Chapter 6

Factorization

Exercise 6.1

1. Find the remainder (without division) on dividing f(x) by (x -2) where

(i) $f(x) = 5x^2 – 7x + 4$

Solution

Let us assume $x – 2 = 0$

Then, $x = 2$

Given, $f(x) = 5x^2 – 7x + 4$

Now, substitute the value of x in f(x),

$f(2) = (5\times 2^2) – (7\times 2) + 4$

$= (5\times 4) – 14 + 4$

$= 20 – 14 + 4$

$= 24 – 14$

$= 10$

Therefore, the remainder is 10.

(ii) $f(x) = 2x^3 – 7x^2 + 3$

Solution

Let us assume $x – 2 = 0$

Then, $x = 2$

Given, $f(x) = 2x^3 – 7x^2 + 3$

Now, substitute the value of x in f(x)

$f(2) = (2 \times 2^3) – (7 \times 2^2) + 3$

$= (2 \times 8) – (7 \times 4) + 3$

$= 16 – 28 + 3$

$= 19 – 28$

$= - 9$

Therefore, the remainder is – 9.

2. using the remainder theorem, find the remainder on dividing f(x) by (x + 3) where

(i) $f(x) = 2x^2 – 5x + 1$

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = 2x^2 – 5x + 1$

Now, substitute the value of x in f(x)

$f(-3) = (2 \times -3^2) – (5 \times (-3)) + 1$

$= (2 \times 9) – (- 15) + 1$

$= 18 + 15 + 1$

$= 34$

Therefore, the remainder is 34.

(ii) $f(x) = 3x^3 + 7x^2 – 5x + 1$

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = 3x^3 + 7x^2 – 5x + 1$

Now, substitute the value of x in f(x)

$f(-3) = (3 \times -3^3) + (7 \times -3^2) – (5 \times -3) + 1$

$= (3 \times -27) + (7 \times 9) – (-15) + 1$

$= - 81 + 63 + 15 + 1$

$= -81 + 79$

$= -2$

Therefore, the remainder is -2.

3. Find the remainder (without division) on dividing f(x) by (2x + 1) where,

(i) f(x) = $4x^2 + 5x + 3$

Solution

Let us assume 2x + 1 = 0

Then, 2x = - 1

$X = -\frac{1}{2}$

Given, f(x) = $4x^2 + 5x + 3$

Now, substitute the value of x in f(x),

$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + 3$

$= \left(4 \times \frac{1}{4}\right) + \left(-\frac{5}{2}\right) + 3$

$= 1 - \frac{5}{2} + 3$

$= 4 - \frac{5}{2}$

$= \frac{8-5}{2}$

$= \frac{3}{2} = 1\frac{1}{2}$

Therefore, the remainder is $1\frac{1}{2}$

(ii) f(x) = $3x^3 - 7x^2 + 4x + 11$

Solution

Let us assume 2x + 1 = 0

Then, 2x = - 1

X = $-\frac{1}{2}$

Given, f(x) = $3x^3 - 7x^2 + 4x + 11$

Now, substitute the value of x in f(x),

$$f\left(-\frac{1}{2}\right) = \left(3 \times \left(-\frac{1}{2}\right)^3\right) - \left(7 \times \left(-\frac{1}{2}\right)^2 + \left(4 \times -\frac{1}{2}\right) + 11\right)$$

$$= 3 \times \left(-\frac{1}{8}\right) - \left(7 \times \frac{1}{4}\right) + (-2) + 11$$

$$= -\frac{3}{8} - \frac{7}{4} - 2 + 11$$

$$= -\frac{3}{8} - \frac{7}{4} + 9$$

$$= \frac{-3 - 14 + 72}{8}$$

$$= \frac{55}{8}$$

$$= 6\frac{7}{8}$$

4. Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$ leaves a remainder 7.

Solution

Let us assume, $x - 2 = 0$

Then, $x = 2$

Given, $2x^3 + 3x^2 - kx + 5$

Now, substitute the value of x in f(x)

$f(2) = (2 \times 2^3) + (3 \times 2^2) - (k \times 2) + 5$

$= (2 \times 8) + (3 \times 4) - 2k + 5$

$= 16 + 12 - 2k + 5$

$= 33 - 2k$

Form the question it is given that, remainder is 7.

So, $7 = 33 - 2k$

$2k = 33 - 7$

$2k = 26$

$K = \frac{26}{2}$

$K = 13$

Therefore, the value of k is 13.

5. Using remainder theorem, find the value of 'a' if the division of $x^3 + 5x^2 - ax + 6$ by $(x - 1)$ leaves the remainder 2a.

Solution

Let us assume $x - 1 = 0$

Then, $x = 1$

Given, $f(x) = x^3 + 5x^2 - ax + 6$

Now, substitute the value of x in f(x),

$f(1) = 1^3 + (5 \times 1^2) - (a \times 1) + 6$

$= 1 + 5 - a + 6$

$= 12 - a$

From the question it is given that, remainder is 2a

So, $2a = 12 - a$

$2a + a = 12$

$3a = 12$

$A = \frac{12}{3}$

$A = 4$

Therefore, the value of a is 4.

6. (i) what number must be divided be subtracted from $2x^2$ – 5x so that the result polynomial leaves the remainder 2, when divided by 2x + 1?

Solution

Let u assume ‘p’ be subtracted from $2x^2 - 5x$

So, dividing $2x^2 - 5x$ by $2x + 1$

$$
\begin{array}{r|l|l}
2x+1 & 2x^2 - 5x - p & x-3 \\
 & 2x^2 + x & \\
 & - \quad - & \\
\hline
 & \quad -6x - p & \\
 & \quad -6x - 3 & \\
 & \quad + \quad + & \\
\hline
 & \quad -p + 3 &
\end{array}
$$

Hence, remainder is $3 - p$

From the question it is given that, remainder is 2.

$3 - p = 2$

$P = 3 - 2$

$P = 1$

Therefore, 1 is to be subtracted.

(ii) What number must be added to $2x^3 - 7x^2 + 2x$ so that the resulting polynomial leaves the remainder – 2 when divided by $2x - 3$?

Solution

Let us assume 'P' be subtracted from $2x^3 - 7x^2 + 2x$,

So, dividing it by $2x - 3$,

$2x-3$	$2x^3 - 7x^2 + 2x + p$	$X^2 - 2x - 2$
	$2x^3 - 3x^2$	
	$-\quad -$	
	$-4x^2 + 2x$	
	$-4x^2 + 6x$	
	$+\quad -$	
	$-4x + p$	
	$-4x + 6$	
	$+\quad -$	
	$P - 6$	

Hence, remainder is $P - 6$

From the question it is given that, remainder is – 2

$P - 6 = -2$

$P = -2 + 6$

$P = 4$

Therefore, 4 is to be added.

7. (i) When divided by x – 3 the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'

Solution

From the question it is given that, by dividing $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ by $x - 3 = 0$, then $x = 3$

Let us assume $p(x) = x^3 - px^2 + x + 6$

Now, substitute the value of x in p(x),

$P(3) = 3^3 - (p \times 3^2) + 3 + 6$

$= 27 - 9p + 9$

$= 36 - 9p$

Then, $q(x) = 2x^3 - x^2 - (p + 3)x - 6$

Now, substitute the value of x in q(x),

$q(3) = (2 \times 3^3) - (3)^2 - (p + 3) \times 3 - 6$

$= (2 \times 27) - 9 - 3p - 9 - 6$

$= 54 - 24 - 3P$

$= 30 - 3p$

Given, the remainder in each case is same,

So, $36 - 9p = 30 - 3p$

$36 - 30 = 9p - 3p$

$6 = 6p$

$p = \frac{6}{6}$

p = 1

therefore, value of p is 1.

(ii) Find 'a' if the two polynomials $ax^3 + 3x^2 – 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by x + 3.

Solution

Let us assume $p(x) = ax^3 + 3x^2 – 9$ and $q(x) = 2x^3 + 4x + a$

From the question it is given that, both p(x) and q(x) leaves the same remainder when divided by x + 3.

Let us assume that, x + 3 = 0

Then, x = - 3

Now, substitute the value of x in p(x) and in q(x),

So, p(-3) = q(-3)

$a(-3)^3 + 3(-3)^2 – 9 = 2(-3)^3 + 4(-3) + a$

- 27 a + 27 – 9 = - 54 – 12 + a

- 27 a – a = -66 – 18

- 28 a = - 84

$a = \frac{84}{28}$

therefore, $a = 3$

(iii) The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $x - 4$ leaves the remainder r_1 and r_2 respectively. If $2r_1 = r_2$, then find the value of a.

Solution

Let us assume $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$

From the question it is given that, both p(x) and q(x) leaves the remainder r_1 and r_2 respectively when divide by $x - 4$.

Also, given relation $2r_1 = r_2$

Let us assume that, $x - 4 = 0$

Then , $x = 4$

Now, substituted the value of x in p(x) and in q(x),

By factor theorem $r_1 = p(x)$ and $r_2 = q(x)$

So, $2 \times p(4) = q(4)$

$2[a(4)^3 + 3(4)^2 - 3] = 128 - 20 + a$

$128a + 96 - 6 = 128 - 20 + a$

$128a + 90 = 108 + a$

$128a - a = 108 - 90$

$127a = 18$

$a = \frac{18}{127}$

therefore, the value of a = $\frac{18}{127}$

8. using remainder thermo, find the remainders obtained when x^3 + (kx + 8)x + k is divided by x + 1 and x – 2. Hence, find k if the sum of two remainders is 1.

Solution

Let us assume p(x) = x^3 + (kx + 8)x + k

From the question it is given that, the sum of the remainders when p(x) is divided by (x +1) and (x – 2) is 1.

Let us assume that, x + 1 = 0

Then, x = -1

Also, when x – 2 = 0

Then, x = 2

Now, by remainder theorem we have

P(-1) + p(2) = 1

$(-1)^3 + [k(-1) + 8](-1) + k + (2)^3 + [\ k\ (2) + 8\ ](2) + k = 1$

- 1 + k – 8 +k + 8 + 4k + 16 + k = 1

7k + 15 = 1

7k = 1 – 15

K = $-\frac{14}{7}$

K = - 2

Therefore, k = -2

9. By factor theorem, show that (x +3) and (2x – 1) are factors of $2x^2 + 5x – 3$.

Solution

Let us assume, x + 3 = 0

Then, x = - 3

Given , $f(x) = 2x^2 + 5x – 3$

Now, substitute the value of x in f(x),

$f(-3) = (2 \times (-3)^2) + (5 \times -3) – 3$

$= (2 \times 9) + (-15) – 3$

$= 18 – 15 – 3$

$= 18 – 18$

$= 0$

Now, 2x – 1 = 0

Then, 2x = 1

$X = \frac{1}{2}$

Given , $f(x) = 2x^2 + 5x – 3$

Now, substitute the value of x in f(x)

$f\left(\frac{1}{2}\right) = \left(2 \times \left(\frac{1}{2}\right)^2\right) + \left(5 \times \frac{1}{2}\right) - 3$

$= \left(2 \times \left(\frac{1}{4}\right)\right) + \frac{5}{2} - 3$

$= \frac{1}{2} + \frac{5}{2} - 3$

$= \frac{1+5}{2} - 3$

$= \frac{6}{2} - 3$

= 3 -3

= 0

Hence, it is proved that (x + 3) and (2x – 1) are factors of $2x^2 + 5x - 3$.

10. Without actual division, prove that $x^4 + 2x^3 - 2x^2 + 2x + 3$ is exactly divisible by $x^2 + 2x - 3$.

Solution

Consider $x^2 + 2x - 3$

By factor method, $x^2 + 3x - x - 3$

= x(x + 3) – 1(x + 3)

= (x -1) (x +3)

So, $f(x) = x^4 + 2x^3 - 2x^2 + 2x + 3$

Now take, x + 3 = 0

X = -3

Then, $f(-3) = (-3)^4 + 2 \times -(3^3) - (2 \times (-3)^2) + (2\times -3) + 3$

$= 81 - 54 - 18 - 6 - 3$

$= 0$

Therefore, $(x + 3)$ is a factor of $f(x)$

And also, take $x - 1 = 0$

X = 1

Then, $f(1) = 1^4 + 2(1)^3 - 2(1)^2 + 2(1) - 3$

$= 0$

Therefore, $(x - 1)$ is a factor of $f(x)$

By comparing both results, $p(x)$ is exactly divisible by $x^2 + 2x - 3$.

11. show that $(x - 2)$ is a factor of $3x^2 - x - 10$. Hence factories $3x^2 - x - 10$.

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 3x^2 - x - 10$

Now, substitute the value of x in f(x),

$f(2) = (3 \times 2^2) - 2 - 10$

$= (3 \times 4) - 2 - 10$

$= 12 – 12$

$= 0$

Therefore, (x -2) is a factor of f(x)

Then, dividing $(3x^2 – x – 10)$ by (x- 2) we get

x-2	$3x^2 – x – 10$ $3x^2 + x$ - +	3x + 5
	5x – 10 5x – 10 - +	
	0	

Therefore, $3x^2 – x – 10 = (x – 2) (3x + 5)$

12. Using the factor theorem, show that(x -2) is a factor of $x^3 + x^2 – 4x – 4$. Hence factorize the polynomial completely.

Solution

Let us assume, x – 2 = 0

Then, x = 2

Given, $f(x) = x^3 + x^2 – 4x – 4$

Now, substitute the value of x in f(x)

$f(2) = (2)^3 + (2)^2 – 4(2) – 4$

$= 8 – 4 – 8 – 4$

$= 0$

Therefore, by factor theorem $(x - 2)$ is a factor of $x^3 + x^2 - 4x – 4$

Then, dividing $f(x)$ by $(x – 2)$, we get

$$
\begin{array}{l|l}
 & X^2 + 3x + 2 \\
\hline
x-2 & X^3 + x^2 - 4x - 4 \\
 & X^3 - 2x^2 \\
 & - \\
\hline
 & 3x^2 \ - 4x \ - 4 \\
 & - \\
 & 3x^2 \ - 6x \\
\hline
 & 2x \ - 4 \\
 & - \\
 & 2x \ - 4 \\
\hline
 & 0
\end{array}
$$

Therefore, $x^3 + x^2 – 4x – 4 = (x - 2) (x^2 + 3x + 2)$

$= (x - 2) (x^2 + 2x + x + 2)$

$= (x – 2) (x(x + 2) + 1(x + 2))$

$= (x - 2) (x + 2) (x + 1)$

13. Show that 2x + 7 is a factor of $2x^3 + 5x^2 - 11x - 14$.hence factorize the given expression completely, using the factor theorem.

Solution

Let us assume 2x + 7 = 0

Then, 2x = - 7

$X = -\frac{7}{2}$

Given, $f(x) = 2x^3 + 5x^2 - 11x - 14$

Now, substitute the value of x in f(x),

$f\left(-\frac{7}{2}\right) = 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 + 11\left(-\frac{7}{2}\right) - 14$

$= 2\left(-\frac{343}{8}\right) + 5\left(\frac{49}{4}\right) + \left(-\frac{77}{2}\right) - 14$

$= -\frac{343}{4} + \frac{245}{4} - \frac{77}{2} - 14$

$= \frac{-343+245+154-56}{4}$

$= -399 + \frac{399}{4}$

= 0

Therefore,(2x + 7) is a factor of $2x^3 + 5x^2 - 11x - 14$

Then, dividing f(x) by (2x + 1) we get

	$X^2 - x - 2$
2x+7	$2x^3 + 5x^2 - 11x - 14$ - $2x^3 + 7x^2$
	$-2x^2 - 11x - 14$ - $-2x^2 - 7x$
	$-4x - 14$ - $-4x - 14$
	0

Therefore, $2x^3 + 5x^2 - 11x - 14 = (2x + 7)(x^2 - x - 2)$

$= (2x + 7)(x^2 - 2x + x - 2)$

$= (2x + 7)(x(x - 2) + 1(x - 2))$

$= (x + 1)(x - 2)(2x + 7)$

14. use factor theorem to factorize the following polynomials completely.

(i) $x^3 + 2x^2 - 5x - 6$

Solution

Let us assume $x = -1$

Given, $f(x) = x^3 + 2x^2 - 5x - 6$

Now, substitute the value of x in f(x)

$f(-1) = (-1)^3 + 2(-1)^2 – 5(-1) – 6$

$= - 1 + 2 (1) + 5 – 6$

$= - 1 + 2 + 5 – 6$

$= - 7 + 7$

$= 0$

Then, dividing f(x) by (x + 1) we get

	$X^2 + x – 6$
X + 1	$X^3 + 2x^2 - 5x – 6$
	-
	$X^3 + x^2$
	$X^2 - 5x – 6$
	-
	$X^2 + x$
	$-6x - 6$
	-
	$-6x - 6$
	0

Therefore, $x^3 + 2x^2 – 5x – 6 = (x + 1)(x^2 + 3x – 2x – 6)$

$= (x + 1) (x(x + 3) – 2(x + 3))$

$= (x + 1) (x – 2) (x + 3)$

(ii) x^3 – 13x – 12

Solution

Let us assume x = -1,

Given, $f(x) = x^3 – 13x – 12$

Now, substitute the value of x in f(x)

$f(-1) = (-1)^3 – 13(-1) – 12$

$= -1 + 13 – 12$

$= -13 + 13$

$= 0$

Then, dividing f(x) by (x + 1), we get

	$X^2 - x - 12$
X + 1	$X^3 + 0x^2 – 13x – 12$
	-
	$X^3 + x^2$
	$-x^2 -13x – 12$
	-
	$- x^2 - x$
	$-12x - 12$
	-
	$- 12x - 12$
	0

Therefore $x^3 – 13x – 12 = (x + 1) (x^2 – x – 12)$

$= (x + 1) (x^2 – 4x + 3x – 12)$

$= (x + 1) (x(x – 4)) + 3(x – 4)$

$= (x + 1) (x + 3) (x – 4)$

15. Use the remainder theorem to factorize the following expression

(i) $2x^3 + x^2 – 13x + 6$

Solution

Let us assume x = 2

Then, $f(x) = 2x^3 + x^2 – 13x + 6$

Now, substitute the value of x in f(x)

$f(2) = (2 \times 2^3) + 2^2 – 13 \times 2 + 6$

$= (2 \times 8) + 4 – 26 + 6$

$= 16 + 4 – 26 + 6$

$= 26 – 26$

$= 0$

Then , dividing f(x) by (x -2) we get

	$2x^2 + 5x - 3$
x - 2	$2x^2 + x^2 – 13x + 6$ - $2x^3 – 4x^2$
	$5x^2 - 13x + 6$ - $5x^2 - 10x$
	$-3x + 6$ - $-3x + 6$
	0

Therefore, $2x^3 + x^2 – 13x + 6 = (x – 2) (2x^2 + 5x – 3)$

$= (x – 2) (2x^2 + 6x – x – 3)$

$= (x – 2) (2x(x+ 3) – 1(x + 3))$

$= (x – 2) (x + 3) (2x -1)$

(ii) $3x^2 + 2x^2 – 19x + 6$

Solution

Given $f(x) = 3x^3 + 2x^2 – 19x + 6$

Let us assume $x = 1$

Then $f(1) = 3(1)^3 + 2(1)^2 - (19 \times 1) + 6$

$= 3 + 2 – 19 + 6$

$= 11 – 19$

$= - 8$

So, $- 8 \neq 0$

Let us assume $x = -1$

Then, $f(-1) = 3(-1)^3 + 2(-1)^2 -(19 \times (-1)) + 6$

$= - 3 + 2 + 19 + 6$

$= - 3 + 27$

$= 24$

So, $24 \neq 0$

Now, assume $x = 2$

Then, $f(2) = 3(2)^3 + 2(2)^2 - (19 \times (2)) + 6$

$= 24 + 8 - 38 + 6$

$= 38 – 38$

So, $0 = 0$

Therefore, $(x -2)$ is a factor of $f(x)$

$f(x) = 3x^3 + 2x^2 – 19x + 6$

$= 3x^3 – 6x^2 + 8x^2 – 16x – 3x + 6$

$= 3x^2 (x -2) + 8x (x -2) – 3(x -2)$

$= (x -2) (3x^2 + 8x -3)$

$= (x -2) (3x^2 + 9x – x – 3)$

$= (x -2) (3x(x + 3) – 1(x + 3)$

$= (x – 2) (3x (x + 3) – 1(x +3)$

$= (x – 2) (x +3) (3x -1)$

(iii) $2x^3 + 3x^2 – 9x – 10$

Solution

Given, $f(x) = 2x^3 + 3x^2 – 9x – 10$

Let us assume, $x = -1$

$= 2(-1)^3 + 3(-1)^2 – 9(-1) – 10$

$= -2 + 3 + 9 – 10$

$= 12 - 12$

$= 0$

Therefore, $(x + 1)$ is the factor of $2x^3 + 3x^2 - 9x - 10$

Then, dividing $f(x)$ by $(x + 1)$, we get

	$2x^2 + x - 10$
$X + 1$	$2x^3 + 3x^2 - 9x - 10$
	-
	$2x^3 + 2x^2$
	$X^2 - 9x - 10$
	-
	$X^2 + x$
	$-10x - 10$
	-
	$- 10x - 10$
	0

Therefore, $2x^3 + 3x^2 - 9x - 10 = 2x^2 + 5x - 4x - 10$

$= x(2x + 5) - 2(2x + 5) - (2x + 5)(x - 2)$

Hence the factors are $(x + 1)(x - 2)(2x + 5)$

(iv) $x^3 + 10x^2 – 37x + 26$

Solution

Given, $f(x) = x^3 + 10x^2 – 37x + 26$

Let us assume, $x = 1$

Then, $f(1) = 1^3 + 10(1)^2 – 37(1) + 26$

$= 1 + 10 – 37 + 26$

$= 37 - 37$

$= 0$

Therefore $x – 1$ is a factor of $x^3 + 10x^2 – 37x + 26$

Then, dividing f(x) by (x -1) we get

	$X^2 + 11x – 26$
$X – 1$	$X^3 + 10x^2 – 37x + 26$ - $X^3 - x^2$
	$11x^2 - 37x + 26$ - $11x^2 - 11x$
	$-26x + 26$ - $-26x + 26$
	0

Therefore , $x^3 + 10x^2 – 37x + 26 = (x -1) (x^2 + 11x – 26)$

$= (x -1) (x^2 + 13x – 2x – 26)$

$= (x -1) (x(x + 13) – 2(x + 13))$

$= (x -1) ((x -2) (x + 13))$

16. if (2x +1) is a factor of $6x^3 + 5x^2 + ax – 2$ find the value of a.

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$X = - \frac{1}{2}$

Given, $f(x) = 6x^3 + 5x^2 + ax – 2$

Now substitute the value of x in f(x)

$f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) - 2$

$= 6\left(-\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) - \frac{1}{2}a - 2$

$= -\frac{3}{4} + \frac{5}{4} - \frac{a}{2} - 2$

$= \frac{-3+4-2a-8}{4}$

$= \frac{-6-2a}{4}$

From the question, (2x +1) is a factor of $6x^3 + 5x^2 + ax – 2$

Then remainder is 0.

So, $\frac{-6-2}{4} = 0$

-6 – 2a = 4 × 0

- 6 – 2a = 0

-2 a = 6

a = $-\frac{6}{2}$

a = - 3

Therefore, the value of a is – 3.

17. if (3x – 2) is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k

Solution

Let us assume 3x – 2 = 0

Then, 3x = 2

X = $\frac{2}{3}$

Given, f(x) = $3x^3 - kx^2 + 21x - 10$

Now, substitute the value of x in f(x)

F $\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10$

= $3\left(\frac{8}{27}\right) - k\left(\frac{4}{9}\right) + 14 - 10$

= $\frac{8}{9} - \frac{4k}{9} + 14 - 10$

$= \frac{8}{9} - \frac{4k}{9} + 4$

$= \frac{8-4k+36}{9}$

$= \frac{44-4k}{9}$

From the question, $(3x - 2)$ is a factor of $3x^3 - kx^2 + 21x - 10$

Then, remainder is 0

So, $\frac{44 - 4k}{9} = 0$

$44 - 4k = 0 \times 9$

$44 = 4k$

$K = \frac{44}{4}$

$K = 11$

18. if $(x - 2)$ is a factor of $2x^3 - x^2 + px - 2$, then (i) find the value of p (ii) with this value of p, factorize the above expression completely.

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 2x^3 - x^2 + px - 2$

Now, substitute the value of x in f(x)

$f(2) = (2 \times 2^3) - 2^2 + (p \times 2) - 2$

$= (2 \times 8) - 4 + 2p - 2$

$= 16 - 4 + 2p - 2$

$= 16 - 6 + 2p$

$= 10 + 2p$

From the question, $(x - 2)$ is a factor of $2x^3 - x^2 + px - 2$

Then, remainder is 0

$10 + 2p = 0$

$2p = -10$

$P = -\frac{10}{2}$

$P = -5$

So $(x - 2)$ is a factor of $2x^3 - x^2 + 5x - 2$

	$2x^2 + 3x + 11$
X - 2	$2x^3 - x^2 + 5x - 2$
	-
	$2x^3 - 4x^2$
	$3x^2 + 5x - 2$
	-
	$3x^2 - 6x$
	$11x - 2$
	-
	$11x - 22$
	20

Therefore, $2x^3 - x^2 + 5x - 2 = (x - 2)(2x^2 + 3x + 1)$

$= (x - 2)(2x^2 + 2x + x + 1)$

$= (x - 2)(2x(x + 1) + 1(x + 1))$

$= (x + 1)(x - 2)(2x + 1)$

19. What number should be subtracted from $2x^3 - 5x^2 + 5x$ so that the resulting polynomial has $2x - 3$ as a factor?

Solution

Let us assume the number to be subtracted from $2x^3 - 5x^2 + 5x$ be p.

Then $f(x) = 2x^3 - 5x^2 + 5x - p$

Given, $2x - 3 = 0$

$X = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = 0$

so, $f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) - p = 0$

$2\left(\frac{22}{7}\right) - 5\left(\frac{9}{4}\right) + \frac{15}{2} - p = 0$

$\frac{22}{4} - \frac{45}{4} + \frac{15}{2} - p = 0$ [multiply by 4 for all numerator]

$27 - 45 + 30 - 4p = 0$

$57 – 45 – 4p = 0$

$12 – 4p = 0$

$P = \frac{12}{4}$

$P = 3$

Therefore, 3 is the number should be subtracted from $2x^3 – 5x^2 + 5x$

20. (i) Find the value of the constants a and b, if (x – 2) and (x + 3) are both factors of the expression $x^3 + ax^2 + bx - 12$

Solution

Let us assume $x – 2 = 0$

Then, $x = 2$

Given, $f(x) = x^3 + ax^2 + bx – 12$

Now, substitute the value of x in f(x)

$f(2) = 2^3 + a(2)^2 + b(2) – 12$

$= 8 + 4a + 2b – 12$

$= 4a + 2b – 4$

From the question, (x- 2) is a factor of $x^3 + ax^2 + bx – 12$

So, $4a + 2b – 4 = 0$

$4a + 2b = 4$

By dividing both the side by 2 we get,

$2a + b = 2$.... [equation (i)]

Now, assume $x + 3 = 0$

Then , $x = - 3$

Given, $f(x) = x^3 + ax^2 + bx – 12$

Now, substitute the value of x in f(x)

$f(-3) = (-3)^3 + a(-3)^2 + b(-3) – 12$

$= - 27 + 9a – 3b – 12$

$= 9a – 3b – 39$

From the question, (x- 3) is a factor of $x^3 + ax^2 + bx – 12$

So, $9a – 3b – 39 = 0$

$9a – 3b = 39$

By dividing both the side by 3 we get,

$3a – b = 13$.....[equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

$(2a + b) + (3a – b) = 2 + 13$

$2a + 3b + b – b = 15$

$5a = 15$

$a = \frac{15}{15}$

$a = 3$

consider the equation (i) to find out ‘b’

$2a + b = 2$

$2(3) + b = 2$

$6 + b = 2$

$B = 2 – 6$

$B = -4$

(ii) if $(x + 2)$ and $(x +3)$ are factors of $x^3 + ax + b$, find the values of a and b.

Solution

Let us assume $x + 2 = 0$

Then, $x = - 2$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in f(x)

$f(-2) = (-2)^3 + a(-2) + b$

$= -8 – 2a + b$

From the question $(x + 2)$ is a factor of $x^3 + ax + b$

Therefore, remainder is 0

$f(x)= 0$

$-8 – 2a + b = 0$

$-2a – b = - 8$ ....[equation (i)]

Let us assume x + 3 = 0

Then, x = - 3

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in f(x)

$f(-2) = (-3)^3 + a(-3) + b$

$= -27 – 3a + b$

From the question, (x + 3) is a factor of $x^3 + ax + b$

Therefore, remainder is 0

f(x) = 0

-27 – 3a + b = 0

3a – b = -27 ....[equation(i)]

Now, subtracting both equation (i) and equation(ii) we get,

(2a – b) – (3a – b) = - 8 – (-27)

2a – 3a – b + b = - 8 + 27

-a = 19

a = -19

consider the equation (i) to find out 'b'.

2a – b = -8

2(-19) – b = -8

-38 – b = -8

b = - 38 + 8

b = -30

21. if (x + 2) and (x- 3) are factors of x^3 + ax + b, find the value of a and b. With these values of a and b, factorize the given expression.

Solution

Let us assume x + 2 = 0

Then, x = -2

Given, f(x) = x^3 + ax + b

Now, substitute the value of x in f(x),

f(-2) = $(-2)^3$ + a(-2) + c

= -8 – 2a + b

From the question, (x + 2) is a factor of x^3 + ax + b

Therefore, remainder is 0.

f(x) = 0

-8 – 2a + b = 0

2a – b = - 8....[equation (i)]

Now, assume x – 3 = 0

Then, x = 3

Given , f(x) = x^3 + ax + b

Now, substitute the value of x in f(x)

f(3) = $(3)^3$ + a(3) + b

now, substitute the value of x in f(x)

f(3) = $(3)^3$ + a(3) + b

= 27 + 3a + b

From the question , (x – 3) is a factor of $x^3 + ax + b$

Therefore, remainder is 0

f(x) = 0

27 + 3a + b = 0

3a + b = - 27 ....[equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

(2a – b) + (3a + b) = - 8 – 27

2a – b + 3a + b = - 35

5a = - 35

$a = -\frac{35}{5}$

a = -7

consider the equation(i) to find out 'b'

2a – b = - 8

2(-7) – b = -8

-14 – b = - 8

b = - 14 + 8

b = - 6

therefore, value of a = - 7 and b = - 6

then, $f(x) = x^3 – 7x – 6$

$(x + 2)\ (x - 3)$

$= x(x - 3) + 2(x – 3)$

$= x^2 – 3x + 2x – 6$

$= x^2 – x – 6$

Dividing f(x) by $x^2 – x – 6$ we get,

	$X + 1$
$X^2 – x – 6$	$X^3 + 0x^2 – 7x – 6$
	-
	$X^3 – x^2 – 6x$
	$X^2 – x – 6$
	-
	$X^2 – x - 6$
	0

Therefore, $x^3 – 7x – 6 = (x + 1)\ (x + 2)\ (x – 3)$

22. (x – 2) is a factor of expression $x^3 + ax^2 + bx + 6$. When this expression is divided by (x- 3), it leaves the remainder 3. Find the value of a and b.

Solution

From the question it is given that, (x- 2) is a factor of the expression $x^3 + ax^2 + bx + 6$

Then, $f(x) = x^3 + ax^2 + bx + 6$ ....[equation (i)]

Let assume $x – 2 = 0$

Then, $x = 2$

Now, substitute the value of x in f(x),

$f(2) = 2^3 + a(2)^2 + 2b\ \ 6$

$= 8 + 4a + 2b + 6$

$= 14 + 4a + 2b$

By dividing the numbers by 2 we get,

$= 7 + 2a + b$

From the question,(x – 2) is a factor of the expression $x^3 + ax^2 + bx + 6$.

So, remainder is 0.

$f(x) = 0$

$7 + 2a + b = 0$

$2a + b = - 7$....[equation(ii)]

Now, expression is divided by (x – 3) it leaves the remainder 3.

	$X^2 + x(a + 3) + 3a + b + 9$
$X - 3$	$X^3 + ax^2 + bx + 6$ - $X^3 \quad -3x^2$
	$X^2(a + 3) + bx + 6$ - $X^2(a + 3) + x(-3a - 9)$
	$X(3a + b + 9) + 6$ - $X(3a + b + 9) + -9a - 3b - 27$
	$9a + 3b + 33$

So, remainder = $33 + 9a + 3b = 3$

$9a + 3b = 3 - 33$

$9a + 3b = - 30$

By dividing the numbers by 3 we get,

$= 3a + b = - 10$....[equation(iii)]

Now, subtracting equation(iii) from equation (ii) we get,

$(3a + b) - (2a + b) = - 10 - (-7)$

$3a - 2a + b - b = -10 + 7$

$a = -3$

consider the equation (ii) to find out ‘b’

$2a + b = - 7$

$2(-3) + b = - 7$

$-6 + b = -7$

b = - 7 + 6

b = - 1

23. If (x- 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52, find the values of a and b.

Solution

From the question it is given that (x -2) is a factor of the expression $2x^3 + ax^2 + bx - 14$

Then, $f(x) = 2x^3 + ax^2 + bx - 14$....[equation(i)]

Let assume x – 2 = 0

Then , x = 2

Now, substitute the value of x in f(x),

$f(2) = 2(2)^3 + a(2)^2 + 2b - 14$

= 16 + 4a + 2b – 14

= 2 + 4a + 2b

By dividing the numbers by 2 we get,

= 1 + 2a + b

From the question, (x-2) is a factor of the expression $2x^3 + ax^2 + bx - 14$

So, remainder is 0.

$f(x) = 0$

$1 + 2a + b = 0$

$2a + b = - 1$ .....[equation(ii)]

Now, expression is divided by (x – 3) it leaves the remainder 52.

	$2x^2 + x(a + 6) + 3a + b + 18$
X - 3	$2x^3 + ax^2 + bx – 14$
	-
	$2x^3 \quad -6x^2$
	$X^2(a + 6) \quad +bx \quad -14$
	-
	$X^2(a + 6) + x (-3a – 18)$
	$X (3a + b + 18) \quad -14$
	-
	$X(3a + b + 18) + -9a -3b – 54$
	$9a + 3b + 40$

So, remainder = $9a + 3b + 40 = 52$

$9a + 3b = 52 – 40$

$9a + 3b = 12$

By dividing the number by 3 we get,

$= 3a + b = 4$....[equation(iii)]

Now , subtracting equation(iii) from equation(ii) we egt,

$(3a + b) – (2a + b) = 4 – (-1)$

$3a – 2a + b – b = 4 + 1$

$a = 5$

a = 5

consider the equation (ii) to find out ‘b’.

2a + b = - 1

2(5) + b = - 1

10 + b = - 1

b = - 1 – 10

b = -11

24. If $ax^3 + 3x^2 + bx - 3$ has a factor (2x + 3) and leaves remainder – 3 when divided by (x + 2), find the value of a and b. With these values of a and b, factorize the given expression.

Solution

Let us assume, 2x + 3 = 0

Then, 2x = - 3

$X = -\frac{3}{2}$

Given, $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of x in f(x),

$f\left(-\frac{3}{2}\right) = a\left(-\frac{3}{2}\right)^3 + 3\left(-\frac{3}{2}\right)^2 + b\left(-\frac{3}{2}\right) - 3$

$= a\left(-\frac{27}{8}\right) + 3\left(\frac{9}{4}\right) - \frac{3b}{2} - 3$

$= -\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$

From the question it is given that, $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$.

So, remainder is 0.

$-\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3 = 0$

$-27a + 54 - 12b - 24 = 0$

$-27a - 12b = -30$

By dividing the numbers by – 3 we get,

$9a + 4b = 10$ [equation(i)]

Now, let us assume $x + 2 = 0$

Then $x = -2$

Given, $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of x in f(x)

$f(2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$

$= - 8a + 12 - 2b - 3$

$= -8a - 2b + 9$

Leaves the remainder -3

So, $-8a - 2b + 9 = -3$

$-8a - 2b = -3 - 9$

$-8a - 2b = - 12$

By dividing both sides by – 2 we get,

4a + b = 6 [equation(ii)]

By multiplying equation(ii) by 4

16a + 4b = 24

Now, subtracting equation(ii) from equation (i) we get.

(16a + 4b) – (9a + 4b) = 24 – 10

16a – 9a + 4b – 4b = 14

7a = 14

a = $\frac{14}{7}$

a = 2

consider the equation (i) to find out 'b'

9a + 4b = 10

9(2) + 4b = 10

18 + 4b = 10

4b = 10 – 18

4b = -8

b = $-\frac{8}{4}$

b = -2

therefore, f(x) = $ax^3 + 3x^2 + bx - 3$

= $2x^3 + 3x^2 - 2x - 3$

Given, $2x + 3$ is a factor of f(x)

So, divide f(x) by $2x + 3$

	$X^2 - 1$
$2x + 3$	$2x^3 + 3x^2 - 2x - 3$
	-
	$2x^3 + 3x^2$
	$0 - 2x - 3$
	-
	$- 2x - 3x$
	0

Therefore, $2x^3 + 3x^2 - 2x - 3 = (2x + 3)(x^2 - 1)$

$= (2x + 3)(x + 1)(x - 1)$

25. Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If $x - 2$ is a factor of f(x) but leaves the remainder – 15 when it divides g(x), find the value of a and b. With these values of a and b, factorize the expression. $f(x) + g(x) + 4x^2 + 7x$

Solution

From the equation it is given that, $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$ and $x – 2$ is a factor of $f(x)$

So, $x = 2$

Now, substitute the value of x in f(x)

$f(2) = 0$

$a(2)^2 + b(2) + 2 = 0$

$4a + 2b + 2 = 0$

By dividing both sides by 2 we get,

$2a + b + 1 = 0$....[equation (i)]

Given, g(x) divide by (x – 2) , leaves remainder – 15

$g(x) = bx^2 + ax + 1$

so, $g(2) = -15$

$b(2)^2 + 2a + 1 = - 15$

$4b + 2a + 1 + 15 = 0$

$4b + 2a + 16 = 0$

By dividing both sides by 2 we get,

$2b + a + 8 = 0$ ....[equation (ii)]

Now, subtracting equation (ii) from equation(i) multiplied by 2,

$(4a + 2b + 2) – (a + 2b + 8) = 0 – 0$

$4a – a + 2b – 2b + 2 – 8 = 0$

$3a – 6 = 0$

$3a = 6$

$a = \frac{6}{3}$

$a = 2$

consider the equation (i) to find out ‘b’

$2a + b + 1 = 0$

$2(2) + b = - 1$

$4 + b = -1$

$b = - 1 – 4$

$b = - 5$

now $f(x) = ax^2 + bx + 2 = 2x^2 – 5x + 2$

$g(x) = bx^2 + ax + 1 = -5x^2 + 2x + 1$

then, $f(x) + g(x) + 4x^2 + 7x$

$= 2x^2 – 5x + 2 – 5x^2 + 2x + 1 + 4x^2 + 7x$

$= x^2 + 4x + 3$

$= x^2 + 3x + x + 3$

$= x(x + 3) + 1(x + 3)$

$= (x + 1) (x + 3)$

Chapter test

1. find the remainder when $2x^3 - 3x^2 + 4x + 7$ is divided by

(i) $x - 2$

(ii) $x + 3$

(iii) $2x + 1$

Solution

From the question it is given that, $f(x) = 2x^3 - 3x^2 + 4x + 7$

(i) consider $x - 2$

Let us assume $x - 2 = 0$

Then, $x = 2$

Now, substitute the value of x in f(x)

$f(2) = 2(2)^3 - 3(2)^2 + 4(2) + 7$

$= 16 - 12 + 8 + 7$

$= 31 - 12$

$= 19$

Therefore, the remainder is 19

(ii) consider x + 3

Let us assume x + 3 = 0

Then, x = - 3

Now, substitute the value of x in f(x)

$f(2) = 2(-3)^3 - 3(-3)^2 + 4(-3) + 7$

$= 2(-27) - 3(9) - 12 + 7$

$= - 54 - 27 - 12 + 7$

$= - 93 + 7$

$= - 86$

Therefore, remainder is -86

(iii) consider 2x + 1

Let us assume 2x + 1 = 0

Then, 2x = -1

$X = -\frac{1}{2}$

Now, substitute the value of x in f(x)

$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7$

$= 2\left(-\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) + 7$

$= -\frac{1}{4} - \frac{3}{4} - 2 + 7$

$= 4$

Therefore, remainder is 4

2. When $2x^3 – 9x^2 + 10x – p$ is divided by (x + 1) the remainder is -24 find the value of p.

Solution

Let us assume x + 1 = 0

Then x = - 1

Given $f(x) = 2x^3 – 9x^2 + 10x – p$

Now, substitute the value of x in f(x)

$f(-1) = 2(-1)^3 – 9(-1)^2 + 10(-1) – p$

= -2 – 9 – 10 + p

= - 21 + p

From the question it is given that the remainder is – 2,

So, - 21 + p = -24

P = - 24 + 21

P = -3

So, $f(x) = 2x^3 – 9x^2 + 10x – (-3)$

$= 2x^3 – 9x^2 + 10x + 3$

Therefore, the value of p is 3

3. If (2x – 3) is a factor of $6x^2 + x + a$, find the value of a. With this value of a, factorise the given expression.

Solution

Let us assume 2x – 3 = 0

Then, 2x = 3

$X = \frac{3}{2}$

Given $f(x) = 6x^2 + x + a$

Now, substitute the value of x in f(x)

$f\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + a$

$= 6\left(\frac{9}{4}\right) + \left(\frac{3}{2}\right) + a$

$= \frac{27}{2} + \frac{3}{2} + a$

$= \frac{30}{2} + a$

= 15 + a

From the question (2x – 3) is a factor of $6x^2 + x + a$

So remainder is 0

Then 15 + a = 0

a = - 15

therefore $f(x) = 6x^2 + x – 15$

dividing f(x) by 2x – 3 we get

	$3x + 5$
X - 3	$6x^2 + x – 15$ - $6x^2$ $-9x$
	$10x - 15$ - $10x - 15$
	0

Therefore, $6x^2 + x – 15 = (2x – 3)(3x + 5)$

4. when $3x^2 – 5x + p$ is divided by (x – 2) the remainder is 3. Find the value of p. Also factorize the polynomial $3x^2 – 5x + p – 3$

Solution

Let us assume $x – 2 = 0$

Then, $x = 2$

Given, $f(x) = 3x^2 – 5x + p$

Now, substitute the value of x in f(x),

So, $f(2) = 3(2)^2 – 5(2) + p$

$-3(4) – 10 + p$

$= 12 – 10 + p$

$= 2 + p$

From the question it is given that, remainder is 3.

So, $2 + p = 3$

$P = 3 - 2$

$P = 1$

Therefore, $f(x) = 3x^2 - 5x + 1$

Consider the polynomial, $3x^2 - 5x + p - 3$

Now, substitute the value of p in polynomial,

$= 3x^2 - 5x + 1 - 3$

$= 3x^2 - 5x - 2$

Now by factorizing the polynomial $3x^2 - 5x - 2$.

Dividing $3x^2 - 5x - 2$ by $x - 2$ we get

	$3x + 1$
$X - 2$	$3x^2 - 5x - 2$
	-
	$3x^2 - 6x$
	$X - 2$
	-
	$X - 2$
	0

Therefore, $3x^2 - 5x - 2 = (x - 2)(3x + 1)$

5. Prove that (5x + 4) is a factor of $5x^3 + 4x^2 - 5x - 4$. Hence factorize the given polynomial completely.

Solution

Let us assume (5x + 4) = 0

Then, 5x = -4

$X = -\frac{4}{5}$

Given, $f(x) = 5x^3 + 4x^2 - 5x - 4$

Now, substitute the value of x in f(x)

So, $f\left(-\frac{4}{5}\right) = 5\left(-\frac{4}{5}\right)^3 + 4\left(-\frac{4}{5}\right)^2 - 5\left(-\frac{4}{5}\right) - 4$

$= 5\left(-\frac{64}{125}\right) + 4\left(\frac{16}{25}\right) + 4 - 4$

$= -\frac{64}{25} + \frac{64}{25}$

$= \frac{-64+64}{25}$

$= \frac{0}{25}$

= 0

Hence, (5x + 4) is a factor of $5x^3 + 4x^2 - 5x - 4$

So, dividing $5x^3 + 4x^2 - 5x - 4$ by 5x + 4 we get,

	$X^2 - 1$
$5x + 4$	$5x^3 + 4x^2 – 5x – 4$ - $5x^3 + 4x^2$
	$0 – 5x – 4$ - $-5x – 4x$
	0

Therefore, $5x^3 + 4x^2 – 5x – 4 = (5x + 4) (x^2 – 1)$

$= (5x + 4) (x^2 – 1^2)$

$= (5x + 4) (x + 1) (x -1)$

6. use factor theorem to factorize the following polynomials completely:

(i) $4x^3 + 4x^2 – 9x – 9$

Solution

Let us assume x = -1

Given, $f(x) = 4x^3 + 4x^2 – 9x – 9$

Now, substitute the value of x in f(x)

$f(-1) = 4(-1)^3 + 4(-1)^2 -9(-1) – 9$

$= - 4 + 4 + 9 – 9$

$= 0$

Therefore, x + 1 is the factor of $4x^3 + 4x^2 – 9x – 9$

Now, dividing $4x^3 + 4x^2 – 9x – 9$ by x + 1 we get,

	$4x^2 - 9$
X + 1	$4x^3 + 4x^2 – 9x – 9$ - $4x^3 + 4x^2$
	$0 – 9x - 9$ - $-9x - 9x$
	0

Therefore , $4x^3 + 4x^2 – 9x – 9 = (x + 1) (4x^2 – 9)$

$= (x + 1) ((2x)^2 – (3)^2)$

$= (x + 1) (2x + 3) (2x – 3)$

(ii) x^3 – 19x – 30

Solution

Let us assume x = -2

Given $f(x) = x^3 – 19x – 30$

Now, substitute the value of x in f(x)

$f(-1) = (-2)^3 – 19(-2) – 30$

$= -8 + 38 – 30$

$= -38 + 38$

$= 0$

Therefore, $x + 2$ is the factor of $x^3 - 19x - 30$

Now, dividing $x^3 - 19x - 30$ by $x + 2$ we get,

	$X^2 - 2x - 15$
$X + 2$	$X^3 + 0x^2 - 19x - 30$
	-
	$X^3 \quad + 2x^2$
	$- \ 2x^2 \ -19x \ - 30$
	-
	$- 2x^2 \ - 4x$
	$-15x \ - 30$
	-
	$- 15x \ - 30$
	0

Therefore, $x^3 - 19x - 30 = (x + 2)(x^2 - 2x - 15)$

$= (x + 2)(x^2 - 5x + 3x - 15)$

$= (x + 2)(x - 5)(x + 3)$

7. if $x^3 – 2x^2 + px + q$ has a factor $(x + 2)$ and leaves a remainder 9, when divided by $(x + 1)$, find the value of p and q with these values of p and q factorize the given polynomial completely.

Solution

From the question it is given that, $(x + 2)$ is a factor of the expression $x^3 – 2x^2 + px + q$

Then, $f(x) = x^3 – 2x^2 + px + q$

Let assume $x + 2 = 0$

Then $x = -2$

Now, substitute the value of x in f(x)

$F(-2) = (-2)^3 -2(-2)^2 + p(-2) + q$

$= - 8 – 8 – 2p + q$

$= -16 – 2p + q$

$2p – q = -16$ .....[equation (i)]

Now, consider$(x+ 1)$

Then, $f(x) = x^3 – 2x^2 + px + q$

Let assume $x + 1 = 0$

Then, $x = -1$

Now, substitute the value of x in f(x)

$F(-1) = (-1)^3 – 2(-1)^2 + p(-1) + q$

$= -1 – 2 – p + q$

$= -3 – p + q$

Given, remainder is 9

So, $-3 – p + q = 9$

$-p + q = 9 + 3$

$-p + q = 12$....[equation(ii)]

Now adding equation (i) and equation (ii) we get,

$(2p –q) + (-p + q) = -16 + 12$

$2p – q – p + q = -4$

$P = -4$

Consider the equation (ii) to find out 'b'

$-p + q = 12$

$-(4) + q = 12$

$4 + q = 12$

$Q = 12 – 4$

$Q = 8$

Therefore, by substituting the value of p and q $f(x) = x^3 – 2x^2 –$

$4x + 8$

Dividing f(x) be (x + 2) we get,

	$X^2 - 4x + 4$
$X + 2$	$X^3 - 2x^2 - 4x + 8$ - $X^3 \ + 2x^2$
	$-4x^2 - 4x + 8$ - $- 4x^2 \ - 8x$
	$4x \ + 8$ - $4x + 8$
	O

$X^3 - 2x^2 - 4x + 8 = (x + 2)\ (x^2 - 4x + 4)$

$= (x + 2)\ (x^2 - 2\times\ x(-2) + 2^2\ )$

$= (x +2)\ (x -2)^2$

8. if (x + 3) and (x -4) are factors of $x^3 + ax^2 - bx + 24$ find the values of a and b : with these values of a and b, factorize the given expression.

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of x in f(x)

$f(-3) = (-3)^3 + a(-3)^2 – b(-3) + 24$

$= -27 + 9a + 3b + 24$

$= 9a + 3b – 3$

Dividing all terms by 3 we get,

$= 3a + b – 1$

From the question(x +3) is a factor of $x^3 + ax^2 – bx + 24$

Therefore, remainder is 0

$f(x) = 0$

$3a + b – 1 = 0$

3a + b = 1....[equation(i)]

Now, assume $x – 4 = 0$

Then, $x = 4$

Given $f(x) = x^3 + ax^2 – bx + 24$

Now substitute the value of x in f(x)

$F(4) = 4^3 + a(4)^2 – b(4) + 24$

$= 64 + 16a – 4b + 24$

$= 88 + 16a – 4b$

Dividing all terms by 4 we get,

$= 22 + 4a – b$

From the question (x -4) is a factor of $x^3 + ax^2 – bx + 24$

Therefore remainder is 0

$f(x) = 0$

$22 + 4a - b = 0$

$4a - b = -22$....[equation(ii)]

Now, adding both equation(i) and equation(ii) we get,

$(3a + b) + (4a - b) = 1 - 22$

$3a + b + 4a - b = -21$

$7a = -21$

$a = -\frac{21}{7}$

$a = -3$

consider the equation (i) to find out 'b'.

$3a + b = 1$

$3(-3) + b = 1$

$-9 + b = 1$

$b = 1 + 9$

$b = 10$

therefore value of a = -3 and b =10

then, by substituting the value of a and b $f(x) = x^3 - 3x^2 - 10x + 24$

$(x + 3)(x - 4)$

$= x(x - 4) + 3(x - 4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$

Dividing f(x) by $x^2 - x - 12$ we get,

	$X - 2$
$X^2 - x - 12$	$X^3 - 3x^2 - 10x + 24$
	-
	$X^3 - x^2 - 12x$
	$-2x^2 + 2x + 24$
	-
	$-2x^2 + 2x + 24$
	0

Therefore, $x^3 - 3x^2 - 10x + 24 = (x^2 - x - 12)(x - 2)$

$= (x + 3)(x - 4)(x - 2)$

9. if (2x + 1) is a factor of both the expressions $2x^2 - 5x + p$ and $2x^2 + 5x + q$, find the value of p and q hence find the other factors of both the polynomials.

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$X = -\frac{1}{2}$

Given $p(x) = 2x^2 - 5x + p$

Now, substituted the value of x in p(x)

$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) + p$

$= 2\left(\frac{1}{4}\right) + \frac{5}{2} + p$

$= \frac{6}{2} + p$

$= 3 + p$

From the question it is given that, $(2x + 1)$ is a factor of both the expressions $2x^2 - 5x + p$

So, remainder is 0

Then, $3 + p = 0$

$P = -3$

Now consider $q(x) = 2x^2 + 5x + q$

Substitute the value of x in q(x)

$q\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + q$

$= 2\left(\frac{1}{4}\right) - \frac{5}{2} + q$

$= \frac{1-5}{2} + q$

$= -\frac{4}{2} + q$

$= q - 2$

From the question it is given that, $(2x + 1)$ is a factor of both the expressions $2x^2 + 5x + q$

So, remainder is 0

$Q - 2 = 0$

$q = 2$

therefore, $p = -3$ and $q = 2$

$p(x) = 2x^2 - 5x - 3$

$q(x) = 2x^2 + 5x + 2$

then, divide p(x) by $2x + 1$

	$X - 3$
$2x + 1$	$2x^2 - 5x - 3$
	-
	$2x^2 + x$
	$-6x - 3$
	-
	$-6x - 3$
	0

Therefore $2x^2 - 5x - 3 = (2x + 1)(x - 3)$

Now, divide q(x) by $2x + 1$

	$X + 2$
$2x + 1$	$2x^2 + 5x + 2$
	-
	$2x^2 + x$
	$4x + 2$
	-
	$4x + 2$
	0

Therefore, $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

10. if a polynomial $f(x) = x^4 – 2x^3 + 3x^2 – ax + b$ leaves remainder 5 and 19 when divided by $(x – 1)$ and $(x + 1)$ respectively. Find the value of a and b. Hence determined the reminder when f(x) is divided by $(x – 2)$

Solution

From the question it is given that

$f(x) = x^4 - 2x^3 + 3x^2 – ax + b$

factor $(x – 1)$ leaves remainder 5,

factor$(x + 1)$ leaves remainder 19,

where $x = 1$ and $x = -1$

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$1 – 2(-1) + 3(1) –a(-1) + b = 19$

$1 + 2 + 3 + a + b = 19$

$6 + a + b = 19$

$A + b = 19 – 6$

$A + b = 13$......[equation (i)]

$f(1) = (1)^4 – 2(1)^3 +3(1)^2 – a(1) + b = 5$

$1 – 2(1) + 3(1) - a(1) + b = 5$

$1 -2 +3 – a + b = 5$

$2 – a + b = 5$

$-a + b = 5 – 2$

$-a + b = 3$.....[equation(ii)]

Now, subtracting equation(ii) from equation (i) we egt,

$(a + b) –(-a + b) = 13 – 3$

$A + b + a – b = 10$

$2a = 10$

$a = \frac{10}{2}$

$a = 5$

to find out the value of b, substitute the value of a in equation (i) we get,

$a + b = 13$

$5 + b = 13$

B= 13 – 5

b = 8

therefore, value of a = 5 and b = 8

11. when a polynomial f(x) is divided by (x -1) the remainder is 5 and when it is, divided by (x -2) the remainder is 7. Find the remainder when it is divided by (x -1) (x -2)

Solution

From the question it is given that,

Polynomial f(x) is divided by (x -1)

Remainder = 5

Let us assume x – 1 = 0

X = 1

f(1) = 5

and the divide be (x – 2) remainder = 7

let us assume x – 2 = 0

x = 2

therefore, f(2) = 7

so, f(x) = (x – 1) (x -2) q(x) + ax + b

where $q(x)$ is the quotient and $ax + b$ is remainder

now put $x = 1$ we get

$f(1) = (1-1)\ (1-2)q(1) + (a \times 1) + b$

$a + b = 5$ .....[equation (i)]

$x = 2$

$f(2) = (2-1)\ (2-2)\ q(2) + (a \times 2) + b$

$2a + b = 7$....[equation(ii)]

Now subtracting equation (i) from equation (ii) we get,

$(2a + b) – (a + b) = 7 – 5$

$2a + b – a – b = 2$

$A = 2$

To find out the value of b, substitute the value of a in equation(i) we get

$a + b = 5$

$2 + b = 5$

$b = 5 – 2$

$b = 3$

therefore, the remainder $= ax + b = 2x + 3$