Factorisation

MATHEMATICAL REASONING

1. Factorisation of $xy-pq+qy-px$ is ______.

(a) $(y-p)(x+q)$ (b) $(y-p)(x-q)$

(c) $(y+p)(x+q)$ (d) $(y+p)(x-q)$

2. If $(x^2+3x+5)(x^2-3x+5)=m^2-n^2$ then m=_____.

(a) x^2-3x (b) $3x$

(c) x^2+5 (d) Both (a) and (b)

3. The factors of $\frac{x^2}{4}-\frac{y^2}{9}$ are ____.

(a) $\left(\frac{x}{4}+\frac{y}{9}\right)\left(\frac{x}{4}-\frac{y}{9}\right)$

(b) $\left(\frac{x}{2}+\frac{y}{9}\right)\left(\frac{x}{2}-\frac{y}{9}\right)$

(c) $\left(\frac{x}{2}+\frac{y}{3}\right)\left(\frac{x}{2}-\frac{y}{3}\right)$

(d) Both (a) and (b)

4. The factors of $15x^2-26x+8$ are __.

(a) $(3x-4)(5x+2)$ (b) $(3x-4)(5x-2)$

(c) $(3x+4)(5x-2)$ (d) $(3x+4)(5x+2)$

5. The factors of x^2-16 are __.

(a) $(x^2+2)(x^2-2)$

(b) $(x+4)(x-4)$

(c) $(x+2)(x-2)$

(d) Does not exist

6. The factors of $\sqrt{3}x^2+11x+6\sqrt{3}$ are __.

(a) $\left(x-3\sqrt{3}\right)\left(\sqrt{3}x+2\right)$

(b) $\left(x-3\sqrt{3}\right)\left(\sqrt{3}x-2\right)$

(c) $\left(x+3\sqrt{3}\right)\left(\sqrt{3}x-2\right)$

(d) $\left(x+3\sqrt{3}\right)\left(\sqrt{3}x+2\right)$

7. Factors of $x^4-(x-z)^4$ are __.

(a) $(2x+z)(2x^3+z^3-2x^2)$

(b) $z(x+2z)(x^2+z^2-x^2)$

(c) $z(2x-z)(2x^2-2xz+z^2)$

(d) $z(x-2z)(2z^2-2xz+x^2)$

8. Factorising $(x-y)^2+4xy-z^2$, we get

(a) $(x+y+z)(x+y-z)$

(b) $(y-y-z)(x+y-z)$

(c) $(x-y+z)(x+y-z)$

(d) None of these

9. The factors of $x^4+y^4+x^2y^2$ are _____.

(a) $(x^2+y^2)(x^2+y^2-xy)$

(b) $(x^2+y^2)(x^2-y^2)$

(c) $(x^2+y^2+xy)(x^2+y^2-xy)$

(d) Factorisation is not possible

10. For x^2+2x+5 to be a factor of $x4+px^2+q$, the values of p and q must be _____.

(a) – 2, 5 (b) 5, 25

(c) 10, 20 (d) 6, 25

11. One of the factors of 4 (x + y) (3a – b) + 6 (x + y) (2b – 3a) is

(a) (2b – 3a) (b) (3a – b)

(c) (4a – 3b) (d) (– 3a + 4b)

12. Divide $(32x^4y^3-16x^3y^4)$ by $(-8x^2y)$

(a) $4x^3y^2+2xy^3$ (b) $4x^3y-2xy^3$

(c) $-4x^2y^2+2xy^3$ (d) $-4xy^2+2xy^3$

13. One of the factors of $((p+q)^2-(a-b)^2+p+q-a+b$ is

(a) (p + q + a + b) (b) (p + q –a + b)

(c) (p – q + a – b) (d) (p – q + a + b)

14. Factorise $(2x+3y)^2-5(2x+3y)$ - 14.

(a) $4(2x+3y)(x+y-2)$

(b) $4(2x+3y)(x+y+2)$

(c) $(2x-3y+7)(2x-3y+2)$

(d) $(2x+3y-7)(2x+3y+2)$

15. Simplify:- $\dfrac{-14x^{12}y+8x^5z}{2x^2}$

(a) $x^3(-7x^7y+4z)$ (b) $x^2(7x^7y-4z)$

(c) $x^2(-7x^6y+2z)$ (d) $x^3(-7x^7y+4z)$

ACHIEVERS SECTION (HOTS)

16. Which of the following is the factor of $12(a^2+7a)^2-8(a^2+7a)(2a-1)-15(2a-1)^2$?

(i) $(2a^2+8a+3)$ (ii) $(6a^2+52a-5)$

(iii) (3a+5)

(a) Only (i) (b) Both (i) and (ii)

(c) Only (ii) (d) All (i), (ii) and (iii)

17. Fill in the blanks.

(i) $\dfrac{a^2-b^2}{a(a-b)}-\dfrac{ab^2+a^2b}{ab^2}$ is equal to $\underline{\ P\ }$.

(ii) $\dfrac{64y^4+8y^3}{4y^3}$ is equal to $\underline{\ Q\ }$.

(iii) When we divide $(38a^3b^3c^2-19a^4b^2c)$ by $19a^2bc$, the result is kab^2c-a^2b. Then $k=\underline{\ R\ }$

	P	Q	R
(a)	$\dfrac{(a+b)(b-a)}{ab}$	3(8y+1)	2
(b)	$\dfrac{(a+b)(b-a)}{ab}$	3(8y+1)	1
(c)	$\dfrac{(a+b)(a-b)}{ab}$	2(8y+1)	1
(d)	$\dfrac{(a+b)(b-a)}{ab}$	2(8y+1)	2

18. Which of the following statements is CORRECT?

(a) The factors of an expression are always either algebraic variable or algebraic expression.

(b) An irreducible factor is a factor that cannot be expressed further as a product of factors.

(c) Every binomial expression can be factorised into two monomial expression.

(d) The process of writing a given expression as the product of two or more factors is called multiplication of factors.

19. Match the expression given in Column-I to one of their factors given in Column-II.

Column – I	Column – II
(P) $9x^2+24x+16$	(i) $(2x-4)$
(Q) $25x^2+30x+9$	(ii) $(4x+1)$
(R) $40x^2+14x+1$	(iii) $(5x+3)$
(S) $4x^2-16x+16$	(iv) $(3x+4)$

(a) P→(iv); Q→(iii); R→(ii); S→(i)

(b) P→(iii): Q→(i); R→(iv); S→(ii)

(c) P→(ii); Q→(i); R→(iv): S→(iii)

(d) P→(iv); Q→(iii); R→(i); S→(ii)

20. Do as directed.

(i) Factorise: $x^2+\dfrac{1}{x^2}-3$

(ii) Find the greatest common factors of $14x^2y^3, 21x^3y^2$ and $35x^4y^5z$.

(iii) Divide $z(5z^2-80)$ by $5z(z+4)$.

	(i)	(ii)	(iii)
(a)	$\left(x-\dfrac{1}{x}\right)\left(x-\dfrac{1}{x}-2\right)$	$7xy^2$	$z-4$
(b)	$\left(x+\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}+2\right)$	$7x^2y$	$z-4$
(c)	$\left(x-\dfrac{1}{x}+1\right)\left(x-\dfrac{1}{x}-1\right)$	$7x^2y^2$	$z-4$
(d)	$\left(x-\dfrac{1}{x}-1\right)\left(x+\dfrac{1}{x}+1\right)$	$7x^2y^2$	$z-2$

ANSWER KEY

1.	A	**2.**	C	**3.**	C	**4.**	B	**5.**	B
6.	D	**7.**	C	**8.**	A	**9.**	C	**10.**	D
11.	D	**12.**	C	**13.**	B	**14.**	D	**15.**	A
16.	B	**17.**	D	**18.**	B	**19.**	A	**20.**	C

HINTS & EXPLANATIONS

1. (a) : We have.
$xy - pq + qy - px = xy - px + qy - pq$
$= x(y-p) + q(y-p) = (y-p)(x+q)$

2. (c) : We have,
$(x^2+3x+5)(x^2-3x+5) = m^2 - n^2$
$\Rightarrow (x^2+5+3x)(x^2+5-3x) = m^2 - n^2$
$\Rightarrow (x^2+5)^2 - (3x)^2 = m^2 - n^2$
$\Rightarrow m^2 = (x^2+5)^2 \Rightarrow m = x^2 + 5$

3. (c) : We have, $\frac{x^2}{4} - \frac{y^2}{9} = \left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2$
$= \left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)$

4. (b) : We have, $15x^2 - 26x + 8$
$= 15x^2 - 20x - 6x + 8 = 5x(3x-4) - 2(3x-4)$
$= (3x-4)(5x-2)$

5. (b)

6. (d) : We have, $\sqrt{3}x^2 + 11x + 6\sqrt{3}$
$= \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$
$= \sqrt{3}x(x+3\sqrt{3}) + 2(x+3\sqrt{3})$
$= (x+3\sqrt{3})(\sqrt{3}x+2)$

7. (c) : $x^4 - (x-z)^4 = (x^2)^2 - [(x-z)^2]^2$
$= [x^2 + (x-z)^2][x^2 - (x-z)^2]$
$= [x^2 + (x-z)^2][\{x + (x-z)\}\{x - (x-z)\}]$
$= [x^2 + (x-z)^2][(2x-z)(z)]$
$= (2x-z)z(2x^2 + z^2 - 2xz)$

8. (a) : We have, $(x-y)^2 + 4xy - z^2$
$= x^2 + y^2 - 2xy + 4xy - z^2 = (x+y)^2 - z^2$
$= (x+y+z)(x+y-z)$

9. (c) : We have, $x^4 + y^4 + x^2y^2 + x^2y^2 - x^2y^2$
$= x^4 + y^4 + 2x^2y^2 - x^2y^2$
$= (x^2+y^2)^2 - (xy)^2 = (x^2+y^2+xy)(x^2+y^2-xy)$

10. (d) : $x^2 + 2x + 5$ be a factor of $x^4 + px^2 + q$
So, other be $x^2 - 2x + 5$
So $(x^2 - 2x + 5)(x^2 + 2x + 5)$
$= (x^2+5)^2 - 4x^2 = x^4 + 10x^2 + 25 - 4x^2$
$= x^4 + 6x^2 + 25$
So, $p = 6$, $q = 25$

11. (d) : We have,
$4(x+y)(3a-b) + 6(x+y)(2b-3a)$
$= 2(x+y)[2(3a-b) + 3(2b-3a)]$
$= 2(x+y)[6a - 2b + 6b - 9a]$
$= 2(x+y)[-3a + 4b]$

12. (c) : $\frac{32x^4y^3 - 16x^3y^4}{-8x^2y} = \frac{32x^4y^3}{-8x^2y} + \frac{16x^3y^4}{8x^2y}$
$= -4x^2y^2 + 2xy^3$

13. (b) : We have.
$(p+q)^2 - (a-b)^2 + p + q - a + b$
$= [(p+q)^2 - (a-b)^2] + (p+q) - (a-b)$
$= [\{(p+q) + (a-b)\}\{(p+q) - (a-b)\}]$
$+\{(p+q) - (a-b)\}$
$= \{(p+q) - (a-b)\}[(p+q) + (a-b) + 1]$
$= (p+q-a+b)(p+q+a-b+1)$

14. (d) : We have, $(2x+3y)^2 - 5(2x+3y) - 14$
$= (2x+3y)^2 - 7(2x+3y) + 2(2x+3y) - 14$
$= (2x+3y)(2x+3y-7) + 2(2x+3y-7)$
$= (2x+3y+2)(2x+3y-7)$

15. (a) : $\dfrac{-14x^{12}y+8x^5z}{2x^2}=\dfrac{2x^2(-7x^{10}y+4x^3z)}{2x^2}$

$=x^3(-7x^7y+4z)$

16. (b) : We have,

$12(a^2+7a)^2-8(a^2+7a)(2a-1)-15(2a-1)^2$

$=12(a^2+7a)^2-18(a^2+7a)(2a-1)$
$\quad +10(a^2+7a)(2a-1)-15(2a-1)^2$

$=6(a^2+7a)[2(a^2+7a)-3(2a-1)]$
$\quad +5(2a-1)[2a^2+7a)-3(2a-1)]$

$=(6a^2+42a)(2a^2+8a+3)$
$\quad +(10a-5)(2a^2+8a+3)$

$=(6a^2+42a+10a-5)(2a^2+8a+3)$

$=(6a^2+52a-5)(2a^2+8a+3)$

17. (d) : (i) Reduce to lowest terms,

$\dfrac{a^2-b^2}{a(a-b)}-\left[\dfrac{ab^2+a^2b}{ab^2}\right]$

$=\dfrac{(a-b)(a+b)}{a(a-b)}-\left[\dfrac{ab(b+a)}{(ab)b}\right]$

$=\dfrac{a+b}{a}-\dfrac{(b+a)}{b}=\dfrac{(a+b)\times b-(a+b)\times a}{ab}$

$=\dfrac{(a+b)(b-a)}{ab}$

(ii) $\dfrac{64y^4+8y^3}{4y^3}=\dfrac{4y^3(16y+2)}{4y^3}$

$=16y+2=2(8y+1)$

(iii) $\dfrac{38a^3b^3c^2-19a^4b^2c}{19a^2bc}$

$=\dfrac{38a^3b^3c^2}{19a^2bc}-\dfrac{19a^4b^2c}{19a^2bc}$

$=2ab^2c-a^2b=kab^2c-a^2b$

$\therefore k=2$

18. (b) :

19. (a) : P. We have,

$9x^2+24x+16=(3x)^2+2(3x)(4)+(4)^2$

$=(3x+4)^2=(3x+4)(3x+4)$

Q. We have,

$25x^2+30x+9=(5x)^2+2(5x)(3)+(3)^2$

$=(5x+3)^2=(5x+3)(5x+3)$

R. We have,

$40x^2+14x+1=40x^2+10x+4x+1$

$=10x(4x+1)+1(4x+1)=(10x+1)(4x+1)$

S. We have,

$4x^2-16x+16=(2x)^2-2(2x)(4)+(4)^2$

$=(2x-4)^2=(2x-4)(2x-4)$

20. (c) : (i) We have, $x^2+\dfrac{1}{x^2}-3$

$=x^2+\dfrac{1}{x^2}-2+2-3=\left(x-\dfrac{1}{x}\right)^2-(1)^2$

$=\left(x-\dfrac{1}{x}+1\right)\left(x-\dfrac{1}{x}-1\right)$

(ii) The greatest common factor of $14x^2y^3, 21x^3y^2$ and $35x^4y^5z$ is $7x^2y^2$

(iii) $z(5z^2-80)=5z(z^2+16)=5z(z+4)(z-4)$

Thus, $z(5z^2-80)\div 5z(z+4)=\dfrac{z(5z^2-80)}{5z(z+4)}$

$=\dfrac{5z(z+4)(z-4)}{5z(z+4)}=z-4$