☆ CONTROL SYSTEMS:

⇒ Books:

Gate ① Control System → NISE

IES ② Control system → Nagrath & Gopal.

③ Automatic Control System → B.C. Kuo.

IES ④ Control System : Principal & Design → M Gopal.

⑤ Modern CS → ogata.

* Topics:

→ TF, BD, SFG → 1 M (or) 2M

→ TDA → Transient Analysis, Steady State Analysis → 2M

→ Stability Techniques → Time Domain tech. ⇒ RH | RL. ; Freq. Domain tech. ⇒ BP | NP. → 4m

→ Compensator | controllers.

→ State Space Analysis. → 2M

☆ Introduction:

* Transfer, Block Diagram & SFG:

⇒ Transfer function is a Mathematical equivalent model for the system.

⇒ $$TF = \underbrace{\frac{1}{S+1}}$$

⇓

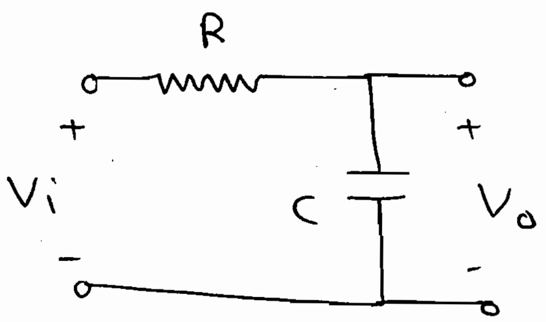
No. of Storage elements

(or)

No. of Time constant.

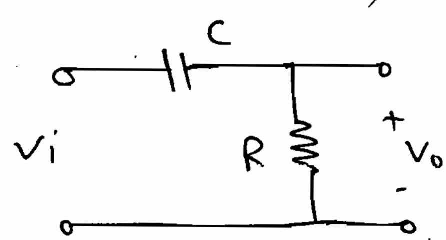
→ Objective of the CS : Set desired / Accurate output.

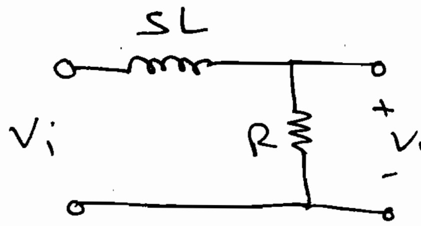
⇒



RC LPF ✓

RC ~~HPF~~





L-R LPF ✓

→ $$\frac{V_o(s)}{V_i(s)} = \frac{1/sc}{R + 1/sc}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{SCR+1}.$$

Let, $\tau = RC$.

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{S\tau+1} \Rightarrow \frac{C(s)}{R(s)} = \left(\frac{1}{S+1}\right).$$

$$\therefore \tau = \underbrace{R}_{1m}\underbrace{C}_{1\mu F} = 1$$

i.e $\frac{C(S)}{R(S)} = \frac{1}{S+1}$ for LPF.

⇒ **Why LPF?**

Noise → Eliminated → LPF

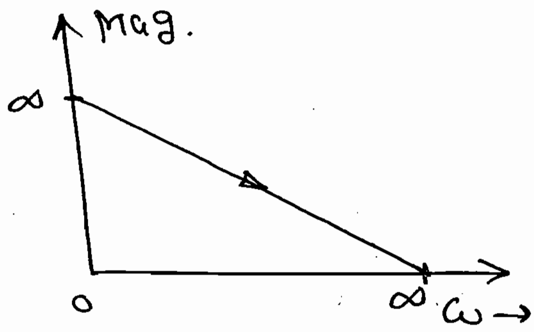
Noise ⇓ High frez. → Eliminated

① Noise get eliminated by LPF.

② Components are more stable at LF.

⇒ System : $= \dfrac{K(1+sT_1)(1+sT_2)\cdots}{s^n(1+sT_a)(1+sT_b)\cdots}$

⇒ Always selecting :



⊙ Poles > Zero ⇒ For LPF

⇒ Then only it is Strictly Proper TF

⊙ When Poles = zeros

⇒ it act as $\left\{ \begin{array}{c} LP \\ HP \\ BP \\ BS \end{array} \right\}$ → Proper TF.

→ zero at origin is not acceptable.

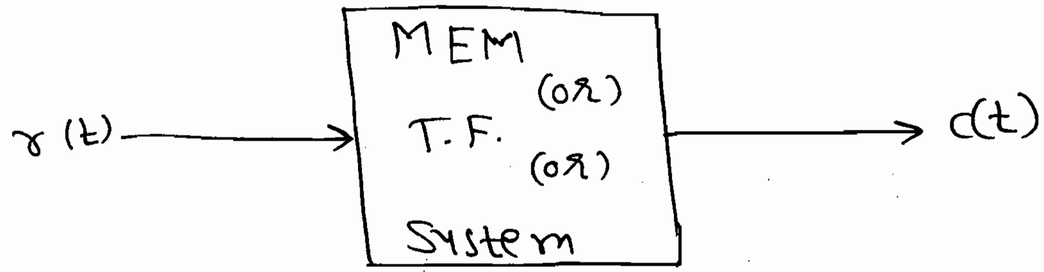
⊙ When Poles < zeros ⇒ Improper T.F. ⇒ HPF.

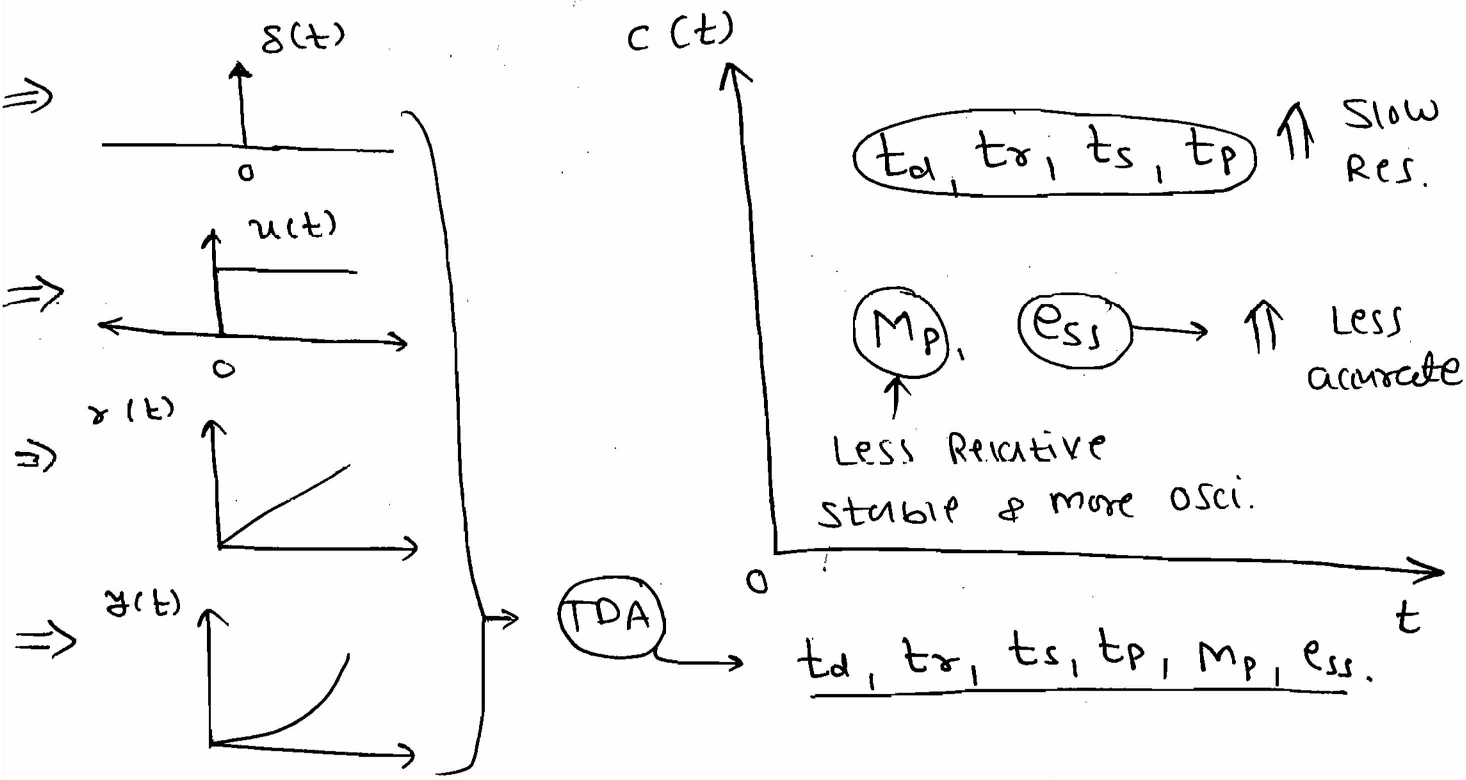
⇒ BD, SFG ⇒ To find the overall TF of the system.

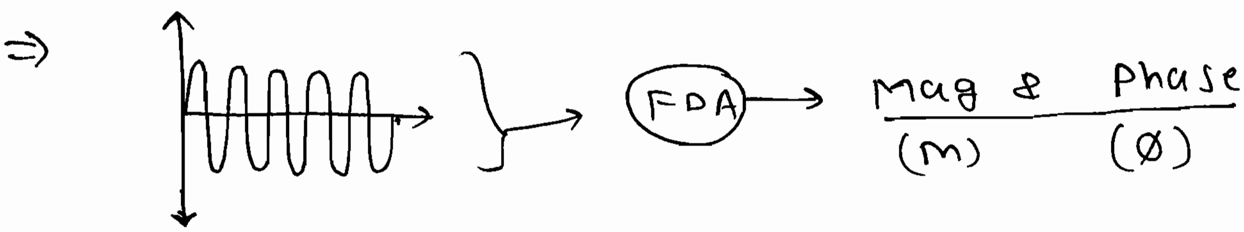
* Time Domain Analysis:

⇒ The objective of the TDA is used to evaluate the performance of the system w.r.t. time.

⇒







⇒ Freq. domain analysis is used to find Gm & Pm.

* Control System Specification:

⇒ Speed → t_r, t_s ↓↓ → Quick Res.

⇒ Accuracy → e_{ss} ↓ → Small → More Accurate.

⇒ Stability ⇒ GM & PM

↑↑ More R.S. (Adv.)
Slow Res. (dis.)

↓↓ Less R.S. (dis.adv.)
More oscillatory (dis.adv.)
↓
t_r ↓, t_s ↑

→ Optimum
Value of GM ⇒ 5 dB to 10 dB
PM ⇒ 30° to 40°.

→ TDA Should be insensitive w.r.t. to unwanted Parameters such as Temperature, Noise & Disturbance.

⇒ e_{ss} : Steady State error.

⇒ M_p : peak overshoot.

⇒ t_d : Delay time.

⇒ t_r : Rise time.

⇒ t_p : propogation time.

⇒ t_s : setting time.

* Stability (for Closed loop system).

OL system

→ Always defined in terms of OLTF.

↓

$G(s)$

(OLTF of a system)

Closed loop system

↓

$G(s)\,[H(s)=1]$

$=$ OLTF of a unity FB sys.

$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

↓

$G(s)\cdot H(s)$

$=$ OLTF of a Non-unity FB sys.

CLTF

$$= \frac{C(s)}{R(s)}$$

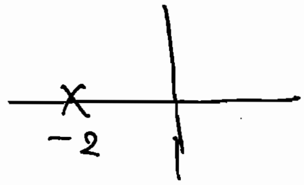
$$= \frac{G(s)}{1+G(s)\cdot H(s)}$$

$\boxed{Q}$ OLTF of a unity FB sys is

$G(s) = \dfrac{10}{s-8}$. Then system is ______ .

Ans:

$$CLTF = \frac{G}{1+G} = \frac{10}{s+2}$$

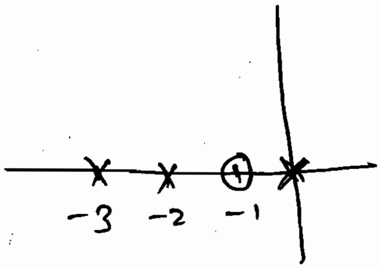


So, $\boxed{\text{Stable}}$.

* Why there is no need of stability technique for OL system?

$\Rightarrow$ OLTF of a system is,

$$G(s) = \frac{s+1}{s^2(s+2)(s+3)}.$$



→ Poles and zeros Location are identified directly from $G(s)$.

⇒ CL SYS.:

OLTF of a unity FB system is,

$$G(s) = \frac{s+1}{s^2(s+2)(s+3)}$$

$$CLTF = \frac{G}{1+G} = \frac{s+1}{s^4+5s^3+6s^2+s+1}$$

⇒ The Feedback changes the locations of the poles. Identification of new location of the poles are very difficult, Hence we need a stability technique for closed loop stability.

⇒ stability Technique:

Priority

1. Nyquist → No. of poles on RL, Range of K, Gm & pm can be find.
2. RL → Nature of the system.
3. BP → Gm & pm.
4. RH.

→ The T-D technique gives the transient Analysis and steady state (ss) Analysis.

⇒ The F-D technique gives the only steady state (ss) Analysis.

⇒ The stability Analysis is a steady state Analysis.

* Transportation Delay | Lag System.

$\Rightarrow \quad L[g(t-\tau)] = e^{-s\tau}\, G(s).$

$\uparrow$ delay

TDA:

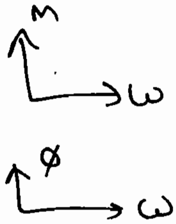
$$e^{-s\tau} \Rightarrow (1-s\tau) + \frac{(s\tau)^2}{2!} + \cdots + \infty .$$

neglected infinite poles.

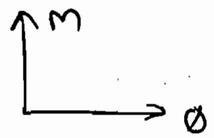
$\Rightarrow$ So, TDA Not gives accurate stability.

FDA:

BP $\rightarrow$ M
BP $\rightarrow$ ϕ



NP $\rightarrow$



$e^{-s\tau} = e^{-j\omega}$, $m = 1$. $\angle\phi = (-\omega\tau)$.

In FDA there is no any approximation.

* Compensators | Controllers:

$\Rightarrow$ It is required to get desired system.

It is a simple electrical N/W which adds the poles and the zeros to the system in order to get the disired performance of the system.

* Steady State Analysis:

$\Rightarrow$ It is only valid for non linear, linear, time variant & time invariant system. It is define for dynamic system.

* System: Ch-1: TF, BD & SFG

⇒ (i/p) → System → Proper (o/p).
May (or) May not be desired o/p.

⇒ A System is a group of elements (or) a physical components arranged in a such a way that it gives the proper output to the given input.

⇒ A Proper output is may (or) may not be the desired output.

⇒ for. e.g.:

→ A FAN w/o Blades ⇒ Not a system
⇒ No proper o/p ⇒ No air flow.

→ A FAN w/o Regulator. ⇒ System
⇒ Proper o/p ⇒ Air flow.

→ A FAN with Regulator ⇒ Controlled system
⇒ Gives Desired o/p.

* Control System:

i/p → Control system → Desired o/p

⇒ A Control System is a group of physical components arranged in a such a way that it gives the desired output by means of control (or) regulate (or) command either direct (or) indirect method to the given input.

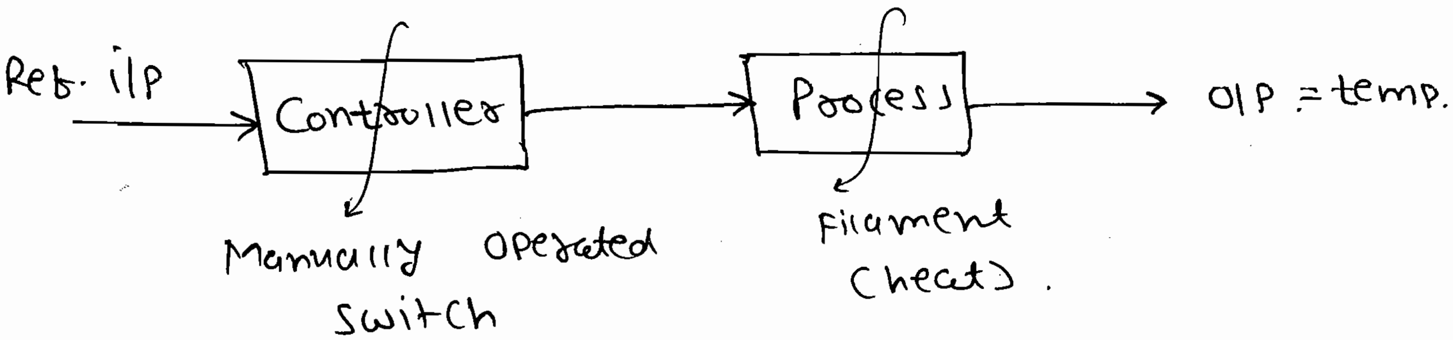
⇒ Control Systems are classified in two ways based on controlling action.

i) open loop control system (OLCS).

ii) Closed loop Control system (CLCS).

① Open Loop Control System : (manual):

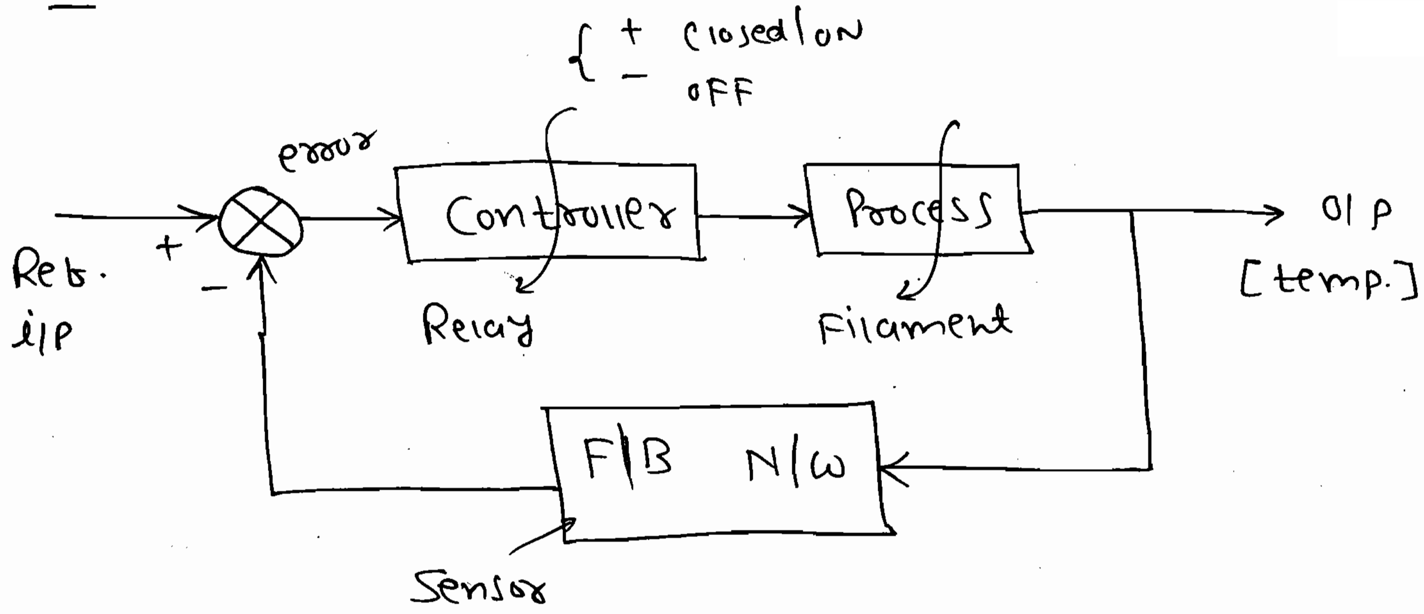
⇒



⇒ Ref. i/p is nothing but a desired o/p. [what we required or what we need].

⇒ A system in which the controller action is completely independent of the output of the system is called open loop control system. e.g. FAN, Lights, cooler, Traffic light.

(ii) Closed loop Control System: (Automatic):



⇒ A system in which the controller action depends on the o/p then it is called closed loop control system.

→ e.g. : AC, Refrigerator, Human beings, Automatic iron box and so on.

→ Any System which is having a sensor and provision to select the reference i/p.

* Feedback Network:

⇒ It is a property of the closed loop system which brings the o/p to i/p and compared with ref. i/p. so that appropriate control action formed to make the error equal to zero.

⇒ Error equal to zero, the system is stable which give the desired o/p.

⇒ The main components in F/B N/w is R, L, C. the maximum gain of F/B N/w ratio is '1'.

⇒ The Best FB is unity (-ve) FB. Because the (-ve) FB improves the relative stability. (loop gain > 0).

⇒ The steady state errors are valid for only unity FB system. If non-unity FB system is given it should be converted into unity FB.

⇒ The FB N/w may consist the transducer which converts the energy from one form to another form.

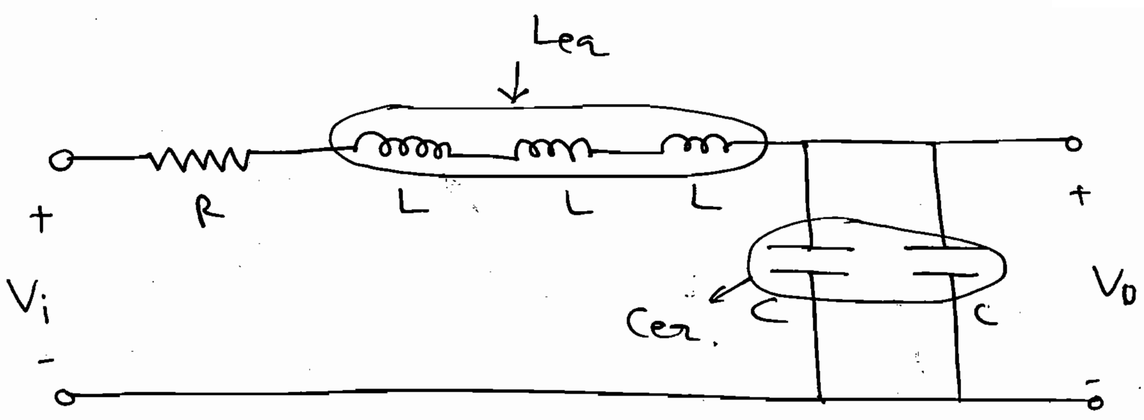
* Transfer Function:

⇒ The transfer function is basically mathamatical equavalent model for the system.

⇒ The order of the transfer f^n represents the no. of storage elements (or) no. of the time constants.

Note: whenever same kind of elements connected either series (or) parallel it should be treated as a single Components.

for e.g.:



2nd order.

⇒ First defination of T.F.:

⇒ A T.F. of a Linear Time invariant system is defined as the ratio of Laplace transform of output to the Laplace transform of input with all initial conditions are zero.

$$T.F. = \left.\frac{L[O/P]}{L[I/P]}\right|_{I_i = 0}$$

⇒ LTI system:

⇒ The LTI system is nothing but RLC ckt because the RLC components gives the Linear Transfer characteresticis and the R, L, C components values are not changes w.r.t. time.

⇒ In TF analysis the initial conditions must be zero because the output should not depends on the past history of the

System. It should depend on the Component values and present i/p.

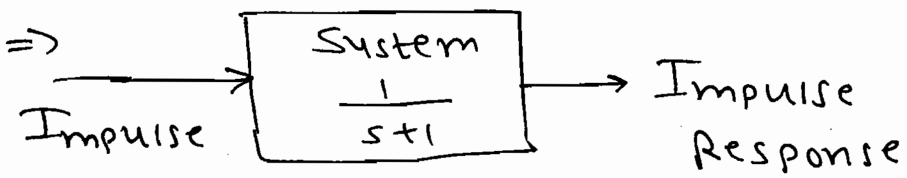
⇒ <u>Second</u> <u>Defⁿ</u> <u>of</u> <u>T.F.</u>:

⇒ The TF of the LTI System is defined as Laplace transform of Impulse Response with all initial conditions are zero.

i.e. $TF = \underbrace{L[\text{Impulse Response}]}_{\text{Sys Res} \mid \text{Natural Res} \mid \text{Free forced Res.}} \Big|_{I_i = 0.}$

$$\Rightarrow T.F. = \frac{L[O/P]}{L[i/P]} = \frac{L[\text{Impulse Res}]}{L[\text{Impulse}]} = \frac{L[I.R.]}{1}$$

$(\because L[\delta(t)] = 1].$

⇒ 

$\frac{C(s)}{R(s)} = \frac{1}{s+1}$

$R(s) = 1$

$\therefore C(s) = \underbrace{\frac{1}{s+1}}_{\text{system comp.}} \cdot 1.$

So, called sys. Res.

⇒ if we take $R(s) = \frac{1}{s}$. i.e. $r(t) = u(t)$.

$\therefore C(s) = \frac{1}{(s+1)} \cdot \frac{1}{s}.$

Response has input ~~cause~~ terms so it is not called sys. Response.

So, finding Sys. Res. we should take $R(s) = 1$.

⇒ The impulse Response gives the system behaviour (or) system characteristics because the Impulse Response consist only system Parameters. No i/p term presents in the impulse Response. Hence the impulse Response is called system Response / Natural Response (or) Free forced Response.

⇒ If the signals are unit step, ramp (or) Parabolic then their response is called Forced Response.

* Transfer Function to Electrical N/w:

⇒ Any system basically defined in terms of OLTF.

⇒ The standard form of the system is described as

Time constant form.

$$\boxed{G(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}}$$

⇒ K & τ are called system Parameters.

K: system Gain.

τ: system Time Constant.

n: Type-n system.

⇒ Type gives the no. of Poles at origin.

⇒ Order gives the total no. of poles ~~and~~ in s-plane.

Q Find the System gain, type and Order to the following system.

OL sys. $\dfrac{C(s)}{R(s)} = \dfrac{10\,(s+5)^2}{s^3\,(s+2)^2\,(s+10)}$ [Pole-zero form]*

Soln:

$$\frac{C(s)}{R(s)} = \frac{10 \times 25\,\left(1+\frac{s}{5}\right)^2}{4 \times 10 \times s^3\left(s/2+1\right)^2\left(1+s/10\right)}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{6.25\,\left(1+\frac{s}{5}\right)^2}{s^3\left(1+\frac{s}{2}\right)^2\left(1+\frac{s}{10}\right)}$$ [Time cons. form or standard form]

Type: → 3

Order: → 6

Gain: → 6.25

** ↓ H.B

$$\boxed{\text{Sys. gain } K = \frac{\text{Nr. Const}}{\text{Dr. const}}}$$

Q Find the type and order to the given CLTF of a unity feedback system.

$$\frac{C(s)}{R(s)} = \frac{2s+5}{s^5+4s^4+6s^3+7s^2+2s+5} = \frac{G(s)}{1+G(s)}$$

Soln: Here, $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)}$

$$\therefore \frac{G(s)}{1+G(s)} = \frac{2s+5}{s^5+4s^4+6s^3+7s^2+2s+5}$$

$$\therefore G(s)\left[s^5+4s^4+6s^3+7s^2+2s+5\right] = 2s+5 + G(s)\left[2s+5\right].$$

$$\therefore G(s) = \frac{2s+5}{s^5+4s^4+6s^3+7s^2}.$$

$$\therefore G(s) = \frac{2s+5}{s^2\left(s^3+4s^2+6s+7\right)}.$$

So, order $\longrightarrow$ 5.

Type $\longrightarrow$ 2.

NOTE: N.B

$\Rightarrow$ The Type & order is not defined for closed loop TF. To get a type and order for CL sys. require OLTF of unity feedback system.

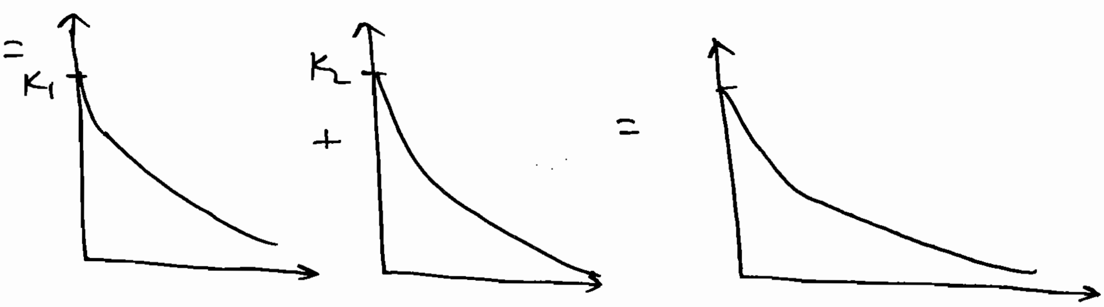
* Characterestics Equation:

$$\Rightarrow \frac{C(s)}{\cancel{R(s)}\ 1} = \frac{(s-10)}{(s+1)(s+5)}.$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+5)}.$$

$$\therefore \frac{C(s)}{R(s)} = K_1 e^{-t} + K_2 e^{-5t}.$$

C(s) =



⇒ Denominator terms decide the Chara. of the system not Numarator term. so for Chara. eqn we take denominator term equal to zero.

⇒ The Denominator of transfer function makes equal to zero then it is called Characterestic equation.

⇒ The chara. eqn gives the system behaviour (or) characterestics of the system.

→ For a CL system, the characterestics equation is $\boxed{1 + G(s) \cdot H(s) = 0}$.

⇒ The roots of Chara. eqn is called Poles.

* Pole:

⇒ The Pole is nothing but the negative of Inverse of system time constant at which magnitude of the TF becomes ∞

$$S_p = -\frac{1}{\tau_a}, -\frac{1}{\tau_b}, \ldots \quad |TF| = \infty.$$

* Zero:

⇒ The zero is nothing but the negative of the inverse of the system time const. at which magnitude of the TF become 0.

$$S_z = -\frac{1}{\tau_1}, -\frac{1}{\tau_2}, \ldots \qquad |TF| = 0.$$

H.B

⇒ The Pole can affect the system response and system stability. but not the zero.

* Time Constant:

⇒ The time constant gives the system behaviour. If the time constant is very very large then it is called slow response system. Because it takes the large time to reach the steady state.

→ Practically any ~~ste~~ system takes the 5τ to reach the steady state.

$$\boxed{\tau = -\frac{1}{\text{Real part of the Dominant pole}}}$$

H.B.

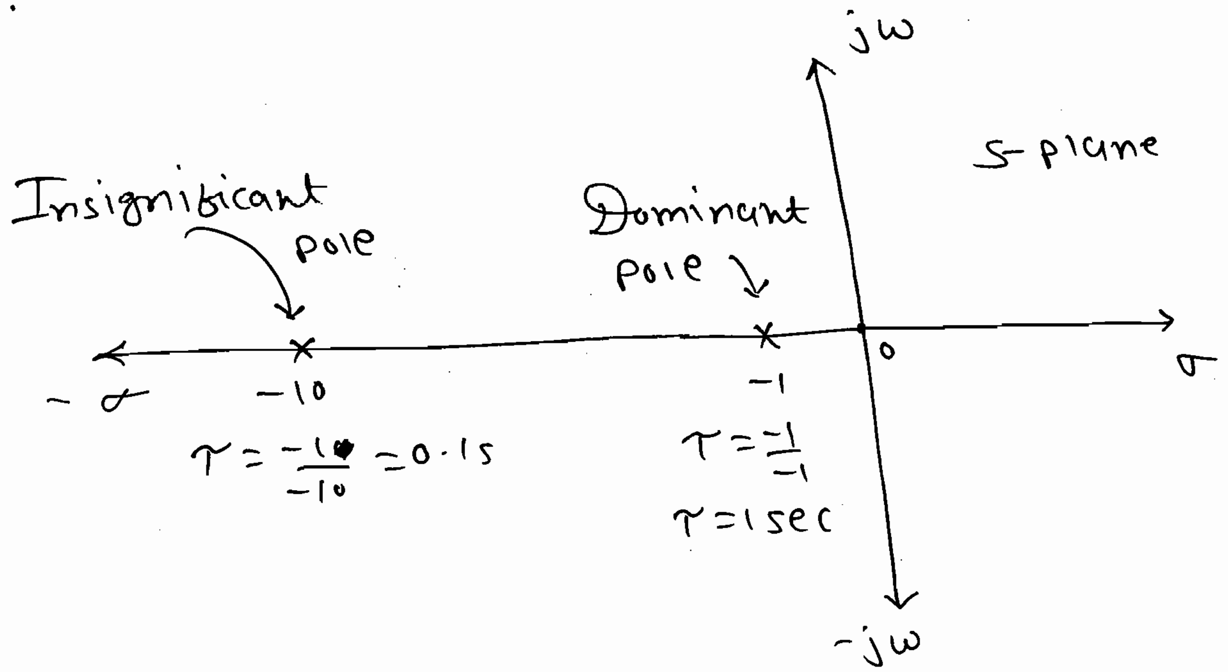
⇒ Dominant Pole: ← H.B

⇒ The Pole which is very close to the imaginary axis is called as dominant pole.

$\boxed{Q}$ ① Find the equivalent 1st order system.
② Find the system time constant for

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}.$$

Solⁿ:



⇒ Insignificant pole has less time constant so good performance and hence best pole.

⇒ Dominant pole has large time constant so bad pole. It affect the system. So we have to compensate it by adding zero at same position. So ~~that~~ we discuss only for DP.

If
⇒ H.B $\boxed{\text{Insignificant Pole} \le 5 \cdot \text{Times of Dominant Pole.}}$

then only it is called insignificant pole.

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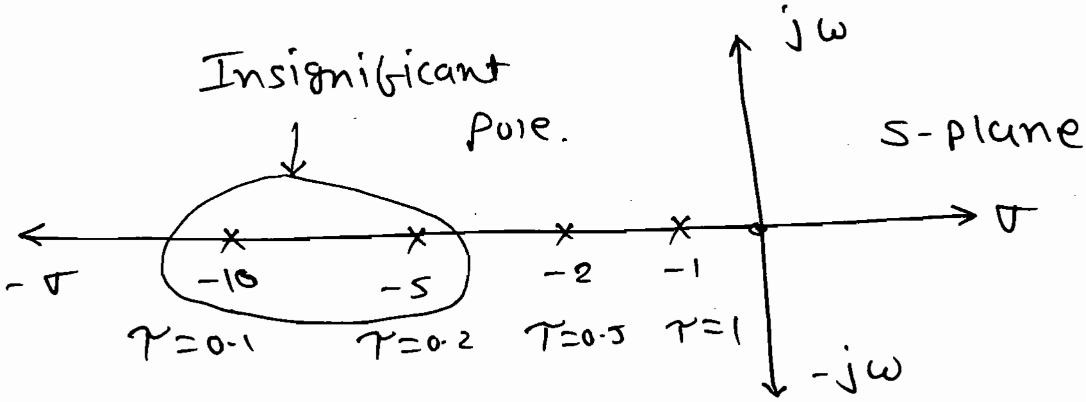
i.e here $-10 \le 5(-1)$

$\Rightarrow -10 < -5$ ✓

Eg.



$$Ins(\tau) \le \frac{DP(\tau)}{5}$$

$$Ins(\tau) \le \left(\frac{1}{5} = 0.2\right)$$

* Insignificant Pole:-

→ The Poles which lies in the left most side.

→ The insignificant Pole time Constant must be less than (OR) equal to 5 times of the dominant Pole time Constant that means insignificant Pole.

$$\boxed{Isp(\tau) \le \frac{DP(\tau)}{5}}$$ ← H.B

→ The best Pole is the insignificant pole.

because it gives the very quick response and more relatively stable. Because of the dominant pole the system response become the slow and the system becomes less relatively stable.

⇒ The insignificant poles are neglected because even if insignificant poles are neglected there is no much change in the system Response.

⇒ $$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$$

System Response | $R(s)=1$.

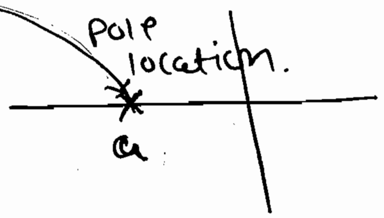
$$\therefore \; C(s) = \frac{1}{(s+1)(s+10)}$$

$$\therefore \; C(s) = \frac{1}{9(s+1)} - \frac{1}{9(s+10)}$$

$$\xrightarrow{ILT} \; c(t) = \underbrace{\frac{1}{9}e^{-t}}_{} \; \underbrace{-\frac{1}{9}\cdot e^{-10t}}_{}$$

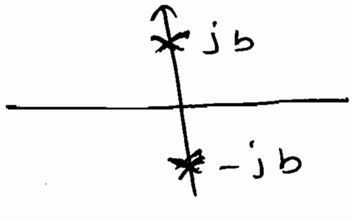
$T = 1s$ Dp Res.   ISp Res. $T = 0.1 sec$.

⇒ $$L[e^{-at}] = \frac{1}{s+a}$$



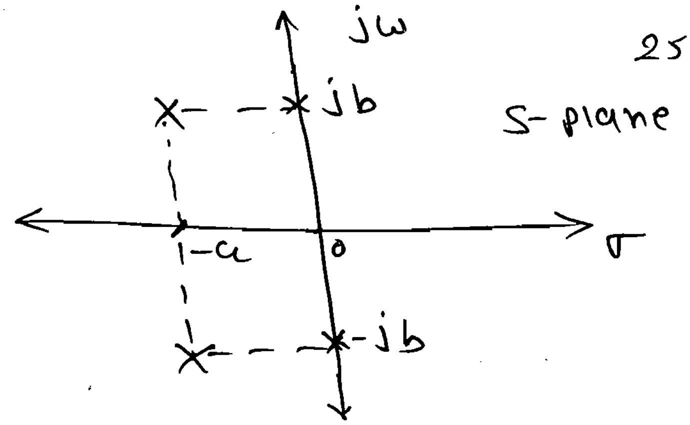
Real part of pole

⇒ $$L[\sin \text{(or)} \cos bt] = \frac{b \text{ or } s}{s^2+b^2}$$

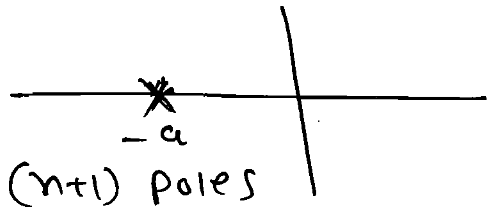


Img part of pole.

$\Rightarrow L[e^{-at} \cdot \sin bt] \Rightarrow$



$\Rightarrow L[t^{n} \cdot e^{-at}] \Rightarrow$



(n+1) poles are repeated.

→ In the response exponential powers are Real part of the poles, sine or cos function are Imaginary part of poles. The t term represents the Repeated nature of the poles.

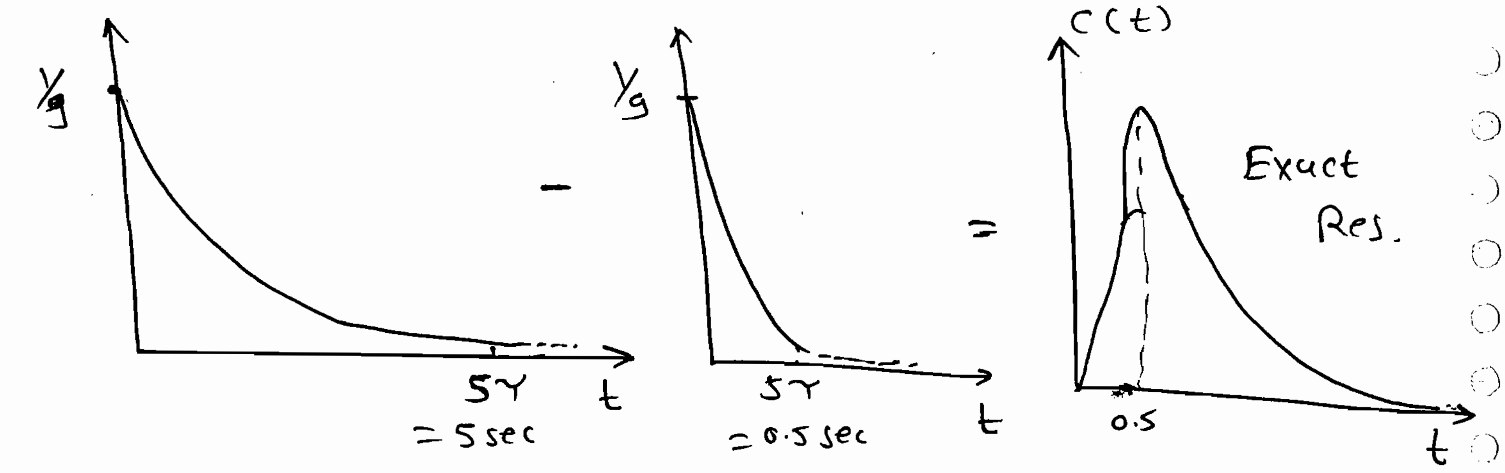
⇒ To get the system time constant from the response, compare the response with $e^{-t/\tau}$.

⇒ The system time constant is nothing but the dominant pole time constant and it should have the largest value.

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{1}{(S+1)(S+10)}$$

(Pole-zero form).

H.B. never neglect pole directly in pole-zero form.



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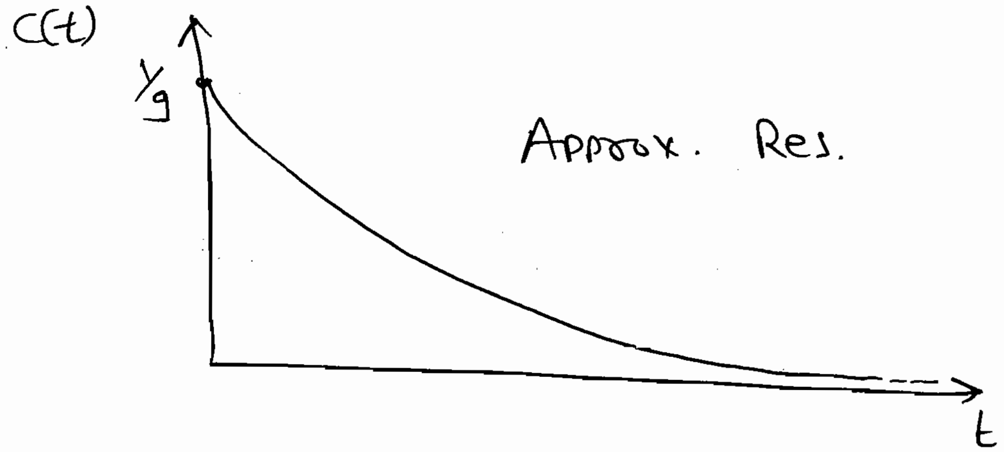


$$\Rightarrow \quad \frac{C(s)}{\cancel{R(s)}\ 1} = \frac{1}{10(s+1)(1+0.1s)}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{0.1}{(s+1)}, \quad \tau = 1 \text{ sec.}$$

System Res. $\Rightarrow R(s) = 1 \Rightarrow \quad c(t) = 0.1e^{-t}$.

NOTE: ✓ (h.B).

$\Rightarrow$ The Insignificant Pole must be neglacted only in the time constant form.

Q Find the equivalent TF of the following system $\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)}$.

(a) $\frac{1}{s+1}$ (b) $\frac{1}{(s+1)(s+10)}$ (c) $\frac{0.01}{(s+1)(s+10)}$ (d) ✓ $\frac{0.001}{(s+1)}$.

* While finding,

Sys. Response.
Sys. Stability.
Sys. Time Constant

We consider only Poles not zeros.

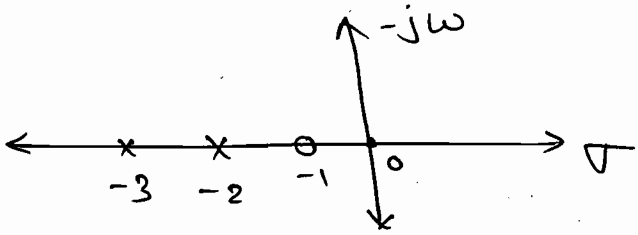
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(s+1)}{(s+2)(s+3)}.$$

Sys. Resp ; $R(s) = 1$.

$$\therefore C(s) = \frac{(s+1)}{(s+2)(s+3)}.$$

$$\therefore C(s) = \frac{-1}{(s+2)} + \frac{2}{(s+3)}.$$

$$\xrightarrow{ILT} c(t) = \left(-e^{-2t} + 2e^{-3t}\right).$$



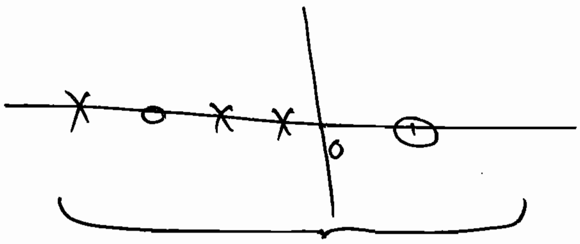
$\Rightarrow$ While finding System Response, system Stability, system time Constant we Consider only Poles but not zeros because the system Response Consist only the Poles response terms there is no zeros response term exist in the System Response.

⇒ Stability ⇒ $t = \infty$ ⇒ Sys. Res.

'∞' value ⇒ Unstable

Finite value Stable.

⇒



Stable (∵ no Pole at RHS).

⇒ $G(s) = K \dfrac{(\text{OL zeros})}{(\text{OL Poles})}$

CL sys: ⟶ $\text{CLTF} = \dfrac{\text{CL Zero}}{\text{CL Poles}}$

CL sys. ⟶ $G(s) = \dfrac{K\,(\text{OL zeros})}{(\text{OL Poles})}$; $H(s) = 1$

CLTF ⟶ $\text{CLTF} = \dfrac{K\,(\text{OL zeros})}{(\text{OL poles}) + K\,(\text{OL zeros})}$

⇒ The OL zeros never affect the OL stability.

⇒ The CL zeros never affect the CL stability.

⇒ The OL zeros affect the CL Stability because the CL Poles are nothing but the Sum of the OL Poles & OL Zeros with the function of Sys. gain K.

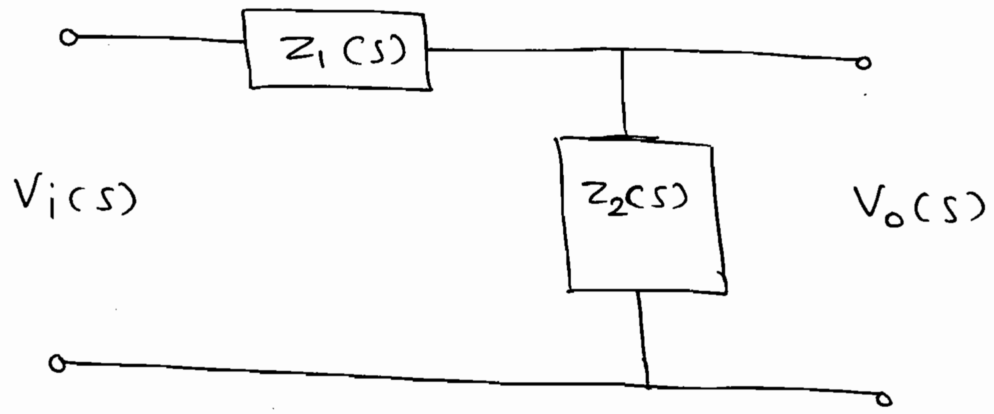
⇒ NOTE:

(i) To get the OLTF from the CLTF, Subtract numerator in the denominator when the feedback is unity.

(ii) To get the CLTF from OLTF, Add the numerator ~~from~~ in the dinominator when the F/B is unity.

* Transfer function of the Electrical N/ws:

⇒

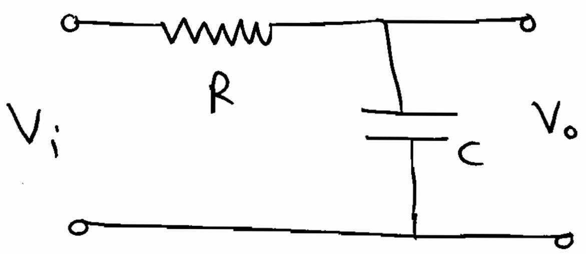


$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\text{Impedence across O/P}}{\text{Total ckt impedence}}$$

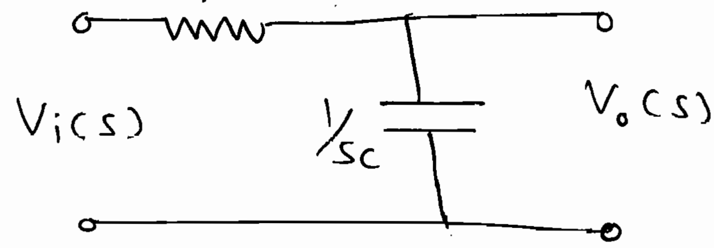
$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}.} \leftarrow \text{H.B}$$

Q (a) Find the TF to the given electrical N/ws & locate the poles in S-plane.

(b) Find the System Response.



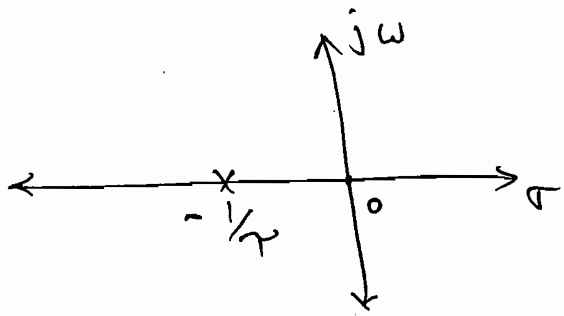
Sol^n:



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sCR}.$$

$\tau = RC$

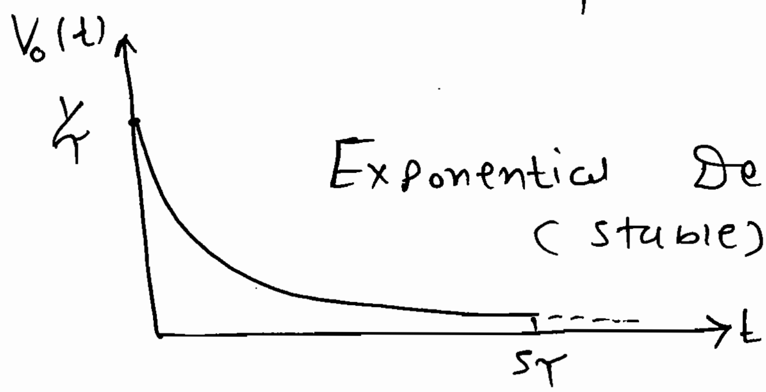
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + s\tau}$$



$\Rightarrow$ For Sys. Res. $V_i(s) = 1$.

$$\therefore V_o(s) = \frac{1}{\tau(s + 1/\tau)}.$$

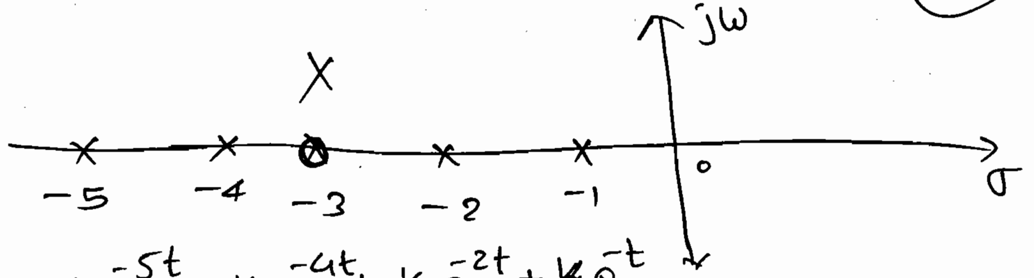
$$\xrightarrow{ILT} V_o(t) = \frac{1}{\tau} e^{-t/\tau}.$$



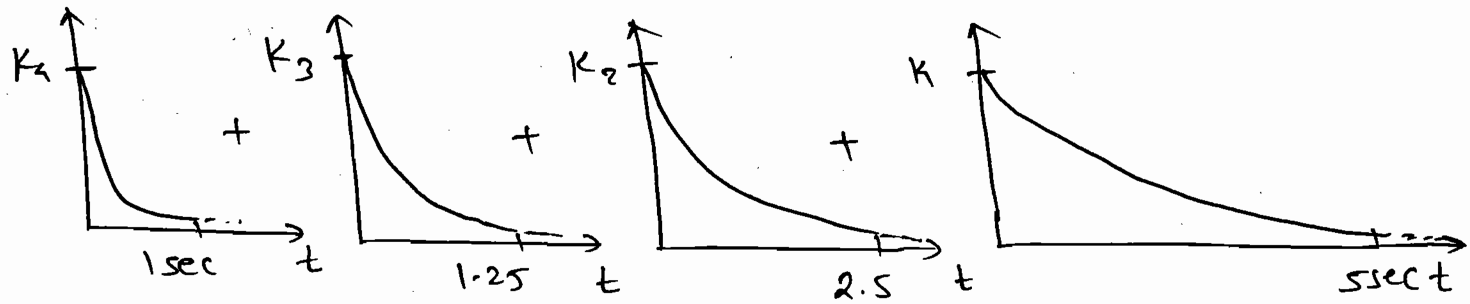
Exponential Decay.
(stable)

NOTE:

⇒ If one (or) more poles lies in the left of s-plane at different locations then the system response is exponential decay irrespective of position of zeros, the system is stable. ⇑ (H.B.)



$K_4 e^{-5t} + K_3 e^{-4t} + K_2 e^{-2t} + K_1 e^{-t}$



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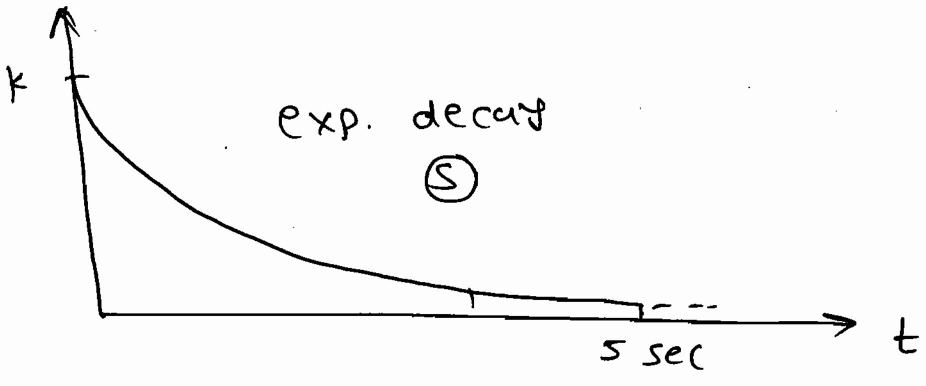


* Stability:

⇒ The movement of the pole in the s-Plane is nothing but varying the system Components (R, L C).

⇒ Absolutely stable system means the system is stable for all the values of the system parameters (or) system components like 'K' from '0 to ∞'.

⇒ Conditional stable system means the system is stable for certain range of system components like 'K' from 0 to 100.

⇒ Addition of poles & zeros to TF means adding RLC's components to the system.

⇒ The RLC components added to the system in a two ways.

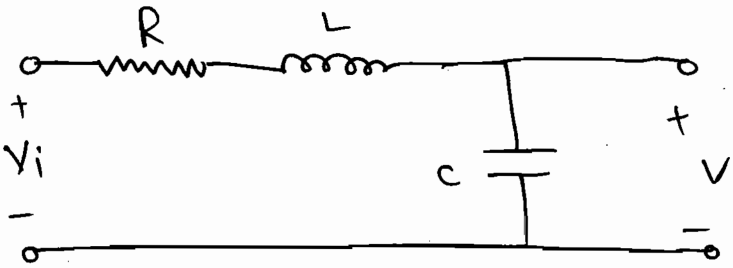
① series connection.

② parallel connection

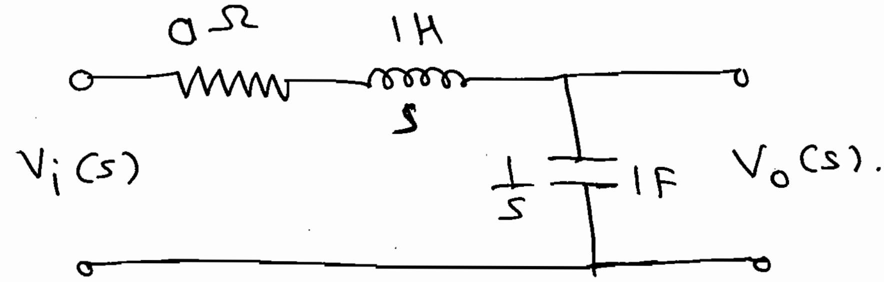
⇒ In series connection the RLC components are added in a forward path.

⇒ In parallel connection the RLC components added in a feedback path.

$\boxed{Q-1}$ Find the T.F. to the given electrical N/W and Locate the Poles in the S-plane by Considering $R=0$, $L=1H$, $C=1F$.



Soln:

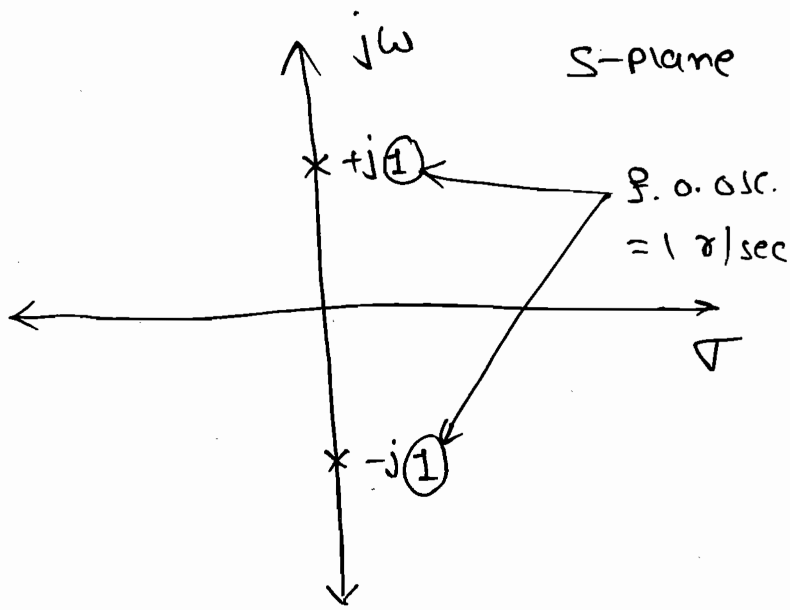


$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} = \frac{1}{s^2LC+sCR+1}.$$

$\Rightarrow$ $R=0\Omega$, $L=1H$, $C=1F$.

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+0+1} = \frac{1}{s^2+1}.$$

Poles: $s^2+1=0 \Rightarrow s=\pm j$.



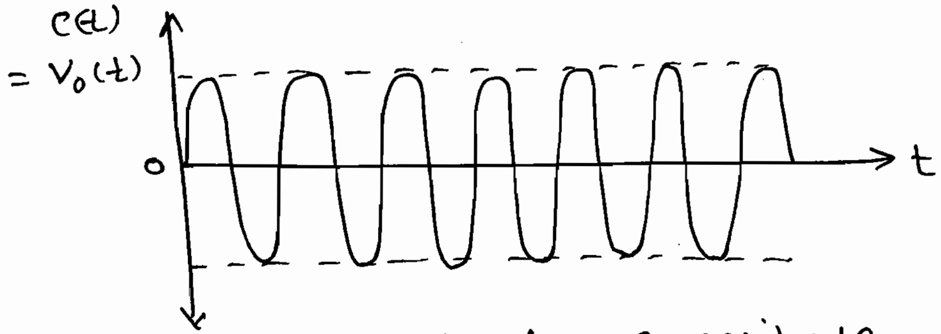
$\Rightarrow$ Non-Repeated Poles on $j\omega$ axis, system marginally stable.

$$\Rightarrow V_o(s) = \frac{1}{(s^2+1)} \cdot V_i(s).$$

for system response $V_i(s)=1$.

$\therefore \quad V_o(s) = \dfrac{1}{s^2+1}$

$\Rightarrow \quad V_o(t) = \sin t.$



Constant amplitude & freq. of oscillation

Undamped oscillation i.e. Marginally Stable.

⇒ When the poles lies on imaginary axis which are non-repeated then the system response is constant amplitude and freq. of oscillation which are called undamped oscillation.

⇒ Any system which produce undamped oscillation is called undamped ~~undamped~~ system. and the system becomes marginal stable.

$\boxed{\text{c}}$ Repeate the above problem by considering $R = 1\,\Omega$, $C = 1F$, $L = 1H$.

Soln:

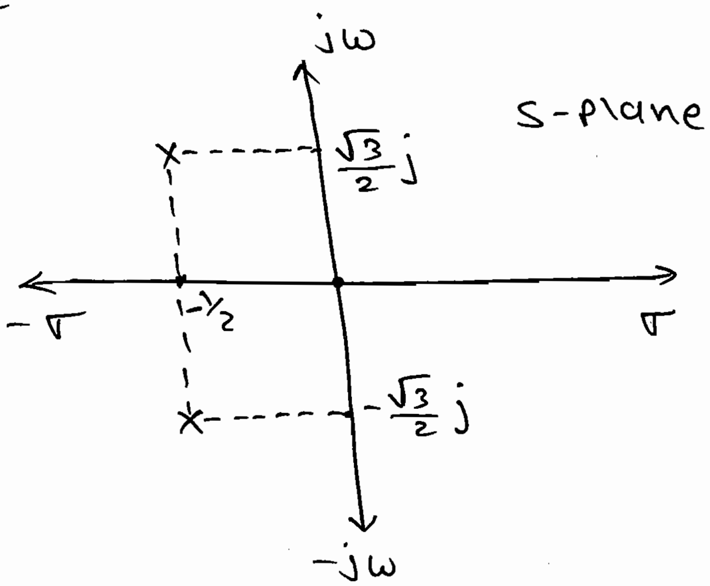
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+s+1}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+s+\frac{1}{4}+\frac{3}{4}}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

Poles: $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$.



∴ Time Constant

$$\boxed{\tau = \frac{-1}{-1/2} = 2\text{ sec.}}$$

$$\therefore j\omega = \frac{\sqrt{3}}{2}j$$

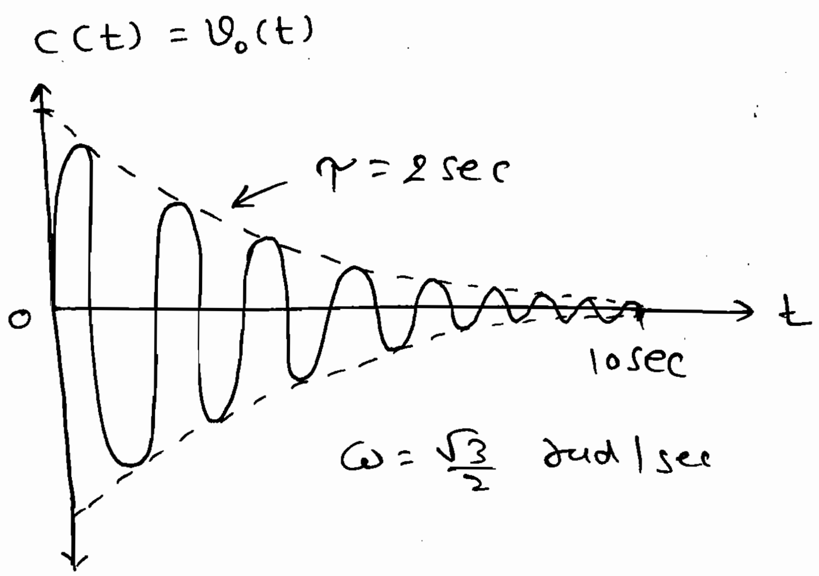
$$\therefore \boxed{\omega = \frac{\sqrt{3}}{2}\text{ rad/sec.}}$$

NOTE: H.B.

⇒ In Complex Conjugate poles, the real part gives the system time constanat and imaginary part gives the freq. of oscillation.

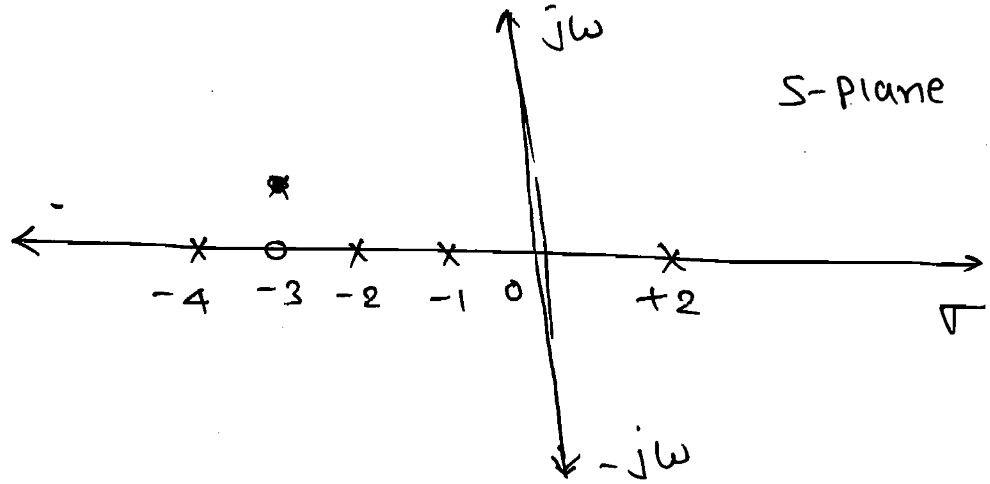
$v_o(t) = c(t)$

$= \frac{2}{\sqrt{3}} \cdot e^{-t/2} \cdot \sin\frac{\sqrt{3}}{2}t$

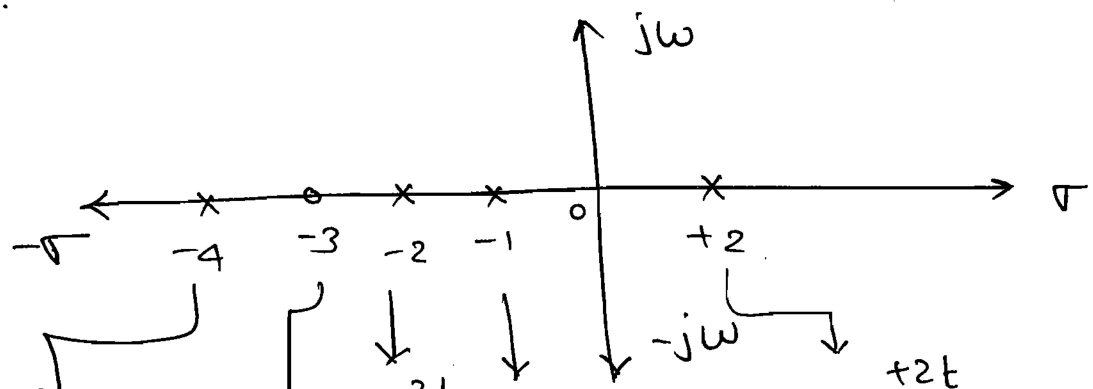


⇒ whenever the poles are complex conjugate in the left of S-plane then the system response is exponetically decay and freq. of oscillation which are called damped oscillations. Any system which produce damped oscillations is called under damped system and the system is stable.

[a] Find the system time const. and system response to the given poles location in the s-plane.



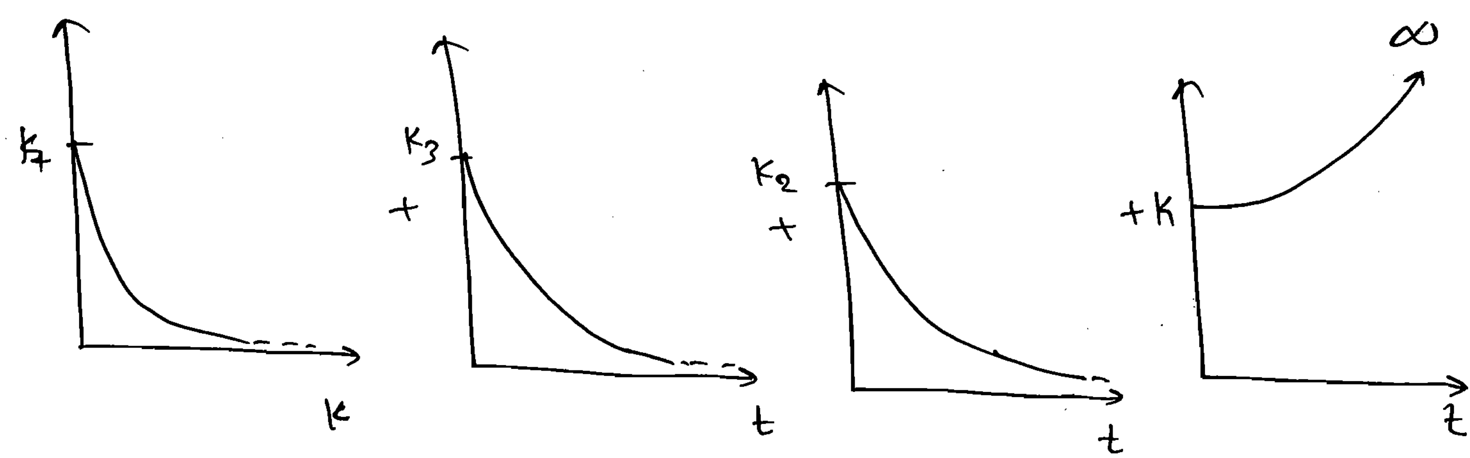
Sol^n:



$K_4 \cdot e^{-4t}$ $\cancel{K_3 e}$ $K_3 e^{-2t}$ $K_2 e^{-t}$ $K_1 e^{+2t}$

$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{(s+3)}{(s+1)(s+2)(s+4)(s-2)}$$

→ A given system is un-stable and the time const. is not defined. The time constant defined for only stable system.



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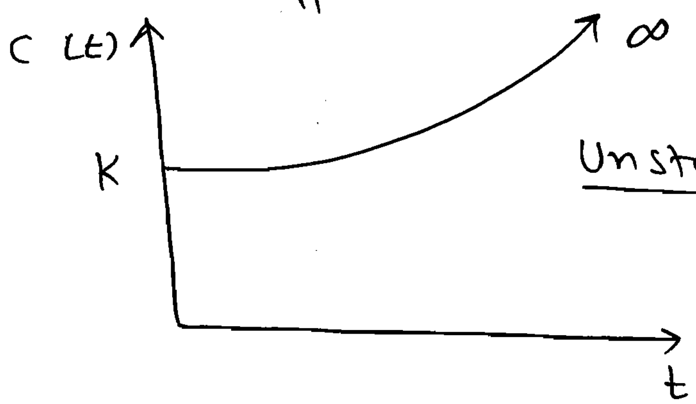


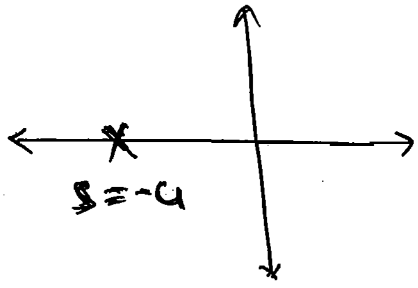
Unstable.

NOTE:

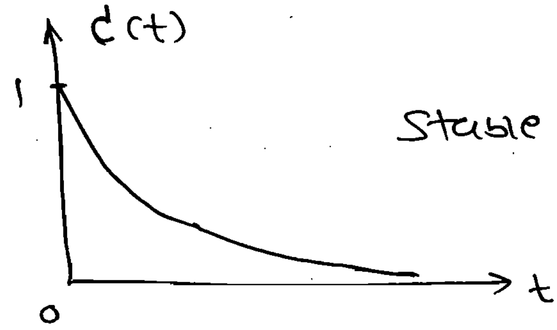
⇒ whenever one (or) more poles lies in the right of the s-plane at different locations on the real - axis then the system is un-stable because the system response expential rised to infinity.

⇒ The system response follows the i/p then the system become stable.

*



Draw the Res.



$\Rightarrow$

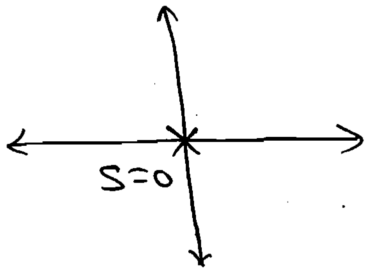
Stable

TF : $\frac{1}{s+a}$

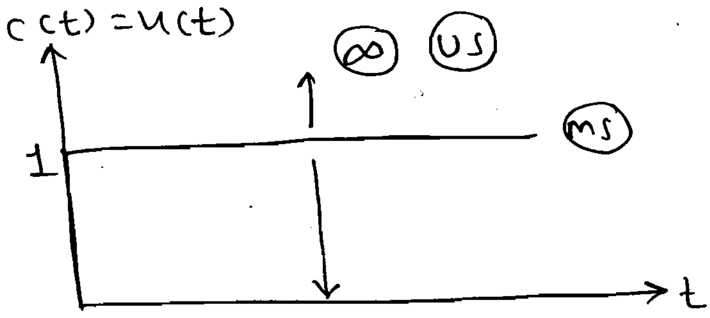
Sys. Res : $1 \cdot e^{-at}$

Sys. Response follows the i/p.

*



$\Rightarrow$



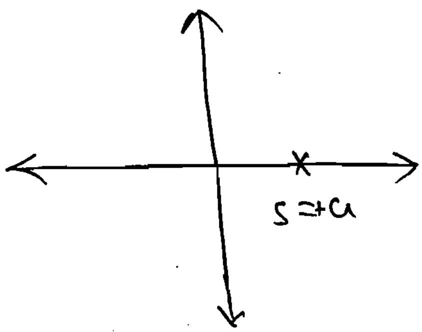
$\rightarrow$ TF = $\frac{1}{s}$.

$\Rightarrow c(t) = u(t)$.

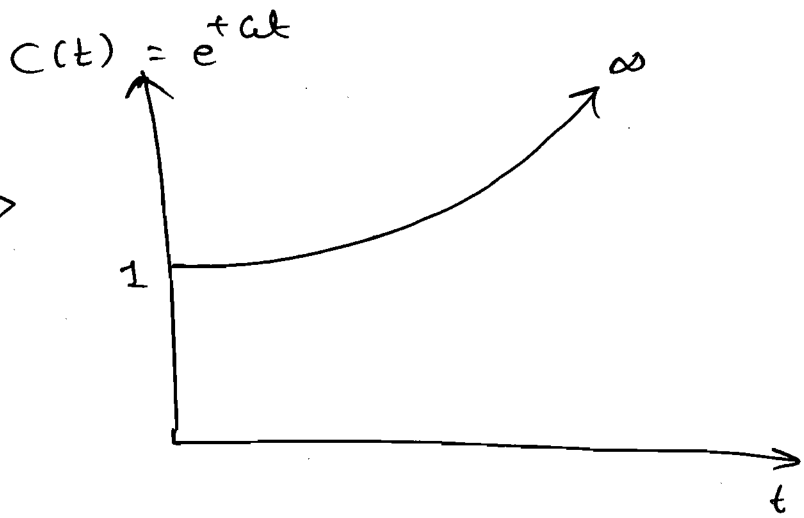
Const. Amp & f.o.o.

MS

*



$\Rightarrow$



TF = $\frac{1}{s-a}$.

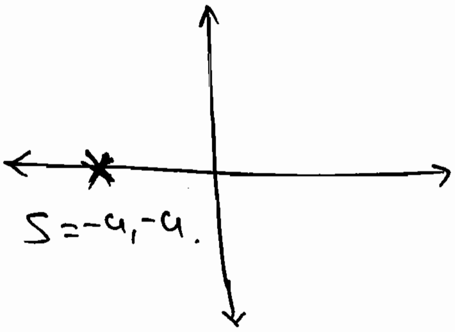
$c(t) = 1 \cdot e^{+at}$

Unstable

exponentially raised

So, unstable system.

*

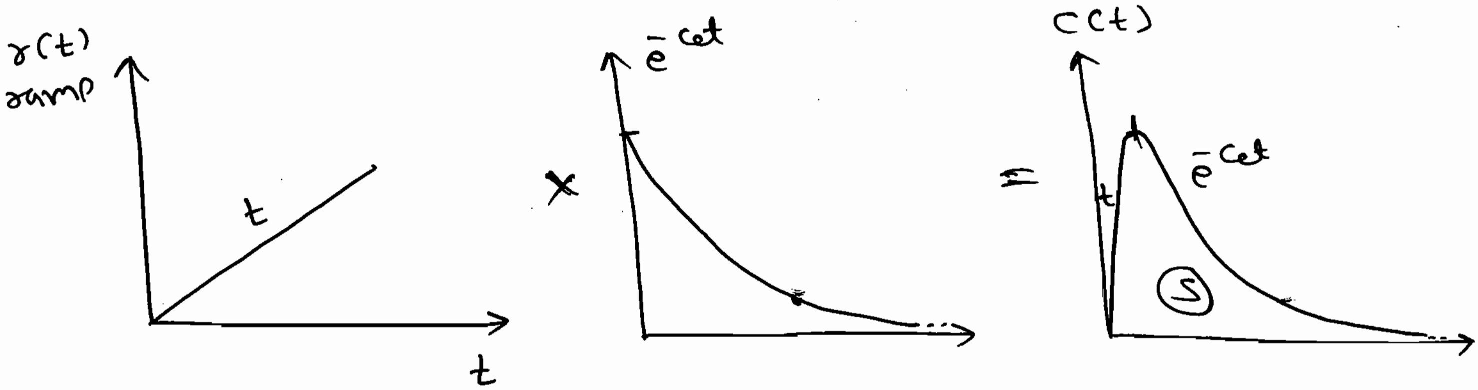


$T.F. = \frac{1}{(s+a)^2}$

$c(t) = t \cdot e^{-at}$.

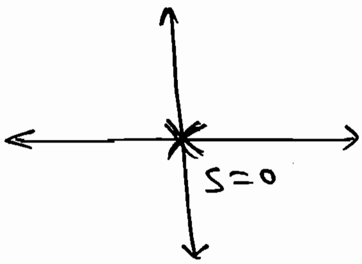
$t=0 \Rightarrow 0$ (t- term).

$t=\infty \Rightarrow 0$ (exp - term).

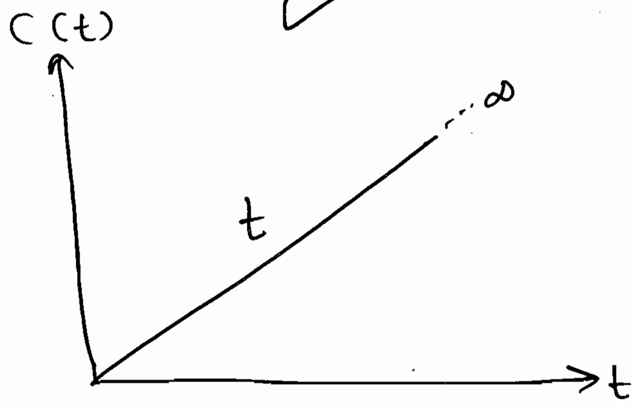


=> for Lower value of t, t term dominant and for higher value of t, e^{-at} term dominant.

*



$\Rightarrow$



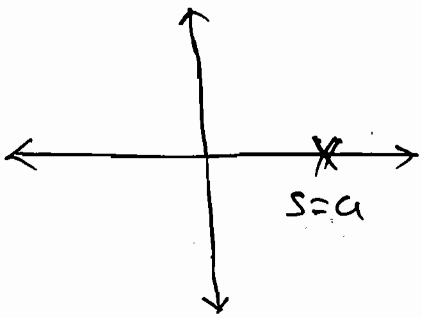
$T.F. = \frac{1}{s^2}$

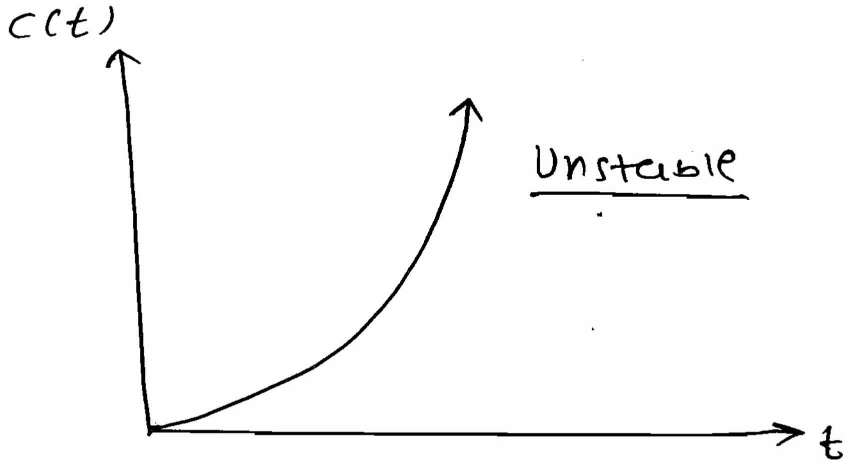
$c(t) = t \cdot u(t)$.

Unstable.

Repeated pole on jω axis unstable.

*





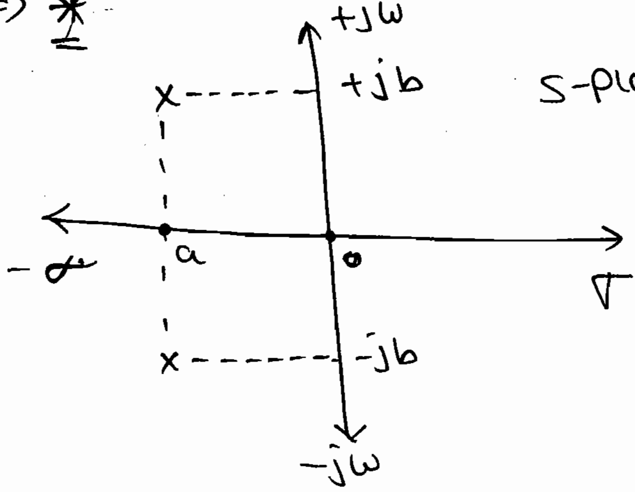
Unstable

$T.F. = \frac{1}{(s-a)^2}$

$c(t) = t \cdot e^{at}$

* Complex Conjugate Poles:

⇒ *

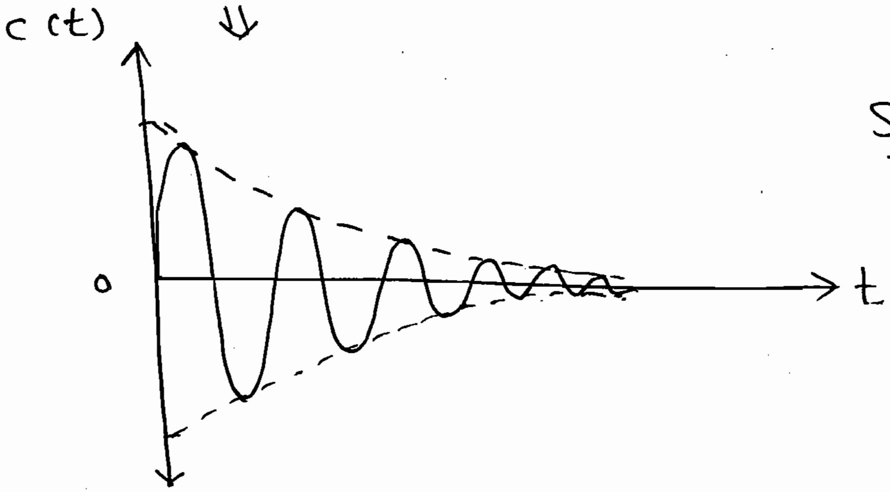


s-plane

$$TF = \frac{1}{(s+a)^2 + b^2}$$

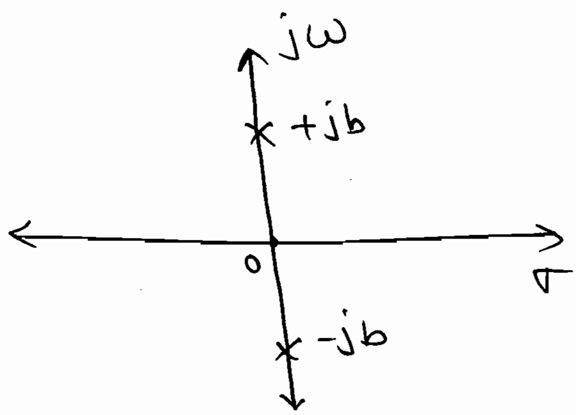
$$c(t) = \frac{1}{b} \cdot e^{-at} \cdot \sin bt.$$

⇓



Stable.

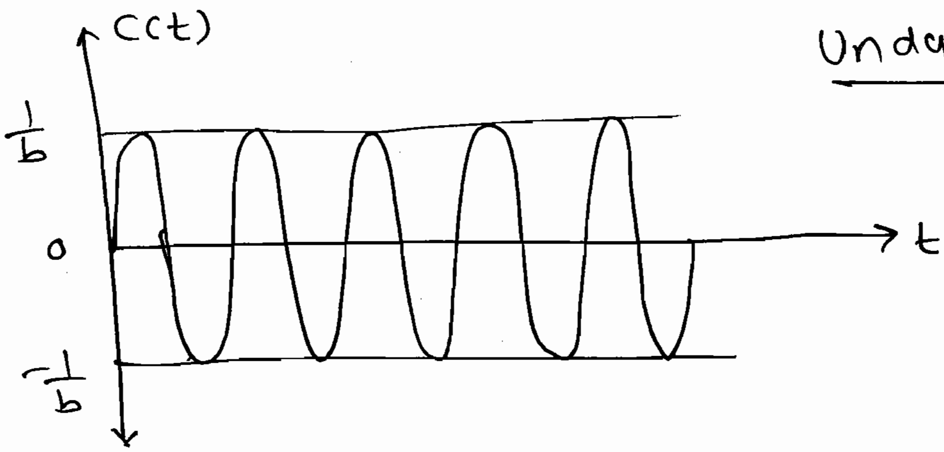
*



$$TF = \frac{1}{s^2+b^2}.$$

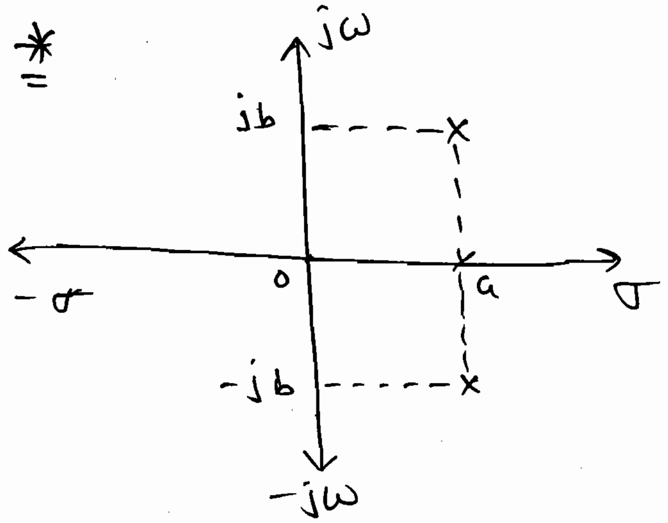
$$c(t) = \frac{1}{b} \cdot \sin bt.$$

⇒



Undamped

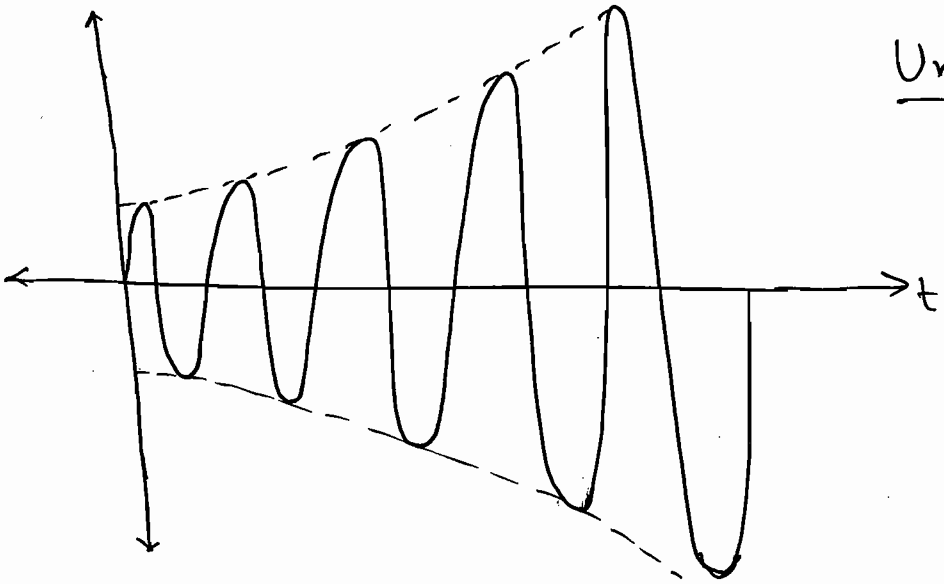
*



$$T.F. = \frac{1}{(s-a)^2 + b^2}$$

$$c(t) = \frac{1}{b} \cdot e^{at} \cdot \sin bt.$$

=>



Unstable

=> S-plane of TF — Sys. Res.

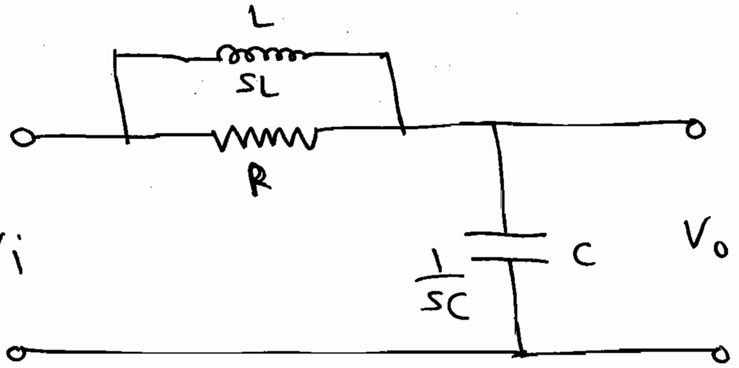
① Real Part ⟶ Exp. term.

② Img. part ⟶ Sine (or) cos term
(cos will come whenever there is a zero at origin)

③ (Real + Img) ⟶ Product of exp. and sine (or) cosine term.

④ (Repeated Pole) ⟶ Product of t and exp. term.

Q. Find the TF to the electrical N/W.



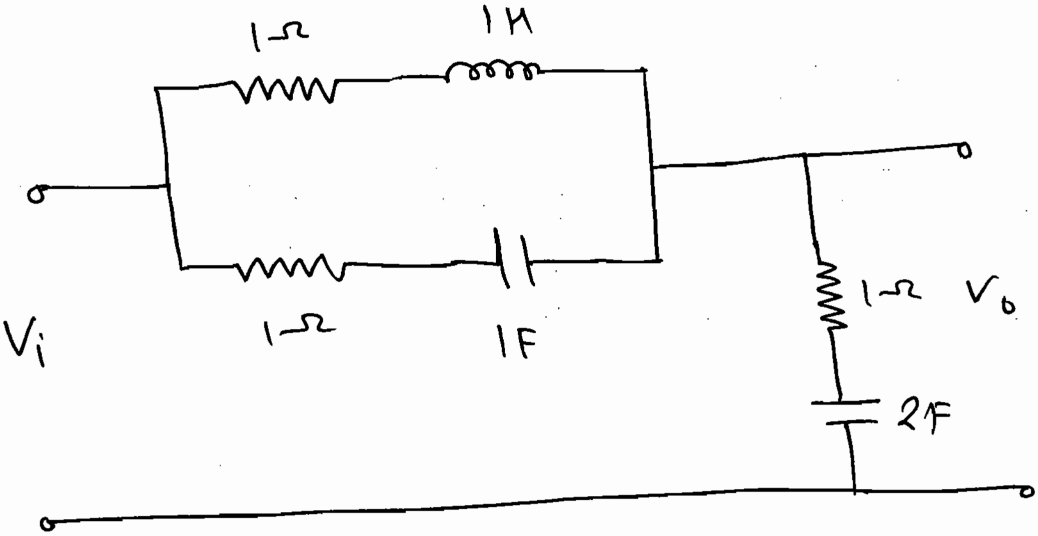
Soln:

$$SL \parallel R = \frac{RSL}{R+SL}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + \frac{RSL}{R+SL}}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{R+SL}{R+SL+s^2RLC}}$$

Q-2



Soln:

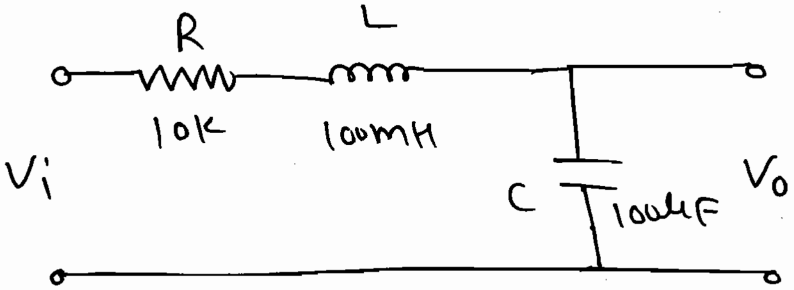
$$\left(1+\frac{1}{s}\right) \parallel (1+s)$$

$$= \frac{\left(1+\frac{1}{s}\right) \times (1+s)}{1+\frac{1}{s}+1+s}$$

$$= \frac{(1+s+\cancel{\frac{1}{s}+1})}{(1+\cancel{\frac{1}{s}+1+s})} = 1.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1 + \frac{1}{2s}}{1 + 1 + \frac{1}{2s}} = \frac{2s+1}{4s+1}.$$

Q-3



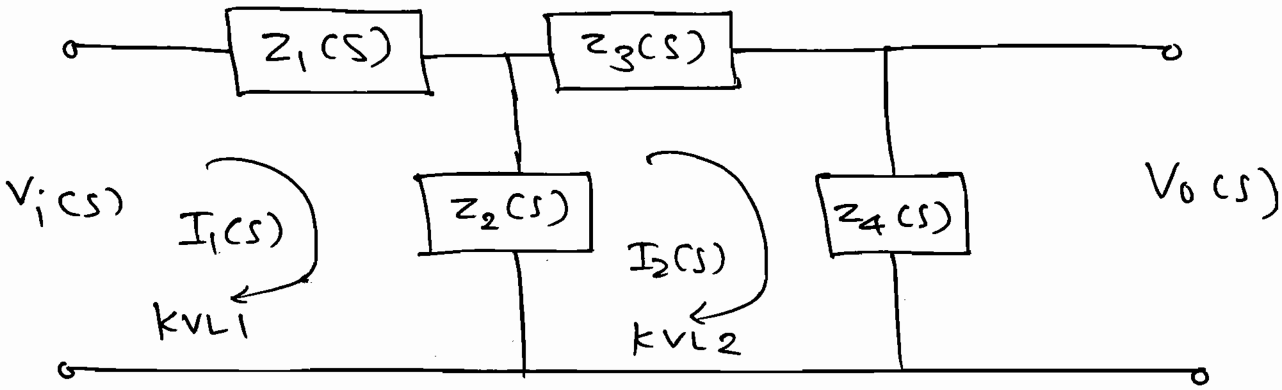
Soln:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}$$

$$= \frac{1}{s^2 \times 10^{-5} + s + 1}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{10^5}{s^2 + 10^5 s + 1}}$$

Q



Soln: By KVL,

$$V_i(s) = Z_1(s) \cdot I_1(s) + Z_2(s) \cdot [I_1(s) - I_2(s)]$$

$$V_i(s) = [Z_1(s) + Z_2(s)]\, I_1(s) - Z_2(s) \cdot I_2(s). \quad -①$$

By KVL,

$$Z_3(s) \cdot I_2(s) + Z_4(s) \cdot I_2(s) + [I_2(s) - I_1(s)]\, Z_2(s) = 0.$$

$$\therefore [Z_3(s) + Z_4(s) + Z_2(s)]\, I_2(s) - Z_2(s)\, I_1(s) = 0 \quad -②$$

$$\Rightarrow \quad V_o(s) = I_2(s) \cdot Z_4(s).$$

$$\begin{bmatrix} V_i \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1(s) + Z_2(s) & -Z_2(s) \\ -Z_2(s) & Z_2(s) + Z_3(s) + Z_4(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}.$$

→ By using Crammer's rule,

$$\rightarrow \quad I_2 = \frac{\Delta_2}{\Delta}$$

$$\cancel{I_2} \;\cancel{=}$$

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_2 & V_i \\ -Z_2 & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 + Z_4 \end{vmatrix}}.$$

$$\therefore \quad I_2 = \frac{+V_i \cdot Z_2}{(Z_1+Z_2)(Z_2+Z_3+Z_4) - Z_2^2}.$$

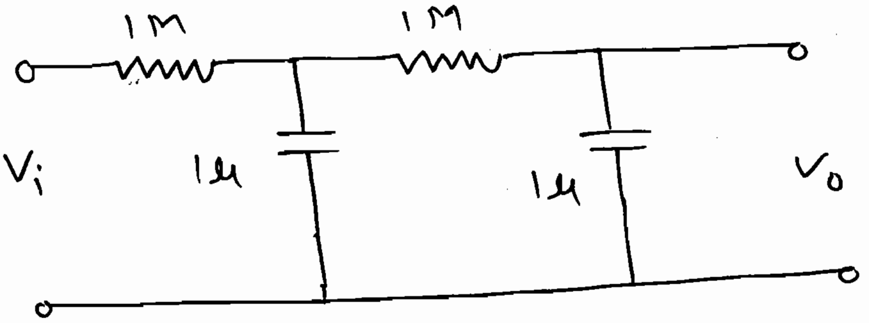
$$\therefore \quad I_2 = \frac{V_i \cdot Z_2}{Z_1 \cdot Z_2 + Z_1 \cdot Z_3 + Z_1 \cdot Z_4 + \cancel{Z_2^2} + Z_2 \cdot Z_3 + Z_2 \cdot Z_4 - \cancel{Z_2^2}}$$

$$\therefore \quad I_2 = \frac{V_i \cdot Z_2}{Z_1 \cdot (Z_2 + Z_3 + Z_4) + Z_2 \cdot (Z_3 + Z_4)}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2 \cdot Z_4}{\cancel{Z_1 \cdot \xi}\; Z_1(Z_2+Z_3+Z_4) + Z_2(Z_3+Z_4)}}.$$

↑
H.B.

[a] Find the TF.



Soln:

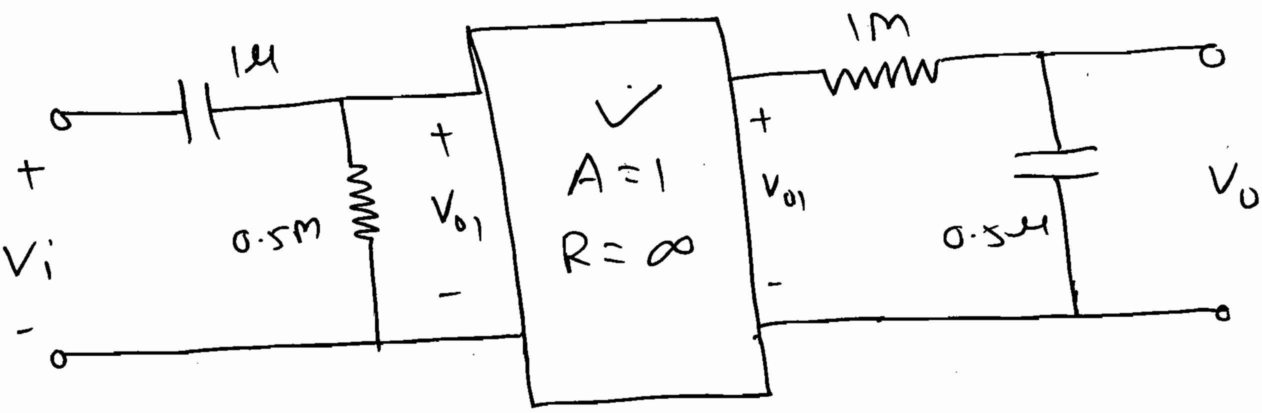
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{s\cdot 1\mu} \times \frac{1}{s\cdot 1\mu}}{1M\left(\frac{1}{s\cdot 1\mu} + 1M + \frac{1}{s\cdot 1\mu}\right) + \frac{1}{s\cdot 1\mu}\left[1M + \frac{1}{s\cdot 1\mu}\right]}$$

$$= \frac{\frac{1}{s^2}}{\left(\frac{1}{s} + 1 + \frac{1}{s}\right) + \left(\frac{1}{s} + \frac{1}{s^2}\right)}$$

$$= \frac{1}{(2+s)\,(s+1)}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+3s+1}}$$

[a] Find the TF.



Soln:

$$\frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_{o1}(s)} \times \frac{V_{o1}(s)}{V_i(s)}$$

$$\Rightarrow \frac{V_{o1}}{V_i} = \frac{R}{R + \frac{1}{sC}} = \frac{RsC}{1+RsC} = \frac{0.5s}{1+0.5s} = \frac{s}{s+2}$$

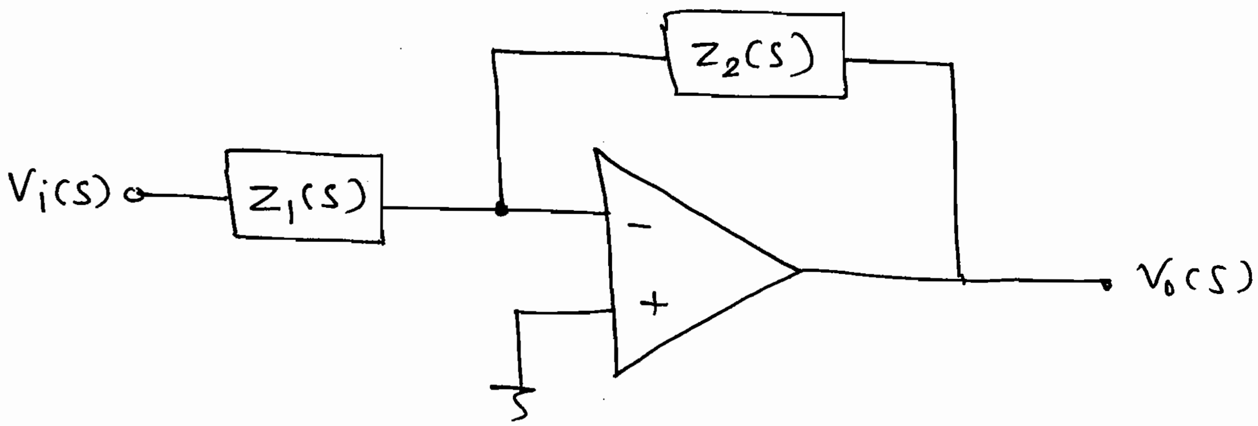
$$\Rightarrow \frac{V_o(s)}{V_{o1}(s)} = \frac{1}{1+sCR} = \frac{1}{1+s/2} = \frac{2}{s+2}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{s}{(s+2)} \times \frac{2}{(s+2)^2} = \frac{2s}{s^2+4s+4}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{2s}{(s+2)^2}}$$

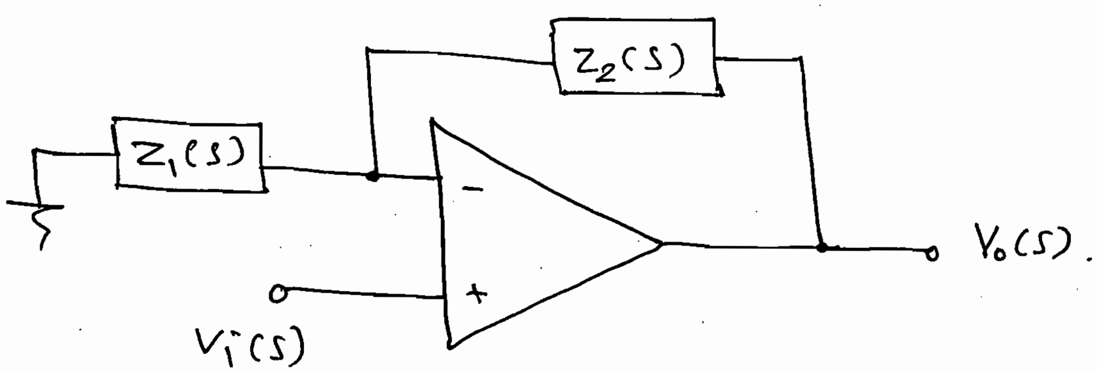
* TF of the OP-AMP :-

① Inverting OP-Amp:



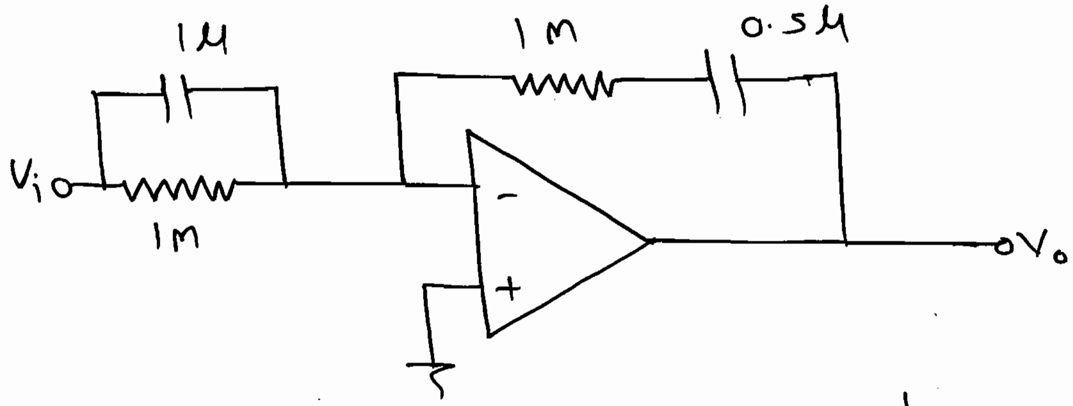
$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}.$$

② Non-inverting OP-Amp:-



$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}}$$

$\boxed{Q}$ Find the TF.



Solⁿ:

$$Z_1(s) = \left(\frac{1}{sC}\right) \| (R) = \frac{\frac{1}{sC} \times R}{R + \frac{1}{sC}} = \frac{R}{1 + sCR}$$

$$\rightarrow Z_1(s) = \frac{1m}{1 + s\,1m \cdot 1\mu} = \frac{1m}{s+1}.$$

$$\rightarrow Z_2(s) = R + \frac{1}{sC} = 1m + \frac{1}{0.5\mu\, s} = 1m + \frac{2}{1\mu s}$$

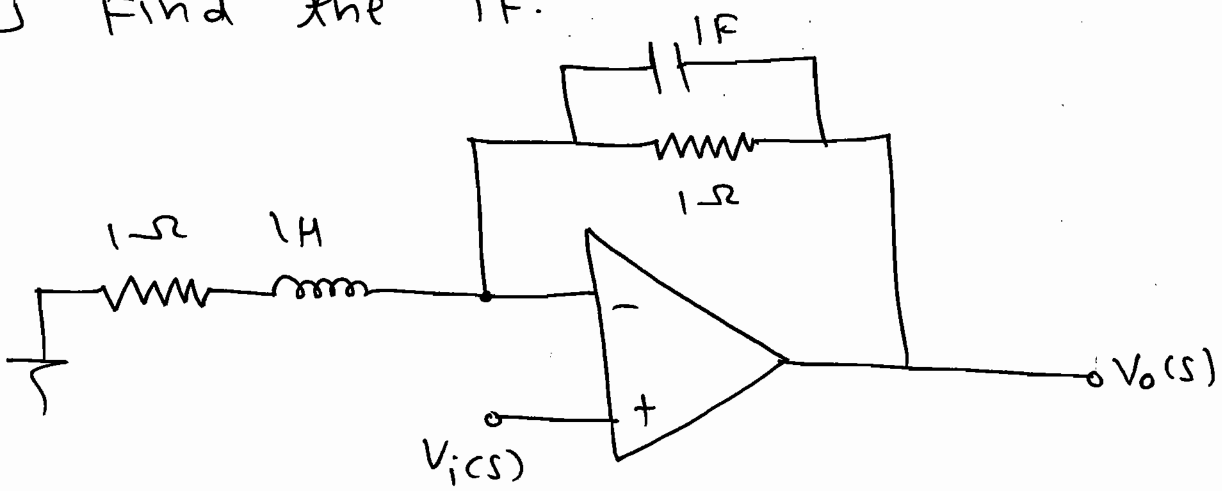
$$= \frac{s+2}{s\,1\mu} = 1m\left(\frac{s+2}{s}\right).$$

$$\therefore \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \frac{-1m\left(\frac{s+2}{s}\right)}{1m/(s+1)}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = -\frac{(s+1)(s+2)}{s}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = -\frac{s^2+3s+2}{s}}$$

$\boxed{Q}$ Find the TF.



Sol^n:

$$Z_1(s) = R + sL = (1+s).$$

$$Z_2(s) = R \parallel \frac{1}{sC} = \frac{R}{1+sCR} = \frac{1}{(s+1)}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)} = 1 + \frac{1}{(s+1)^2}.$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^2+2s+2}{s^2+2s+1}}$$

* TF to the Differential equations:-

⇒ Write the TF to the given system where x is ~~output~~ input and y is o/p.

(1) $$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{dx}{dt} + x(t-T).$$

Sol^n: Take L.T.

$$\therefore s^3 Y(s) + 5s^2 Y(s) + 7sY(s) + 9Y(s) = 2sX(s) + e^{-sT} X(s).$$

$$\therefore (s^3 + 5s^2 + 7s + 9)\,Y(s) = (2s + e^{-sT})\,X(s).$$

$$\therefore \boxed{TF = \frac{Y(s)}{X(s)} = \frac{2s + e^{-s\tau}}{s^3 + 5s^2 + 7s + 9}}.$$

$\dfrac{(\text{i/p related term})}{(\text{o/p related term})}$

(2) $$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + y\frac{dy}{dt} + 10 = \frac{dx}{dt} + x.$$

Soln: The given system is non-linear hence TF is not define. H.S.

[a] Write the differential eqn to the given TF.

$$\frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+5s+6}.$$

Soln:

$$Y(s)\,(s^2+5s+6) = (2s+3)\,X(s).$$

$$\Rightarrow s^2Y(s) + 5s\,Y(s) + 6Y(s) = 2s\,X(s) + 3X(s).$$

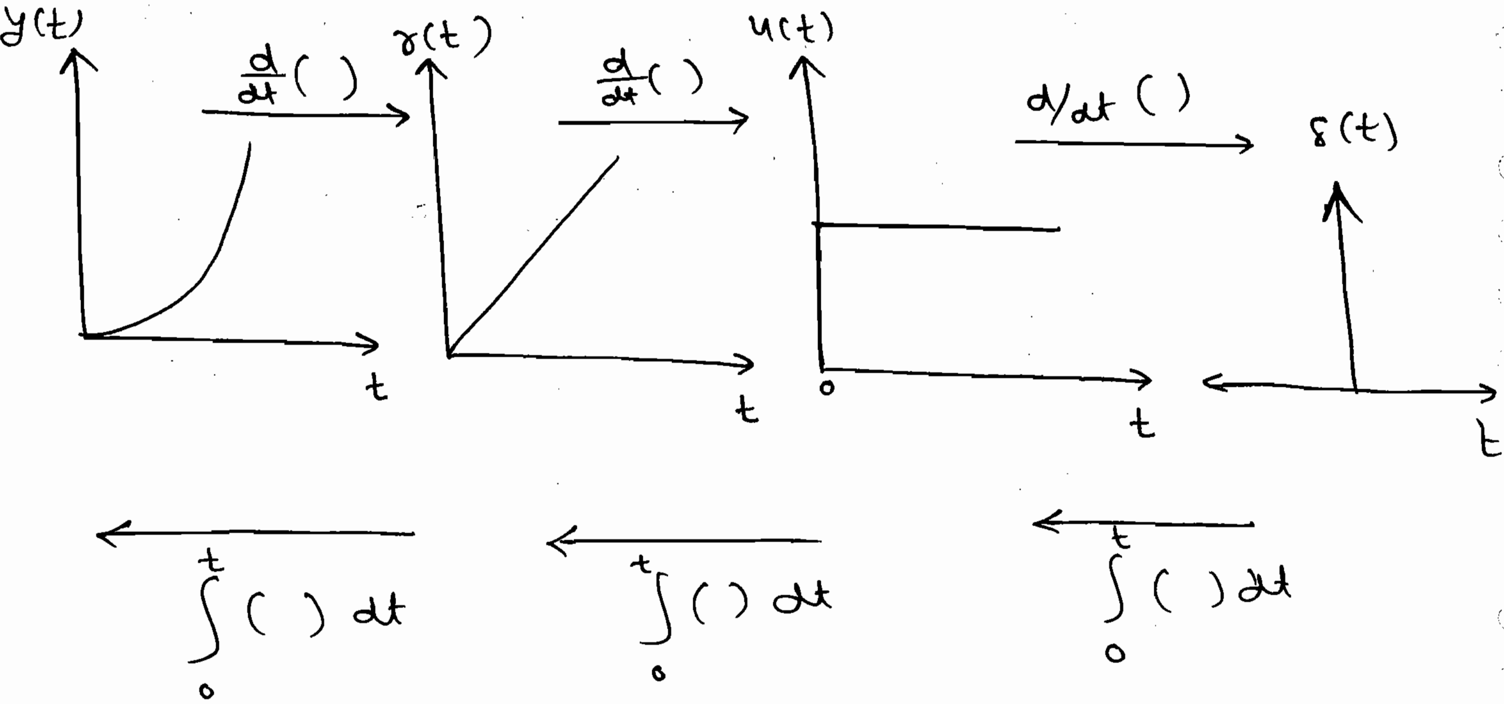
$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\frac{dx}{dt} + 3x}.$$

* TF to the signal Response:-

⇒ To get the TF from the signal response used the following formula.

$$\boxed{TF = \left.\frac{L[\text{o/p}]}{L[\text{i/p}]}\right|_{I_i = 0}}.$$

$\Rightarrow$ Conversion of Responses: (1.B.)



$\boxed{a}$ The unit step response of the system is

$y(t) = \left(\frac{5}{2} - \frac{5}{2}e^{-2t} + 5t\right)$, $t \geq 0$ its TF is __?

Sol^n:

$$TF = \frac{L[\text{Unit step Res.}]}{L[\text{Unit step}]}.$$

$$\therefore TF = \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$= \frac{5s(s+2) - 5s^2 + 5(s+2)2}{2(s+2) \times s^2 . s}$$

$$\therefore TF = \frac{10s+10}{2s(s+2)}.$$

$$\therefore \boxed{TF = \frac{5(s+1)}{s(s+2)}.}$$

(a) The impulse response of the system is $c(t) = (-4e^{-t} + 6e^{-2t})$. $t \geq 0$. The equivalent step response is ?

Soln: Step response is $\int_0^t (-4e^{-t} + 6e^{-2t})\,dt$.

$$= \left[4e^{-t} - 3e^{-2t}\right]_0^t$$

$$= \left[4e^{-t} - 3e^{-2t} - 4 + 3\right]$$

$$\boxed{r(t) = 4e^{-t} - 3e^{-2t} - 1.}$$

* Sensitivity:

⇒ The Sensitivity gives the relative variations in the output due to parameter variations in (i) G(s) (ii) H(s).

⇒ Sensitivity of the TF w.r.t.

$G(s)$ ⇒ $S_G^T = \dfrac{\text{\% Change in TF}}{\text{\% Change in G}}$.

$$\therefore \boxed{S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{G}{T} \times \frac{\partial T}{\partial G}.}$$

Simillarly,

$$\boxed{S_H^T = \frac{H}{T} \times \frac{\partial T}{\partial H}.}$$

* Find the Sensitivity of the OL and CL sys. w.r.t. Variations. (i) G(s) (ii) H(s).

Soln:

① OL sys.

$$S_G^T = \frac{G}{T} \times \frac{\partial T}{\partial G} = \frac{G}{G} \cdot \frac{\partial G}{\partial G} = 1.$$

② CL sys.

$$T = \frac{G}{1+GH}.$$

$$\Rightarrow S_G^T = \frac{G}{T} \times \frac{\partial T}{\partial G}$$

$$= \frac{\cancel{G}}{\cancel{G}} \times \cancel{(1+GH)} \times \frac{(1+GH)(1) - (G)(H)}{(1+GH)^{\cancel{2}}}$$

$$\therefore \boxed{S_G^T = \frac{1}{1+GH}} \leftarrow \text{H.B}$$

$$\Rightarrow S_H^T = \frac{H}{T} \times \frac{\partial T}{\partial H}$$

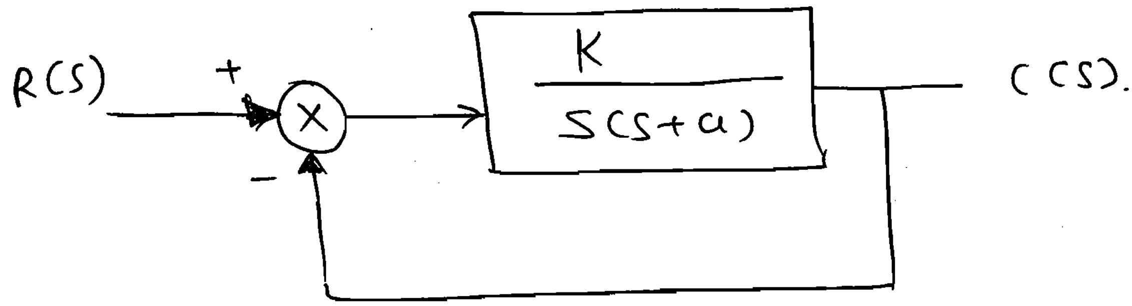
$$= \frac{H}{\cancel{G}} \times \cancel{(1+GH)} \times \frac{-G^{\cancel{2}}}{(1+GH)^{\cancel{2}}}$$

$$\therefore \boxed{S_H^T = \frac{-GH}{(1+GH)}} \checkmark \text{ H.B.}$$

$$\Rightarrow \boxed{S_H^T > S_G^T}$$

⇒ feedback is more sensitive than the forward path.

Q Find the sensitivity of the system w.r.t. variations in ① K ② a



Soln: $\frac{C(s)}{R(s)} = \frac{K}{s(s+a)+K} = \frac{K}{s^2+sa+K}$.

(i) $S_K^T = \frac{K}{T} \times \frac{\partial T}{\partial K}$

$= \frac{\cancel{K}}{\cancel{K}} \times \cancel{(s^2+sa+K)} \times \frac{(s^2+sa+K)(1) - (K)(1)}{(s^2+sa+K)^{\cancel{2}}}$

$$\boxed{S_K^T = \frac{s^2+as}{s^2+as+K}}$$

(ii) $S_a^T = \frac{\cancel{K}a}{T} \times \frac{\partial T}{\partial a}$.

$S_a^T = \frac{\cancel{K}a}{\cancel{K}} \times (s^2+a\cancel{s+K}) \times \frac{-\cancel{K} \times s}{(s^2+sa+K)^2}$

$$\therefore \boxed{S_a^T = \frac{-as}{(s^2+as+K)^2}}$$