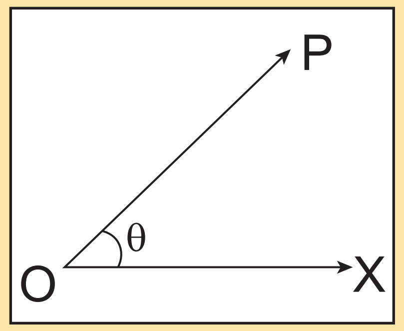
BASIC MATHEMATICS

Trigonometry

- **Angle : The angle covered by the revolving line OP is** $\theta = \angle POX$



$1° = 60'$ (minute) ; $1' = 60''$ (second)

$\Rightarrow 1 \text{ rad} = \frac{180°}{\pi} \approx 57.3°$

$\Rightarrow$ Angle θ° to Radian multiply it by $\frac{\pi}{180°}$.

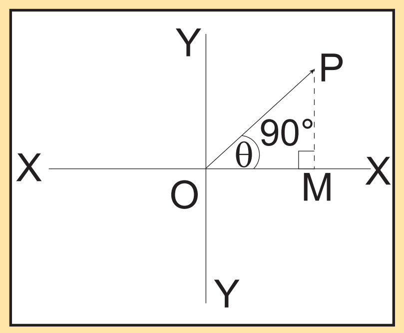
$\Rightarrow$ Angle Radian to θ° multiplying it by $\frac{180°}{\pi}$.

- **Trigonometrical Ratios :**

$\sin\theta = \frac{P}{H} = \frac{MP}{OP}$ $\quad \cos\theta = \frac{B}{H} = \frac{OM}{OP}$

$\tan\theta = \frac{P}{B} = \frac{MP}{OM}$ $\quad \cot\theta = \frac{B}{P} = \frac{OM}{MP}$

$\sec\theta = \frac{H}{B} = \frac{OP}{OM}$ $\quad \text{cosec}\,\theta = \frac{H}{P} = \frac{OP}{MP}$



$\Rightarrow \sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2$; $1 + \cot^2\theta = \text{cosec}^2\theta$

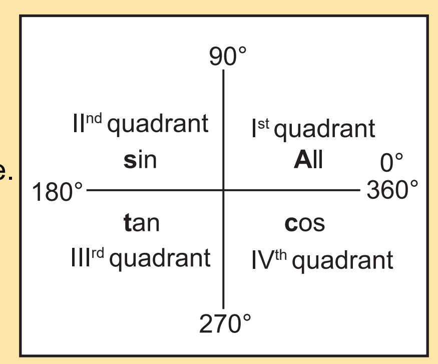
- **Table : Trigonometry standard angles from 0° to 180°**
- **Four Quadrans and ASTC Rule :**

$\Rightarrow$ In Ist quadrant, all trigonometric ratios are positive.

$\Rightarrow$ In IInd quadrant, only sin θ and cosec θ are positive.

$\Rightarrow$ In IIIrd quadrant, only tan θ and cot θ are positive.

$\Rightarrow$ In IVth quadrant, only cos θ and sec θ are positive.



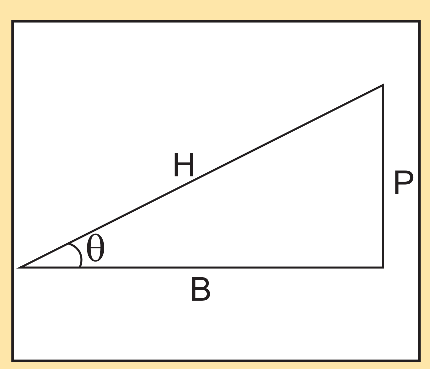
- **Important trigonometric formula :**
- **Range of trigonometric functions :**

$\Rightarrow \sin\theta = \frac{P}{H}$ and $P \le H$

So, $-1 \le \sin\theta \le 1$

$\Rightarrow \cos\theta = \frac{B}{H}$ and $B \le H$, So $-1 \le \cos\theta \le 1$

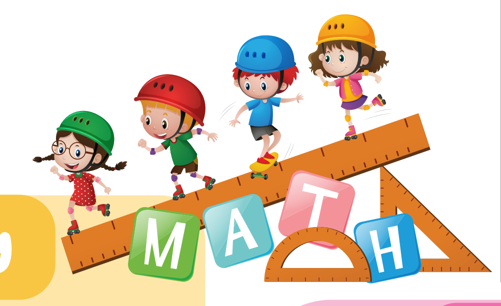
$\Rightarrow \tan\theta = \frac{P}{B}$ So, $-\infty < \tan\theta < \infty$



- **Small Angle Approximation :**

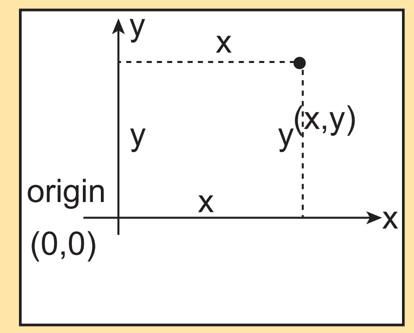
If θ is small (θ < 45°)

$\sin\theta \approx \theta$; $\cos \approx 1$ and $\tan\theta \approx \theta$



Coordinate Geometry

- **Origin : Any fixed point from which all measurements are taken from this.**
- **Axis : Any fixed direction passing through origin.**



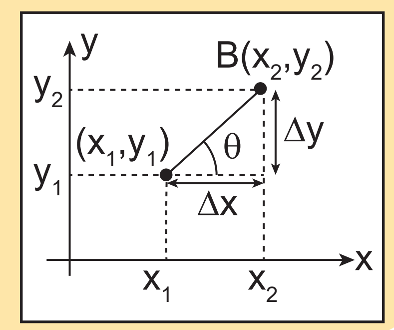
- **Distance Formula :**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

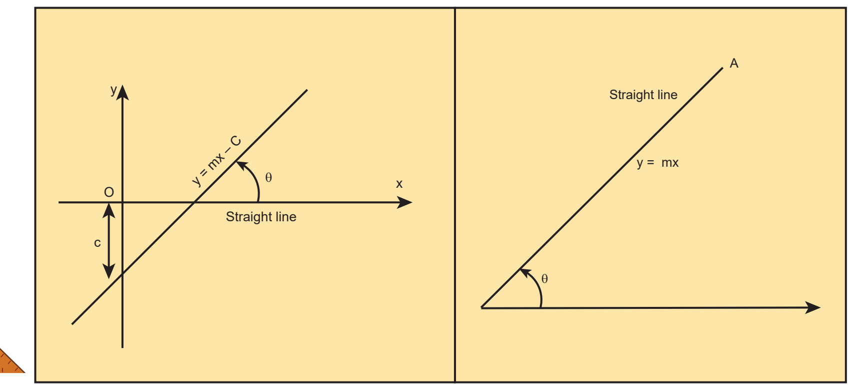
In 3-d (space) – $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

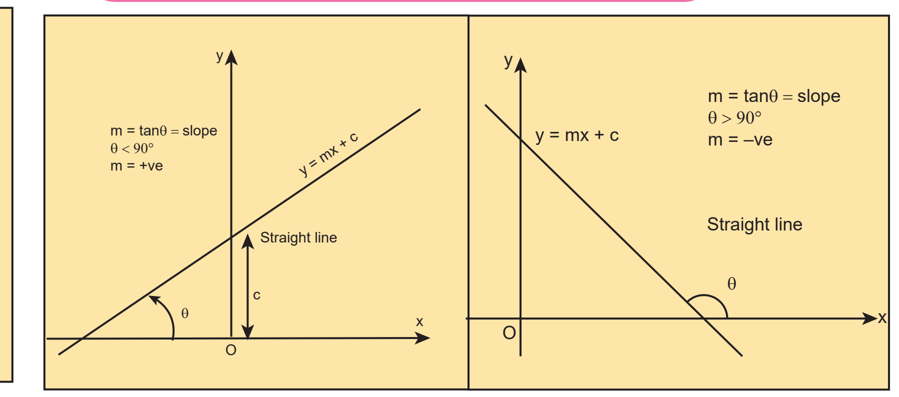
- **Slope of a linen :**

$m = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



Standard Graphs and their Equations





Differentiation

- **Physical Meaning of $\frac{dy}{dx} = 1$**

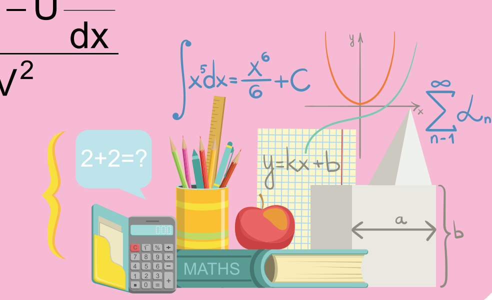
(i) The ratio of small change in the function y and the variable x is called the average rate of change y w.r.t. x.

(ii) When $\Delta x \to 0$. The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim_{\Delta x \to \infty} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

- **Main Formulas of Differentiation**

1. $\frac{d}{dx}(K) = 0$ $\quad$ K = constant
2. $\frac{d}{dx}(KU) = K\frac{dU}{dx}$ $\quad$ [U is a function of x]
3. $\frac{d}{dx}(U \pm V \pm W) = \frac{dU}{dx} \pm \frac{dV}{dx} \pm \frac{dW}{dx}$

   where U, V and W are functions of x.
4. $\frac{d}{dx}(UV) = U\frac{dV}{dx} + V\frac{dV}{dx}$
5. $\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$
6. $\frac{d}{dx}(x^n) = nx^{n-1}$

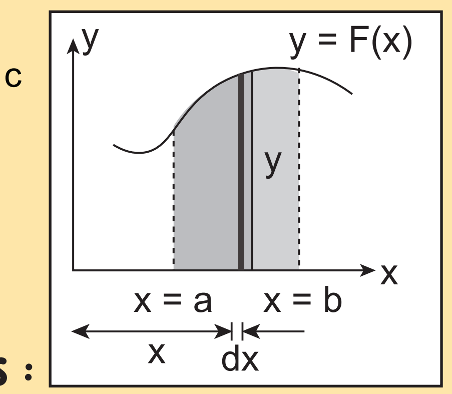


Integration

If I is the integration of f(x) with respect to x then I = $\int f(x)dx$

- **Main formulae of integration :**

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \ne -1$
2. $\int \sin x\, dx = -\cos x + c$, c = constant
3. $\int \cos x\, dx = \sin x + c$
4. $\int \frac{1}{x} dx = \log_e x + c$
5. $\int e^x\, dx = e^x + c$



- **Define Integrals :**

If $\frac{d}{dx}(f(x)) = f'(x)$ then $\int_a^b f'(x)dx = |f(x)|_a^b$

is called definite integral.

- **Area Under Curve :**

$\int_a^b f(x)dx$ = Shaded area between curve and x-axis.

- **Important formulae of Differentiation :**

$\Rightarrow \frac{d}{dx}(\sin x) = \cos x \Rightarrow \frac{d}{dx}(\cos x) = -\sin x$ $\quad$ (#) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

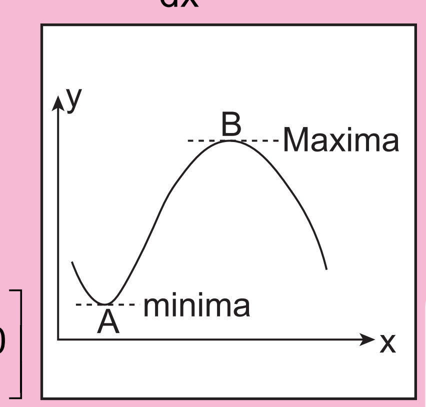
$\Rightarrow \frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \frac{d}{dx}(\cot x) = -\text{cosec}^2 x$ $\quad$ (#) $\frac{d}{dx}(e^x) = e^x$

$\Rightarrow \frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$ $\quad$ (#) $\frac{d}{dx}(e^{ax}) = ae^x$

- **Concept of Maxima and Minima**

$\Rightarrow$ Condition for minima : $\left[\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} > 0\right]$

$\Rightarrow$ Condition for maxima : $\left[\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0\right]$



Algebra

- **Quadratic Equation and its Solutions :**
An algebraic equation of 2nd order is called a quadratic equation. Equation = $ax^2 + bx + c = 0$

General Solution $\Rightarrow X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

gives $x_1 = \frac{b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $b^2 - 4ac \ge 0$ For real roots.

$\Rightarrow$ $b^2 - 4ac < 0$ For Imaginary roots.

- **Binomial Expression :**

An algebraic expression having two terms only.Lorem ipsum

Example : (a + b), $(a + b)^4$, $(2x - 3y)^2$ etc.

- **Binomial Theorem :**

$(a + b)^n = a^n + na^{n-1}b' + \frac{n(n-1)}{2\times1}a^{n-2}b^2 + ...$

$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2\times1} + ...$

- **Logarithm Main formulae :**

log mn = log m + log n $\quad$ log m^n = n log m

$\log\frac{m}{n} = \log m - \log n$ $\quad$ $\log_e m = 2.303 \log_{10} m$

- **Arithmetic Progression (AP) :**

(AP) = a + a + d + a + 2d + ... + a + (n – 1)d

where a = first term d = common difference

$\Rightarrow$ Sum of n term's $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + n^{th} \text{ term}]$

(i) Sum of first n natural number's — $S_n = \frac{n(n+1)}{2}$;

(ii) Sum of squares of first n $\rightarrow S_{n^2} = \frac{n(n+1)(2n+1)}{6}$

- **Geometric Progression (GP)**

(GP) = a, ar, ar^2, ar^3 .... ar^{n-1}; a = first term, r = common ratio

Sum of n terms $\rightarrow S_n = \frac{a(1-r^n)}{1-r}$; For $0 \le |r| < 1$

Sum of ∞ terms $S_\infty = \frac{a}{1-r}$