ m}$$

Distance between the handpump =  $10\text{m} = 2c$

$$\Rightarrow c = 5 \text{ m}$$

$$c^2 = a^2 - b^2$$

$$\Rightarrow 25 = 169 - b^2$$

$$\Rightarrow b^2 = 144$$

Equation is  $\frac{x^2}{169} + \frac{y^2}{144} = 1$

- iii. Equation of ellipse is  $\frac{x^2}{169} + \frac{y^2}{144} = 1$  comparing with standard equation of ellipse  $a=13$ ,  $b=12$  and  $c=5$  (given)

Length of major axis =  $2a = 2 \times 13 = 26$

Length of minor axis =  $2b = 2 \times 12 = 24$

$$\text{eccentricity } e = \frac{c}{a} = \frac{5}{13}$$

**OR**

Equation of the ellipse is  $\frac{x^2}{169} + \frac{y^2}{144} = 1$  hence  $a = 13$  and  $b = 12$

length of latus rectum of ellipse is given by  $\frac{2b^2}{a} = \frac{2 \times 144}{13}$

37. i. We make the table from the given data.

Class	$f_i$	cf	Mid-point( $x_i$ )	$ x_i - M $	$f_i x_i - M $
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
	50				508

Here,  $\frac{N}{2} = \frac{50}{2} = 25$

Here, 25th item lies in the class 20-30. Therefore, 20-30 is the median class.

Here,  $l = 20$ ,  $cf = 13$ ,  $f = 15$ ,  $b = 10$  and  $N = 50$

$$\therefore \text{Median, } M = l + \frac{\frac{N}{2} - cf}{f} \times b$$

$$\Rightarrow M = 20 + \frac{25 - 13}{15} \times 10 = 20 + 8 = 28$$

Thus, mean deviation about median is given by

$$MD(M) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - M| = \frac{1}{50} \times 508 = 10.16$$

Hence, mean deviation about median is 10.16.

ii. Here,  $l = 20$ ,  $cf = 13$ ,  $f = 15$ ,  $b = 10$  and  $N = 50$

$$\therefore \text{Median, } M = l + \frac{\frac{N}{2} - cf}{f} \times b$$

$$\Rightarrow M = 20 + \frac{25-13}{15} \times 10 = 20 + 8 = 28$$

iii.  $MD = \frac{\sum f_i |x_i - M|}{N}$

OR

$$M = l + \frac{\frac{N}{2} - cf}{f} \times h$$

38. i. Given Raju and Ravi are at the extreme positions

**Case 1** Raju \_\_\_\_\_ Ravi

**Case 2** Ravi \_\_\_\_\_ Raju

So remaining 5 places are filled in 5! Ways in both cases

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Hence total number of arrangements =  $120 \times 2 = 240$  ways

ii. \_\_\_\_\_ **Joseph** \_\_\_\_\_

So here middle place is occupied by Joseph remaining 6 places are filled by remaining 6 students in 6! Ways

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ ways}$$

iii. When all girls are together let's consider them as a single unit. So four 4 boys with single group of girls can be arranged in  $4 + 1 = 5!$  Ways

\_\_\_\_\_ Sangeeta Priya Meena

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

But all the tree girls can be arranged in themselves in 3! Ways =  $3 \times 2 \times 1 = 6$

Hence total number of ways =  $5! \times 3! = 120 \times 6 = 720$

**OR**

When Aman and Ravi are together let's consider them as a single unit. So remaining 5 students with single group of Aman and Ravi can be arranged in  $5 + 1 = 6!$  Ways

\_\_\_\_\_ Aman Ravi

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

But Aman and Ravi can be arranged in themselves in 2! Ways =  $2 \times 1 = 2$

Hence total number of ways =  $6! \times 2! = 720 \times 2 = 1440$  ways ...(i)

Total number of sitting arrangements of all 7 students without restriction

\_\_\_\_\_

All seven students can fill seven seats in 7! Ways

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \text{ ways ...}(ii)$$

But here we need the number of arrangements so that Aman and Ravi are not together = Total number of sitting arrangements of all 7 students without restriction - Number of arrangements so that Aman and Ravi are together.

From (i) and (ii) we have

The number of arrangements so that Aman and Ravi are not together =  $5040 - 1440 = 3600$