

- c) A is true but R is false. d) A is false but R is true.
14. **Assertion:** When a bottle of cold carbonated drink is opened, a slight fog forms around the opening. [1]
Reason: Adiabatic expansion of the gas causes lowering of temperature which start condensation of water vapours.
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement. d) Assertion is wrong statement but reason is correct statement.
15. **Assertion (A):** The time period of revolution of a satellite close to the surface of the earth is smaller than that revolving away from surface of earth. [1]
Reason (R): The square of time period of revolution of a satellite is directly proportional to the cube of its orbital radius.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.
16. **Assertion (A):** Angle between two vectors $\hat{j} + \hat{k}$ and \hat{j} is 45° . [1]
Reason (R): Vector $\hat{j} + \hat{k}$ is equally inclined to both Y and Z axes and angle between \hat{j} and \hat{k} is 90° .
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.

Section B

17. State the principle of superposition of waves. [2]
18. The depth x to which a bullet penetrates a human body depends upon [2]
- i. coefficient of elasticity η and
- ii. kinetic energy E_k . By the method of dimensions, show that: $x \propto \left[\frac{E_k}{\eta} \right]^{1/3}$
19. The velocity of a body that has fallen freely under gravity varies as $g^p h^q$, where g is the acceleration due to gravity at the place and h is the height through which the body has fallen. Determine the values of p and q . [2]
20. Two particles of masses m_1 and m_2 in projectile motion have velocities \vec{v}_1 and \vec{v}_2 respectively at time $t = 0$. [2]
They collide at time t_0 . Their velocities become \vec{v}'_1 and \vec{v}'_2 at time $2t_0$ while still moving in air. The value of $\left| \left(m_1 \vec{v}'_1 + m_2 \vec{v}'_2 \right) - \left(m_1 \vec{v}_1 + m_2 \vec{v}_2 \right) \right|$.
21. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30} kg. The mean distance of Jupiter from the Sun is 7.8×10^{11} m. Calculate gravitational force which sun exerts on Jupiter, and the speed of Jupiter. [2]

OR

An apple of mass 0.25 kg falls from a tree. What is the acceleration of the apple towards the earth? Also, calculate the acceleration of the earth towards the apple. Mass of the earth = 5.983×10^{24} kg, Radius of the earth = 6.378×10^6 m and $G = 6.67 \times 10^{-11}$ Nm² kg⁻².

Section C

22. Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area. [3]
23. What is meant by a triple point of water? What is the advantage of taking the triple point of water as the fixed point for a temperature scale? [3]
24. A balloon is ascending at the rate of 9.8 ms^{-1} at a height of 39.2 m above the ground when a food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground? Take $g = 9.8 \text{ ms}^{-2}$. [3]
25. A body weighing 0.4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1.2 m, find the tension in the string, when body is [3]
- at the bottom of the circle,
 - at the top of the circle.
26. One kilogram molecule of a gas at 400k expands isothermally until its volume is doubled. Find the amount of work done and heat produced. [3]
27. A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2} . The crew and the passengers weigh 300 kg. Give the magnitude and direction of the [3]
- force on the floor by the crew and passengers,
 - action of the rotor of the helicopter on the surrounding air,
 - force on the helicopter due to the surrounding air.
28. If a 5 cm long capillary tube with 0.1 mm internal diameter opens at both ends and is slightly dipped in water [3]
- having surface tension 75 dyne cm^{-1} , state whether
- water will rise halfway in the capillary
 - water will rise up to the upper end of the capillary
 - water will overflow out of the upper end of the capillary.
- Explain your answer.

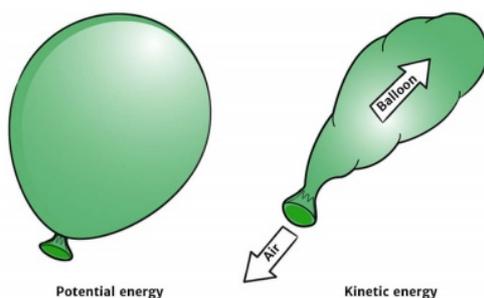
OR

The flow rate of water from a tap of diameter 1.25 cm is $0.48 \frac{\text{L}}{\text{min}}$. The coefficient of viscosity of water is 10^{-3} Pa s . After some time the flow rate is increased to 3 L/min. Characterise the flow for both the flow rates.

Section D

29. **Read the text carefully and answer the questions:** [4]
- Potential energy is the energy stored within an object, due to the object's position, arrangement or state. Potential energy is one of the two main forms of energy, along with kinetic energy. Potential energy depends on the force acting on the two objects.

Potential and Kinetic Energy



- (i) A body is falling freely under the action of gravity alone in a vacuum. Which of the following quantities

remain constant during the fall?

- a) mechanical energy
 - b) none of these
 - c) potential energy
 - d) kinetic energy
- (ii) Work done by a conservative force is positive, if
- a) potential energy decreases
 - b) kinetic energy increases
 - c) potential energy increases
 - d) kinetic energy decreases
- (iii) When does the potential energy of a spring increase?
- a) only when spring is compressed
 - b) none of these
 - c) both only when spring is stretched and compressed
 - d) only when spring is stretched

OR

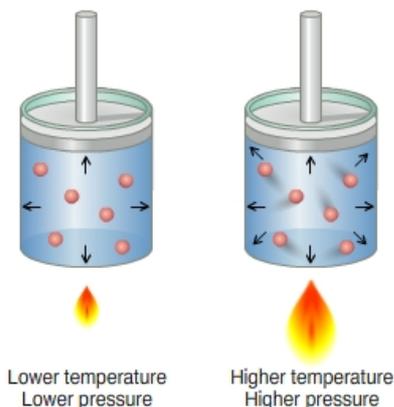
A vehicle of mass 5000 kg climbs up a hill of 10 m. The potential energy gained by it

- a) 5×10^4
 - b) 5×10^5 J
 - c) 500 J
 - d) 5 J
- (iv) Dimension of k/m is, here k is the force constant
- a) T^2
 - b) T^{-2}
 - c) T^{-1}
 - d) T^1

30. **Read the text carefully and answer the questions:**

[4]

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) If the temperature of the gas increases from 300 K to 600 K then the average kinetic energy becomes:
- a) same
 - b) becomes double
 - c) becomes half
 - d) none of these
- (ii) What is the average velocity of the molecules of an ideal gas?
- a) Infinite
 - b) Same
 - c) None of these
 - d) Zero
- (iii) Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas

molecules inside will _____.

- a) decrease
- b) none of these
- c) increase
- d) remains same

(iv) Find the ratio of average kinetic energy per molecule of Oxygen and Hydrogen:

- a) 1:1
- b) 4:1
- c) 1:2
- d) 1:4

OR

The velocities of the three molecules are $3v$, $4v$, and $5v$. calculate their root mean square velocity?

- a) $4.0 v$
- b) $4.02 v$
- c) $4.08 v$
- d) $4.04 v$

Section E

31. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion. [5]

OR

Which of the following functions of time represent

- i. simple harmonic motion,
- ii. periodic but not simple harmonic and
- iii. non-periodic motion? Find the period of each periodic motion. Here m is a positive real constant.

- a. $\sin \omega t + \cos \omega t$.
- b. $\sin \pi t + 2 \cos 2\pi t + 3 \sin 3\pi t$.
- c. $\cos(2\omega t + \frac{\pi}{3})$
- d. $\sin^2 \omega t$
- e. $\cos \omega t + 2 \sin^2 \omega t$

32. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 16 m high at a horizontal distance of 32 m ? Take $g = 10 \text{ ms}^{-2}$. [5]

OR

A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of the hill and can be moved on the ground at a speed of 2 m/s ; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10 \text{ m/s}^2$.

33. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. [5]

- a. What is the angular speed of the two-disc system?
- b. Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.

OR

- a. A cat is able to land on its feet after a fall. Why?

b. If angular momentum moment of inertia is decreased, will its rotational K.E. be also conserved? Explain.

Solution

Section A

- (c) Resistance

Explanation: [Resistance] = $[ML^2T^{-3}A^{-2}]$

\therefore Unit of resistance = $kg\ m^2A^{-2}s^{-3}$
- (a) there is no transfer of heat

Explanation: There is no transfer of heat from compression to rarefaction as air is a bad conductor of heat. And time of compression/rarefaction is too small.
- (b) 5 : 7

Explanation: For a solid sphere, $k^2 = \frac{2R^2}{5}$

$$a_{\text{slipping}} = g \sin \theta$$

$$a_{\text{rolling}} = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}} = \frac{g \sin \theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin \theta$$

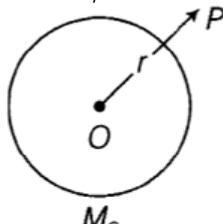
$$\therefore \frac{a_{\text{rolling}}}{a_{\text{slipping}}} = \frac{5}{7}$$

= 5 : 7
- (d) $6.92 \times 10^5 \frac{N}{m^2}$

Explanation: $p = \frac{f}{a} = \frac{F}{A} = \frac{3000 \times 9.8}{4.25 \times 10^{-2}}$

= $6.92 \times 10^5 \frac{N}{m^2}$
- (d) $\frac{-GM_e}{r}$

Explanation: The gravitational potential at a distance r from the centre of the earth.

$$v(r) = \frac{-GM_e}{r}$$

- (b) 6

Explanation: Fundamental frequency is $\nu = \frac{v}{4l} = \frac{340\text{ ms}^{-1}}{4 \times 0.85\text{ m}} = 100\text{ Hz}$

Only odd harmonics are present in the pipe closed at one end. So the possible harmonics less than 12500 Hz are 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz

These are six in number.
- (c) $\frac{2}{n^2} - \frac{1}{n^2}$

Explanation: $S_{\text{nth}} = 0 + \frac{a}{2}(2n - 1)$

$$S_n = 0 + \frac{1}{2}an^2$$

$$\therefore \frac{S_{\text{eth}}}{S_n} = \frac{2^a(2n - 1)}{\frac{1}{2}an^2} = \frac{2}{n} - \frac{1}{n^2}$$

8.

(c) $\frac{2\pi}{3}$

Explanation: After 2 seconds, the two pulses will nullify each other. As the string now becomes straight, there will be no deformation in the string. In such a situation, the string will not have any potential energy.

9.

(d) 28 N

Explanation: Force required to separate the plates,

$$F = \frac{2TA}{x} = \frac{2 \times 70 \times 10^{-3} \times 10^{-2}}{0.05 \times 10^{-3}} \text{ N} = 28 \text{ N}$$

10.

(c) $\frac{-3Gm^2}{L}$

Explanation: $\frac{-3Gm^2}{L}$

11.

(d) $t = 1 \text{ s}$

Explanation: $\theta(t) = 2t^3 - 6t^2$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\alpha = \frac{d\omega}{dt} = 12t - 12 = 0$$

$$\Rightarrow t = 1 \text{ s}$$

12.

(a) $\frac{2\lambda}{3}$

Explanation: By Wien's law,

$$\lambda'_m T' = \lambda_m T$$

$$\therefore \lambda'_m = \frac{\lambda T}{T'}$$

$$= \frac{\lambda \times 2000}{3000} = \frac{2\lambda}{3}$$

13.

(d) A is false but R is true.

Explanation: In case of the uniform circular motion, the tangential force is zero, only force is the centripetal force. Since the velocity of the body along the direction of the centripetal force is zero, so the power developed by the centripetal force is zero or in term of work done which is zero in circular motion (because displacement is zero) thus the power which is work done per unit time is also zero.

In the case of the non-uniform circular motion, the body has velocity in the direction of the tangential force. That is why, we say that this force develops power. In uniform motion $\alpha = 0$, $\tau = 0$. No work is done.

14. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: Assertion and reason both are correct statements and reason is correct explanation for assertion.

15. (a) Both A and R are true and R is the correct explanation of A.

Explanation: The time period of satellite, $T \propto r^{3/2}$ or, $T \propto (R_e + h)^{3/2}$

For a satellite revolving close to surface of earth, $h \approx 0$

$\therefore T \propto R_c^{3/2}$. It is evident that the period of revolution of a satellite depends upon its height above the earth's surface.

16. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

17. The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y_1 = y_1 + y_2 + y_3 + \dots + y_n$$

18. Let $x = K\eta^a [E_k]^b$, then

$$L^1 = [ML^{-1} T^{-2}]^a [ML^2 T^{-2}]^b$$

$$= M^{a+b} L^{-a+2b} T^{-2a-2b}$$

$$\therefore a + b = 0, -a + 2b = 1, -2a - 2b = 0$$

On solving, $a = -\frac{1}{3}$, $b = \frac{1}{3}$

$$\text{Hence } x = K\eta^{-1/3}E_k^{1/3} = K\left[\frac{E_k}{\eta}\right]^{1/3}$$

19. Let $v = K g^p h^q$,

where $K =$ a dimensionless constant. Putting the dimensions of various quantities, we get

$$LT^{-1} = [LT^{-2}]^p [L]^q$$

$$\text{or } L^1T^{-1} = L^{p+q}T^{-2p}$$

Equating the powers of L and T on both sides, we get:

$$p + q = 1, -2p = -1$$

$$\text{On solving, } p = \frac{1}{2}, q = \frac{1}{2}$$

20. As there is no external force in the horizontal direction, the momentum is changed in the vertical direction only by the gravitational force in time 0 to $2t_0$.

Change in momentum = external force \times time interval

$$\therefore \left| \left(m_1 \vec{v}'_1 + m_2 \vec{v}'_2 \right) - \left(m_1 \vec{v}_1 + m_2 \vec{v}_2 \right) \right|$$

$$= (m_1 + m_2)g \times (2t_0 - 0) = 2(m_1 + m_2)gt_0.$$

21. The mass of planet Jupiter is $(m_2) = 1.9 \times 10^{27}$ kg

The mass of sun is 1.99×10^{30} kg.

The mean distance of Jupiter from the Sun is $(r) = 7.8 \times 10^{11}$ m

$$F = \frac{GMm_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})^2}$$

$$F = 4.1 \times 10^{23} \text{ N}$$

$$\therefore F = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2} \times \frac{r}{m}}$$

$$v = 1.3 \times 10^4 \text{ ms}^{-1}$$

OR

Here $m = 0.25$ kg, $M = 5.983 \times 10^{24}$ kg, $R = 6.378 \times 10^6$ m

Force of gravitation between earth and apple,

$$F = \frac{GMm}{R^2}$$

Acceleration of the apple towards the earth,

$$a = \frac{F}{m} = \frac{GM}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.983 \times 10^{24}}{(6.378 \times 10^6)^2} = 9.810 \text{ ms}^{-2}$$

Acceleration of the earth towards the apple,

$$a' = \frac{F}{m} = \frac{Gm}{R^2} = \frac{6.67 \times 10^{-11} \times 0.25}{(6.378 \times 10^6)^2}$$

$$= 4.099 \times 10^{-25} \text{ ms}^{-2}$$

Section C

22. The physical significance of Reynold's number. Consider a narrow tube having a cross-sectional area A Suppose a fluid flows through it with a velocity v for a time interval Δt .

Length of the fluid = Velocity \times time = $v \Delta t$

The volume of the fluid flowing through the tube in time $\Delta t = Av \Delta t$

Mass of the fluid,

$$\Delta m = \text{Volume} \times \text{density} = Av \Delta t \times \rho$$

The inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A}$$

$$= \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av \Delta t \rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta \frac{v}{D}$$

$$\frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta v/D} = \frac{\rho v D}{\eta} = R_e$$

Thus Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

23. **The triple point of water:** The triple point of water is the state at which the three phases of water namely ice, liquid water, and water vapor are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column. Thus for water.

$$P_{tr} = 0.46 \text{ cm of Hg}$$

$$T_{tr} = 273.16 \text{ K or } 0.01^\circ\text{C}$$

In modern thermometry, the triple point of water is chosen to be one of the fixed points. As it is characterized by a unique temperature and pressure, it is preferred over the conventional fixed points namely the melting point of ice and boiling point of water. The melting point of ice and the boiling point of water both change with pressure. Moreover, the presence of impurities changes their values. But the triple point of water is independent of external factors.

In the absolute Kelvin scale, the triple point of water is assigned the value 273.16 K. The absolute zero is taken as the other fixed point on this scale.

24. Initially, the food packet shares the upward velocity of the balloon, so,

$$u = 9.8 \text{ ms}^{-1}, g = -9.8 \text{ ms}^{-2}, s = -39.2 \text{ m}$$

Here s is taken negatively because it is in the opposite direction of the initial velocity.

Using, $s = ut + \frac{1}{2}gt^2$, we get

$$-39.2 = 9.8 t - \frac{1}{2} \times 9.8 t^2$$

$$\text{or } 4.9 t^2 - 9.8 t - 39.2 = 0 \text{ or } t^2 - 2t - 8 = 0$$

$$\text{or } (t - 4)(t + 2) = 0 \text{ or } t = 4 \text{ s or } -2 \text{ s}$$

As time is never negative, so $t = 4 \text{ s}$

The velocity with which the food packet reaches the ground is

$$v = u + gt = 9.8 - 9.8 \times 4 = -29.4 \text{ ms}^{-1}$$

A negative sign shows that the velocity is directed vertically downwards.

25. Here $m = 0.4 \text{ kg}$, $r = 1.2 \text{ m}$, $\nu = 2 \text{ rps}$

$$\text{Angular speed, } \omega = 2\pi\nu = 2\pi \times 2 = 4\pi \text{ rad s}^{-1}$$

- i. When body is at bottom of the circle. Let T_1 be tension in the string. Then

$$T_1 - mg = \text{Centripetal force} = mr\omega^2$$

$$\text{or } T_1 = m(r\omega^2 + g) = 0.4 [1.2 \times (4\pi)^2 + 9.8]$$

$$= 0.4 [1.2 \times 16 \times 9.87 + 9.8]$$

$$= 0.4 [189.5 + 9.8] = 79.32 \text{ N}$$

- ii. When the body is at the top of the circle. Let T_2 be the tension in the string. Then

$$T_2 + mg = \text{Centripetal force} = mr\omega^2$$

$$\text{or } T_2 = m(r\omega^2 - g) = 0.4 [1.2 \times (4\pi)^2 - 9.8]$$

$$= 0.4 [189.5 - 9.8] = 71.88 \text{ N}$$

26. Initial volume of gas, $V_1 = V$

$$\text{Final volume of gas, } V_2 = 2V$$

Initial temperature of gas $T =$ Final temperature of gas $= 400 \text{ K}$ (\therefore process is isothermal)

Universal gas constant, $R = 8.3 \text{ kJ/mole/K} = 8.3 \times 10^{-3} \text{ J/mole/K}$

$$\text{Work done during isothermal process} = w = 2.3026 RT \log_{10} \left(\frac{V_2}{V_1} \right)$$

$$W = 2.3026 \times 8.3 \times 10^{-3} \times 400 \times \log_{10} \left(\frac{2V}{V} \right)$$

$$W = 2.3026 \times 8.3 \times 10^{-3} \times 400 \times \log_{10} (2)$$

$$W = 2.3016 \text{ J}$$

If H is the heat produced then,

$$H = \frac{W}{J} = \frac{2.3016}{4.2} = 0.548 \text{ cal}$$

27. Mass of the helicopter, $m_h = 1000 \text{ kg}$, total mass of the system $= 1300 \text{ kg}$

i. The mass of crew plus passengers $m_1 = 300$ kg. When the helicopter is rising up then apparent weight is

$$W = m_1(g + a) = m_1(g + 15) = 300(10 + 15) = 7500 \text{ N}$$

Since the helicopter is accelerating vertically upward, the reaction force will also be directed upward. Therefore, as per the Newton's third law of motion the force acting on the floor of the helicopter is 7500 N acting downwards.

ii. The mass of the helicopter will also be included. Therefore $m_2 = 1300$ kg. The force acting on the surrounding air by the rotor = $(1300)(10 + 15) = 32500$ N acting downward.

iii. By Newton's Third Law of motion, the magnitude of the force acting on the helicopter due to the surrounding air is the same as in case (ii) above i.e., 32500 N. But the direction of the force will be opposite i.e., in the upward direction.

28. Radius, $r = \frac{0.1}{2} \text{ mm} = 0.05 \text{ mm} = 0.005 \text{ cm}$

Surface tension, $\sigma = 75 \text{ dyne cm}^{-1}$

density, $\rho = 1 \text{ gcm}^{-3}$; angle of contact, $\theta = 0^\circ$

Let h be the height to which water rises in the capillary tube. Then

$$h = \frac{2\sigma \cos \theta}{r\rho g} = \frac{2 \times 75 \times \cos 0^\circ}{0.005 \times 1 \times 981} = 30.58 \text{ cm}$$

Given the length of the capillary tube, $h' = 5 \text{ cm}$

i. As $h > \frac{h'}{2}$, so the first possibility is ruled out.

ii. As the tube is of insufficient length, the water will rise upto the upper end of the tube.

iii. The water will not overflow out of the upper end of the capillary. It will rise only up to the upper end of the capillary. The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \quad [\because hR = \frac{2\sigma}{\rho g} = \text{constant}]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore R' = \frac{Rh}{h'} = \frac{rh}{h'} = \frac{0.005 \times 30.58}{5} = 0.0306 \text{ cm} \quad [\because R = \frac{r}{\cos \theta} = \frac{r}{\cos 0^\circ} = r]$$

OR

Let the speed of the flow be v .

Given, diameter of tap = $d = 1.25 \text{ cm}$

Volume of water flowing out per second.

$$Q = v \times \frac{\pi d^2}{4} \Rightarrow v = \frac{4Q}{d^2 \pi}$$

Estimate Reynold's number, $R_e = \frac{4\rho Q}{\pi d \eta}$

$$Q = 0.48 \frac{\text{L}}{\text{min}} = 8 \times 10^{-3} \frac{\text{L}}{\text{s}} = 8 \times 10^{-6} \frac{\text{m}^3}{\text{s}}$$

$$R_e = \frac{4 \times 10^3 \times 8 \times 10^{-6}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}}$$

$R_e = 815$ [i.e. below 1000, the flow is steady] After some time, when

$$Q = 3 \frac{\text{L}}{\text{min}} = 5 \times 10^5 \frac{\text{m}^3}{\text{s}}$$

$$R_e = \frac{4 \times 10^3 \times 5 \times 10^{-5}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 5095$$

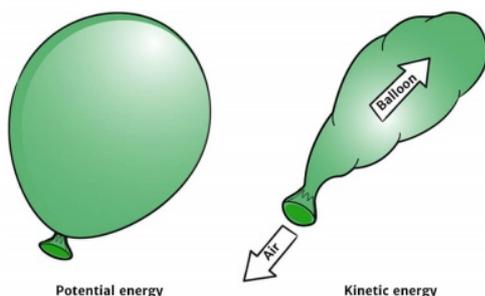
\therefore The flow will be turbulent.

Section D

29. Read the text carefully and answer the questions:

Potential energy is the energy stored within an object, due to the object's position, arrangement or state. Potential energy is one of the two main forms of energy, along with kinetic energy. Potential energy depends on the force acting on the two objects.

Potential and Kinetic Energy



(i) (a) mechanical energy

Explanation: mechanical energy

- (ii) (a) potential energy decreases

Explanation: potential energy decreases

- (iii) (c) both only when spring is stretched and compressed

Explanation: both only when spring is stretched and compressed

OR

- (b) 5×10^5 J

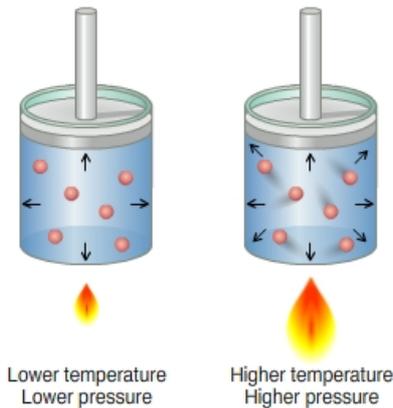
Explanation: 5×10^5 J

- (iv) (b) T^{-2}

Explanation: T^{-2}

30. Read the text carefully and answer the questions:

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) (b) becomes double

Explanation: becomes double

- (ii) (d) Zero

Explanation: Zero

- (iii) (d) remains same

Explanation: remains same

- (iv) (a) 1:1

Explanation: 1:1

OR

- (c) 4.08 v

Explanation: 4.08 v

Section E

31. For calculation of this problem we can proceed in following manner-

Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

$F = -(\text{Volume} \times \text{Density} \times g)$

$F = -(A \times 2h \times \rho \times g) = -2\rho gh = -k \times \text{Displacement in one of the arms (h)}$

Where, $2h$ is the height of the mercury column in the two arms k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where, m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, $m = \text{Volume of mercury} \times \text{Density of mercury}$

$= Al\rho$

$$\therefore T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

Hence, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\frac{l}{2g}}$.

OR

a. $\sin \omega t + \cos \omega t = \sqrt{2} \sin(\omega t + \frac{\pi}{4})$, $T = \frac{2\pi}{\omega}$

b. Each term represents S.H.M.

Period of $\sin \pi t$, $T = \frac{2\pi}{\pi} = 2s$

Period of $2 \cos 2\pi t = \frac{2\pi}{2\pi} = 1s = \frac{T}{2}$

Period of $2 \cos 3\pi t = \frac{2\pi}{2\pi} = \frac{2}{3}s = \frac{T}{3}$

The sum is not simple harmonic but periodic with

$T = 2s$.

c. $\cos(2\omega t + \frac{\pi}{3})$ represents S.H.M. with

$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

d. $\sin^2 \omega t = \frac{1}{2} - (\frac{1}{2}) \cos 2\omega t$

The function does not represent S.H.M. but is periodic with $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$.

e. $\cos \omega t + 2 \sin^2 \omega t = \cos \omega t + 1 - \cos 2\omega t$

$= 1 + \cos \omega t - \cos 2\omega t$

$\cos \omega t$ represents S.H.M. with $T = \frac{2\pi}{\omega}$

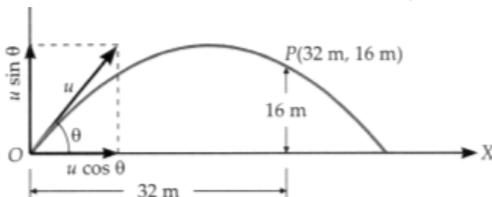
$\cos 2\omega t$ represents S.H.M. with period

$= \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{T}{2}$

The combined function does not represent S.H.M. but is periodic with $T = \frac{2\pi}{\omega}$.

32. As shown in figure, if point of projection is taken as the origin of the coordinate system, the projected body must pass through a point having coordinates (32 m, 16 m). If u be the initial velocity of the projectile and θ the angle of projection, then
Horizontal component of initial velocity, $u_x = u \cos \theta$

Vertical component of initial velocity, $u_y = u \sin \theta$



If the body passes through point P after time t , then horizontal distance covered,

$x = (u \cos \theta)t$

or $32 = (24 \cos \theta)t$ (i)

Similarly, vertical distance covered,

$y = (u \sin \theta)t - \frac{1}{2}gt^2$

or $16 = (24 \sin \theta)t - \frac{1}{2} \times 10 \times t^2$

From equation (i), $t = \frac{32}{24 \cos \theta}$

Putting this value of t in equation (ii), we get

$16 = (24 \sin \theta) \frac{32}{24 \cos \theta} - \frac{1}{2} \times 10 \times \left(\frac{32}{24 \cos \theta}\right)^2$

or $16 = 32 \tan \theta - 5 \times \frac{16}{9 \cos^2 \theta}$

or $1 = 2 \tan \theta - \frac{5}{9} \sec^2 \theta$

or $9 = 18 \tan \theta - 5(1 + \tan^2 \theta)$

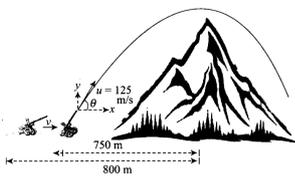
or $5 \tan^2 \theta - 18 \tan \theta + 14 = 0$

$\therefore \tan \theta = \frac{18 \pm \sqrt{(18)^2 - 4 \times 5 \times 14}}{10}$

$= 2.462$ or 1.37

Hence $\theta = 67^\circ 54'$ or $48^\circ 40'$

OR



Speed of packets, $u = 125 \text{ m/s}$

Height of hill, $h = 500 \text{ m}$

For a packet to cross the hill the vertical components of the speed of packet (125 ms^{-1}) must be minimized so that it can attain a height of 500 m and the distance between hill and cannon must be half the range of packet.

$$v^2 = u^2 + 2gh$$

$$0 = u_y^2 - 2gh$$

$$u_y = \sqrt{2gh} = \sqrt{2 \times 10 \times 500} = \sqrt{10000}$$

$$u_y = 100 \text{ m/s}$$

$$u^2 = u_x^2 + u_y^2$$

$$(125)^2 = u_x^2 + 100^2 \Rightarrow u_x^2 = 125^2 - 100^2$$

$$u_x^2 = (125 - 100)(125 + 100) = 25 \times 225$$

$$u_x = 5 \times 15 \Rightarrow [u_x = 75 \text{ m/s}]$$

Vertical motion of packet

$$v_y = u_y + gt$$

$$0 = 100 - 10t$$

Total time to reach the top of hill, $t = 10 \text{ s}$

\therefore Total time of flight = $2 \times 10 = 20 \text{ s}$

So the canon must be at $= u_x \times 10 = 75 \times 10 \text{ m} = 750 \text{ m}$

Hence, the distance between hill and canon = 750 m

So the distance to which canon must move toward the hill = $800 - 750 = 50 \text{ m}$

Time taken to move canon in 50 m = $\frac{\text{distance}}{\text{speed}} = \frac{50}{2} = 25 \text{ sec}$

Hence, the total time taken by packet from 800 m away from hill to reach other side,

$$T_{\text{total}} = 25 \text{ s} + 10 \text{ s} + 10 \text{ s} = 45 \text{ Seconds.}$$

33. 1. **Conservation of angular momentum** is a physical property of a spinning system such that its spin remains constant unless it is acted upon by an external torque; put another way, the speed of rotation is constant as long as net torque is zero.

Moment of inertia of disc I = I_1

Angular speed of disc I = ω_1

Moment of inertia of disc II = I_2

Angular speed of disc II = ω_2

Angular momentum of disc I, $L_1 = I_1\omega_1$

Angular momentum of disc II, $L_2 = I_2\omega_2$

Total initial angular momentum, $L_i = I_1\omega_1 + I_2\omega_2$

When the two discs are joined together, their moments of inertia get added up.

Moment of inertia of the system of two discs, $I = I_1 + I_2$

Let ω be the angular speed of the system.

Total final angular momentum, $L_f = (I_1 + I_2)\omega$

Using the law of conservation of angular momentum, we have:

$$L_i = L_f$$

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\therefore \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

2. **Mechanical energy** is the sum of the potential and kinetic energies in a system. The principle of the conservation of mechanical energy states that the total mechanical energy in a system (i.e., the sum of the potential plus kinetic energies) remains constant as long as the only forces acting are conservative forces.

Kinetic energy of disc I, $E_1 = \frac{1}{2}I_1\omega_1^2$

Kinetic energy of disc II, $E_2 = \frac{1}{2}I_2\omega_2^2$

Total initial kinetic energy, $E_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$

When the discs are joined, their moments of inertia get added up.

Moment of inertia of the system, $I = I_1 + I_2$

Angular speed of the system = ω

Final kinetic energy E_f :

$$= \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2) \left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \right)^2 = \frac{1}{2} \frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$$

$\therefore E_i - E_f$

$$= \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2) - \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$$

$$= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2} \frac{I_1^2\omega_1^2}{(I_1 + I_2)} - \frac{1}{2} \frac{I_2^2\omega_2^2}{(I_1 + I_2)} - \frac{1}{2} \frac{2I_1I_2\omega_1\omega_2}{(I_1 + I_2)}$$

$$= \frac{1}{2(I_1 + I_2)} \left[\frac{1}{2}I_1^2\omega_1^2 + \frac{1}{2}I_1I_2\omega_1^2 + \frac{1}{2}I_1I_2\omega_2^2 + \frac{1}{2}I_2^2\omega_2^2 - \frac{1}{2}I_1^2\omega_1^2 - \frac{1}{2}I_2^2\omega_2^2 - I_1I_2\omega_1\omega_2 \right]$$

$$= \frac{I_1I_2}{2(I_1 + I_2)} [\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2]$$

$$= \frac{I_1I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

All the quantities on RHS are positive.

$\therefore E_i - E_f > 0$

Thus, $E_f < E_i$

Hence, the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs.

The loss of KE can be attributed to the frictional force that comes into play when the two discs come in contact with each other.

OR

a. When cat lands on the ground, it stretches its tail as a result its moment of inertia increases

As $I\omega = \text{constant}$ (In the absence of external torque, angular momentum of the system remains constant)

\therefore Angular speed will be small due to increase in moment of inertia and the cat is able to land on its feet without any harm as it provides enough time for cat to land on the ground.

b. Let moment of inertia of a system decrease from I to I'

Then angular speed increase from ω to ω'

$\Rightarrow I\omega = I'\omega'$ ($\because I\omega = \text{constant}$ because in the absence of external torque angular momentum of the system remains conserved)

$$\omega' = \frac{I\omega}{I'}$$

Kinetic Energy of rotation of the system

$$KE = \frac{1}{2}I'\omega'^2$$

$$KE = \frac{1}{2}I' \left(\frac{I\omega}{I'} \right)^2$$

$$K.E = \frac{1}{2} \frac{I^2\omega^2}{I'}$$

As $I' < I$

\therefore Kinetic Energy of the system has increased which means it will not remain constant.