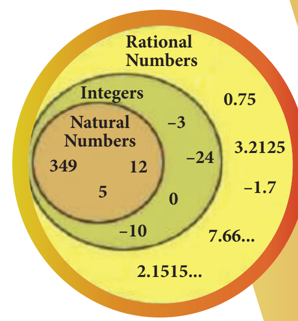


1 NUMBERS

Learning Objectives

- To understand the necessity for extending fractions to rational numbers, to represent rational numbers on the number line and to know that between any two given rational numbers, there lies many rational numbers.
- To learn and perform the four basic arithmetic operations and solve word problems on rational numbers and simplify expressions with atmost three brackets.
- To understand the properties of rational numbers.
- To compute the square, the square root, the cube and the cube root of numbers.
- To make a rough estimate of the square roots and the cube roots of numbers.
- To express numbers in exponential form and understand the laws of exponents with integral powers.
- To identify and express the numbers in scientific notation.

B7XMD8

1.1 Introduction

Let us recall the different types of numbers which we have already learnt in our earlier classes. When we want to count, it is natural to start with numbers 1, 2, 3, 4, 5, … Isn't it?

These are all called as **Counting numbers** or **Natural numbers** and their collection is denoted by $\mathbb{N}$. The use of three dots at the end of the list is a notation to show that the list keeps going forever.

The natural numbers can be visualized as a ray marked with these numbers:

Fig 1.1

Consider the situation that yesterday my purse had money, say ₹8, but today the purse may be empty. How many rupees are there in the purse now? How to denote this emptiness? Here comes the concept of zero which evolved to symbolize the idea of emptiness. The concept of zero, though quite natural now, was not normal to early humans. Only after hundreds of years people started thinking of it as an actual number. The difficulty was solved when the Indian Mathematicians provided the symbol for zero. The natural numbers system with this additional number zero became **Whole numbers.** The whole numbers can be visualized now as follows:

Fig 1.2

The system of whole numbers is denoted by $\mathbb{W}$.

Even zero was not sufficient to solve all problems. Think what happens when 4 is taken away from 6?

Draw a number line up to, say 9. Mark 6 by a dot on it.

Fig 1.3

We know that, to subtract 4, we need to go 4 steps to the left side from 6. We will land on 2 and so the answer is 2. But what will happen if we want to subtract 6 from 4? This situation is where the humans needed (and created) negative numbers.

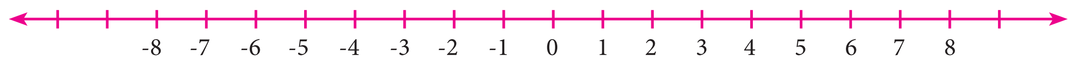
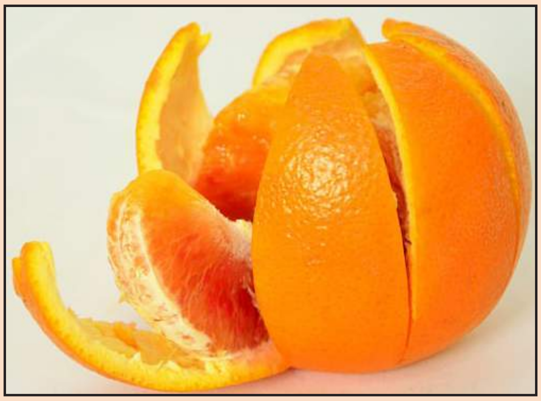
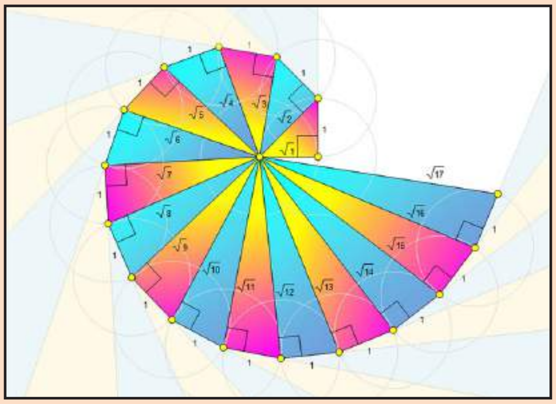


Fig 1.4

But, how can a number be negative? Simple! Just think of them as numbers less than zero. Including the negative integers with the whole numbers, we get the list of numbers called the **Integers.** The integers consist of zero, the natural numbers and the negatives of the natural numbers and it consists a list of numbers that stretch in either direction without end. The entire collection of integers is denoted by $\mathbb{Z}$.

MATHEMATICS ALIVE - NUMBERS IN REAL LIFE	
	
If an orange is peeled off and 8 carpels are found, then one carpel represents the rational number $\frac{1}{8}$.	Finding the square root of numbers in the form spiral using the Pythagoras Theorem from Geometry

1.2 Rational Numbers

Even after coining integers, one could not relax! $10 \div 5$ is no doubt fine, giving the answer 2 but is $8 \div 5$ comfortable? Numbers between numbers are needed. $8 \div 5$ seen as 1.6, is a number between 1 and 2. But, where does $(-3) \div 4$ lie? Between 0 and -1. Similarly, where do you find

$\frac{-12}{5}$ on the number line? Between –2 and –3. Thus, a ratio made by dividing an integer by another integer is called a **rational number**. (Remember, we should not divide by zero!)

Formally speaking, a rational number is a number of the fractional (ratio) form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. The collection of all rational numbers is denoted by $\mathbb{Q}$. Non-negative rational numbers may be thought of as fractions. They can also be expressed as decimals and percents.

Since operations on rational numbers demand a basic knowledge of operations on fractions, let us recall through an exercise some of the basic ideas related to fractions.

Recap Exercise

1. The simplest form of $\frac{125}{200}$ is __________.
2. Which of the following is not an equivalent fraction of $\frac{8}{12}$?
   (A) $\frac{2}{3}$ (B) $\frac{16}{24}$ (C) $\frac{32}{60}$ (D) $\frac{24}{36}$
3. Which is bigger: $\frac{4}{5}$ or $\frac{8}{9}$?
4. Add the fractions: $\frac{3}{5}+\frac{5}{8}+\frac{7}{10}$
5. Simplify: $\frac{1}{8}-\left(\frac{1}{6}-\frac{1}{4}\right)$
6. Multiply: $2\frac{3}{5}$ and $1\frac{4}{7}$.
7. Divide: $\frac{7}{36}$ by $\frac{35}{81}$.
8. Fill in the boxes: $\frac{\square}{66}=\frac{70}{\square}=\frac{28}{44}=\frac{\square}{121}=\frac{7}{\square}$
9. In a city $\frac{7}{20}$ of the population are women and $\frac{1}{4}$ are children. Find the fraction of the population of men.
10. Represent $\left(\frac{1}{2}+\frac{1}{4}\right)$ by a diagram.

Try these

1. Is the number –7 a rational number? Why?
2. Write any 6 rational numbers between 0 and 1.

Note

The word 'ratio' in Math refers to the comparison of the sizes of two different quantities of any kind. For example, if there is one teacher for every 20 students in a class, then the ratio of teachers to students is 1:20. Ratios are often written as fractions and so $1:20=\frac{1}{20}$. For this reason, numbers in the fractions form are called rational numbers.

1.2.1 Rational numbers on a number line

Locating the rational numbers on a number line is an important skill. For example, to represent the number $\frac{-3}{4}$ on the number line, $\frac{-3}{4}$ being negative would be marked to the left of 0 and it is between 0 and –1. We know that the integers, 1 and –1 are equidistant from 0 and so are the numbers 2 and –2, 3 and –3 from 0. This concept remains the same for rational numbers too. Now, as we mark $\frac{3}{4}$ to the right of zero, at 3 parts out of 4 between 0 and 1, the same way, we will mark $\frac{-3}{4}$ to the left of zero, at 3 parts out of 4 between 0 and –1 as shown below.

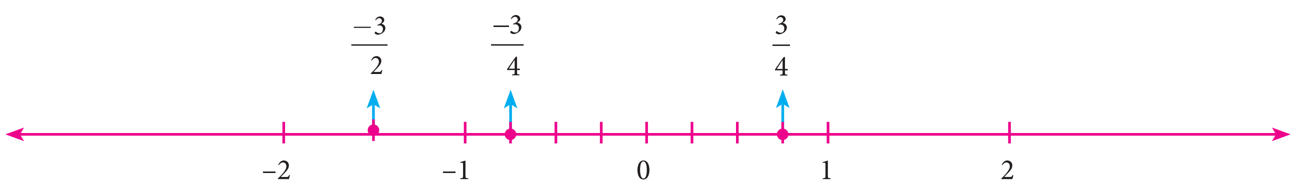


Fig. 1.5

Similarly, it is easy to find $\frac{-3}{2}$ between –1 and –2 since $\frac{-3}{2} = -1\frac{1}{2}$.

Now, on the following number line what rational numbers do the letters A and B represent?

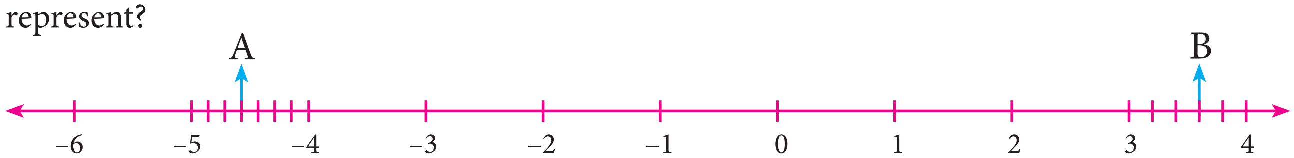


Fig. 1.6

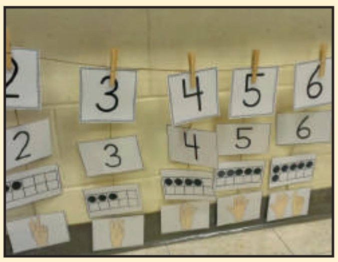
You will now be able to say easily the rational numbers marked by A and B on the number line as shown above. Isn't it? Here, A represents the rational number $-4\frac{4}{7}\left(\frac{-32}{7}\right)$ and B represents the rational number $3\frac{3}{5}\left(\frac{18}{5}\right)$.

1.2.2 Decimal representation of a rational number

A rational number can be nicely represented in decimal form rather than in the usual fractional form. Given a rational number in the form $\frac{a}{b}$ $(b \neq 0)$, just divide the numerator a by the denominator b and we can see that it can be expressed as a terminating or non-terminating, recurring decimal.

Activity

Use a string as a number line and fix it on the wall, for the length of the class room. Just mark the integers spaciously and ask the students to pick the rational number cards from a box and fix it roughly at the right place on the number line string. This can be played between teams and the team which fixes more number of cards correctly (by marking) will be the winner.



Example 1.1

Write the decimal forms of the following rational numbers:

i) $\frac{1}{4}$ ii) $1\frac{3}{20}$ iii) $-5\frac{4}{5}$ iv) 3 v) $\frac{1}{3}$

Solution:

(i) $\frac{1}{4}=\frac{25}{100}=0.25$ (ii) $1\frac{3}{20}=\frac{23}{20}=\frac{115}{100}=1.15$ (iii) $-5\frac{4}{5}=\frac{-29}{5}=\frac{-58}{10}=-5.8$

(iv) $3=\frac{3}{1}=\frac{30}{10}=3.0$ (v) $\frac{1}{3}=0.3333...$ (by actual division and it is recurring and non-terminating)

Note

❖ The above examples show how a rational number may be given in decimal form. The reverse process of converting the decimal form of a rational number to the fractional form may be seen in the higher classes.

❖ There are decimal numbers which are non-terminating and non-recurring such as

π = 3.141592653589793238462643…..

$\sqrt{2}$ = 1.41421356237309504880168……. etc.

They are not rational numbers and one can study more about them in the higher classes.

Try these

Write the decimal forms of the following rational numbers:

1. $\frac{4}{5}$ 2. $\frac{6}{25}$ 3. $\frac{486}{1000}$ 4. $\frac{1}{9}$ 5. $3\frac{1}{4}$ 6. $-2\frac{3}{5}$

1.2.3 Positive and Negative rational numbers

Rational numbers may be classified as positive and negative rational numbers. If both the numerator and the denominator of the fraction representing a rational number are of the same sign, then the rational number is positive.

For example, numbers like $\frac{3}{4}, \frac{-11}{-6}$ etc., are positive rational numbers.

If either the numerator or the denominator of the fraction representing a rational number is negative, then the rational number is negative.

For example, numbers like $\frac{-3}{4}, \frac{11}{-6}$ etc., are negative rational numbers.

Note

❖ 0 is a rational number which is neither positive nor negative.

❖ Note that $\frac{-11}{6}=\frac{11}{-6}=-\frac{11}{6}$

1.2.4 Equivalent rational numbers

We know how to write equivalent fractions when a fraction is given. Since a rational number can be represented by a fraction, we can think of equivalent rational numbers, duly obtained through equivalent fractions.

Suppose a rational number is in fractional form. Multiply its numerator and denominator by the same non-zero integer to obtain a rational number which is equivalent to it.

For example,

$$-\frac{2}{3} \text{ is equivalent to } \frac{-6}{9} \text{ since } \frac{-2}{3}=\frac{-2\times 3}{3\times 3}=\frac{-6}{9}$$

$$-\frac{2}{3} \text{ is also equivalent to } \frac{10}{-15} \text{ since } \frac{2}{-3}=\frac{2\times 5}{-3\times 5}=\frac{10}{-15}$$

Thus, $-\frac{2}{3}=\frac{-6}{9}=\frac{10}{-15}$.

Try these

1. $\frac{7}{3}=\frac{?}{9}=\frac{49}{?}=\frac{-21}{?}$
2. $\frac{-2}{5}=\frac{?}{10}=\frac{6}{?}=\frac{-8}{?}$

1.2.5 Rational numbers in Standard form

Observe the following rational numbers: $\frac{4}{5},\frac{-3}{7},\frac{1}{6},\frac{-4}{13},\frac{-50}{51}$. Here, we see that

i) the denominators of these rational numbers are all positive integers
ii) 1 is the only common factor between the numerator and the denominator of each of them and
iii) the negative sign occurs only in the numerator.

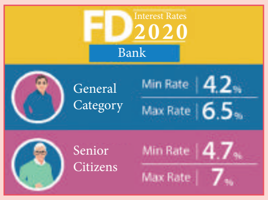
Such rational numbers are said to be in standard form.

A rational number is said to be in standard form, if its denominator is a positive integer and both the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be simplified to arrive at the standard form.

DO YOU KNOW?

The collection of rational numbers is denoted by the letter $\mathbb{Q}$ because it is formed by considering all quotients, except those involving division by 0. Decimal numbers can be put in quotient form and hence they are also rational numbers.



Example 1.2

Reduce to the standard form: (i) $\frac{48}{-84}$ (ii) $\frac{-18}{-42}$

Solution:

(i) **Method 1:**

$$\frac{48}{-84}=\frac{48\div(-2)}{-84\div(-2)}=\frac{-24\div 2}{42\div 2}=\frac{-12\div 3}{21\div 3}=\frac{-4}{7} \text{ (dividing by } -2, 2 \text{ and } 3 \text{ successively)}$$

Method 2:

The HCF of 48 and 84 is 12 (Find it!). Thus, we can get its standard form by dividing it by −12.

$$\frac{48}{-84}=\frac{48\div(-12)}{-84\div(-12)}=\frac{-4}{7}$$

(ii) **Method 1:**

$$\frac{-18}{-42}=\frac{-18\div(-2)}{-42\div(-2)}=\frac{9\div 3}{21\div 3}=\frac{3}{7} \quad \text{(dividing by −2 and 3 successively)}$$

Method 2:

The HCF of 18 and 42 is 6 (Find it!). Thus, we can get its standard form by dividing it by 6.

$$\frac{-18}{-42}=\frac{-18\times(-1)}{-42\times(-1)}=\frac{18}{42}=\frac{18\div 6}{42\div 6}=\frac{3}{7}$$

Try these

1. Which of the following pairs represents equivalent rational numbers?

   (i) $\frac{-6}{4}, \frac{18}{-12}$ (ii) $\frac{-4}{-20}, \frac{1}{-5}$ (iii) $\frac{-12}{-17}, \frac{60}{85}$

2. Find the standard form of:

   (i) $\frac{36}{-96}$ (ii) $\frac{-56}{-72}$ (iii) $\frac{27}{18}$

3. Mark the following rational numbers on a number line.

   (i) $\frac{-2}{3}$ (ii) $\frac{-8}{-5}$ (iii) $\frac{5}{-4}$

1.2.6 Comparison of rational numbers

It is useful to remember the following points:

- Every positive number is greater than zero.
- Every negative number is smaller than zero.
- Every positive number is greater than every negative number.
- Every number on the right of a number on a number line is greater than that number.

When two integers or fractions are given, we know how to compare them and say which is greater or smaller. Now, in the same way, we can compare a pair of rational numbers.

Type 1 : Comparing two rational numbers with opposite signs

Example 1.3

Compare $\frac{5}{17}$ and $\frac{-10}{19}$.

Solution:

Since every positive number is greater than every negative number,

we conclude that $\frac{5}{17} > \frac{-10}{19}$.

Type 2 : Comparing two rational numbers represented by two fractions with same denominators

Example 1.4

Compare $\frac{1}{3}$ and $\frac{4}{3}$.

Solution:

Since the denominators are the same, just compare the numerators.

Since $1 < 4$, we conclude that $\frac{1}{3} < \frac{4}{3}$.

Type 3 : Comparing two rational numbers represented by two fractions with different denominators

Example 1.5

Compare $\frac{3}{4}$ and $\frac{5}{6}$.

Solution:

The LCM of the denominators is 12 (Find it!). Consider for each rational number an equivalent rational number with the LCM 12 as denominator.

We get $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$, which become like fractions now.

Here, $\frac{9}{12} < \frac{10}{12}$. Hence, we conclude that $\frac{3}{4} < \frac{5}{6}$.

Type 4 : Comparing two rational numbers that are not in standard form

Example 1.6

Compare $\frac{9}{-4}$ and $\frac{-2}{3}$.

Solution:

The number $\frac{9}{-4}$ is not in standard form. First put it in the standard form.

$$\frac{9}{-4} = \frac{9}{-4} \times \frac{-1}{-1} \text{(to get a positive denominator)} = \frac{-9}{4}$$

Now, we shall compare the fractions $\frac{-9}{4}$ and $\frac{-2}{3}$. We find that these two fractions are unlike fractions. To make them as like fractions, we make use of their LCM, which is 12.

We can now compare their equivalent fractions $\frac{-9}{4} = \frac{-27}{12}$ and $\frac{-2}{3} = \frac{-8}{12}$ (How?). We find that the denominators are the same and so just comparing the numerators –27 and –8 are enough.

Visualizing these numbers on the number line, we see that

Fig. 1.7

−8 is to the right of −27 and hence (−8) > (−27). This leads to the result that $\frac{-8}{12} > \frac{-27}{12}$ and consequently we conclude that $\frac{-2}{3} > \frac{9}{-4}$.

Example 1.7

Write the following rational numbers in ascending and descending order.

$$\frac{-3}{5}, \frac{7}{-10}, \frac{-15}{20}, \frac{14}{-30}, \frac{-8}{15}$$

Solution:

First make the denominators to be positive and write the numbers in standard form as $\frac{-3}{5}, \frac{-7}{10}, \frac{-15}{20}, \frac{-14}{30}, \frac{-8}{15}$. Here, the LCM of 5,10,15,20 and 30 is 60 (Find it!). Change the given rational numbers in equivalent form with common denominator 60.

$\frac{-3}{5}$	$\frac{-7}{10}$	$\frac{-15}{20}$	$\frac{-14}{30}$	$\frac{-8}{15}$
$= \frac{-3}{5} \times \frac{12}{12}$	$= \frac{-7}{10} \times \frac{6}{6}$	$= \frac{-15}{20} \times \frac{3}{3}$	$= \frac{-14}{30} \times \frac{2}{2}$	$= \frac{-8}{15} \times \frac{4}{4}$
$= \frac{-36}{60}$	$= \frac{-42}{60}$	$= \frac{-45}{60}$	$= \frac{-28}{60}$	$= \frac{-32}{60}$

Comparing the numerators alone, that is, $-36, -42, -45, -28$ and -32 we see that

$$-45 < -42 < -36 < -32 < -28$$

Hence, $\frac{-45}{60} < \frac{-42}{60} < \frac{-36}{60} < \frac{-32}{60} < \frac{-28}{60}$ and so, $\frac{-15}{20} < \frac{7}{-10} < \frac{-3}{5} < \frac{-8}{15} < \frac{14}{-30}$.

So, the ascending order is $\frac{-15}{20}, \frac{7}{-10}, \frac{-3}{5}, \frac{-8}{15}$ and $\frac{14}{-30}$.

Also, its reverse order gives the descending order as $\frac{14}{-30}, \frac{-8}{15}, \frac{-3}{5}, \frac{7}{-10}$ and $\frac{-15}{20}$.

1.2.7 Rational numbers between any two given rational numbers

Consider the integers 4 and 10. We can locate five integers namely 5,6,7,8 and 9 (shown in dark dots) between them. Isn't it?

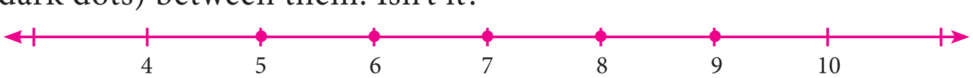


Fig. 1.8

How many integers can you find between 3 and −2? List them.

Are there any integers between −5 and −4? No, is the answer.

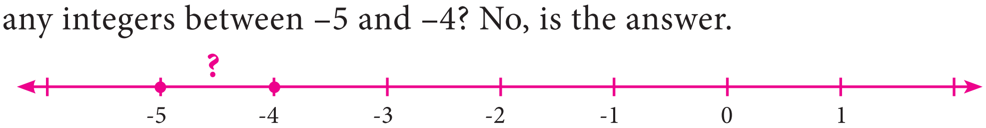


Fig. 1.9

This shows that the choice of integers between two given integers is limited. They are finite in number or may be nothing between them. Let us think what will happen, if we consider rational numbers instead of integers? We will see that we can have many rational numbers between any two rational numbers. There are at least two methods to find more rational numbers between any two rational numbers.

Method of Averages:

We know that that the average of any two numbers always lies at the middle of them.

For example, the average of 2 and 8 is $\frac{2+8}{2}=5$ and this 5 lies at the middle of 2 and 8 as shown in the following number line.

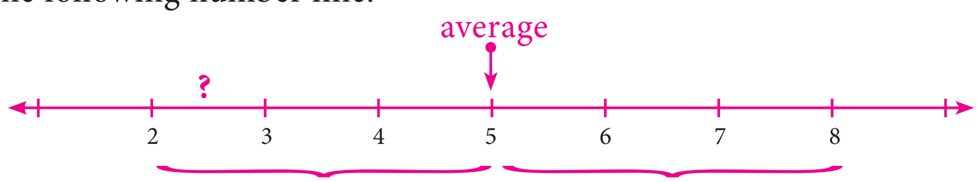


Fig. 1.10

We use this idea to find more rational numbers between any two rational numbers.

Example 1.8

Find a rational number between $\frac{1}{3}$ and $\frac{5}{9}$.

Solution:

The average of $\frac{1}{3}$ and $\frac{5}{9}=\frac{1}{2}\left(\frac{1}{3}+\frac{5}{9}\right)=\frac{1}{2}\left(\frac{1}{3}\times\frac{3}{3}+\frac{5}{9}\right)$ (Why?)

$$=\frac{1}{2}\left(\frac{3}{9}+\frac{5}{9}\right)=\frac{1}{2}\times\frac{8}{9}=\frac{4}{9}$$

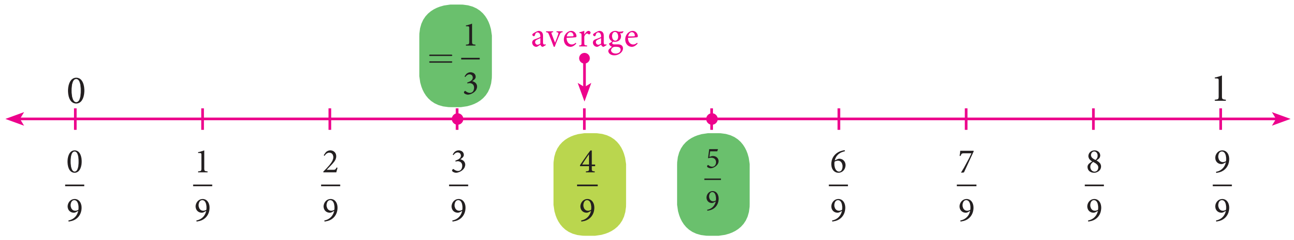


Fig. 1.11

Note that $\frac{4}{9}$ is one rational number we have found in between $\frac{1}{3}$ and $\frac{5}{9}$ and we can find many such numbers in between $\frac{1}{3}$ and $\frac{5}{9}$. This shows that between any two rational numbers there lie an unlimited number of rational numbers! Mathematically, we say that there lie an infinite number of rational numbers between any two rational numbers.

Think

Are there any rational numbers between $\frac{-7}{11}$ and $\frac{6}{-11}$?

Method of Equivalent rational numbers:

We can use the idea of equivalent fractions to get more rational numbers between any two rational numbers. This is clearly explained in the following illustration.

Illustration:

Let us now try to find more rational numbers say between $\frac{3}{7}$ and $\frac{4}{7}$ by the following visual explanation on the number line. If we get the multiples of the denominator of the equivalent rational numbers (the easy one will be to multiply by 10), then we can insert as many rational numbers as we want. We shall write $\frac{3}{7}$ as $\frac{30}{70}$ and $\frac{4}{7}$ as $\frac{40}{70}$ and see that there are 9 rational numbers between $\frac{3}{7}$ and $\frac{4}{7}$ as given in the number line below.

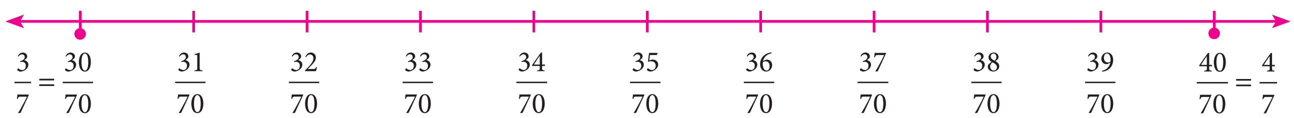


Fig. 1.12

Now, if we want more rational numbers between say $\frac{37}{70}$ and $\frac{38}{70}$ we can write $\frac{37}{70}$ as $\frac{370}{700}$ and $\frac{38}{70}$ as $\frac{380}{700}$. Then again, we will get nine rational numbers between $\frac{37}{70}$ and $\frac{38}{70}$ as $\frac{371}{700}, \frac{372}{700}, \frac{373}{700}, \frac{374}{700}, \frac{375}{700}, \frac{376}{700}, \frac{377}{700}, \frac{378}{700}$ and $\frac{379}{700}$.

The following diagram helps us to understand this nicely with a magnifying lens used between 0 and 1 and further zoomed into the fractional parts also.

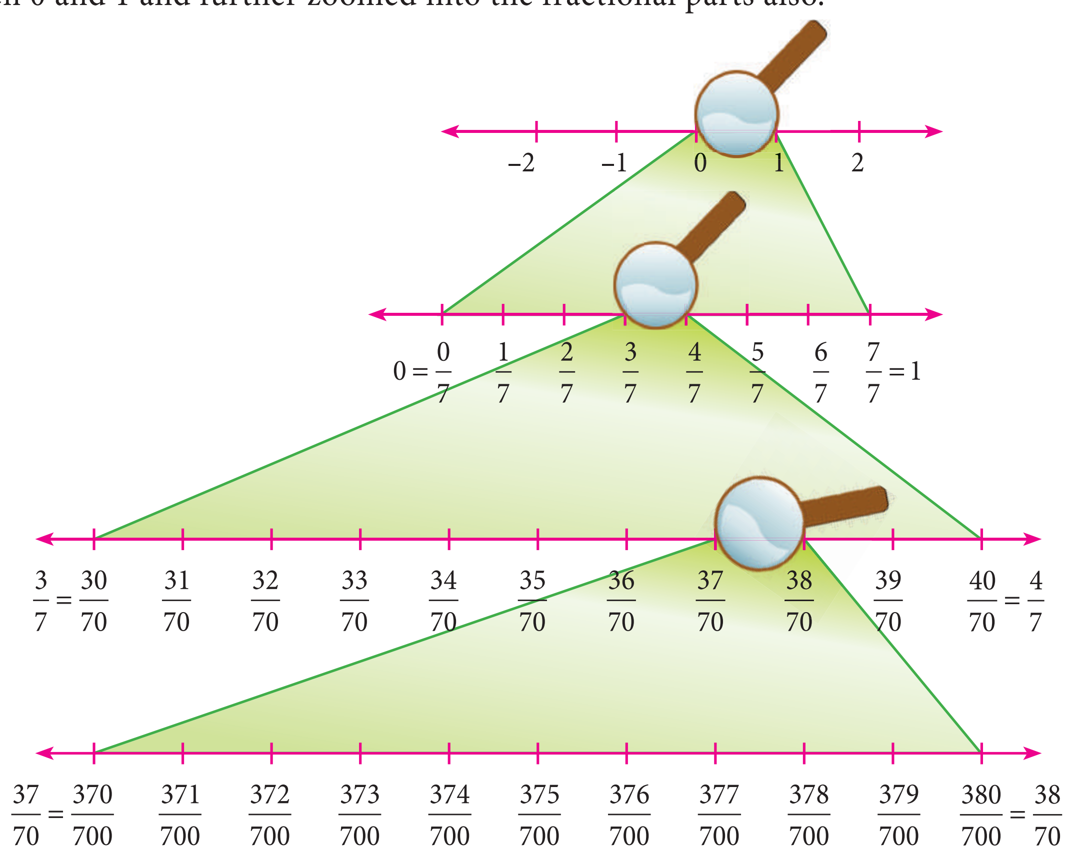


Fig. 1.13

Thus, we can see that there are an unlimited number of rational numbers between any two given rational numbers.

Example 1.9

Find atleast two rational numbers between $\frac{-3}{4}$ and $\frac{-2}{5}$.

Solution:

The denominators are different for the given rational numbers. The LCM of the denominators 4 and 5 is 20. Make the rational numbers such that they have common denominators as 20. Here,

$\frac{-3}{4} = \frac{-3}{4} \times \frac{5}{5} = \frac{-15}{20}$ and $\frac{-2}{5} = \frac{-2}{5} \times \frac{4}{4} = \frac{-8}{20}$.

It is easy now to find and insert rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$ as shown below.

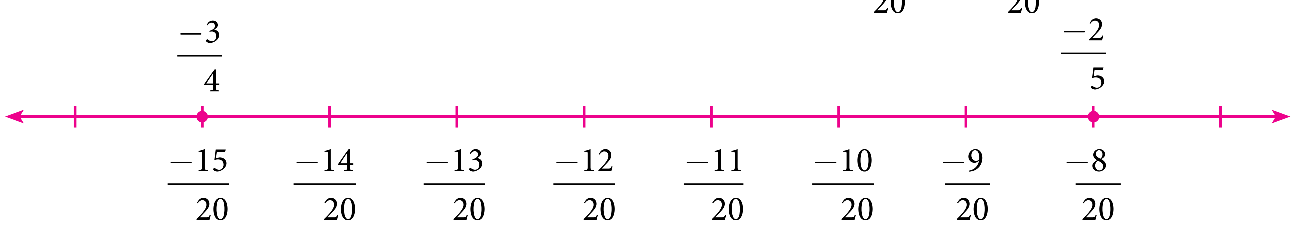


Fig. 1.14

We can list a few rational numbers as $\frac{-14}{20}, \frac{-13}{20}, \frac{-12}{20}, \frac{-11}{20}, \frac{-10}{20}$ and $\frac{-9}{20}$ between $\frac{-15}{20}$ and $\frac{-8}{20}$.

Are these the only rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$? Think! Try to find 10 more rational numbers between them, if possible!

Note

We can find many rational numbers between $\frac{-7}{11}$ and $\frac{5}{-9}$ quickly as given below: The range of rational numbers can be got by the cross multiplication of denominators with the numerators after writing the given fractions in standard form. The cross multiplication here $\frac{-7}{11} \times \frac{-5}{9}$ gives the range of rational numbers from −63 to −55 with the denominator 99. This is nothing but making the given rational numbers equivalent with the denominator 99!

Exercise 1.1

1. **Fill in the blanks:**

(i) $\frac{-19}{5}$ lies between the integers ___________ and ___________.

(ii) The decimal form of the rational number $\frac{15}{-4}$ is ___________.

(iii) The rational numbers $\frac{-8}{3}$ and $\frac{8}{3}$ are equidistant from ___________.

(iv) The next rational number in the sequence $\frac{-15}{24}, \frac{20}{-32}, \frac{-25}{40}$ is ___________.

(v) The standard form of $\frac{58}{-78}$ is ___________.

2. **Say True or False:**

(i) 0 is the smallest rational number.

(ii) $\frac{-4}{5}$ lies to the left of $\frac{-3}{4}$.

(iii) $\frac{-19}{5}$ is greater than $\frac{15}{-4}$.

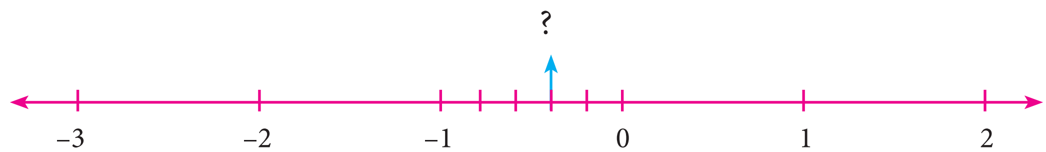
(iv) The average of two rational numbers lies between them.

(v) There are an unlimited number of rational numbers between 10 and 11.

3. Find the rational numbers represented by each of the question marks marked on the following number lines.

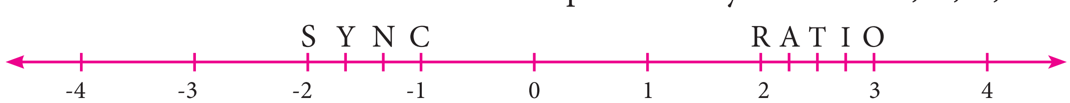
(i)

(ii)



(iii)

4. The points S, Y, N, C, R, A, T, I and O on the number line are such that CN=NY=YS and RA=AT=TI=IO. Find the rational numbers represented by the letters Y, N, A, T and I.



5. Draw a number line and represent the following rational numbers on it.

(i) $\frac{9}{4}$ (ii) $\frac{-8}{3}$ (iii) $\frac{-17}{-5}$ (iv) $\frac{15}{-4}$

6. Write the decimal form of the following rational numbers.

(i) $\frac{1}{11}$ (ii) $\frac{13}{4}$ (iii) $\frac{-18}{7}$ (iv) $1\frac{2}{5}$ (v) $-3\frac{1}{2}$

7. List any five rational numbers between the given rational numbers.

(i) −2 and 0 (ii) $\frac{-1}{2}$ and $\frac{3}{5}$ (iii) $\frac{1}{4}$ and $\frac{7}{20}$ (iv) $\frac{-6}{4}$ and $\frac{-23}{10}$

8. Use the method of averages to write 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$.

9. Compare the following pairs of rational numbers.

(i) $\frac{-11}{5}, \frac{-21}{8}$ (ii) $\frac{3}{-4}, \frac{-1}{2}$ (iii) $\frac{2}{3}, \frac{4}{5}$.

10. Arrange the following rational numbers in ascending and descending order.

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$ (ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

Objective Type Questions

11. The number which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is ________.

(A) $\frac{34}{99}$ (B) $\frac{-142}{99}$ (C) $\frac{142}{99}$ (D) $\frac{-34}{99}$

12. Which of the following pairs is equivalent?

(A) $\frac{-20}{12}, \frac{5}{3}$ (B) $\frac{16}{-30}, \frac{-8}{15}$ (C) $\frac{-18}{36}, \frac{-20}{44}$ (D) $\frac{7}{-5}, \frac{-5}{7}$

13. $\frac{-5}{4}$ is a rational number which lies between ________ .

(A) 0 and $\frac{-5}{4}$ (B) –1 and 0 (C) –1 and –2 (D) –4 and –5

14. Which of the following rational numbers is the greatest?

(A) $\frac{-17}{24}$ (B) $\frac{-13}{16}$ (C) $\frac{7}{-8}$ (D) $\frac{-31}{32}$

15. The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$ is ________ .

(A) 4 (B) 5 (C) 6 (D) 7

1.3 Basic Arithmetic Operations on Rational Numbers

All the rules and principles that govern fractions in the basic operations apply to rational numbers also.

1.3.1 Addition

There can be four different situations while doing addition.

Type 1 : Adding numbers that have same denominators

This is simply like adding like fractions and the result is the sum of the numerators divided by their common denominator.

Example 1.10

Add : $\frac{-6}{11}, \frac{8}{11}, \frac{-12}{11}$

Solution:

Write the given rational numbers in the standard form and then add them.

$$\text{So, } \frac{-6}{11} + \frac{8}{11} + \frac{-12}{11} = \frac{-6+8-12}{11} = \frac{-10}{11}$$

Type 2 : Adding numbers that have different denominators

After writing the given rational numbers in the standard form, use the LCM of their denominators to convert the numbers into equivalent rational numbers with a common denominator so that this reduces to Type1.

Example 1.11

Add : $\frac{-5}{9}, \frac{-4}{3}, \frac{7}{12}$

Solution:

LCM of $9, 3, 12 = 36$

$$\text{So, } \frac{-5}{9} + \frac{-4}{3} + \frac{6}{12} = \frac{-5}{9} \times \frac{4}{4} + \frac{-4}{3} \times \frac{12}{12} + \frac{7}{12} \times \frac{3}{3}$$

$$= \frac{-20}{36} + \frac{-48}{36} + \frac{21}{36} = \frac{-20-48+21}{36} = \frac{-47}{36}$$

1.3.2 Additive Inverse

The additive inverse of a rational number is another rational number which when added to the given number, gives zero.

For example, $\frac{4}{3}$ and $\frac{-4}{3}$ are additive inverses of each other, since their sum is zero.

Think

Is zero a rational number? If so, what is its additive inverse?

1.3.3 Subtraction

Subtraction is simply adding the additive inverse.

Example 1.12

Subtract : $\frac{9}{17}$ from $\frac{-12}{17}$

Solution:

$$\text{Now, } \frac{-12}{17} - \frac{9}{17} = \frac{-12}{17} + \left(\frac{-9}{17}\right) = \frac{-12-9}{17} = \frac{-21}{17}$$

Example 1.13

Subtract : $\left(-2\frac{6}{11}\right)$ from $\left(-4\frac{5}{22}\right)$

Solution:

$$\text{Now, } \left(-4\frac{5}{22}\right) - \left(-2\frac{6}{11}\right)$$

$$= \frac{-93}{22} - \left(\frac{-28}{11}\right)$$

$$= \frac{-93}{22} + \frac{28}{11} = \frac{-93+28\times 2}{22}$$

$$= \frac{-93+56}{22} = \frac{-37}{22} = -1\frac{15}{22}$$

1.3.4 Multiplication

Product of two or more rational numbers is found by multiplying the corresponding numerators and denominators of the numbers and then writing them in the standard form.

Example 1.14

Evaluate : (i) $\frac{-5}{8}\times\frac{7}{3}$ (ii) $\frac{-6}{-11}\times(-4)$

Solution:

(i) $\frac{-5}{8}\times\frac{7}{3}=\frac{-5\times7}{8\times3}=\frac{-35}{24}$ (ii) $\frac{-6}{-11}\times(-4)=\frac{6}{11}\times\frac{(-4)}{1}=\frac{6\times(-4)}{11\times1}=\frac{-24}{11}$

1.3.5 Multiplicative Inverse

If the product of two rational numbers is 1, then each of them is said to be the reciprocal or the multiplicative inverse of the other.

Think

What is the multiplicative inverse of 1 and –1?

For the rational number a, its reciprocal is $\frac{1}{a}$ and vice versa since $a\times\frac{1}{a}=\frac{1}{a}\times a=1$.

For the rational number $\frac{a}{b}$, its multiplicative inverse is $\frac{b}{a}$ and vice versa since $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$.

1.3.6 Division

The idea of reciprocals of fractions is extended to the division of rational numbers also. To divide a given rational number by another rational number, we have to multiply the given rational number by the reciprocal of the second rational number. That is, division is simply multiplying by the multiplicative inverse of the divisor.

Example 1.15

Divide : $\frac{7}{-8}$ by $\frac{-3}{4}$

Solution:

Here, $\frac{7}{-8}\div\frac{-3}{4}=\frac{-7}{8}\times\frac{-4}{3}=\frac{7}{6}$

Try these

Divide :

(i) $\frac{-7}{3}$ by 5 (ii) 5 by $\frac{-7}{3}$ (iii) $\frac{-7}{3}$ by $\frac{35}{6}$

Note

For any non-zero b, c, and d, we have

(i) $\left(\frac{a}{b}\right)\div c=\frac{a}{bc}$ (ii) $a\div\left(\frac{b}{c}\right)=\frac{ac}{b}$ (iii) $\left(\frac{a}{b}\right)\div\left(\frac{c}{d}\right)=\frac{ad}{bc}$

1.4 Word Problems on the basic operations

Example 1.16

The sum of two rational numbers is $\frac{4}{5}$. If one number is $\frac{2}{15}$, then find the other.

Solution:

Let the other number be x.

Given, $\frac{2}{15}+x=\frac{4}{5}$

$$\Rightarrow x=\frac{4}{5}-\frac{2}{15}=\frac{12-2}{15}=\frac{10}{15}\Rightarrow x=\frac{2}{3}$$

Example 1.17

The product of two rational numbers is $\frac{-2}{3}$. If one number is $\frac{3}{7}$, then find the other.

Solution:

Let the other number be x.

Given, $\frac{3}{7} \times x = \frac{-2}{3}$

Multiplying by the reciprocal of $\frac{3}{7}$, that is $\frac{7}{3}$ on both sides,

$$\Rightarrow \frac{7}{3} \times \frac{3}{7} \times x = \frac{7}{3} \times \frac{-2}{3}$$

$$\Rightarrow x = \frac{-14}{9}$$

Aliter

$$\frac{3}{7} \times x = \frac{-2}{3}$$

$$\Rightarrow x = \frac{-2}{3} \times \frac{7}{3} = \frac{-14}{9}$$

(by cross multiplication)

Example 1.18

One roll of ribbon is $18\frac{3}{4}$ m long. Sankari has four full rolls and one–third of another roll. How many metres of ribbon does Sankari have in total?

Fig. 1.15

Solution:

Number of metres of ribbon Sankari has in total

$$= 18\frac{3}{4} \times 4\frac{1}{3}$$

$$= \frac{75}{4} \times \frac{13}{3} = \frac{325}{4} = 81\frac{1}{4} \text{ m}$$

Example 1.19

Find the rational numbers that should be added and subtracted so that they will make the sum $3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$ to the nearest whole number.

Solution:

Now, $3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$

$$= \frac{7}{2} + \frac{7}{4} + \frac{19}{8} = \frac{7 \times 4 + 7 \times 2 + 19 \times 1}{8} = \frac{28 + 14 + 19}{8}$$

$$= \frac{61}{8} = 7\frac{5}{8}$$, which lies between the whole numbers 7 and 8.

If we subtract $\frac{5}{8}$ from $7\frac{5}{8}$, it becomes 7. If we add $\frac{3}{8}$ to $7\frac{5}{8}$, it becomes $7+\frac{5}{8}+\frac{3}{8}=7+1=8$.

Example 1.20

A student instead of multiplying a number by $\frac{8}{9}$, by mistake divided it by $\frac{8}{9}$. If the difference between the correct answer and the answer got by him is 34, then find the number.

Solution:

Let the number be x.

The student had to find $\frac{8x}{9}$ but, he had found $\frac{x}{\left(\frac{8}{9}\right)}$, that is $\frac{9x}{8}$.

$$\text{Now, } \frac{9x}{8} - \frac{8x}{9} = 34 \text{ (given)}$$

$$\frac{81x - 64x}{72} = 34 \Rightarrow \frac{17x}{72} = 34$$

$$x = \frac{34 \times 72}{17} = 144$$

Example 1.21

Simplify: $\left(\frac{4}{3} - \left(\frac{-3}{2}\right)\right) + \left(\frac{-5}{3} \div \frac{30}{12}\right) + \left(\frac{-12}{9} \times \frac{-27}{16}\right)$

Solution:

Here, $\left(\frac{4}{3} - \left(\frac{-3}{2}\right)\right) + \left(\frac{-5}{3} \div \frac{30}{12}\right) + \left(\frac{-12}{9} \times \frac{-27}{16}\right) = \left(\frac{4}{3} + \frac{3}{2}\right) + \left(\frac{-5}{3} \times \frac{12}{30}\right) + \left(\frac{-12}{9} \times \frac{-27}{16}\right)$

$$= \left(\frac{8}{6} + \frac{9}{6}\right) + \left(\frac{-1}{1} \times \frac{4}{6}\right) + \left(\frac{-3}{1} \times \frac{-3}{4}\right)$$

$$= \left(\frac{17}{6}\right) + \left(\frac{-4}{6}\right) + \left(\frac{9}{4}\right)$$

$$= \left(\frac{17-4}{6}\right) + \frac{9}{4} = \frac{13}{6} + \frac{9}{4}$$

$$= \frac{26+27}{12} = \frac{53}{12}$$

Exercise 1.2

1. **Fill in the blanks:**

   (i) The value of $\frac{-5}{12} + \frac{7}{15} =$ ________.

   (ii) The value of $\left(\frac{-3}{6}\right) \times \left(\frac{18}{-9}\right)$ is ________.

   (iii) The value of $\left(\frac{-15}{23}\right) \div \left(\frac{30}{-46}\right)$ is ________.

   (iv) The rational number ________ does not have a reciprocal.

   (v) The multiplicative inverse of -1 is ________.

2. **Say True or False:**

   (i) All rational numbers have an additive inverse.

   (ii) The rational numbers that are equal to their additive inverses are 0 and -1.

   (iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$.

(iv) The rational number which is its own reciprocal is –1.

(v) The multiplicative inverse exists for all rational numbers.

3. Find the sum:

(i) $\frac{7}{5}+\frac{3}{5}$ (ii) $\frac{7}{5}+\frac{5}{7}$ (iii) $\frac{6}{5}+\left(\frac{-14}{15}\right)$ (iv) $-4\frac{2}{3}+7\frac{5}{12}$

4. Subtract : $\frac{-8}{44}$ from $\frac{-17}{11}$.

5. Evaluate : (i) $\frac{9}{132}\times\frac{-11}{3}$ (ii) $\frac{-7}{27}\times\frac{24}{-35}$

6. Divide : (i) $\frac{-21}{5}$ by $\frac{-7}{-10}$ (ii) $\frac{-3}{13}$ by –3 (iii) –2 by $\frac{-6}{15}$

7. Find $(a + b) \div (a - b)$ if

(i) $a=\frac{1}{2}, b=\frac{2}{3}$ (ii) $a=\frac{-3}{5}, b=\frac{2}{15}$

8. Simplify : $\frac{1}{2}+\left(\frac{3}{2}-\frac{2}{5}\right)\div\frac{3}{10}\times 3$ and show that it is a rational number between 11 and 12.

9. Simplify :

(i) $\left[\frac{11}{8}\times\left(\frac{-6}{33}\right)\right]+\left[\frac{1}{3}+\left(\frac{3}{5}\div\frac{9}{20}\right)\right]-\left[\frac{4}{7}\times\frac{-7}{5}\right]$ (ii) $\left[\frac{4}{3}\div\left(\frac{8}{-7}\right)\right]-\left[\frac{3}{4}\times\frac{4}{3}\right]+\left[\frac{4}{3}\times\left(\frac{-1}{4}\right)\right]$

10. A student had multiplied a number by $\frac{4}{3}$ instead of dividing it by $\frac{4}{3}$ and got 70 more than the correct answer. Find the number.

Objective Type Questions

11. The standard form of the sum $\frac{3}{4}+\frac{5}{6}+\left(\frac{-7}{12}\right)$ is ________ .

(A) 1 (B) $\frac{-1}{2}$ (C) $\frac{1}{12}$ (D) $\frac{1}{22}$

12. $\left(\frac{3}{4}-\frac{5}{8}\right)+\frac{1}{2}=$ ________ .

(A) $\frac{15}{64}$ (B) 1 (C) $\frac{5}{8}$ (D) $\frac{1}{16}$

13. $\frac{3}{4}\div\left(\frac{5}{8}+\frac{1}{2}\right)=$ ________ .

(A) $\frac{13}{10}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{8}$

14. $\frac{3}{4}\times\left(\frac{5}{8}\div\frac{1}{2}\right)=$ ________ .

(A) $\frac{5}{8}$ (B) $\frac{2}{3}$ (C) $\frac{15}{32}$ (D) $\frac{15}{16}$

15. Which of these rational numbers which have additive inverse?

(A) 7 (B) $\frac{-5}{7}$ (C) 0 (D) all of these

Note

Why $\frac{a}{b}+\frac{c}{d} \neq \frac{a+c}{b+d}$?

A mathematical statement is true only if it is 100% true without any exception. If we do the addition, say $\frac{3}{4}+\frac{1}{4}$ as $\frac{3+1}{4+4}=\frac{4}{8}=\frac{1}{2}$. This is wrong, since $\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1$.

1.5 Properties of Rational Numbers

Some properties listed here below will be of good use in solving problems.

1.5.1 Closure property/law for the collection $\mathbb{Q}$ of rational numbers

i) Closure property for Addition

For any two rational numbers ***a*** and ***b***, the sum ***a + b*** is also a rational number.

ii) Closure property for Multiplication

For any two rational numbers ***a*** and ***b***, the product ***ab*** is also a rational number.

Illustration

Take $a=\frac{3}{4}$ and $b=\frac{-1}{2}$

Now, $a+b=\frac{3}{4}+\frac{-1}{2}=\frac{3}{4}+\frac{-2}{4}=\frac{3-2}{4}=\frac{1}{4}$ is in $\mathbb{Q}$

Also, $a \times b=\frac{3}{4}\times\frac{-1}{2}=\frac{-3}{8}$ is in $\mathbb{Q}$

Try this

The closure property on integers holds for subtraction and not for division. What about rational numbers? Verify.

1.5.2 Commutative property/law for the collection $\mathbb{Q}$ of rational numbers

i) Commutative property for Addition

For any two rational numbers ***a*** and ***b***, ***a + b = b + a***.

ii) Commutative property for Multiplication

For any two rational numbers ***a*** and ***b***, ***ab = ba*** (*ab* means $a \times b$ and *ba* means $b \times a$).

Illustration

Take $a=\frac{-7}{8}$ and $b=\frac{3}{5}$

Now, $a+b=\frac{-7}{8}+\frac{3}{5}=\frac{-7\times5+3\times8}{40}=\frac{-35+24}{40}=\frac{-11}{40}$

Also, $b+a=\frac{3}{5}+\frac{-7}{8}=\frac{3\times8+-7\times5}{40}=\frac{24-35}{40}=\frac{-11}{40}$

Here, we find that $a+b=b+a$ and hence addition is commutative.

Operations	Fill in the blanks in the table given below of properties of Integers. (If a, b, c are integers, then $-a$, $-b$, $-c$ are also integers)					
	Closure	**Commutative**	**Associative**	**Identity**	**Inverse**	**Distributive**
Addition	$a+b$ is in $\mathbb{Z}$ E.g. $5+(-3)=2$ $\Rightarrow 2$ is in $\mathbb{Z}$	$a+b=b+a$ E.g. $5+(-3)=(-3)+5$ $\Rightarrow 2=2$	$(a+b)+c$ $=a+(b+c)$ E.g. $(2+3)+(-4)=1$ $2+[3+(-4)]=1$	$a+0$ $=0+a=a$ E.g. $(-4)+0$ $=0+(-4)=-4$	$a+(-a)$ $=(-a)+a=0$ E.g. $5+(-5)$ $=(-5)+5=0$	$a\times(b+c)$ $=(a\times b)+(a\times c)$ E.g. $2\times[3+(-5)]=-4$ $(2\times3)+[2\times(-5)]$ $=-4$
Multiplication	ab is in $\mathbb{Z}$ E.g.________	$a\times b=b\times a$ E.g.________	$(a\times b)\times c$ $=a\times(b\times c)$ E.g. $(2\times3)\times(-6)=-36$ $2\times[3\times(-6)]=-36$	$a\times1$ $=1\times a=a$ E.g.________	Does not exist	Not Applicable
Subtraction	$a-b$ is in $\mathbb{Z}$ E.g.________	Fails $a-b\neq b-a$ E.g.________	Fails $(a-b)-c$ $\neq a-(b-a)$ E.g.________	Fails $a-0\neq 0-a$ E.g. $5-0=5$ $0-5=-5$ $5\neq-5$	Fails $a-(-a)$ $\neq(-a)-a$ E.g. $2-(-2)=4$ $(-2)-2=-4$ $4\neq-4$	$a\times(b-c)$ $=(a\times b)-(a\times c)$ E.g.________
Division	Fails $a\div b$ is not in $\mathbb{Z}$ E.g. $3\div5=\frac{3}{5}$ does not belong to $\mathbb{Z}$	Fails	Fails	Fails	Fails	Not applicable

Further,

$$a \times b = \frac{-7}{8} \times \frac{3}{5} = \frac{-7 \times 3}{8 \times 5} = \frac{-21}{40}$$

Also, $$b \times a = \frac{3}{5} \times \frac{-7}{8} = \frac{3 \times -7}{5 \times 8} = \frac{-21}{40}$$

Here, we find that $a \times b = b \times a$ and hence multiplication is commutative.

Try these

(i) Is $\frac{3}{5} - \frac{7}{8} = \frac{7}{8} - \frac{3}{5}$?

(ii) Is $\frac{3}{5} \div \frac{7}{8} = \frac{7}{8} \div \frac{5}{3}$?

So, what do you conclude?

1.5.3 Associative property/law for the collection $\mathbb{Q}$ of rational numbers

i) Associative property for Addition

For any three rational numbers ***a***, ***b***, and ***c***, ***a* + (*b* + *c*) = (*a* + *b*) + *c***

ii) Associative property for Multiplication

For any three rational numbers ***a***, ***b***, and ***c***, ***a*(*bc*) = (*ab*)*c***

Illustration

Take rational numbers a,b,c as $a = \frac{-1}{2}$, $b = \frac{3}{5}$ and $c = \frac{-7}{10}$

Now, $a + b = \frac{-1}{2} + \frac{3}{5} = \frac{-5}{10} + \frac{6}{10}$ (equivalent rationals with common denominators)

$$a + b = \frac{-5+6}{10} = \frac{1}{10}$$

$$(a+b)+c = \frac{1}{10} + \left(\frac{-7}{10}\right) = \frac{1-7}{10} = \frac{-6}{10} = \frac{-3}{5} \quad ...(1)$$

Also, $$b + c = \frac{3}{5} + \frac{-7}{10} = \frac{6}{10} + \frac{-7}{10} = \frac{6-7}{10} = \frac{-1}{10}$$

$$a+(b+c) = \frac{-1}{2} + \frac{-1}{10} = \frac{-5}{10} + \frac{-1}{10} = \frac{-5-1}{10} = \frac{-6}{10} = \frac{-3}{5} \quad ...(2)$$

(1) and (2) shows that $(a+b)+c = a+(b+c)$ is true for rational numbers.

Similarly, $$a \times b = \frac{-1}{2} \times \frac{3}{5} = \frac{-1 \times 3}{2 \times 5} = \frac{-3}{10}$$

$$(a \times b) \times c = \frac{-3}{10} \times \frac{-7}{10} = \frac{-3 \times -7}{10 \times 10} = \frac{21}{100} \quad ...(3)$$

Also, $$b \times c = \frac{3}{5} \times \frac{-7}{10} = \frac{3 \times -7}{5 \times 10} = \frac{-21}{50}$$

$$a \times (b \times c) = \frac{-1}{2} \times \frac{-21}{50} = \frac{-1 \times -21}{2 \times 50} = \frac{21}{100} \quad ...(4)$$

(3) and (4) shows that

$(a \times b) \times c = a \times (b \times c)$ is true for rational numbers.

Thus, the associative property is true for addition and multiplication of rational numbers.

Try this

Check whether associative property holds for subtraction and division.

1.5.4 Identity property/law for the collection $\mathbb{Q}$ of rational numbers

i) Identity property for Addition

For any rational number $\boldsymbol{a}$, there exists a unique rational number $\mathbf{0}$ such that $\mathbf{0} + \boldsymbol{a} = \boldsymbol{a} = \mathbf{0} + \boldsymbol{a}$.

ii) Identity property for Multiplication

For any rational number $\boldsymbol{a}$, there exists a unique rational number $\mathbf{1}$ such that $\mathbf{1} \times \boldsymbol{a} = \boldsymbol{a} = \boldsymbol{a} \times \mathbf{1}$.

Illustration

Take $a=\frac{3}{-7}$ that is, $a=\frac{-3}{7}$. Now $\frac{-3}{7}+0=\frac{-3}{7}=0+\frac{-3}{7}$ (Isn't it?)

Hence, 0 is the additive identity for $\frac{-3}{7}$

Also, $\frac{-3}{7}\times 1=\frac{-3}{7}=1\times\frac{-3}{7}$ (Isn't it?)

Hence, 1 is the multiplicative identity for $\frac{-3}{7}$

1.5.5 Inverse property/law for the collection $\mathbb{Q}$ of rational numbers

i) Additive Inverse property

For any rational number $\boldsymbol{a}$, there exists a unique rational number $-\boldsymbol{a}$ such that $\boldsymbol{a} + (-\boldsymbol{a}) = (-\boldsymbol{a}) + \boldsymbol{a} = \mathbf{0}$. Here, 0 is the additive identity.

ii) Multiplicative Inverse property

For any rational number $\boldsymbol{b}$, there exists a unique rational number $\frac{\mathbf{1}}{\boldsymbol{b}}$ such that $\boldsymbol{b}\times\frac{\mathbf{1}}{\boldsymbol{b}}=\frac{\mathbf{1}}{\boldsymbol{b}}\times \boldsymbol{b}=\mathbf{1}$. Here, 1 is the multiplicative identity.

Illustration

Take $a=\frac{-11}{23}$ Now, $-a=-\left(\frac{-11}{23}\right)=\frac{11}{23}$

So, $a + (-a) = \frac{-11}{23}+\frac{11}{23}=\frac{-11+11}{23}=\frac{0}{23}=0$

Also, $(-a)+ a = \frac{11}{23}+\frac{-11}{23}=\frac{11-11}{23}=\frac{0}{23}=0$

$\therefore\ a + (-a) = (-a)+ a = 0$ is true.

Also, take $b=\frac{-17}{29}$. Now, $\frac{1}{b}=\frac{29}{-17}=\frac{-29}{17}$

$b\times\frac{1}{b} = \frac{-17}{29}\times\frac{-29}{17}=1$. Also, $\frac{1}{b}\times b = \frac{-29}{17}\times\frac{-17}{29}=1$

$\therefore\ b\times\frac{1}{b}=\frac{1}{b}\times b=1$ is true.

1.5.6 Distributive property/law for the collection $\mathbb{Q}$ of rational numbers

Multiplication is distributive over addition for the collection of rational numbers.

For any three rational numbers ***a***, ***b*** and ***c***, the distributive law is $a\times(b+c)=(a\times b)+(a\times c)$

Illustration

Take rational numbers a,b,c as $a=\frac{-7}{9}$, $b=\frac{11}{18}$ and $c=\frac{-14}{27}$

Now, $b+c \;=\; b+c=\frac{11}{18}+\frac{-14}{27}=\frac{33}{54}+\frac{-28}{54}=\frac{33-28}{54}=\frac{5}{54}$

(equivalent rationals with common denominators)

$\therefore\ a\times(b+c) \;=\; \frac{-7}{9}\times\frac{5}{54}=\frac{-7\times5}{9\times54}=\frac{-35}{486}$ ...(1)

Also, $a\times b \;=\; \frac{-7}{9}\times\frac{11}{18}=\frac{-7\times11}{9\times18}=\frac{-77}{9\times9\times2}$

$a\times c \;=\; \frac{-7}{9}\times\frac{-14}{27}=\frac{7\times14}{9\times9\times3}=\frac{98}{9\times9\times3}$

$\therefore\ (a\times b)+(a\times c) \;=\; \frac{-77}{9\times9\times2}+\frac{98}{9\times9\times3}$

$=\frac{-77\times3+98\times2}{9\times9\times2\times3}=\frac{-231+196}{486}=\frac{-35}{486}$ ...(2)

(1) and (2) shows that $a\times(b+c)=(a\times b)+(a\times c)$.

Hence, multiplication is distributive over addition for the collection $\mathbb{Q}$ of rational numbers.

Exercise 1.3

1. Verify the closure property for addition and multiplication for the rational numbers $\frac{-5}{7}$ and $\frac{8}{9}$.
2. Verify the commutative property for addition and multiplication for the rational numbers $\frac{-10}{11}$ and $\frac{-8}{33}$.

N4JH34

3. Verify the associative property for addition and multiplication for the rational numbers $\frac{-7}{9},\frac{5}{6}$ and $\frac{-4}{3}$.
4. Verify the distributive property $a\times(b+c)=(a\times b)+(a+c)$ for the rational numbers $a=\frac{-1}{2}$, $b=\frac{2}{3}$ and $c=\frac{-5}{6}$.
5. Verify the identity property for addition and multiplication for the rational numbers $\frac{15}{19}$ and $\frac{-18}{25}$.

6. Verify the additive and multiplicative inverse property for the rational numbers $\frac{-7}{17}$ and $\frac{17}{27}$.

Objective Type Questions

7. Closure property is not true for division of rational numbers because of the number

   (A) 1 (B) −1 (C) 0 (D) $\frac{1}{2}$

8. $\frac{1}{2}-\left(\frac{3}{4}-\frac{5}{6}\right) \neq \left(\frac{1}{2}-\frac{3}{4}\right)-\frac{5}{6}$ illustrates that subtraction does not satisfy the ________ property for rational numbers.

   (A) commutative (B) closure (C) distributive (D) associative

9. Which of the following illustrates the inverse property for addition?

   (A) $\frac{1}{8}-\frac{1}{8}=0$ (B) $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ (C) $\frac{1}{8}+0=\frac{1}{8}$ (D) $\frac{1}{8}-0=\frac{1}{8}$

10. $\frac{3}{4}\times\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{3}{4}\times\frac{1}{2}-\frac{3}{4}\times\frac{1}{4}$ illustrates that multiplication is distributive over

    (A) addition (B) subtraction (C) multiplication (D) division

DO YOU KNOW?

We know that different operations with the same pair of rational numbers usually give different answers. Check the following calculations which are some interesting exceptions in rational numbers.

(i) $\frac{13}{4}+\frac{13}{9}=\frac{13}{4}\times\frac{13}{9}$ (ii) $\frac{169}{30}+\frac{13}{15}=\frac{169}{30}\div\frac{13}{15}$

Amazing …! Isn't it? Try a few more like these, if possible.

Think

Observe that, $\frac{1}{1.2}+\frac{1}{2.3}=\frac{2}{3}$

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}=\frac{3}{4}$$

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}=\frac{4}{5}$$

Use your reasoning skills, to find the sum of the first 7 numbers in the pattern given above.

1.6 Introduction to Square Numbers

1	1 2 3 4	1 2 3 4 5 6 7 8 9
This is a square of side 1 unit. It is 1 squared. We write this as 1^2 $\mathbf{1^2 = 1 \times 1 = 1}$	This is a square of side 2 units. It is 2 squared. We write this as 2^2 $\mathbf{2^2 = 2 \times 2 = 4}$	This is a square of side 3 units. It is 3 squared. We write this as 3^2 $\mathbf{3^2 = 3 \times 3 = 9}$

More often we write like this:

$$4^2 = 16$$

This says "4 squared is 16"

The 2 at the top stands for squared and it indicates the number of times the number 4 appears in the product ($4^2 = 4 \times 4 = 16$).

The numbers 1, 4, 9, 16, ... are all square numbers (also called perfect square numbers). Each of them is made up of the product of same two factors.

A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.

Is 49 a square number? Yes, because it can be written as 7^2. Is 50 a square number?

The following table gives the squares of numbers up to 20.

Number	Its square	Number	Its square	Number	Its square	Number	Its square
1	1	6	36	11	121	16	256
2	4	7	49	12	144	17	289
3	9	8	64	13	169	18	324
4	16	9	81	14	196	19	361
5	25	10	100	15	225	20	400

Try to extend the table up to 50 numbers!

We can now easily verify the following properties of square numbers by referring the table given above:

- The square numbers end with 0, 1, 4, 5, 6 and 9 only.
- If a number ends with 1 or 9, its square ends with 1.
- If a number ends with 2 or 8, its square ends with 4.
- If a number ends with 3 or 7, its square ends with 9.
- If a number ends with 4 or 6, its square ends with 6.
- If a number ends with 5 or 0, its square also ends with 5 or 0 respectively.
- Square of an odd number is always odd and the square of an even number is always even.
- Numbers that end with 2,3,7 and 8 are not perfect squares.

Think

1. Is the square of a prime number, prime?
2. Will the sum of two perfect squares always be a perfect square? What about their difference and their product?

Try these

1. Which among 256, 576, 960, 1025, 4096 are perfect square numbers? (Hint: Try to extend the table of squares already seen).
2. One can judge just by look that each of the following numbers 82, 113, 1972, 2057, 8888, 24353 is not a perfect square. Explain why?

1.6.1 Some more special properties of square numbers

(i) The square of a natural number other than 1, is either a multiple of 3 or exceeds a multiple of 3 by 1.

(ii) The square of a natural number, other than 1, is either a multiple of 4 or exceeds a multiple of 4 by 1.

(iii) The remainder of a perfect square when divided by 3, is either 0 or 1 but never 2.

(iv) The remainder of a perfect square, when divided by 4, is either 0 or 1 but never 2 and 3.

(v) When a perfect square number is divided by 8, the remainder is either 0 or 1 or 4, but will never be equal to 2, 3, 5, 6 or 7.

DO YOU KNOW?

Perfect numbers such as 6, 28, 496, 8128 etc., are not square numbers.

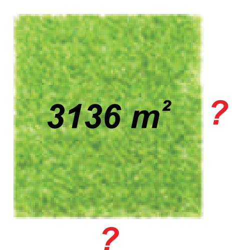
Note

If a perfect square number ends in zero, then it must end with even number of zeroes always. We can verify this for a few numbers in the table given below.

Number	10	20	30	40	...	90	100	110	...	2000	...
Its Square	100	400	900	1600	...	8100	10000	12100	...	4000000	...

Example 1.22

The area of a square field is 3136 m^2. Find its perimeter.



Solution:

Given that the area of the square field = 3136 m^2.

$\therefore$ The side of square field = $\sqrt{3136}$ = 56 m

$\therefore$ The perimeter of the square field $= 4 \times \text{side}$

$= 4 \times 56$

$= 224$ m

```
        5  6
    -----------
  5 | 31 36
    | 25
    -----------
106 |  6 36
    |  6 36
    -----------
    |     0
```

Think

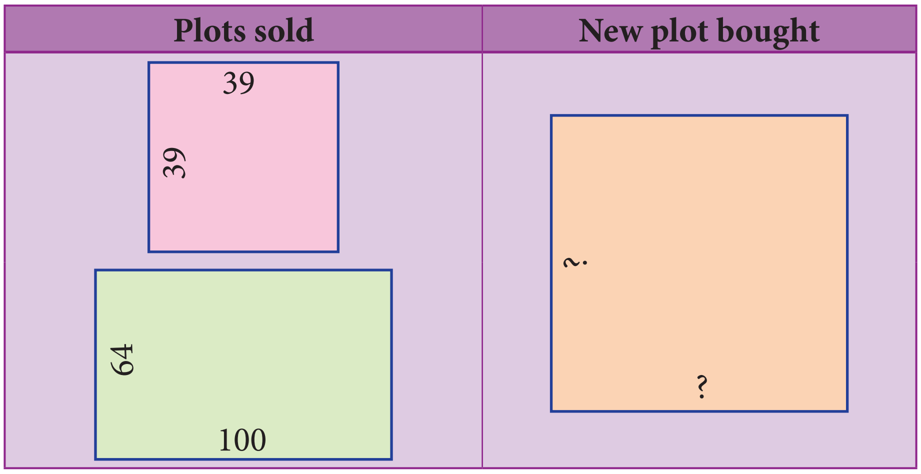
Consider the claim: "Between the squares of the consecutive numbers n and $(n+1)$, there are $2n$ non-square numbers". Can it be true? How many non-square numbers are there between 2500 and 2601? Verify the claim.

Example 1.23

A real estate owner had two plots, a square plot of side 39 m and a rectangular plot of dimensions 100 m length and 64 m breadth. He sells both of these plots and acquires a new square plot of the same area. What is the length of side of his new plot?

Solution:

The transactions can be visualised as follows:



Area of the square plot bought	=	Area of the square plot sold	+	Area of the rectangular plot sold
	=	39×39	+	100×64
	=	1521	+	6400
	=	$7921\ m^2$		

Length of a side of the new square plot = $\sqrt{7921}$ = 89 m

$$
\begin{array}{r|l}
 & 8\ 9 \\ \hline
8 & \overline{79}\ \overline{21} \\
 & 64 \\ \hline
169 & 15\ 21 \\
 & 15\ 21 \\ \hline
 & 0
\end{array}
$$

1.7 Square Root

Squaring a number is another mathematical operation just like addition, subtraction, multiplication etc., Most mathematical operations have 'inverse' (meaning 'opposite') operations. For example, subtraction is the inverse of addition, division is the inverse of multiplication etc., Squaring also has an inverse operation namely finding the Square root.

The square root of a number n, written $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself. For example, $\sqrt{81}$ is 9, because $9 \times 9 = 81$.

In the adjacent table, we have square roots of all the perfect squares starting from 1 to 100.

Square root	Reason
$\sqrt{1} = 1$	$1^2 = 1$
$\sqrt{4} = 2$	$2^2 = 4$
$\sqrt{9} = 3$	$3^2 = 9$
$\sqrt{16} = 4$	$4^2 = 16$
$\sqrt{25} = 5$	$5^2 = 25$
$\sqrt{36} = 6$	$6^2 = 36$
$\sqrt{49} = 7$	$7^2 = 49$
$\sqrt{64} = 8$	$8^2 = 64$
$\sqrt{81} = 9$	$9^2 = 81$
$\sqrt{100} = 10$	$10^2 = 100$

If $11^2 = 121$, what is $\sqrt{121}$? If $529 = 23^2$, what is the square root of 529? If we know that $324 = 18^2$, we can immediately tell that $\sqrt{324}$ is 18.

We have, $1^2 = 1$ and so 1 is a square root of 1. Similarly, $(-1)^2 = 1$. So, (-1) is also a square root of 1.

$2^2 = 4$ and so 2 is a square root of 4. Similarly, $(-2)^2 = 4$. So, (-2) is also a square root of 4.

This continues as $3^2 = 9$ and so 3 is a square root of 9. Similarly, $(-3)^2 = 9$. So, (-3) is also a square root of 9.

The above examples suggest that there are two integral square roots for a perfect square number. However, in working out the problems, we will take up only positive integral square root. The positive square root of a number is always denoted by the symbol $\sqrt{\ }$. Thus, $\sqrt{4}$ is 2 (and not –2). Also, $\sqrt{9}$ is 3 (and not –3). We have to remember that this is a universally accepted notation.

1.7.1 Square root through Prime Factorisation

Study the following table giving the prime factors of numbers and those of their squares.

Numbers	Prime factorisation of the numbers	The square of given numbers	Prime factorisation of their squares
6	$6 = 2 \times 3$	$6^2 = 36$	$36 = 2 \times 2 \times 3 \times 3 = (2 \times 3)^2$
8	$8 = 2 \times 2 \times 2$	$8^2 = 64$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2 \times 2 \times 2)^2$
12	$12 = 2 \times 2 \times 3$	$12^2 = 144$	$44 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3)^2$
15	$15 = 3 \times 5$	$15^2 = 225$	$225 = 3 \times 3 \times 5 \times 5 = (3 \times 5)^2$

Look at 6 and its prime factors. How many times do 2 and 3 occur in 36 in the list? Now, look at its square 36 and its prime factors. How many times do 2 and 3 occur in 36 here?

Repeat the above task in the case of other numbers 8, 12, and 15 also. (We may also choose our own numbers and their squares). What do we find? We find that,

The number of times a prime factor occurs in the square of a number = **twice the number of times it occurs in the prime factorisation of the number.**

We use this idea to find the square root of a square number. First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

Example 1.24

Find the square root of 324 by prime factorisation.

Solution:

First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

$$\text{Now, } 324 = 2\times2\times3\times3\times3\times3$$
$$= 2^2\times3^2\times3^2$$
$$= (2\times3\times3)^2$$
$$\therefore \sqrt{324} = \sqrt{(2\times3\times3)^2}$$
$$= 2\times3\times3$$
$$\therefore \sqrt{324} = 18$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 1.25

Find the least number by which 250 is to be multiplied (or) divided so that the resulting number is a perfect square. Also, find the square root in that case.

Solution:

5	250
5	50
5	10
2	2
	1

Here, $250 = 5 \times 5 \times 5 \times 2$

$= 5^2 \times 5 \times 2$

Here, the prime factors 5 and 2 do not have pairs.

Therefore, we can either divide 250 by 10 (5×2) or multiply 250 by 10.

(i) If we multiply 250 by 10, we get $2500 = 5^2 \times 5 \times 2 \times 5 \times 2$ and therefore the square root of 2500 would be $5 \times 5 \times 2 = 50$.

(ii) If we divide 250 by 10, we get 25 and in this case we get $\sqrt{25} = \sqrt{5^2} = 5$.

Example 1.26

Is 108 a perfect square number?

Solution:

2	108
2	54
3	27
3	9
3	3
	1

Here, $108 = 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^2 \times 3$

Here, the prime factor 3 does not have a second pair. Hence, 108 is not a perfect square number.

Think

In this case, if we want to find the smallest factor with which we can multiply or divide 108 to get a square number, what should we do?

1.7.2 Finding the square root of a number by Long Division Method

When we come across numbers with large number of digits, finding their square roots by factorisation becomes lengthy and difficult. Use of long division helps us in such cases. Let us look into the method with a couple of illustrations.

Illustration 1

Find the square root of 576 by long division method.

Step 1:

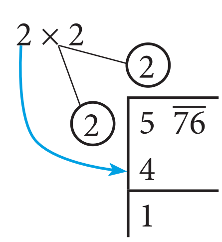
Group the digits in pairs, starting with the digit in the unit's place. Each pair and the remaining digit (if any) is called a period. Put a bar over every pair of digits starting from the right of the given number. If there are odd number of digits, the extreme left digit will be without a bar sign above it.

$5\ \overline{76}$

So, here we have $5\ \overline{76}$

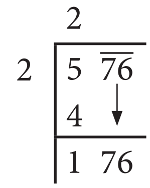
Step 2:

Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as the quotient. The left extreme number here is 5. The largest number whose square is less than or equal to 5 is 2. This is our divisor and the quotient.



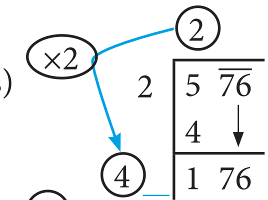
Step 3:

Bring 76 down and write it down to the right of the remainder 1.
Now, the new dividend is 176.



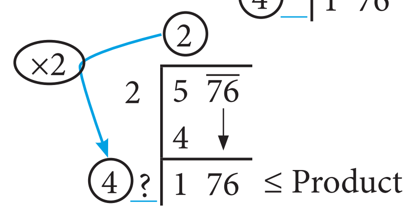
Step 4:

To find the new divisor, multiply the earlier quotient (2) by 2 (always) and write it leaving a blank space next to it.



Step 5:

The new divisor is 4 followed by a digit. We should choose this digit next to 4 such that the new quotient multiplied by the new divisor will be less than or equal to 176.



Step 6:

Clearly, the required digit here has to be 4 or 6. (Why?)

When we calculate, $4\underline{6}\times\underline{6}=276$ whereas $4\underline{4}\times\underline{4}=176$.

Therefore, we put 4 in the blank space and write $4\underline{4}\times\underline{4}=176$ below 176 and subtract to get the remainder 0 and the quotient at the top, that is 24 is the square root of 576.

$\therefore \sqrt{576}=24$.

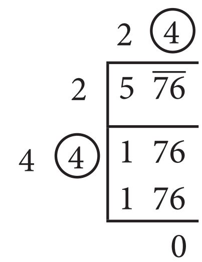
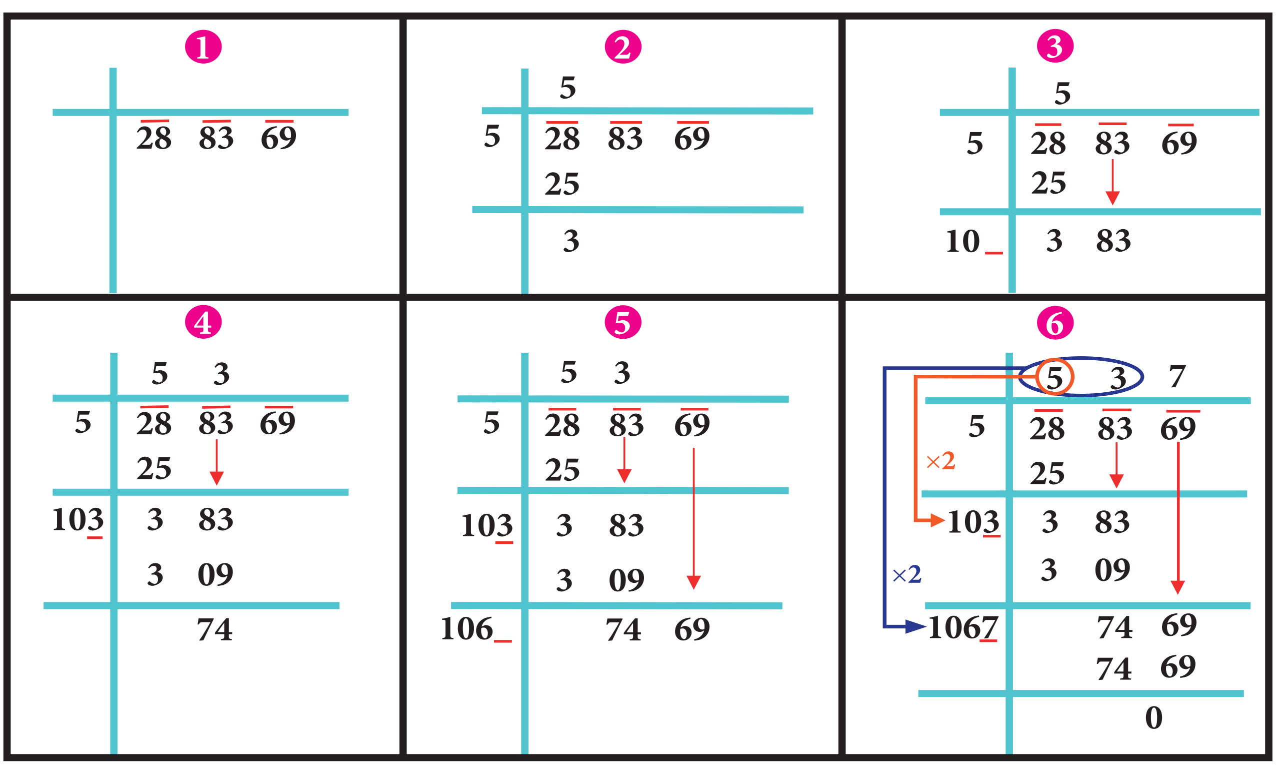


Illustration 2

In the following example, follow the figures one after another and try to understand what each figure explains, the stage by stage and the gradual computation of computing the square root of 288369.



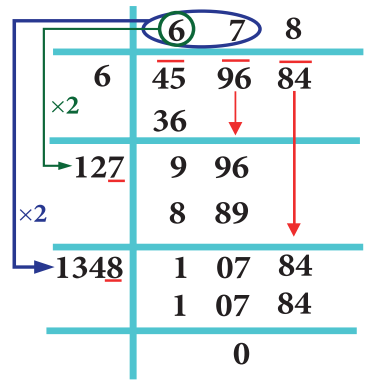
We find that $\sqrt{288369}=537$.

Example 1.27

Find the square root of 459684 by long division method.

Solution:

By long division method, we can find the square root of 459684 as given below:



$\therefore \sqrt{459684} = 678$

Try these

Find the square root by long division method.

1. 400 2. 1764 3. 9801

1.7.3 Number of digits in the square root of a perfect square number

We made use of bars to find the square root in the division method. This marking bars help us to find the number of digits in the square root of a perfect square number. Observe the following examples (with bars shown as if we compute square root by division procedure).

Square root of numbers	No. of bars	No. of digits in the square root	Square root of numbers	No. of bars	No. of digits in the square root
$\sqrt{169} = 13$	2	2	$\sqrt{4356} = 66$	2	2
$\sqrt{441} = 21$	2	2	$\sqrt{6084} = 78$	2	2
$\sqrt{12544} = 112$	3	3	$\sqrt{27225} = 165$	3	3

Hence, we conclude that the number of bars indicates the number of digits in the square root.

Try these

Without calculating the square root, guess the number of digits in the square root of the following numbers:

1. 14400 2. 390625 3. 100000000

1.7.4 Square root of decimal numbers

To compute the square root of numbers in the decimal form, we simply follow the following steps:

Step 1:

Make even number of decimal places even by affixing a zero on the extreme right of the digit in the decimal part (only if required).

To find $\sqrt{42.25}$

Step 2:

The number has an integral part and a decimal part. In the integral part, mark the bars as done in the case of division method to find the square root of a perfect square number.

We put the bars as $\sqrt{\overline{42}.\overline{25}}$

Step 3:

In the decimal part, mark the bars on every pair of digits beginning with the first decimal place.

Step 4:

Now, calculate the square root by long division method.

	6 5
6	$\overline{42}$ $\overline{25}$
	36
125	6 25
	6 25
	0

Step 5:

Put the decimal point in the square root as soon as the integral part is exhausted. $\therefore \sqrt{42.25} = 6.5$

1.7.5 Square root of product and quotient of numbers

For any two positive numbers a and b. we have

(i) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$

Try these

Find the square root of

1. 5.4756 2. 19.36 3. 116.64

Example 1.28

Find the value of $\sqrt{256}$

Solution:

$\sqrt{256} = \sqrt{16 \times 16} = \sqrt{16} \times \sqrt{16} = 4 \times 4 = 16$ (or) $\sqrt{256} = \sqrt{64 \times 4} = \sqrt{64} \times \sqrt{4} = 8 \times 2 = 16.$

Think

Try to fill in the blanks using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.

$\sqrt{36} = 6$	$\sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$	Is $\sqrt{36} = \sqrt{9} \times \sqrt{4}$?	$\sqrt{81} = ?$	$\sqrt{9} \times \sqrt{9} =$ __ $\times$ __ $=$ __	Is $\sqrt{81} = \sqrt{9} \times \sqrt{9}$?
$\sqrt{144} = ?$	$\sqrt{9} \times \sqrt{16} =$ __ $\times$ __ $=$ __	Is $\sqrt{144} = \sqrt{9} \times \sqrt{16}$?	$\sqrt{144} = ?$	$\sqrt{36} \times \sqrt{4} =$ __ $\times$ __ $=$ __	Is $\sqrt{144} = \sqrt{36} \times \sqrt{4}$?
$\sqrt{100} = ?$	$\sqrt{25} \times \sqrt{4} =$ __ $\times$ __ $=$ __	Is $\sqrt{100} = \sqrt{25} \times \sqrt{4}$?	$\sqrt{1225} = ?$	$\sqrt{25} \times \sqrt{49} =$ __ $\times$ __ $=$ __	Is $\sqrt{1225} = \sqrt{25} \times \sqrt{49}$?

Activity

Attempt to prepare a table of square root problems as in the above case to show that if a and b are two perfect square numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$. We can use this idea to compute certain square-root problems easily.

Example 1.29

Find the value of $\sqrt{42.25}$.

Solution:

We can write this as $\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}}$

Now, it is easy to compute the square root of the whole number 4225 by long division method as $\sqrt{4225} = 65$ and so, we now get $\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}} = \frac{65}{10} = 6.5$

This is another way of tackling problems of square root of decimal numbers without any botheration of decimal symbol.

> **Try these**
>
> Using this method, find the square root of the numbers 1.2321 and 11.9025.

Example 1.30

Simplify: (i) $\sqrt{12} \times \sqrt{3}$ (ii) $\sqrt{\frac{98}{162}}$

Solution:

(i) Remembering the rule, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$$\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = \sqrt{6^2} = 6$$

(ii) Remembering the rule, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\ (b \neq 0)$

$$\sqrt{\frac{98}{162}} = \sqrt{\frac{2 \times 49}{2 \times 81}} = \sqrt{\frac{49}{81}} = \sqrt{\frac{7^2}{9^2}} = \frac{7}{9}$$

Example 1.31

Simplify: (i) $\sqrt{2\frac{7}{9}}$ (ii) $\sqrt{1\frac{9}{16}}$

Solution:

(i) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \sqrt{\frac{5^2}{3^2}} = \frac{5}{3} = 1\frac{2}{3}$ (ii) $\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \sqrt{\frac{5^2}{4^2}} = \frac{5}{4} = 1\frac{1}{4}$

Remark: In the case of the (ii) problem one may be tempted to give the answer immediately as $1\frac{3}{4}$, but this is not correct since you have to convert the mixed fraction into an improper fraction and then use the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\ (b \neq 0)$

1.7.6 Approximating square roots

Can you write the given numbers $\sqrt{40}$, 6 and 7 in ascending order? Here $\sqrt{40}$ is not a square number and so we cannot determine its root easily. However, we can estimate an approximation to $\sqrt{40}$ and use it here.

> **Try these**
>
> Write the numbers in ascending order.
>
> 1. 4, $\sqrt{14}$, 5   2. 7, $\sqrt{65}$, 8

We know that the two closest squares surrounding 40 are 36 and 49.

Thus, $36 < 40 < 49$ which can be written as $6^2 < 40 < 7^2$.

Considering the square root, we have $6 < \sqrt{40} < 7$.

Exercise 1.4

1. **Fill in the blanks:**
   (i) The ones digit in the square of 77 is__________.
   (ii) The number of non-square numbers between 24^2 and 25^2 is ______.
   (iii) The number of perfect square numbers between 300 and 500 is ______.
   (iv) If a number has 5 or 6 digits in it, then its square root will have __________ digits.
   (v) The value of $\sqrt{180}$ lies between integers ______ and ______.

2. **Say True or False:**
   (i) When a square number ends in 6, its square root will have 6 in the unit's place.
   (ii) A square number will not have odd number of zeros at the end.
   (iii) The number of zeros in the square of 91000 is 9.
   (iv) The square of 75 is 4925.
   (v) The square root of 225 is 15.

3. Find the square of the following numbers.
   (i) 17 (ii) 203 (iii) 1098

4. Examine if each of the following is a perfect square.
   (i) 725 (ii) 190 (iii) 841 (iv) 1089

5. Find the square root by prime factorisation method.
   (i) 144 (ii) 256 (iii) 784 (iv) 1156 (v) 4761 (vi) 9025

6. Find the square root by long division method.
   (i) 1764 (ii) 6889 (iii) 11025 (iv) 17956 (v) 418609

7. Estimate the value of the following square roots to the nearest whole number:
   (i) $\sqrt{440}$ (ii) $\sqrt{800}$ (iii) $\sqrt{1020}$

8. Find the square root of the following decimal numbers and fractions.
   (i) 2.89 (ii) 67.24 (iii) 2.0164 (iv) $\frac{144}{225}$ (v) $7\frac{18}{49}$

9. Find the least number that must be subtracted to 6666 so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

10. Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Objective Type Questions

11. The square of 43 ends with the digit ______.
    (A) 9 (B) 6 (C) 4 (D) 3

12. _______ is added to 24^2 to get 25^2.

(A) 4^2 (B) 5^2 (C) 6^2 (D) 7^2

13. $\sqrt{48}$ is approximately equal to ______.

(A) 5 (B) 6 (C) 7 (D) 8

14. $\sqrt{128} - \sqrt{98} + \sqrt{18} =$ ______.

(A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) $\sqrt{48}$ (D) $\sqrt{32}$

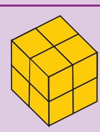
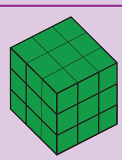
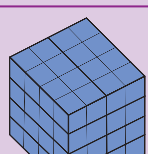
15. The number of digits in the square root of 123454321 is ______.

(A) 4 (B) 5 (C) 6 (D) 7

1.8 Cubes and Cube Roots

If you multiply a number by itself and then by itself again, the result is a cube number. This means that a cube number is a number that is the product of three identical numbers. If n is a number, its cube is represented by n^3.

Cube numbers can be represented visually as 3D cubes comprising of single unit cubes. Cube numbers are also called as perfect **cubes**. The perfect cubes of natural numbers are 1, 8, 27, 64, 125, 216, ... and so on.

Geometrical Representation	Product Representation	Notation	Perfect cube
	$1 \times 1 \times 1$	1^3	1
	$2 \times 2 \times 2$	2^3	8
	$3 \times 3 \times 3$	3^3	27
	$4 \times 4 \times 4$	4^3	64

DO YOU KNOW?

Ramanujan Number - $1729 = 12^3+1^3 = 10^3+9^3$

Once Professor Hardy went to see Ramanujan when he was ill at Putney, riding in taxi cab number 1729 and said that the number seemed a dull one, and hoped it was not an unfavourable omen. "No," replied Ramanujan and he completed saying "It is a very interesting number. Infact, it is the smallest number expressible as the sum of two cubes in two different ways." 4104, 13832, 20683 are a few more examples of Ramanujan-Hardy numbers.

1.8.1 Properties of cubes of numbers

S. No.	Properties	Examples
1.	The cube of a positive number is positive.	$6^3 = 6 \times 6 \times 6 = 216$.
2.	The cube of a negative number is negative.	$(-7)^3 = (-7) \times (-7) \times (-7) = -343$
3.	The cube of every even number is even.	$8^3 = 8 \times 8 \times 8 = 512$ is even
4.	The cube of every odd number is odd.	$9^3 = 9 \times 9 \times 9 = 729$ is odd
5.	If a natural number ends with 0, 1, 4, 5, 6 or 9, its cube also ends with the same 0,1, 4, 5, 6 or 9 respectively.	$10^3 = 1000$, $11^3 = 1331$, $14^3 = 2744$ $15^3 = 3375$, $16^3 = 4096$, $19^3 = 6859$
6.	If a natural number ends with 2 or 8, its cube ends with 8 or 2 respectively.	$12^3 = 1728$, $18^3 = 5832$
7.	If a natural number ends with 3 or 7, its cube ends with 7 or 3 respectively.	$13^3 = 2197$, $17^3 = 4913$
8.	The sum of the cubes of first n natural numbers is equal to the square of their sum.	$1^3 + 2^3 + 3^3 + ... + n^3 = (1+2+3+.....+n)^2$ Check that, $1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$

Note

- A perfect cube does not end with two zeroes.
- The cube of a two digit number may have 4 or 5 or 6 digits in it.

Try these

Find the ones digit in the cubes of each of the following numbers.

1. 12 2. 27 3. 38
4. 53 5. 71 6. 84

1.8.2 Cube root

The cube root of a number is the value that when cubed gives the original number.
For example, the cube root of 27 is 3 because when 3 is cubed we get 27.

Notation:

The cube root of a number x is denoted as

$$\sqrt[3]{x} \text{ (or) } x^{\frac{1}{3}}.$$

Here are some more cubes and cube roots:

$\sqrt[3]{1} = 1$ since $1^3 = 1$, $\sqrt[3]{8} = 2$ since $2^3 = 8$,
$\sqrt[3]{27} = 3$ since $3^3 = 27$, $\sqrt[3]{64} = 4$ since $4^3 = 64$,
$\sqrt[3]{125} = 5$ since $5^3 = 125$ and so on.

Cubes	Cube Roots	Cubes	Cube Roots
1	1	729	9
8	2	1000	10
27	3	1331	11
64	4	1728	12
125	5	2197	13
216	6	2744	14
343	7	3375	15
512	8	4096	16

Example 1.32

Is 400 a perfect cube?

Solution:

By prime factorisation, we have $400 = \underbrace{2 \times 2 \times 2} \times 2 \times 5 \times 5$.

There is only one triplet. To make further triplets, we will need two more 2's and one more 5. Therefore, 400 is not a perfect cube.

Example 1.33

Find the smallest number by which 675 must be multiplied to obtain a perfect cube.

Solution:

3	675
3	225
3	75
5	25
5	1
	1

We find that, $675 = 3 \times 3 \times 3 \times 5 \times 5$ ....(1)

Grouping the prime factors of 675 as triplets, we are left over with 5×5.

We need one more 5 to make it a perfect cube.

To make 675 a perfect cube, multiply both sides of (1) by 5.

$675 \times 5 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Now, 3375 is a perfect cube. Thus, the smallest required number to multiply 675 such that the new number perfect cube is 5.

Think

In this question, if the word 'multiplied' is replaced by the word 'divided', how will the solution vary?

1.8.3 Cube root of a given number by Prime Factorisation

Step 1: Resolve the given number into the product of prime factors.

Step 2: Make triplet groups of same primes.

Step 3: Choosing one from each triplet, find the product of primes to get the cube root.

Example 1.34

Find the cube root of 27000.

Solution:

By prime factorisation, we have $27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$$\therefore \sqrt[3]{27000} = 2 \times 3 \times 5 = 30.$$

Example 1.35

Evaluate: (i) $\sqrt[3]{\dfrac{9261}{8000}}$ (ii) $\sqrt[3]{\dfrac{1728}{729}}$

Solution:

(i) $\sqrt[3]{\dfrac{9261}{8000}} = \dfrac{\sqrt[3]{9261}}{\sqrt[3]{8000}} = \dfrac{(21 \times 21 \times 21)^{\frac{1}{3}}}{(20 \times 20 \times 20)^{\frac{1}{3}}} = \dfrac{(21^3)^{\frac{1}{3}}}{(20^3)^{\frac{1}{3}}} = \dfrac{21}{20} = 1\dfrac{1}{20}$

(ii) $\sqrt[3]{\dfrac{1728}{729}} = \dfrac{\sqrt[3]{1728}}{\sqrt[3]{729}} = \dfrac{(12 \times 12 \times 12)^{\frac{1}{3}}}{(9 \times 9 \times 9)^{\frac{1}{3}}} = \dfrac{(12^3)^{\frac{1}{3}}}{(9^3)^{\frac{1}{3}}} = \dfrac{12}{9} = \dfrac{4}{3} = 1\dfrac{1}{3}$

Exercise 1.5

1. **Fill in the blanks:**
   - (i) The ones digits in the cube of 73 is____________.
   - (ii) The maximum number of digits in the cube of a two digit number is ________.
   - (iii) The smallest number to be added to 3333 to make it a perfect cube is ____________.
   - (iv) The cube root of 540×50 is ____________.
   - (v) The cube root of 0.000004913 is ____________.

Q5F3Z8

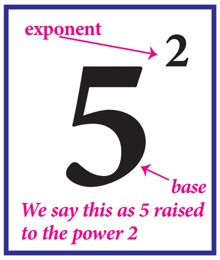
2. **Say True or False:**
   - (i) The cube of 24 ends with the digit 4.
   - (ii) Subtracting 10^3 from 1729 gives 9^3.
   - (iii) The cube of 0.0012 is 0.000001728.
   - (iv) 79570 is not a perfect cube.
   - (v) The cube root of 250047 is 63.
3. Show that 1944 is not a perfect cube.
4. Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.
5. Find the smallest number by which 200 should be multiplied to make it a perfect cube.
6. Find the cube root of $24 \times 36 \times 80 \times 25$.
7. Find the cube root of 729 and 6859 by prime factorisation.
8. What is the square root of cube root of 46656?
9. If the cube of a squared number is 729, find the square root of that number.
10. Find the two smallest perfect square numbers which when multiplied together gives a perfect cube number.

Activity

Observe that $2^3 - 1^3 = 1 + 2 \times 1 \times 3$ $3^3 - 2^3 = 1 + 3 \times 2 \times 3$ $4^3 - 3^3 = 1 + 4 \times 3 \times 3$ Find the value of $15^3 - 14^3$ in the above pattern.	Observe that $1^3 = 1 = 1$ $2^3 = 8 = 3 + 5$ $3^3 = 27 = 7 + 9 + 11$ Continue this pattern to find the value of 7^3 as the sum of consecutive odd numbers.

1.9 Exponents and Powers

We know how to express some numbers as squares and cubes. For example, we write 5^2 for 25 and 5^3 for 125.



In general terms, an expression that represents repeated multiplication of the same factor is called a **power**. The number 5 is called the **base** and the number 2 is called the **exponent** (more often called as power). The exponent corresponds to the number of times the base is used as a factor.

1.9.1 Powers with positive exponents

Value of powers given by positive whole number exponents quite often increase rapidly. Observe the following example:

$$2^1 = 2$$
$$2^2 = 2\times2=4$$
$$2^3 = 2\times2\times2=8$$
$$2^4 = 2\times2\times2\times2 =16$$
$$2^5 = 2\times2\times2\times2\times2=32$$
$$2^6 = 2\times2\times2\times2\times2\times2=64$$
$$2^7 = 2\times2\times2\times2\times2\times2\times2=128$$
$$2^8 = 2\times2\times2\times2\times2\times2\times2\times2=256$$
$$2^9 = 2\times2\times2\times2\times2\times2\times2\times2\times2=512$$
$$2^{10} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2=1024$$

At this rate of increase, what do you think 2^{100} will be?

In fact, $\mathbf{2^{100}=1267650600228229401496703205376}$

Thus, we understand that the positive exponential notation with positive power could be useful when we come across with large numbers.

1.9.2 Powers with zero and negative exponents

Observe this pattern:

$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = ?$

Starting from the beginning, what happens in the successive steps? We find that the result is half of that of the previous step. So, what can we say about 2^0? If we prepare a table like this for 3^5, 3^4, 3^3, and so on what will it tell us about 3^0? We can use the same process as in this pattern, to conclude that any non-zero number raised to the zero exponent must result in 1. Thus,

$$a^0 = 1, \text{ where } a \neq 0$$

Let us see what happens if we extend the above pattern further downward.

As before, starting from the beginning, in the successive steps, we find that the result is half of that of the previous step. Since $2^0 = 1$, the next step is 2^{-1}, whose value is the previous step's value 1, divided by 2, that is $\frac{1}{2}$. Next is 2^{-2}, which is the same as $\frac{1}{2}$ divided by 2, that is $\frac{1}{4}$ and so on. Thus,

$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$
$2^{-3} = \frac{1}{8}$

In general, $a^{-m} = \frac{1}{a^m}$, where m is an integer

1.9.3 Expanded form of numbers using exponents

In the lower classes, we have learnt how to write a whole number in the expanded form. For example, $5832 = 5\times1000 + 8\times100 + 3\times10 + 2\times1$

$= 5\times10^3 + 8\times10^2 + 3\times10^1 + 2$ (when we use exponential notation).

What shall we do if we get decimal places? Powers of 10 with negative exponents come to our rescue!

Thus, $58.32 = 50 + 8 + \frac{3}{10} + \frac{2}{100}$

$= 5\times10 + 8\times1 + 3\times\frac{1}{10} + 2\times\frac{1}{100}$

$= 5\times10^1 + 8\times10^0 + 3\times10^{-1} + 2\times10^{-2}$

Try these

Expand the following numbers using exponents:

1. 8120
2. 20305
3. 3652.01
4. 9426.521

1.9.4 Laws of Exponents

Laws of exponents arise out of certain basic ideas. A **positive exponent** of a number indicate how many times we use that number in a multiplication whereas a **negative exponent** suggests us how many times we use that number in a division, since the opposite of multiplying is dividing.

- **Product law**

According to this law, when multiplying two powers that have the same base, we can add the exponents. That is,

$$a^m \times a^n = a^{m+n}$$

where a ($a\neq0$), m, n are integers. Note that the base should be the same in both the quantities.

Examples:

a)	$2^3\times2^2 = 2^5$ by law(meaning $8\times4 = 32$ and note that it is not $2^{3\times2}$)
b)	$(-2)^{-4}\times(-2)^{-3} = (-2)^{(-4)+(-3)}$ by law and so $= (-2)^{-7}$ (or) $(-2)^{-4}\times(-2)^{-3} = \frac{1}{(-2)^4}\times\frac{1}{(-2)^3} = \frac{1}{(-2)^4\times(-2)^3} = \frac{1}{(-2)^{4+3}} = \frac{1}{(-2)^7} = (-2)^{-7}$
c)	$(-5)^3\times(-5)^{-3} = (-5)^{3-3}$ by law and so $= (-5)^0 = 1$

- **Quotient law**

According to this law, when dividing two powers that have the same base we can subtract the exponents. That is,

$$\frac{a^m}{a^n} = a^{m-n}$$

where a ($a \neq 0$), m, n are integers. Note that the base should be the same in both the quantities. How does it work? Study the following examples.

Examples:

a)	$\frac{(-3)^5}{(-3)^2} = (-3)^{5-2}$ by law and $(-3)^3 = -27$ (or) $\frac{(-3)^5}{(-3)^2} = (-3)^5 \times (-3)^{-2} = (-3)^{5-2} = (-3)^3$ (or) $\frac{(-3)^5}{(-3)^2} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)} = (-3)\times(-3)\times(-3) = (-3)^3$
b)	$\frac{(-7)^{100}}{(-7)^{98}} = (-7)^{100-98}$ by law and $(-7)^2 = 49$ (or) $\frac{(-7)^{100}}{(-7)^{98}} = \frac{(-7)\times(-7)\times(-7)\times\ldots 100 \text{ times}}{(-7)\times(-7)\times(-7)\times\ldots\ 98 \text{ times}} = (-7)\times(-7) = 49$

- **Power law**

According to this law, when raising a power to another power, we can just multiply the exponents.

$$(a^m)^n = a^{mn}$$

where a ($a \neq 0$), m, n are integers.

Examples:

$[(-2)^3]^2 = (-2)^{3\times 2}$ by law and $(-2)^6 = 64$

(or)

$[(-2)^3]^2 = [(-2)\times(-2)\times(-2)]^2 = [-8]^2 = 64$

Try these

Verify the following rules (as we did above). Here, a,b are non-zero integers and m, n are any integers.

1. Product of same powers to power of product rule: $a^m \times b^m = (ab)^m$
2. Quotient of same powers to power of quotient rule: $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
3. Zero exponent rule: $a^0 = 1$.

Example 1.36

Find the value of (i) 4^{-3} (ii) $\frac{1}{2^{-3}}$ (iii) $(-2)^5 \times (-2)^{-3}$ (iv) $\frac{3^2}{3^{-2}}$

Solution:

(i) $4^{-3} = \frac{1}{4^3} = \frac{1}{4\times 4\times 4} = \frac{1}{64}$ (ii) $\frac{1}{2^{-3}} = 2^3 = 2\times 2\times 2 = 8$

(iii) $(-2)^5 \times (-2)^{-3} = (-2)^{5-3} = (-2)^2 = -2 \times -2 = 4$ (iv) $\frac{3^2}{3^{-2}} = 3^2 \times 3^2 = 9 \times 9 = 81$

Example 1.37

Simplify and write the answer in exponential form:

(i) $(3^5 \div 3^8)^5 \times 3^{-5}$ (ii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Solution:

(i) $\left(\frac{3^5}{3^8}\right)^5 \times 3^{-5} = (3^{5-8})^5 \times 3^{-5} = (3^{-3})^5 \times 3^{-5} = 3^{-3\times5} \times 3^{-5} = 3^{-15} \times 3^{-5} = 3^{-15-5} = 3^{-20}$

(ii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = 3^4 \times \frac{5^4}{3^4} = 5^4$

Example 1.38

Find x so that $(-7)^{x+2} \times (-7)^5 = (-7)^{10}$

Solution:

$$(-7)^{x+2} \times (-7)^5 = (-7)^{10}$$

$$(-7)^{x+2+5} = (-7)^{10}$$

Since the bases are equal, we equate the exponents to get

$$x + 7 = 10$$

$$\Rightarrow x = 10 - 7 = 3$$

1.9.5 Standard Form and Scientific Notation

Standard form of a number is just the number as we normally write it. We use expanded notation to show the value of each digit. That is, it is exhibited as a sum of each digit duly multiplied by its matching place value (like ones, tens, hundreds etc.,). For example, **195** is in standard form. It can be expanded as **195 = 1 × 100 + 9 × 10 + 5 × 1**.

Astronomers, biologists, engineers, physicists and many others come across quantities whose measures require very small or very large numbers. If they write the numbers in standard form, it may not help us to understand or make computations easily. **Scientific notation** is a way to make these numbers easier to work with.

To write in scientific notation, follow the form $\mathbf{S \times 10^a}$. where **S** is a number (integer or integer with decimal) between 1 and 10, but not 10 itself, and a is a positive or negative integer. Thus, a number in scientific notation is written as the product of a number (integer or integer with decimal) and a power of 10. We move the decimal place forward or backward until we have a number from 1 to 9. Then, we add a power of ten that tells how many places you moved the decimal forward or backward.

Examples:

Standard Form	Scientific Notation	Standard Form	Scientific Notation
0.00123	1.23×10^{-3}	123	1.23×10^{2}
0.0123	1.23×10^{-2}	1230	1.23×10^{3}
0.123	1.23×10^{-1}	12300	1.23×10^{4}
1.23	1.23×10^{0}	123000	1.23×10^{5}
12.3	1.23×10^{1}	1230000	1.23×10^{6}

Some more examples:

(a) The diameter of the earth is 12756000 miles. This can be easily written in scientific form as 1.2756×10^{7} miles.

(b) The volume of Jupiter is about 1433000000000000 km^{3}. This can be easily written in scientific form as 1.433×10^{14} km^{3}.

(c) The size of a bacterium is 0.00000085 mm. This can be easily written in scientific form as 8.5×10^{-7} mm.

Note

1. The positive exponent in 1.3×10^{12} indicates that it is a large number.
2. The negative exponent in 7.89×10^{-21} indicates that it is a small number.

Example 1.39

Combine the scientific notations: (i) $(7\times10^{2})(5.2\times10^{7})$ (ii) $(3.7\times10^{-5})(2\times10^{-3})$

Solution:

(i) $(7\times10^{2})(5.2\times10^{7}) = 36.4 \times 10^{9} = 3.64 \times 10^{10}$

(ii) $(3.7\times10^{-5})(2\times10^{-3}) = 7.4 \times 10^{-8}$

Example 1.40

Write the following scientific notations in standard form:

(i) 2.27×10^{-4} (ii) Light travels at 1.86×10^{5} miles per second.

Solution:

(i) $2.27\times10^{-4} = 0.000227$.

(ii) Light travels at 1.86×10^{5} miles per second = 186000 miles per second

Try these

1. Write in standard form: Mass of planet Uranus is 8.68×10^{25} *kg*.
2. Write in scientific notation: (i) 0.000012005 (ii) 4312.345 (iii) 0.10524
   (iv) The distance between the Sun and the planet Saturn 1.4335×10^{12} miles.

Exercise 1.6

1. **Fill in the blanks:**

   (i) $(-1)^{\text{even integer}}$ is ______________.

   (ii) For $a \neq 0$, a^0 is ______________.

   (iii) $4^{-3} \times 5^{-3} =$ ______________.

   (iv) $(-2)^{-7} =$ ______________.

   (v) $\left(-\frac{1}{3}\right)^{-5} =$ ______________.

2. **Say True or False:**

   (i) If $8^x = \frac{1}{64}$, the value of x is -2.

   (ii) The simplified form of $(256)^{\frac{-1}{4}} \times 4^2$ is $\frac{1}{4}$.

   (iii) Using the power rule, $\left(3^7\right)^{-2} = 3^5$.

   (iv) The standard form of 2×10^{-4} is 0.0002.

   (v) The scientific form of 123.456 is 1.23456×10^{-2}.

3. Evaluate: (i) $\left(\frac{1}{2}\right)^3$ (ii) $\left(\frac{1}{2}\right)^{-5}$ (iii) $\left(\frac{-5}{6}\right)^{-3}$ (iv) $\left(2^{-5} \times 2^7\right) \div 2^{-2}$ (v) $\left(2^{-1} \times 3^{-1}\right) \div 6^{-2}$

4. Evaluate: (i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$ (ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$ (iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

5. Evaluate: (i) $\left(5^0 + 6^{-1}\right) \times 3^2$ (ii) $\left(2^{-1} + 3^{-1}\right) \div 6^{-1}$ (iii) $\left(3^{-1} + 4^{-2} + 5^{-3}\right)^0$

6. Simplify: (i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$ (ii) $\frac{9^2 \times 7^3 \times 2^5}{84^3}$ (iii) $\frac{2^8 \times 2187}{3^5 \times 32}$

7. Solve for x : (i) $\frac{2^{2x-1}}{2^{x+2}} = 4$ (ii) $\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$

8. Expand using exponents: (i) 6054.321 (ii) 897.14

9. Find the number in standard form for the following expansions:

   (i) $8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$

   (ii) $5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$

   (iii) The radius of a hydrogen atom is 2.5×10^{-11} m

10. Write the following numbers in scientific notation:

    (i) 467800000000 (ii) 0.000001972 (iii) 1642.398

    (iv) Earth's volume is about 1,083,000,000,000 cubic kilometres

    (v) If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.0000000000000000000000016 kg

Objective Type Questions

11. By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1}?

(A) $\frac{2}{3}$ (B) $\frac{-2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{-5}{2}$

12. $(-2)^{-3} \times (-2)^{-2} =$ ______________.

(A) $\frac{-1}{32}$ (B) $\frac{1}{32}$ (C) 32 (D) –32

13. Which is not correct?

(A) $\left(\frac{-1}{4}\right)^2 = 4^{-2}$ (B) $\left(\frac{-1}{4}\right)^2 = \left(\frac{1}{2}\right)^4$ (C) $\left(\frac{-1}{4}\right)^2 = 16^{-1}$ (D) $-\left(\frac{1}{4}\right)^2 = 16^{-1}$

14. If $\frac{10^x}{10^{-3}} = 10^9$, then x is ______________.

(A) 4 (B) 5 (C) 6 (D) 7

15. 0.0000000002020 in scientific form is ______________.

(A) 2.02×10^9 (B) 2.02×10^{-9} (C) 2.02×10^{-8} (D) 2.02×10^{-10}

Exercise 1.7

Miscellaneous Practice Problems

1. If $\frac{3}{4}$ of a box of apples weighs 3 kg and 225 gm, how much does a full box of apples weigh?
2. Mangalam buys a water jug of capacity $3\frac{4}{5}$ litre. If she buys another jug which is $2\frac{2}{3}$ times as large as the smaller jug, how many litre can the larger one hold?
3. Ravi multiplied $\frac{25}{8}$ and $\frac{16}{15}$ and he says that the simplest form of this product is $\frac{10}{3}$ and Chandru says the answer in the simplest form is $3\frac{1}{3}$. Who is correct? (or) Are they both correct? Explain.
4. Find the length of a room whose area is $\frac{153}{10}$ sq.m and whose breadth is $2\frac{11}{20}$ m.
5. There is a large square portrait of a leader that covers an area of 4489 cm^2. If each side has a 2 cm liner, what would be its area?
6. A greeting card has an area 90 cm^2. Between what two whole numbers is the length of its side?
7. 225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped verandah. How long is each side of the square shaped verandah?
8. If $\sqrt[3]{1906624} \times \sqrt{x} = 3100$, find x.
9. If $2^{m-1} + 2^{m+1} = 640$, then find m.

10. Give the answer in scientific notation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in i) an hour? ii) a day? iii) a year? iv) 100 years?

Challenging Problems

11. In a map, if 1 inch refers to 120 km, then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.
12. Give an example and verify each of the following statements.
    (i) The collection of all non-zero rational numbers is closed under division.
    (ii) Subtraction is not commutative for rational numbers.
    (iii) Division is not associative for rational numbers.
    (iv) Distributive property of multiplication over subtraction is true for rational numbers. That is, $a(b-c)=ab-ac$.
    (v) The mean of two rational numbers is rational and lies between them.
13. If $\frac{1}{4}$ of a *ragi adai* weighs 120 grams, what will be the weight of $\frac{2}{3}$ of the same *ragi adai*?
14. If $p+2q=18$ and $pq=40$, find $\frac{2}{p}+\frac{1}{q}$.
15. Find x if $5\frac{x}{5}\times 3\frac{3}{4}=21$.
16. By how much does $\frac{1}{\left(\frac{10}{11}\right)}$ exceed $\frac{\left(\frac{1}{10}\right)}{11}$?
17. A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible, how many more cadets would be required?
18. Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25}+\sqrt{28.6225}$
19. Simplify: $(3.769\times 10^5)+(4.21\times 10^5)$
20. Order the following from the least to the greatest: $16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}$

SUMMARY

- A number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b\neq 0$ is called a rational number.
- All natural numbers, whole numbers, integers and fractions are rational numbers.
- Every rational number can be represented on a number line.
- 0 is neither a positive nor a negative rational number.
- A rational number $\frac{a}{b}$ is said to be in the standard form if its denominator b is a positive integer and HCF $(a,b)=1$

- There are unlimited numbers of rational numbers between two rational numbers.
- Subtracting two rational numbers is the same as adding the additive inverse of the second number to the first rational number.
- Multiplying two rational numbers is the same as multiplying their numerators and denominators separately and then writing the product in the standard form.
- Dividing a rational number by another rational number is the same as multiplying the first rational number by the reciprocal of the second rational number.
- The following table is about the properties of rational numbers($\mathbb{Q}$).

$\mathbb{Q}$	Closure	Commutative	Associative	Multiplication is distributive over +/ −
+	✓	✓	✓	✓
−	✓	×	×	✓
×	✓	✓	✓	-
÷	×	×	×	-

- 0 and 1 are respectively the additive and the multiplicative identities of rational numbers.
- The additive inverse for $\frac{a}{b}$ is $\frac{-a}{b}$ and vice – versa.
- The reciprocal or the multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$ since $\frac{a}{b}\times\frac{b}{a}=1$.
- A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.
- The square root of a number n, written as $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself.
- The number of times a prime factor occurs in the square is equal to twice the number of times it occurs in the prime factorization of the number.
- For any two positive numbers a and b. we have

  (i) $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$ and (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$
- If you multiply a number by itself and then by itself again, the result is a cube number.
- The cube root of a number is the value that when cubed gives the original number.
- An expression that represents repeated multiplication of the same factor is called a power.
- The exponent corresponds to the number of times the base is used as a factor.
- Laws of Exponents: (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$
- Other results: (i) $a^0 = 1$ (ii) $a^{-m} = \frac{1}{a^m}$ (iii) $a^m \times b^m = (ab)^m$ (iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- To write in scientific notation, we follow the form $S \times 10^a$ where S is a number (integer or integer with decimal) between 1 and 10, but not 10 itself, and a is a positive or negative integer.

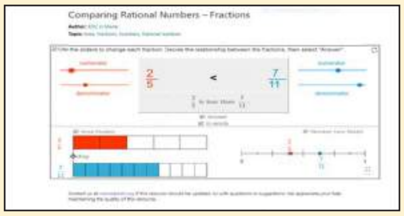
ICT CORNER

Step-1 Open the Browser and type www.Geogebra.com (or) scan the QR CODE given below.

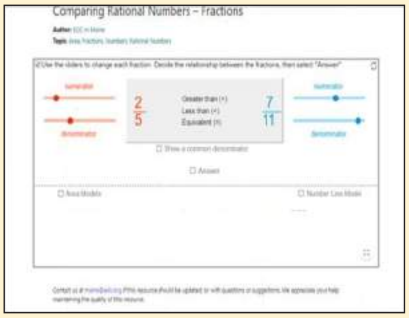
Step-2 Type Rational Numbers on the search column

Step-3 Move the numerator and denominator slide to show the different rational numbers and click the number line models to show the number line.

Through this activity you will know about the rational numbers, operations on them and study their properties as well.



B358_8_MATHS_EM



Step 1

Step 2

Web URL rational numbers (rational number links)

Web URL https://www.geogebra.org/m/n92AKzBF#material/ca5D7VbZ

*Pictures are indicatives only.

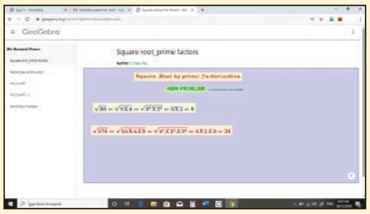
*If browser requires, allow Flash Player or Java Script to load the page.

ICT CORNER

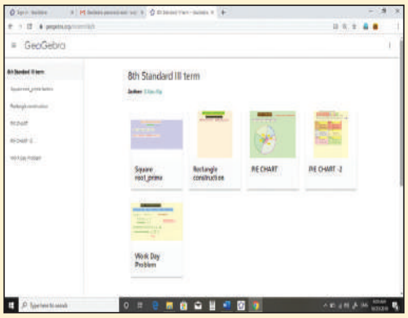
Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Square root_prime factors"

Step-2 Click on " NEW PROBLEM" and check the calculation.

Expected Outcome



B358_8_MATHS_EM



Step 1

Step 2

Browse in the link

Numbers:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.