Playing with numbers

- Any two-digit number *ab*, made of digits *a* and *b*, can be written in general form as $ab = 10a + b$ whereas the number *ba* can be written as $ba = 10b + a$.

  For example, $45 = 4 \times 10 + 5$ and $54 = 5 \times 10 + 4$

- A three-digit number *abc*, made of digits, *a*, *b*, and *c*, can be written in general form as $abc = 100a + 10b + c$.

  For example, $456 = 4 \times 100 + 5 \times 10 + 6$

- The sum of a two-digit number and the number formed by reversing the digits is always a multiple of 11.

  For example, $28 + 82 = 110 = 11 \times 10$ is a multiple of 11.

- The difference between a two-digit number and the number formed by reversing the digits is always a multiple of 9.

  For example, $91 – 19 = 72 = 9 \times 8$ is a multiple of 9.

- The difference between a three-digit number and the number formed by reversing the digits is always a multiple of 99.

  For example,**:** $923 – 329 = 594 = 99 \times 6$ is a multiple of 99.

- If *abc* is a three-digit number, then ($abc + cab + bca$) will be a multiple of 3 and 37.

- Solving puzzles related to addition and multiplication in which some of the digits are denoted by letters and finding the number by which the letters are replaced.

  Rules of solving such puzzles are as follows:

  1. Each letter in the puzzle stands for just one digit.

2. The first digit of a number cannot be zero.
3. The puzzle has just one answer.

Example:

Find the digits, A, B, and C, in the addition.

$$\begin{array}{r} C\ 9\ C \\ +5\ B\ 3 \\ \hline A\ 7\ 6 \end{array}$$

Solution:

Studying the addition in ones column, we find that the addition of C and 3 gives a number whose units digit is 6. Therefore, the only possibility for the letter C is the digit 3 as C is a one-digit number.

Then, studying the addition in the tens column, we find that the addition of 9 and B gives a number whose units digit is 7.

This is possible only when B = 8.

In the hundreds column, there is a carry of 1. Therefore, we should have 1 + C + 5 = A, where C = 3.

∴ A = 9

Thus, the puzzle can now be solved as shown below

$$\begin{array}{r} 3\,9\,3 \\ +5\,8\,3 \\ \hline 9\,7\,6 \end{array}$$

Example:

Find the digits, P and Q, in the given puzzle.

$$\begin{array}{r} P\ \ 2 \\ \times\ Q \\ \hline 360 \end{array}$$

Solution:

The units digit of the number should be obtained as 0 on multiplying 2 by Q.

Therefore, the possibility for Q is either 0 or 5.

However, Q cannot be 0 because then, the result obtained after multiplication will be 0. Therefore, Q = 5

In the tens column, there will be a carry of 1, which was obtained as tens digit on multiplication of 2 by 5 in the ones column.

Therefore, we should obtain,

$1 + P \times 5 = 36$

$\Rightarrow P \times 5 = 35$

$\Rightarrow P = 7$

Therefore, the only possibility for P is 7.

Thus, the puzzle can now be solved as below.

$$\begin{array}{r} 7\ 2 \\ \times\ 5 \\ \hline 360 \\ \hline \end{array}$$

- A number is divisible by 2, if the digit in one's place is either 0, 2, 4, 6, or 8.

  For example, the numbers 9218, 6054, 932 are divisible by 2.

- A number with two or more digits is divisible by 4, if the number formed by its last two digits (one's and ten's) is either 00 or divisible by 4.

  For example, the last two digits of 9584 is 84, which is divisible by 4. So, 9584 is divisible by 4.

- A number with three or more digits is divisible by 8, if the number formed by its last three digits (one's, ten's and hundred's) is either 000 or divisible by 8.

  For example, the last three digits of 9368 is 368, which is divisible by 8. So, 9368 is divisible by 8.

- A number is divisible by 3, if the sum of its digits is divisible by 3.

  For example, 231456 is divisible by 3, since 2 + 3 + 1 + 4 + 5 + 6 = 21 is divisible by 3.

- A number is divisible by 9, if the sum of its digits is divisible by 9.

  For example, 253674 is divisible by 9, since 2 + 5 + 3 + 6 + 7 + 4 = 27 is divisible by 9.

- A number which is divisible by 9 is also divisible by 3.

  For example, in 252, the sum of its digits is 9, which is divisible by 3 and 9 both.

- A number is divisible by 6, if it is divisible by both 2 and 3.

  For example, 39612 is divisible by 2, since it is an even number. Sum of the digits of 39612 is 3 + 9 + 6 + 1 + 2 = 21, which is a multiple of 3. So, 39612 is divisible by 3. Now, 39612 is divisible by both 2 and 3. So, it is divisible by 6. 3513 is not divisible by 2, as it is an odd umber. But it is divisible by 3 (∵ 3 + 5 + 1+ 3 = 12, which is a multiple of 3). So, 3513 is not divisible by 6.

  Example:

  Write the smallest digit and the greatest digit in the blank space of the following number, so that the number is divisible by 6.

  931_

  Solution:

  Since the number is divisible by 6, it has to be divisible by 2. So, the unit's digit can be 0, 2, 4, 6 or 8. Also, the number has to be divisible by 3. For this, the sum of the digits should be a multiple of 3.
  Now, 9 + 3 + 1 = 13. If we add 2, 5, or 8 to 13, then we get a number which is a multiple of 3.
  If the digit in blank space is 2 or 8, then the obtained number will be divisible by 2 as well as 3. So, the required smallest number is 2 and the largest number is 8.

- A number is divisible by 10, if the digit in one's place is zero.

  For example, the numbers 9520, 67120, 830, 1200, etc., are divisible by 10.

- A number is divisible by 5, if the digit in one's place is either 0 or 5.

  For example, 3615, 92185, 370 are divisible by 5.

- A number is divisible by 11, if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) is either 0 or multiple of 11.

  For example, for the number 82918, the sum of the digits at odd places = 8 + 9 + 8 = 25 and the sum of the digits at even places = 1 + 2 = 3. Now, 25 – 3 = 22, which is a multiple of 11. Hence, 82918 is divisible by 11.