# 39. Alternating Current

## Short Answer

### 1. Question

What is the reactance of a capacitor connected to a constant DC source?

**Answer**

Infinite.

The reactance of a capacitor is given by $X_c = \frac{1}{\omega C}$... (i), where ω = angular frequency of oscillation of current, C = capacitance.

For a constant DC source, there are no oscillations in current and hence ω = 0.

Therefore, from (i), reactance $X_c$ becomes $Xc = \frac{1}{0 \times C}$ = infinite. (Ans)

### 2. Question

The voltage and current in a series AC circuit are given by

$V = V_0 \cos \omega t$ and $i = i_0 \sin \omega t$.

What is the power dissipated in the circuit?

**Answer**

Zero.

Given:

Voltage $V = V_0 \cos \omega t$

Current $i = i_0 \sin \omega t$.

Formula used:

The power dissipated in a series AC circuit is given by

$P = V_{rms} i_{rms} \cos\phi$... (i),

where $V_{rms}$ = root mean square value of voltage,

$i_{rms}$ = root mean square value of current,

ϕ = phase difference between voltage and current

Now, $i(current) = i_0 sin\omega t = i_0 cos(\omega t - \frac{\pi}{2})$... (ii), where $i_0$ = peak value of current, ω = angular frequency of oscillation, t = time

Since $V = V_0 \cos \omega t$ we can say that the phase difference between the voltage and the current is $\phi = \frac{\pi}{2}$

Substituting this value in (i), we get

$$P = V_{rms} i_{rms} \cos\left(\frac{\pi}{2}\right)$$

P = 0 (since $\cos\left(\frac{\pi}{2}\right) = 0$ )

Hence, the power dissipated in the given ac circuit is zero. (Ans)

**3. Question**

Two alternating currents are given by

$i_1 = i_0 sin\ \omega t$ and $i_2 = i_0\ sin\left(\omega t + \frac{\pi}{3}\right)$

Will the rms values of the currents be equal or different?

**Answer**

Equal.

The rms value of current is given by $i_{rms} = \frac{i}{\sqrt{2}}$, where $i_0$ = peak value of current.

Since $i_1$ and $i_2$ have the same peak value of current = $i_0$, their rms values will also be equal. (Ans)

**4. Question**

Can the peak voltage across the inductor be greater than the peak voltage of the source in an LCR circuit?

**Answer**

Yes.

Let the LCR circuit be connected across an AC supply of voltage $V = V_0 sin\omega t$ ... (i)

Now, the impedance in an AC circuit is given by

$$Z = \sqrt{r^2 + (X_L - X_c)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \text{ .... (ii)}$$

where R = resistance, $X_L$ = inductive reactance,

$X_c$ = capacitive reactance,

ω = angular frequency of oscillation of current,

L = inductance,

C = capacitance

Hence, the current in the circuit is given by $I = \frac{V}{Z}$ ... (iii),

where V = voltage,

Z = impedance

$\Rightarrow I = \frac{V_0 sin\omega t}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$... (iv)

Now, at resonance, $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ --------------(v)

Substituting (v) in (iv), we get

Current at resonance : $I_{res} = \frac{V_0 sin\omega t}{R}$... (vi)

Hence, at resonance, voltage across the inductor

$V_L = I_{res} \times X_L \Rightarrow V_L = \frac{V_0 sin\omega t}{R} \frac{sin\omega t}{R} x \times \omega L \frac{V}{R} x\, \omega L$ ... (vii)

where $I_{res}$ = current at resonance, $X_L$ = inductive reactance, V = source voltage = $V_0$sinωt, where $V_0$ = peak voltage, ω = angular frequency, t = time, R = resistance, L = inductance

Since ω = angular frequency is always greater than equal to 1,

Therefore, voltage across the inductor $V_L$ will be greater than the source voltage V if $\frac{L}{R} > 1$. (Ans)

**5. Question**

In a circuit containing a capacitor and an AC source, the current is zero at the instant the source voltage is maximum. Is it consistent with Ohm’s law?

**Answer**

No.

Ohm’s law states that the current flowing through a circuit is directly proportional to the potential difference applied across its ends.

Therefore, V = IR, where V = voltage, I = current, R = resistance.

Now, Ohm's law is valid only in purely resistive circuits (without inductors or capacitors), where there is a linear relationship between voltage and current.

However, Ohm's law is not valid in case of circuits with non-linear elements, for example those containing inductors or capacitors or a combination of both.

Hence, in the given circuit, Ohm's law is not consistent. (Ans).

**6. Question**

An AC source is connected to a capacitor. Will the rms current increase, decrease or remain constant if a dielectric slab is inserted into the capacitor?

**Answer**

Increase.

The reactance of a capacitor is given by $X_C = \frac{1}{\omega C}$ ... (i)

where ω = angular frequency of oscillation of current, C = capacitance.

Now, the capacitance of a parallel plate capacitor is given by

$C = \frac{K\epsilon_0 A}{d}$ (ii), where k = dielectric constant, A = area of plates,

$\epsilon_0$ = electric permittivity of vacuum, d = distance between plates.

For vacuum, the dielectric constant k = 1. Let the capacitance in vacuum be $C = C_0 = \frac{A\epsilon_0}{d}$... (iii) (from (ii))

For any other medium, k>1. Hence, capacitance of this slab is given by $C = \frac{K\epsilon_0 A}{d} = kC_0$ ... (iv), where k = dielectric constant, A = area of plates, $\epsilon_0$= electric permittivity of vacuum, d = distance between plates, $C_0$= capacitance in vacuum

Hence, the reactance of the capacitor in vacuum will be

$X_{C1} = \frac{1}{\omega C_0}$ ... (v), where ω = angular frequency, $C_0$ = capacitance in vacuum

And, the reactance of the capacitor in the dielectric slab will be $X_{C2} = \frac{1}{\omega C} = \frac{1}{\omega k C_0}$... (vi) (from (iv)), where k = dielectric constant of slab

Since the dielectric constant k is greater than 1, it becomes clear that $X_{C1} > X_{C2}$, where $X_{C1}$ = reactance of capacitor in vacuum, $X_{C2}$ = reactance of capacitor in dielectric slab

Now, the rms value of current is given by

$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_C}$... (vii), where $i_0$ = peak value of current, $V_0$ = peak value of voltage, $X_C$ = capacitive reactance

Let the rms value of current initially be $i_1$ and then be $_{i2}$ after insertion of dielectric slab.

Therefore, $i_1 = \frac{i_{01}}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_{C1}}$, where $i_{01}$ = peak value of current initially, $V_0$ = peak value of voltage, $X_{C1}$ = reactance of capacitor in vacuum

and $i_2 = \frac{i_{02}}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_{C2}}$, where $i_{02}$ = peak value of current after insertion of slab, $V_0$ = peak value of voltage, $X_{C1}$ = reactance of capacitor after insertion of slab

Since we found out that $X_{C1} > X_{C2}$, it is obvious that

$\frac{1}{X_{C1}} < \frac{1}{X_{C2}} \Rightarrow i_2 > i_1$.

Therefore, the rms current increases. (Ans)

**7. Question**

When the frequency of the AC source in an LCR circuit equals the resonant frequency, the reactance of the circuit is zero. Does it mean that there is no current through the inductor or the capacitor?

**Answer**

No, current will flow through both of them.

At resonance, we know that $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ ... (i), where $X_L$ = inductive reactance, $X_C$ = capacitive reactance, ω = angular frequency, L = inductance, C = capacitance

Current through an LCR circuit is given by $ii_0 = \frac{VV_0}{\sqrt{RR^2 + (\frac{1}{\omega C} - \omega L)^2}}$... (ii), where $i_0$ = peak value of current, $V_0$ = peak value of voltage, R = resistance, ω = angular frequency, L = inductance, C = capacitance

From (i) and (ii), at resonance, the peak value of current is given by $i_{res} = \frac{V_0}{R}$ ... (iii)

This current will flow through all circuit elements. However, since the inductive reactance and the capacitive reactance are equal, the potential difference across the inductor and capacitor will be equal and opposite and they will cancel each other out. (Ans)

**8. Question**

When an AC source is connected to a capacitor there is a steady-state current in the circuit. Does it mean that the charges jump from one plate to the other to complete the circuit?

**Answer**

No.

When an AC source is connected to a capacitor, there is a steady-state current in the circuit which transfers charges smoothly between the plates of the capacitor. This results in a potential difference across the plates of the capacitor. The direction of current is alternatively reversed every half-cycle and this is leads to alternating charging and discharging of capacitor.

**9. Question**

A current $i_1 = i_0 \sin \omega t$ passes through a resistor of resistance R. How much thermal energy is produced in one-time period? A current $i_2 = -i_0 \sin \omega t$ passes through the resistor. How much thermal energy is produced in one-time period? If $i_1$ and $i_2$ both pass through the resistor simultaneously, how much thermal energy is produced? Is the principle of superposition obeyed in this case?

**Answer**

Same thermal energy, principle of superposition is obeyed.

Given:

Current $i_1 = i_0 \sin \omega t$

Current $i_2 = -i_0 \sin \omega t$

Formula used:

The thermal energy produced in one time period due to a current i is given by $H = (i_{rms})^2 \times R \times \frac{2\pi}{\omega}$... (i), where $i_{rms}$ = rms value of current, R = resistance, ω = angular frequency of oscillation of current

Now, the rms current $i_{rms}$ in both cases is given by $\frac{i_0}{\sqrt{2}}$, where $i_0$ is the peak current.

Therefore, for current $i_1$, thermal energy produced is

$H_1 = \frac{{i_0}^2}{2} \times R \times \frac{2\pi}{\omega}$ and that produced for current $i_2$ is also $H_2 = \frac{{i_0}^2}{2} \times R \times \frac{2\pi}{\omega}$ where $i_0$ = peak value of current, R = resistance, ω = angular frequency of oscillation

Hence, the same thermal energy is produced due to both the currents individually. (Ans)

Since $i_1$ and $i_2$ have peak values $i_0$ and $-i_0$, they are equal and opposite in value. Hence, the net current through the resistor will be 0 when both pass through the resistor simultaneously. In this case, the thermal energy produced will be 0. (Ans)

Yes, the principle of superposition is obeyed in this case. (Ans)

**10. Question**

Is energy produced when a transformer steps up the voltage?

**Answer**

No.

When a transformer steps up the voltage, the voltage increases but the current decreases in the process since the supplied power remains constant.

Now, the power is given by $P = VI$

where V = voltage, I = current.

Since the voltage and current increase and decrease in proportion to each other, the value of power remains constant.

Hence, energy is not produced. Energy = power x time remains constant. (Ans)

**11. Question**

A transformer is designed to convert an AC voltage of 220 V to an AC voltage of 12 V. If the input terminals are connected to a DC voltage of 220 V, the transformer usually burns. Explain.

**Answer**

Ideally, we can consider a transformer to be a purely inductive circuit with inductance L.

Hence, the voltage in this case is given by $V = L\,\frac{di}{dt}$, where i = current, t = time, L = inductance.

⇒

Integrating on both sides, we get

$$\int di = \int \frac{\mathrm{V}}{\mathrm{L}} dt \Rightarrow i = \frac{Vt}{L}$$

For a DC source, the current across the inductor increases with time and after a certain amount of time, can reach a very large value. This burns the transformer. (Ans)

**12. Question**

Can you have an AC series circuit in which there is a phase difference of (a) 180° (b) 120° between the emf and the current?

**Answer**

(a) No. (b) No.

For an LCR circuit with angular frequency ω, the impedance is given by the formula $Z = \sqrt{RR^2 + (\omega L - \frac{1}{\omega C})^2}$

where R = resistance, L = inductance, C = capacitance.

Now, the phase difference between the current and the voltage is given by the formula $\tan \emptyset = \omega L - \frac{1}{\omega C}$, where ∅ = phase difference, ω = angular frequency, L = inductance, C = capacitance, R = resistance.

Since there are no restrictions on the values of L, C or R, it is obvious that tan ∅ can take any value between -∞ to +∞, that is ∅ can take any value between $-90^0$ to $+90^0$ (since tan $90^0$ = ∞)

Since both $120^0$ and $180^0$ fall beyond the permitted range of values, we cannot have an AC circuit with any of the given phase differences. (Ans)

**13. Question**

A resistance is connected to an AC source. If a capacitor is included in the series circuit, will the average power absorbed by the resistance increase or decrease? If an inductor of small inductance is also included in the series circuit, will the average power absorbed increase or decrease further?

**Answer**

Power after addition of capacitor will decrease. Power after addition of inductor will increase.

The impedance of the circuit after introduction of capacitor is given by

$Z_1 = \sqrt{R^2 + X_C{}^2}$... (i), where R = resistance, $X_C$ = capacitive reactance.

Now, average power is given by $P = i_{rms}{}^2 \times R$ ... (ii), where $i_{rms}$ = rms value of current, R = resistance

Since $i_{rms}$ decreases with increase in impedance, hence on the introduction of a capacitor, the average power absorbed by the resistance will also decrease. (Ans)

Now, the impedance of an LCR circuit is given by $Z_2 = \sqrt{RR^2 + (X_C - X_L)^2}$ ... (iii), where R = resistance, $X_L$ = inductive reactance, $X_C$ = capacitive reactance.

Hence, compared to $Z_1$, the value of $Z_2$ is less and the rms value of current is greater in this case. Therefore, if a small inductance is also introduced in the circuit, the average power absorbed increases. (Ans)

**14. Question**

Can a hot-wire ammeter be used to measure a direct current having a constant value? Do we have to change the graduations?

**Answer**

(i) Yes (ii) No

A hot wire ammeter only measures the root mean square(rms) value of alternating current. Hence, when it is used to measure a direct current, it will not show the fluctuating value of ac, but only show a constant current equal to the rms value of the current. Thus, it can be used to measure direct current having constant value. (Ans)

No, we do not need to change the graduations since the rms value of current is the same as the direct current.

## Objective I

**1. Question**

A capacitor acts as an infinite resistance for

A. DC

B. AC

C. DC as well as AC

D. neither AC nor DC

**Answer**

The resistance of a capacitor is given by $X_C = \frac{1}{\omega C}$, where ω = angular frequency of oscillation of current C = capacitance.

Now, in case of DC current, ω = 0.

Hence, the resistance for DC becomes $X_C = \frac{1}{0 \times C}$ = infinite. (Ans)

**2. Question**

An AC source producing emf

$\in = \in_0 [\cos(100\pi S^{-1})t + \cos(500\pi S^{-1})t]$

$\in = \in_0 [\cos(100\pi S^{-1})t + \cos(500\pi S^{-1})t]$ is connected in series with a capacitor and a resistor. The steady-state current in the circuit is found to be

$i = i_1 \cos[(100\pi S^{-1})t + \emptyset_1] + i_2 \cos[(500\pi S^{-1})t + \emptyset_2]$

A. $i_1 > i_2$

B. $i_1 = i_2$

C. $i_1 < i_2$

D. The information is insufficient to find the relation between $i_1$ and $i_2$.

**Answer**

Given:

Emf $\in = \in_0 [\cos(100\pi s^{-1})t + \cos(500\pi s^{-1})t]$

Steady state current $i = i_1 \cos[(100\pi S^{-1})t + \emptyset_1] + i_2 \cos[(500\pi S^{-1})t + \emptyset_2]$

Formula used:

Charge in steady state will be given by

$Q = C \in = C \in_0 [\cos(100\pi s^{-1})t + \cos(500\pi s^{-1})t]$ ... (i),

where C = capacitance, ε = emf, t = time

Hence, current is given by $i = \frac{dQ}{dt}$ ... (ii), where Q = charge, t = time

$\Rightarrow i\ i = -100C\varepsilon \in_0 \pi sin(100\pi ss^{-1})t - 500\pi sin(500\pi ss^{-1})t$, from (i)

Comparing this with $i = i_1 \cos[(100\pi S^{-1})t + \emptyset_1] + i_2 \cos[(500\pi S^{-1})t + \emptyset_2]$, we get $i_1 = 100\pi C\varepsilon_0$ and $i_2 = 500\pi C\varepsilon_0$.

Hence, we find that $i_1 < i_2$ (Ans).

**3. Question**

The peak voltage in a 220 V AC source is

A. 220 V

B. about 160 V

C. about 310 V

D. 440 V

**Answer**

Given:

Rms value of AC source voltage $V_{rms}$ = 220 V

Hence, peak value of voltage = $\sqrt{2}V_{rms} = (\sqrt{2} \times 220)$V which is around 310V. (Ans)

**4. Question**

An AC source is rated 220V, 50 Hz. The average voltage is calculated in a time interval of 0.01 s. It

A. must be zero B. many be zero C. is never zero D. is (200/√2) V

**Answer**

The frequency of the AC source is 50 Hz.

which means the time period of the wave is, 1/50 = 002 sec

Now the average voltage of the wave at the time interval 0.01 sec

If the interval lies between π/2 and 3π/2, the average voltage will be zero. In all the other case it will not be zero.

Hence, it may be zero. Thus B is the correct answer.

**5. Question**

The magnetic field energy in an inductor changes from maximum value of minimum value in 5.0 ms when connected to an AC source. The frequency of the source is

A. 20 Hz

B. 50 Hz

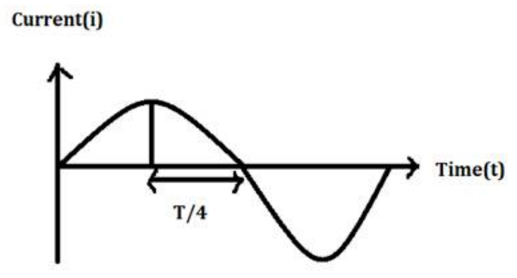
C. 200 Hz

D. 500 Hz

**Answer**

The magnetic field energy in an inductor is given by $E = \frac{Li^2}{2}$... (i), where L = inductor, i = current.

The graph of alternating current is given by:




From (i), we can see that the magnetic field energy reached its maximum and minimum value when the current is maximum and 0 respectively.

Also, from the graph we can see that the time taken by the current to change from its maximum to zero value is T/4, where T = time period.

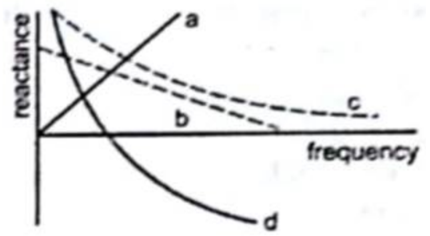
Now, from the given problem, $\frac{T}{4}$ = 5 ms = 0.005 s

Hence, time period T = (0.005 x 4) s = 0.02 s

Therefore, frequency of oscillation is equal to $\frac{1}{T} = \frac{1}{0.02} Hz$ = 50 Hz. (Ans)

**6. Question**

Which of the following plots may represent the reactance of a series LC combination?




**Answer**

The reactance of a series LC combination is given by

$$X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f c}$$

where $X_L$ = resistance of inductor, $X_C$ = resistance of capacitor, f = frequency, L = inductance, C = capacitance.

Also, we can see that when X = 0(point intersecting on the graph), $X_L = X_C$.

This is correctly represented in graph (d). (Ans)

**7. Question**

A series AC circuit has a resistance of 4 Ω and a reactance of 3 Ω. The impedance of the circuit is

A. 5Ω

B. 7Ω

C. 12/7Ω

D. 7/12 Ω

**Answer**

The impedance of an AC circuit is given by $Z = \sqrt{R^2 + X^2}$... (i), where R = resistance, X = impedance.

Given: Resistance R = 4 Ω and Impedance X = 3 Ω

Substituting these values in (i), we get

Impedance Z = $\sqrt{4^2 + 3^2}\Omega = \sqrt{16+9} = \sqrt{25}$ = 5 Ω (Ans)

**8. Question**

Transformers are used

A. in DC circuits only

B. in AC circuits only

C. in both DC and AC circuits

D. neither in DC nor in AC circuits.

**Answer**

Transformers only work in AC circuits where they step up or step down the voltage.

In a DC circuit, there is no change in flux with time across the coils of the conductor, since the current is constant. So, there will be no induced emf in the secondary coil due to the change in flux from changing current in primary coil. For this reason, DC circuits do not obey the principle of transformers.

**9. Question**

An alternating current is given by

i = $i_1$ cos ωt + $i_2$ sin ωt.

The rms current is given by

A. $\frac{i_1 + i_2}{\sqrt{2}}$

B. $\frac{|i_1 + i_2|}{\sqrt{2}}$

C. $\sqrt{\frac{i_1^2 + i_2^2}{2}}$

D. $\sqrt{\frac{i_1^2 + i_2^2}{\sqrt{2}}}$

**Answer**

The rms value of current is given by

$i_{rms}^2 = \frac{\int_0^T i_0^2 sin^2 \omega t\, dt}{\int_0^T i dt}$... (i), where

i = current = $i_1$ cos ωt + $i_2$ sin ωt .... (ii)(given), t = time, ω = angular frequency, T = time period.

Now, squaring on both sides of (ii), we get

$i^2 = i_1^2\cos^2\omega t + 2i_1i_2\sin\omega t\cos\omega t + i_2^2\sin^2\omega t$ ... (iii)

$$= \frac{\left[\int_0^T \frac{i_1^2}{2(\cos 2\omega t + 1)dt} + \int_0^T i_1 i_2 \sin 2\omega t\, dt + \int_0^T \frac{i_1^2}{2(1-\cos 2\omega t)dt}\right]}{T} =$$

$$\frac{\left(\left\{\frac{i_1^2}{2\left[\frac{\sin 2\omega t}{2\omega}+t\right]_0^T} - i_1 i_2\left[\frac{\cos 2\omega t}{2\omega}\right]\ [^T]_0 + \frac{i_2^2}{2\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T}\right\}\right)}{T}$$

ow, $\omega T = \omega \times \frac{2\pi}{\omega} = 2\pi$ ... (v), where ω = angular frequency, T = time period = 2π/ω

Therefore, (iv) becomes:

=>$\frac{i_1^2 + i_2^2}{2}$ [since sin nπ = 0, cos 0 = cos 2nπ = 1]

Hence, rms value of current i is $\sqrt{\frac{i_1^2 + i_2^2}{2}}$ (Ans)

**10. Question**

An alternating current peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is

A. 14 A

B. about 20 A

C. 7 A

D. about 10 A

**Answer**

To produce the same heating effect, the constant current required(i) will be the root mean square(rms) current(irms)

Hence, $i = i_{rms} = \frac{i_0}{2}$ where $i_0$ = peak current = 14 A

=>i = (14/√2) A = 9.899 A which is about 10 A. (Ans)

**11. Question**

A constant current of 2.8 A exists in a resistor. The rms current is

A. 2.8 A

B. about 2 A

C. 1.4 A

D. undefined for a direct current.

**Answer**

The rms current is equal to the value of the constant current.

Hence, the rms current is also 2.8 A. (Ans)

## Objective II

### 1. Question

An inductor, a resistor and a capacitor are joined in series with an AC source. As the frequency of the source is slightly increased from a very low value, the reactance

| A. of the inductor increases

B. of the resistor increases

C. of the capacitor increases

D. of the circuit increases

**Answer**

The reactance of an inductor is given by $X_L = 2\pi f L$, where f = frequency, L = inductance.

Hence, if the frequency is increased, the reactance of the inductor also increases. Therefore, option (A) is correct. (Ans)

The resistance remains unchanged with change in frequency. Hence, option (B) is incorrect.

The reactance of a capacitor is given by $X_C = \frac{1}{2\pi f C}$, where f = frequency, C = capacitance.

Hence, if the frequency increases, the reactance of the capacitor decreases. Hence, option (C) is incorrect.

Now, the reactance of the circuit is given by

$$X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f c}$$

Since on increasing the frequency, $X_L$ increases but $X_C$ decreases, their overall difference, that is reactance of the circuit, also increases. Hence option (D) is also correct. (Ans)

### 2. Question

The reactance of a circuit is zero. It is possible that the circuit contains.

A. an inductor and a capacitor

B. an inductor but no capacitor

C. a capacitor but no inductor

D. neither an inductor nor a capacitor.

**Answer**

The reactance of a circuit is given by $X = X_L - X_C$, where $X_L$ = reactance of inductor, $X_C$ = reactance of capacitor.

X can be 0 in two cases:

(i) When $X_L = X_C$, that is both inductor and capacitor are present. Hence option (A) is correct. (Ans)

(ii) When both $X_L$ and $X_C = 0$, that is, there is neither an inductor nor a capacitor. Hence, option (D) is correct. (Ans)

When either an inductor or a capacitor is present, either $X_L$ or $X_C$ are non-zero and hence the reactance cannot be 0. Therefore, options (B) and (C) are incorrect.

**3. Question**

In an AC series circuit, the instantaneous current is zero when the instantaneous voltage is maximum. Connected to the source may be a

A. pure inductor

B. pure capacitor

C. pure resistor

D. combination of an inductor and a capacitor.

**Answer**

In a pure inductive circuit, the voltage leads the current by a phase difference of $90^0$. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (A) is correct. (Ans)

In a pure capacitive circuit, the voltage lags behind the current by a phase difference of $90^0$. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (B) is correct. (Ans)

In case of a combination of an inductor and a capacitor also, the current may lead or lag behind the voltage by $90^0$, depending on whether the voltage across the inductor or capacitor is greater. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (D) is correct. (Ans)

Option (C) is incorrect because in a pure resistor circuit, the current and voltage are in phase with each other. Hence, when the voltage is maximum, the current is also maximum and vice versa.

**4. Question**

An inductor-coil having some resistance is connected to an AC source. Which of the following quantities have zero average value over a cycle?

A. Current

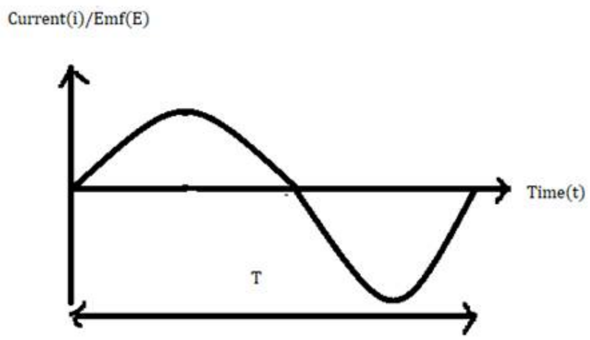
B. Induced emf in the inductor

C. Joule heat

D. Magnetic energy stored in the inductor.

**Answer**

For an inductor-coil circuit with some resistance, the current and the induced emf in the inductor are of the sinusoidal form. It is shown in the diagram below:




Here, T is the time period.

From the graph, we can see that the average value of current or induced emf over a cycle is 0.

Mathematically, we can see it in the following way:

Let the emf be of the form E = $E_0$sinωt, where $E_0$ = peak value of emf, ω = angular frequency, t = time.

Average emf over an entire cycle

$$E_{avg} = \frac{\int_0^T E dt}{\int_0^T dt} = \frac{\int_0^T E_0 sin\omega t\, dt}{T} = -\frac{E_0}{\omega T[cos\omega t]\,[^T{}_0} = -\frac{E_0}{\omega T[\cos 2\pi - \cos 0]}$$
$$= 0$$

Where T = time period

Similarly, we can also show that the average value of current over a full time period is also 0.

Hence, options (A) and (B) are correct.

Joule heat is given by H = $i_{rms}^2$ x R, where $i_{rms}$ = rms value of current, and R = resistance, which is non zero. Hence, option (C) is incorrect.

Magnetic energy stored in inductor is given by $E = \frac{Li_{rms}^2}{2}$, where L = inductance, $i_{rms}$ = rms value of current, which is non zero. Hence, option (D) is incorrect.

**5. Question**

The AC voltage across a resistance can be measured using

A. a potentiometer

B. a hot-wire voltmeter

C. a moving-coil galvanometer.

D. a moving-magnet galvanometer

**Answer**

Only a hot-wire voltmeter can be used to measure AC voltage across a resistance. Normal ammeters cannot be used for this purpose due to the changing value and direction of alternating current. All other devices can measure only the DC voltage. Hence only option (B) is correct. (Ans)

**6. Question**

To convert mechanical energy into electrical energy, one can use

A. DC dynamo

B. AC dynamo

C. motor

D. transformer.

**Answer**

Both DC and AC dynamo can be used to convert mechanical energy into electrical energy using wire coils rotating in a magnetic field. Hence, options (A) and (B) are correct. (Ans)

A motor is used to convert electrical energy to mechanical energy and not the other way around. Hence, option (C) is incorrect.

A transformer is used to step up or step down the voltage or to transfer electrical energy only. It cannot be used for transformation of energy from one type to another. Hence, option (D) is also incorrect.

**7. Question**

An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source.

A. must be 1000 W

B. may be 1000 W

C. may be greater than 1000 W

D. may be less than 1000 W.

**Answer**

Given:

Rms value of voltage($V_{rms}$) = 100 V

Rms value of current($i_{rms}$) = 10 A

Formula used:

The average power is given by $P = V_{rms} i_{rms} cos\Phi$, where $V_{rms}$ = rms value of voltage, $i_{rms}$ = rms value of current, $\emptyset$ = phase difference between current and voltage.

Substituting the given values, we get $P = 1000 cos\emptyset$.

Now, $cos\emptyset$ can have any value between 0 to 1. (since $\emptyset$can have any value between $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$)

Therefore, the range of values for P is:

$0 \leq P \leq 1000$ W.

Therefore, the power can be less than 1000 W, or may be equal to 1000 W. Hence, options (B) and (D) are correct. (Ans)

Options (A) and (C) are incorrect since P can have any value between 0 and 1000 W, and can never be greater than 1000 W.

## Exercises

### 1. Question

Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

**Answer**

I is current at any time 't',

$I_0$ is maximum value of the current in the circuit,

F is the frequency of the alternating current=

Current at any time is given as

$$I = I_0 \sin 2\pi ft$$

Since, $I_{rms} = \frac{I_0}{\sqrt{2}}$

hence , $\frac{I_0}{\sqrt{2}} = I_0 \sin 2\pi ft$

$$\frac{1}{\sqrt{2}} = \sin 2\pi ft$$

$$= 2\pi ft = \frac{\pi}{4}$$

$$= 2 \times 50 \times t = \frac{1}{4}$$

$$= t = \frac{1}{400}$$

$$t = 2.5 \times 10^{-3} sec$$

**2. Question**

The household supply of electricity is at 220 V (rms value) and 50 Hz. Find the peak voltage and the least possible time in which the voltage can change from the rms value to zero.

**Answer**

Given that $E_{rms} = 220V$

Frequency= 50 Hz

Then, Peak voltage $(E_0)$ is given as

$$E_0 = \sqrt{2} \times E_{rms}$$

$$E_0 = \sqrt{2} \times 220 = 1.414 \times 220$$

$$E_0 = 311.08V \approx 311V$$

Now, time taken for the current to reach the peak value= time taken to reach the zero value from rms

$$I = \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t$$

$$\omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f}$$

$$t = \frac{\pi}{8\pi \times 50} = \frac{1}{400} = 2.5 \times 10^{-3} \text{ sec.}$$

Where, f is the frequency, t is time taken and $\omega$ is angular velocity.

**3. Question**

A bulb rated 60W at 220V is connected across a household supply of alternating voltage of 220V. Calculate the maximum instantaneous current through the filament

**Answer**

Given that: Power (P) = 60W

Alternate Voltage (V) =220V

Now, power can be expressed as

$$P = \frac{V^2}{R}$$

Therefore, $R = \frac{V^2}{P} = \frac{220 \times 220}{60} = 806.67$

Then instantaneous voltage is

$$\varepsilon_0 = \sqrt{2} \times V = \sqrt{2} \times 220 = 311.08$$

Thus, the maximum instantaneous current through the filament

$$I_0 = \frac{\varepsilon_0}{R} = \frac{311.08}{806.67} = 0.385 = 0.39 \text{ A}$$

**4. Question**

An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and given normal brightness, what would be the peak voltage of the source?

**Answer**

Given that an electric bulb is designed to operate at voltage = 12V.

If the bulb is connected to an AC source and given normal brightness, then the peak voltage will be

$$E_0 = \sqrt{2} \times E$$

$$E_0 = \sqrt{2} \times 12$$

$= 1.414 \times 12 = 16.97$

$\text{Peak voltage} = 17\text{V}$

**5. Question**

The peak power consumed by a resistive coil when connected to an AC source is 80W. Find the energy consumed by the coil in 100 seconds which is many times larger than the time period of the source.

**Answer**

Given that: Peak power ($P_0$) = 80 W

Then, instantaneous power is

$$P_{rms} = \frac{P_0}{2} = 40\text{W}$$

The energy consumed by the coil in time t=100 seconds will be

= P × t = 40 × 100

= 4000J

Energy consumed = 4.0KJ

**6. Question**

The dielectric strength of air is 3.0 × $10^6$ V/m. A parallel-plate air-capacitor has area 20 $cm^2$ and plate separation 0.10 mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.

**Answer**

Given: Dielectric strength of air (E) = 3.0 × $10^6$ V/m,

Area (A) =20 $cm^2$ and separation width (d) = 0.10 mm.

Potential difference (V) across the capacitor is

$$V = E \times d = 3.0 \times 10^6 \times 1 \times 10^{-4}$$

$$V = 300V$$

The maximum rms voltage of an AC source which can be safely connected to this capacitor will be given as

$$V_{rms} = \frac{V}{\sqrt{2}}$$

$$V_{rms} = \frac{300}{\sqrt{2}} = 212V$$

**7. Question**

The current in a discharging LR circuit is given by $i = i_0 e^{-t/\tau}$ where $\tau$ is the time constant of the circuit. Calculate the rms current for the period t = 0 to t = $\tau$.

**Answer**

The current in a discharging LR circuit is given by

$I = I_0 e^{-t/T}$

Then, the rms current for the period t=0 to t= T can be obtained by:

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 e^{-2t/T}\,dt = \frac{I_0^2}{T}\int_0^T e^{-2t/T}\,dt$$

$$= \frac{I_0^2}{T} \times \left[\frac{T}{2} e^{-2t/T}\right]_0^T = \frac{I_0^2}{T} \times \frac{T}{2} \times [e^{-2T/T} - 1]$$

$$I_{rms}^2 = \frac{I_0^2}{T} \times \left(1 - \frac{1}{e^2}\right)$$

So the rms current is

$$I_{rms} = \frac{I_0}{e}\left(\sqrt{\frac{e^2 - 1}{2}}\right)$$

**8. Question**

A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage $\epsilon$ = (10V) sin ωt. Find the peak currents in the circuit for ω = 10 $s^{-1}$, 100 $s^{-1}$, 500 $s^{-1}$, 1000 $s^{-1}$.

**Answer**

Capacitance of the capacitor C=10 μF,

Output voltage of the oscillator $\epsilon$ = (10V) sin ωt.

On comparing the output voltage of the oscillator with

$\epsilon = \epsilon_0$

We get Peak voltage $\epsilon_0 = 10V$

For a capacitive circuit,

Reactance, $X_c = \frac{1}{\omega C}$

Here, $\omega$ is angular frequency,

C is capacitance of capacitor,

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

{a} At $\omega = 10\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/10 \times 10^{-5}}$$

$$I_0 = 10^{-3}\ A$$

{b} At $\omega = 100\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/100 \times 10^{-5}}$$

$$I_0 = 10^{-2}\ A$$

{c} At $\omega = 500\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/500 \times 10^{-5}}$$

$$I_0 = 5 \times 10^{-2}\ A = 0.05A$$

{d} At $\omega = 1000\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/_{\omega C}}$$

$$I_0 = \frac{10}{1/_{1000 \times 10^{-5}}}$$

$$I_0 = 10^{-1}\,A = 0.1A$$

**9. Question**

A coil of inductance 5.0 mH and negligible resistance is connected to the oscillator of the previous problem. Find the peak currents in the circuit for ω= 100 $s^{-1}$, 500 $s^{-1}$, 1000$s^{-1}$.

**Answer**

Given: Inductance of a coil= 5.0 mH and

Peak voltage $\epsilon_0 = 10V$

{a} At ω= 100 $s^{-1}$

Reactance of a coil is given by $X_L$

Then, $X_L = \omega L$

$$X_L = 0.005 \times 100$$

$$X_L = 0.5\ ohm$$

Here $\omega$ is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{0.5} = 20A$$

{b} At ω= 500 $s^{-1}$

Reactance of a coil is given by $X_L$

Then, $X_L = \omega L$

$$X_L = 0.005 \times 500$$

$X_L = 2.5\ ohm$

Here $\omega$ is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{2.5} = 4A$$

{c} At ω= 1000 $s^{-1}$

Reactance of a coil is given by $X_L$

Then, $X_L = \omega L$

$X_L = 0.005 \times 1000$

$X_L = 5\ ohm$

Here $\omega$ is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{5} = 2A$$

**10. Question**

A coil has a resistance of 10 Ω and an inductance of 0.4 Henry. It is connected to an AC source of 6.5 V, 30/π Hz. Find the average power consumed in the circuit.

**Answer**

Given: Resistance (R) = 10 Ω,

Inductance (L) = 0.4 Henry,

It is connected to an AC source (E) of 6.5 V and

Frequency (f) = 30/π Hz.

Then, Impedance (Z) of a coil is given by

$$Z = \sqrt{\{R^2 + X_L^2\}}$$

$Z = \sqrt{\{R^2 + (2\pi f L)^2\}}$

Hence, the rms current will be $I_{rms} = \frac{6.5}{Z}$

And $\cos \emptyset = \frac{R}{Z}$

Thus, the average power consumed in the circuit will be

$$Power = V_{rms} I_{rms} \cos \emptyset$$

$$= 6.5 \times \frac{6.5}{Z} \times \frac{R}{Z}$$

$$= \frac{(6.5)^2 \times R}{\left(\sqrt{\{R^2 + (2\pi f L)^2\}}\right)^2}$$

$$= \frac{6.5 \times 6.5 \times 10}{(10 \times 10) + \left(2\pi \times 0.4 \times \frac{30}{\pi}\right)^2}$$

$$= \frac{6.5 \times 65}{100 + 576} = \frac{422.5}{676}$$

$$= 0.625 \text{ W}$$

**11. Question**

A resistor of resistance 100 Ω is connected to an AC source ϵ = (12 V) sin (250 π $s^{-1}$) t. Find the energy dissipated as heat during t = 0 to t = 1.0 ms.

**Answer**

Given: Resistance (R) =100 Ω

Source emf ϵ = (12 V) sin (250 π $s^{-1}$) t

T=1.0 ms =$10^{-3}$ s.t

Then, energy dissipated (E) will be

$$E = \int_0^t \frac{\epsilon^2}{R}$$

$$E = \int_0^t \frac{[(12\ V) \sin(250\ \pi\ t)]^2\ dt}{R}$$

Since, $\sin^2 \theta = (1 - \cos^2 \theta)\frac{1}{2}$

$$E = \frac{144}{100}\int_0^t \frac{1 - cos\ 2 \times 250\pi t}{2}\ dt$$

$$E = \frac{144}{100}(1 - cos\ 500\pi t\ )dt$$

On integrating,

$$E = \frac{144}{100}\left[t - \frac{sin\ 500\pi t}{500\pi}\right]$$

$$At\ t = 10^{-3}s$$

$$E = \frac{144}{100}\left[10^{-3} - \frac{1}{500\pi}\right]$$

$$= \frac{144}{100}\left[\frac{1}{1000} - \frac{1}{500 \times 3.14}\right]$$

$$E = 2.61 \times 10^{-4} J$$

**12. Question**

In a series RC circuit with an AC source R = 300Ω, C = 25 μF, $\epsilon_0$ = 50 V and v = 50/π Hz. Find the peak current and the average power dissipated in the circuit.

**Answer**

Given: Resistance (R) = 300Ω,

Capacitance (C) = 25 μF =$25 \times 10^{-6}\ F$,

Rms voltage ($\epsilon_0$) = 50 V and

Frequency (v) = 50/π Hz.

Then, Reactance $(X_C)$ will be

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{2\pi v C}$$

$$= \frac{1}{2\pi \times 25 \times 10^{-6} \times \frac{50}{\pi}} = \frac{10^4}{25}$$

Now, Impedance (Z) will be

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{300^2 + \left[\frac{10^4}{25}\right]^2}$$

$$Z = \sqrt{300^2 + 400^2}$$

$$Z = 500$$

So, Peak current $(I_0)$ will be

$$I_0 = \frac{\epsilon_0}{Z}$$

$$I_0 = \frac{50}{500} = 0.1\,A$$

And average power dissipated will be

$$\text{Power} = \mathrm{V_{rms} I_{rms}} \cos \emptyset$$

$$= \frac{\mathrm{E_0}}{\sqrt{2}} \times \frac{\mathrm{E_0}}{\sqrt{2} \times \mathrm{Z}} \times \frac{\mathrm{R}}{\mathrm{Z}}$$

$$= \frac{\mathrm{E_0^2 \times R}}{2 \times \mathrm{Z^2}}$$

$$= \frac{50 \times 50 \times 300}{2 \times 500 \times 500} = \frac{3}{2} = 1.5\mathrm{W}$$

**13. Question**

An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage?

**Answer**

Given: Power (P) = 55W,

Bulb operated at voltage (V) = 110V,

Voltage supplied (E) = 220V

Resistance (R) will be

$$R = \frac{V^2}{P} = \frac{110 \times 110}{55} = 220\ ohm$$

Frequency (f) =50Hz,

Angular velocity $(\omega)$ will be

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi$$

Current (I) in the circuit will be

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (\omega L)^2}}$$

Voltage drop across the resistor will be

$$V = IR = \frac{ER}{\sqrt{R^2 + (\omega L)^2}}$$

$$110 = \frac{220 \times 220}{\sqrt{(220)^2 + (100\pi L)^2}}$$

$$220 \times 2 = \sqrt{(220)^2 + (100\pi L)^2}$$

$$(440)^2 = (220)^2 + (100\pi L)^2$$

$$48400 + 10^4(\pi L)^2 = 193600$$

$$10^4(\pi L)^2 = 193600 - 48400$$

$$L^2 = \frac{142500}{10^4\pi^2}$$

$$L^2 = 1.4726$$

$$L = \sqrt{1.4726} = 1.2135$$

$$L = 1.2\ Hz$$

Where, L is the inductance of the coil for which the bulb gets correct voltage.

**14. Question**

In a series LCR circuit with an AC source, R = 300Ω, C = 20 μF, L = 1.0 Henry, $\epsilon_{rms}$ = 50 V and v = 50/π Hz. Find

(a) The rms current in the circuit and

(b) The rms potential differences across the capacitor, the resistor and the inductor. Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.

**Answer**

Given: resistance R = 300Ω,

Capacitance of capacitor C = 20 μF,

Inductance of inductor L = 1.0 Henry,

Voltage across the circuit $\epsilon_{rms}$ = 50 V and

Frequency v = 50/π Hz

{a} rms current $(I_{rms})$ in the circuit will be

$$I_{rms} = \frac{\epsilon_{rms}}{Z}$$

Where Z is impedance in the circuit

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2\right\}}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1\right)^2\right\}}$$

$$Z = \sqrt{\left\{(300)^2 + \left(\frac{10^4}{20} - 100\right)^2\right\}}$$

$$Z = 500$$

Then, $I_0 = \frac{50}{500} = 0.1\ A$

{b} potential difference across the capacitor will be

$$V_c = I_0 \times X_c = 0.1 \times 500 = 50V$$

Potential difference across the resistor will be

$$V_R = I_0 \times R = 0.1 \times 300 = 30V$$

Potential difference across the inductor will be

$$V_L = I_0 \times X_L = 0.1 \times 100 = 10V$$

Net sum of all potential drops = $V_c + V_L + R = 50 + 10 + 30 = 90V$

Rms voltage, $\epsilon_{rms} = 50V$

Hence, sum of all potential drops > rms potential applied.

**15. Question**

Consider the situation of the previous problem. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

**Answer**

Given: resistance R = 300Ω,

Capacitance of capacitor C = 20 μF,

Inductance of inductor L = 1.0 Henry,

Voltage across the circuit $\epsilon_{rms}$ = 50 V and

Frequency v = 50/π Hz

Then the rms current $(I_{rms})$ across the circuit

$$I_{rms} = \frac{\epsilon_{rms}}{Z}$$

Where Z is impedance in the circuit

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2\right\}}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1\right)^2\right\}}$$

$$Z = \sqrt{\left\{(300)^2 + \left(\frac{10^4}{20} - 100\right)^2\right\}}$$

$$Z = 500$$

Then, $I_0 = \frac{50}{500} = 0.1\,A$

Electric energy $(E_c)$ stored in capacitor will be

$$E_C = \frac{1}{2} CV^2$$

$$E_C = \frac{1}{2} \times 20 \times 10^{-6} \times 50 \times 50$$

$$E_C = 25 \times 10^{-3} J = 25 mJ$$

Magnetic field energy $(E_M)$ stored in the coil will be

$$E_M = \frac{1}{2} L I_0^2$$

$$E_M = \frac{1}{2} \times 1 \times (0.1)^2 \times 5 \times 10^{-3} J$$

$$E_M = 5 mJ$$

**16. Question**

An inductance of 2.0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20 V with adjustable frequency.

(a) What frequency should be chosen to maximize the current in the circuit?

(b) What is the value of this maximum current?

**Answer**

{a} for current to be maximum in a circuit

$$X_L = X_c \ (resonant\ condition)$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC} = \frac{1}{2 \times 18 \times 10^{-6}} = \frac{10^6}{36}$$

$$\omega = \frac{10^3}{6} = 2\pi f$$

$$f = \frac{1000}{6 \times 2\pi}$$

$$f = 26.537\ Hz = 27\ Hz$$

Where, f is frequency, $\omega$ is angular velocity, L is inductance of inductor, C is capacitance of capacitor, $X_L$ is resonance across inductor and $X_c$ is resonance across capacitor.

{b} maximum current (I) will be

$$I = \frac{E}{R}$$

$I = \frac{20}{10\times10^3} = \frac{2}{10^3}$ A

$I = 2\ mA$

**17. Question**

An inductor-coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the source is varied, a maximum rms current of 6.0 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4.0Ω, what will be the current?

**Answer**

Given that rms voltage $E_{rms}$ =24V

Internal resistance, r= 4 ohm

Rms current $I_{rms}$ =6A

Then, rms resistance will be

$$R = \frac{E_{rms}}{I_{rms}}$$

$$R = \frac{24}{6} = 4\ ohm$$

If this inductor coil is connected to a battery of emf E= 12 V and internal resistance r' = 4.0Ω, then the steady current will be

$$I = \frac{E}{R'}$$

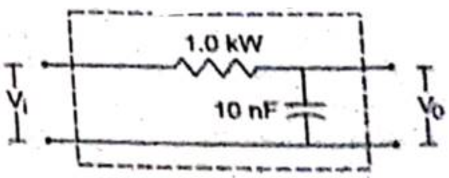
Since, net resistance R' = (R + r') = 4+4 =8 ohm.

Therefore, the steady current (I) is

$$I = \frac{12}{8} = 1.5\ A$$

**18. Question**

Figure shows a typical circuit for low-pass filter. An AC input $V_i$ = 10 mV is applied at the left end and the output V0 is received at the right end. Find the output voltages for v = 10 kHz, 100 k Hz, 1.0 MHz and 10.0 MHz Note that as the frequency is increased the output decreases and hence the name low-pass filter.



**Answer**

Given: Voltage $V_1$=10×$10^{-3}$V,

Resistance (R) =1×$10^3$ ohm,

Capacitance (C) =10×$10^{-9}$ F

Angular velocity $(\omega)$ will be

$\omega = 2\pi f$

{a} At frequency (f) =10kHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-4}} = \frac{10^4}{2\pi}$$

$$X_c = \frac{5000}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{5000}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}$$

Then, Current ($I_0$) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}}$$

Thus, Output voltage ($V_0$) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}} \times \frac{5000}{\pi}$$

$$V_0 = 16.124V = 16.1mV$$

{b} At frequency (f) =100kHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^5 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$$

$$X_c = \frac{500}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{500}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}$$

Then, Current ($I_0$) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}}$$

Thus, Output voltage ($V_0$) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}} \times \frac{500}{\pi}$$

$$V_0 = 1.6124V = 1.6mV$$

{c} At frequency (f) =1 MHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^6 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-2}} = \frac{10^2}{2\pi}$$

$$X_c = \frac{50}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{50}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}$$

Then, Current ($I_0$) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}}$$

Thus, Output voltage ($V_0$) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}} \times \frac{50}{\pi}$$

$$V_0 = 0.16124V = 0.16mV$$

{a} At frequency (f) =10 MHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^7 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10} = \frac{10}{2\pi}$$

$$X_c = \frac{5}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{5}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}$$

Then, Current ($I_0$) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}}$$

Thus, Output voltage ($V_0$) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}} \times \frac{5}{\pi}$$

$$V_0 = 0.016124V = 16\mu V$$

**19. Question**

A transformer has 50 turns in the primary and 100 in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary?

**Answer**

Given that a transformer has 50 turns in the primary and 100 in the secondary.

If the primary is connected to a 220 V DC supply, the voltage across the secondary is zero because a transformer does not work on DC.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and the transformer will not work.