13. Method of Integration

Exercise 13A

1. Question

Evaluate the following integrals:

$$\int (2x+9)^5 \, dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x + 9 = t ⇒ 2 dx = dt

$$\int t^5 \left(\frac{dt}{2}\right) = \frac{1}{2}\int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{t^6}{12} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

2. Question

Evaluate the following integrals:

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 7 – 3x = t ⇒ -3 dx = dt

$$\int t^4 \left(\frac{dt}{-3}\right) = \frac{1}{-3}\int t^4 dt = \frac{1}{-3}\frac{t^5}{5} + c = -\frac{t^5}{15} + c$$

$$= -\frac{(7-3x)^5}{15} + c$$

3. Question

Evaluate the following integrals:

$$\int \sqrt{3x-5} \, dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3x - 5 = t ⇒ 3 dx = dt

$$\int t^{0.5} \left(\frac{dt}{3}\right) = \frac{1}{3}\int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c$$

$$= \frac{2(3x-5)^5}{9} + c$$

4. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{4x+3}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 4x + 3 = t ⇒ 4 dx = dt

$$\int t^{-0.5}\left(\frac{dt}{4}\right) = \frac{1}{4}\int t^{-0.5}dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c$$

$$= \frac{\sqrt{4x+3}}{2} + c$$

5. Question

Evaluate the following integrals:

$$\int \frac{1}{\sqrt{3-4x}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3 – 4x = t ⇒ -4 dx = dt

$$\int t^{-0.5}\left(\frac{dt}{-4}\right) = \frac{1}{-4}\int t^{-0.5}dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c$$

$$= -\frac{\sqrt{3-4x}}{2} + c$$

6. Question

Evaluate the following integrals:

$$\int \frac{1}{(2x-3)^{3/2}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x – 3 = t ⇒ 2 dx = dt

$$\int t^{-\frac{3}{2}}\left(\frac{dt}{2}\right) = \frac{1}{2}\int t^{-\frac{3}{2}}dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c$$

$$= -\frac{1}{\sqrt{2x-3}} + c$$

7. Question

Evaluate the following integrals:

$\int e^{(2x-1)} dx$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 2x – 1 = t ⇒ 2 dx = dt

$$\int e^t (\frac{dt}{2}) = \frac{1}{2}\int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c$$

$$= \frac{e^{(2x-1)}}{2} + c$$

8. Question

Evaluate the following integrals:

$\int e^{(1-3x)} dx$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 1 – 3x = t ⇒ -3 dx = dt

$$\int e^t (\frac{dt}{-3}) = \frac{1}{-3}\int e^t dt = \frac{1}{-3} \times e^t + c = \frac{e^{1-3x}}{-3} + c$$

$$= -\frac{e^{(1-3x)}}{3} + c$$

9. Question

Evaluate the following integrals:

$\int 3^{(2-3x)} dx$

Answer

Formula = $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put 2 – 3x = t ⇒ -3 dx = dt

$$\int 3^t (\frac{dt}{-3}) = \frac{1}{-3}\int 3^t dt = \frac{1}{-3} \times (\frac{3^t}{\log 3}) + c = \frac{3^t}{-3\log 3} + c$$

$$= -\frac{3^{(2-3x)}}{3\log 3} + c$$

10. Question

Evaluate the following integrals:

$\int \sin 3x \, dx$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put 3x = t ⇒ 3 dx = dt

$$\int \sin t\,(\frac{dt}{3}) = \frac{1}{3}\int \sin t\,dt = \frac{1}{3}\times(-\cos t)+c = \frac{-\cos 3x}{3}+c$$

$$= -\frac{\cos 3x}{3}+c$$

11. Question

Evaluate the following integrals:

$$\int \cos(5+6x)\,dx$$

Answer

Formula = $\int \cos x\,dx = \sin x + c$

Therefore ,

Put 5 + 6x = t ⇒ 6 dx = dt

$$\int \cos t\,(\frac{dt}{6}) = \frac{1}{6}\int \cos t\,dt = \frac{1}{6}\times(\sin t)+c = \frac{\sin 5+6x}{6}+c$$

$$= \frac{\sin(5+6x)}{6}+c$$

12. Question

Evaluate the following integrals:

$$\int \sin x\sqrt{1+\cos 2x}\,dx$$

Answer

Formula $\int \cos x\,dx = \sin x + c$

$1+\cos 2x = 2cos^2 x$

Therefore ,

$$\int \sin x\sqrt{1+\cos 2x}\,dx = \int \sin x\sqrt{2}\cos x + c$$

$$\int \sqrt{2}\sin x\cos x\ dx$$

Put sin x =t ⇒ cos x dx = dt

$$\int \sqrt{2}\sin x\cos x\ dx = \int \sqrt{2}t\,dt = \sqrt{2}\,\frac{t^2}{2}+c$$

$$= \frac{(\sin x)^2}{\sqrt{2}}+c$$

13. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^2(2x+5)\,dx$$

Answer

Formula $\int cosec^2 x\,dx = -\cot x + c$

Therefore ,

Put 2x + 5 =t ⇒ 2 dx = dt

$$\int cosec^2 t \,\frac{dt}{2} = -\frac{1}{2}\cot t + c = -\frac{1}{2}\cot(2x+5) + c$$

$$= -\frac{1}{2}\cot(2x+5) + c$$

14. Question

Evaluate the following integrals:

$$\int \sin x \cos x \, dx$$

Answer

Formula $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} + c$$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x \, dx$$

Answer

Formula $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t^3 \, dt = \frac{t^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

16. Question

Evaluate the following integrals:

$$\int \left(\sqrt{\cos x}\right) \sin x \, dx$$

Answer

Formula $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put cos x =t ⇒ -sin x dx = dt

$$\int t^{0.5}(-1) dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^{\frac{3}{2}}}{3} + c$$

17. Question

Evaluate the following integrals:

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Answer

Formula $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$ $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Therefore ,

Put $\sin^{-1} x$ =t ⇒ $\frac{1}{\sqrt{1-x^2}}$ dx = dt

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

18. Question

Evaluate the following integrals:

$$\cdot \int \frac{\sin\left(2\tan^{-1} x\right)}{\left(1+x^2\right)} dx \cdot$$

Answer

Formula $\int \sin t \, dx = -\cos t + c$ $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

Therefore ,

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\int \sin 2t \, dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2\tan^{-1} x)}{2} + c$$

19. Question

Evaluate the following integrals:

$$\int \frac{\cos(\log x)}{x} dx$$

Answer

Formula $\int \cos t \, dx = \sin t + c$ $\frac{d(log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \cos t\, dt = \sin t + c$$

$$= \sin(\log x) + c$$

20. Question

Evaluate the following integrals:

$$\int \frac{\operatorname{cosec}^2(\log x)}{x} dx$$

Answer

Formula $\int cosec^2 x\, dx = -\cot x + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int cosec^2 t \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$$

$$= -\cot(\log x) + c$$

21. Question

Evaluate the following integrals:

$$\int \frac{1}{x \log x} dx$$

Answer

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$$

$$= \log(\log x) + c$$

22. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx$$
$$= \int \left(1+\frac{1}{x}\right) \times \frac{(x+\log x)^2}{1} dx$$

Therefore ,

Put $x + \log x = t \Rightarrow (1+\frac{1}{x}) dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x + \log x)^3}{3} + c$$

23. Question

Evaluate the following integrals:

$$\int \frac{(\log x)^2}{x} dx$$

Answer

Formula $\frac{d(logx)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$$= \frac{(\log x)^3}{3} + c$$

24. Question

Evaluate the following integrals:

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

Answer

Formula $\int \cos t \, dx = \sin t + c \; \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Therefore ,

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\int \cos t \; 2dt = 2 \sin t + c$$

$$= 2 \sin(\sqrt{x}) + c$$

25. Question

Evaluate the following integrals:

$$\int e^{\tan x} \sec^2 x \, dx$$

Answer

Formula = $\int e^x dx = e^x + c \; \frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

Put tan x = t $\Rightarrow sec^2 x$ dx = dt

$$\int e^t dt = e^t + c$$

$= e^{\tan x} + c$

26. Question

Evaluate the following integrals:

$$\int e^{\cos^2 x} \sin 2x \, dx$$

Answer

Formula = $\int e^x dx = e^x + c$ $\frac{d(cos^2 x)}{dx} = 2\cos x\,(-\sin x) = -\sin 2x$

Therefore ,

Put $cos^2 x$ = t ⇒ $-\sin 2x$ dx = dt

$$\int -e^t dt = -e^t + c$$

$= -e^{cos^2 x} + c$

27. Question

Evaluate the following integrals:

$$\int \sin(ax + b)\cos(ax + b)dx$$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put ax+b = t ⇒ adx = dt

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a}\int \sin t \cos t \, dt$$

Put sin t = z � cos t dt = dz

$$\frac{1}{a}\int z dz = \frac{1}{a} \times \frac{z^2}{2} + c$$

$$= \frac{(\sin ax + b)^2}{2a} + c$$

28. Question

Evaluate the following integrals:

$$\int \cos^3 x \, dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

$\cos 3x = 3\cos x - 4cos^3 x$

Therefore ,

$$\int (\frac{3\cos x}{4} - \frac{\cos 3x}{4})dx = \frac{3\sin x}{4} - \frac{\sin 3x}{4 \times 3} + c$$

$$= \frac{3\sin x}{4} - \frac{\sin 3x}{12} + c$$

29. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} e^{-1/x} dx$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int e^t (dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$$

$$= e^{-\frac{1}{x}} + c$$

30. Question

Evaluate the following integrals:

$$\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

Answer

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int \cos t\,(dt) = \int \cos t\, dt = \sin t + c = \sin(-\frac{1}{x}) + c$$

$$= -\sin\frac{1}{x} + c$$

31. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(e^x + e^{-x}\right)}$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

$$\int \frac{e^x}{1 + e^{2x}} dx$$

Put e^x = t ⇒ $e^x dx = dt$

$$\int \frac{1}{1+t^2}(dt) = \int \frac{1}{1+t^2} dt = \tan^{-1} t + c$$

$$= \tan^{-1}(e^x) + c$$

32. Question

Evaluate the following integrals:

$$\int \frac{e^{2x}}{\left(e^{2x}-2\right)} dx$$

Answer

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $e^{2x} - 2$ = t ⇒ $2e^{2x} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(e^{2x} - 2) + c$$

33. Question

Evaluate the following integrals:

$$\int \cot x \log(\sin x) dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x \, dx = dt$

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log \sin x)^2}{2} + c$$

34. Question

Evaluate the following integrals:

$$\int \frac{\cot x}{\log(\sin x)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x \, dx = dt$

$$\int \frac{1}{t} dt = \log t + c$$

$$= \log(\log \sin x) + c$$

35. Question

Evaluate the following integrals:

$$\int 2x \sin\left(x^2 + 1\right) dx$$

Answer

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put $x^2 + 1 = t \Rightarrow 2x\, dx = dt$

$$\int \sin t \;\; dt = -\cos t + c$$

$= -\cos(x^2 + 1) + c$

36. Question

Evaluate the following integrals:

$$\int \sec\,x \log\left(\sec x + \tan x\right) dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sec x + tan x)= t

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + sec^2 x)\; dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x\; (\sec x\; + \tan x)\; dx = dt$$

Sec x dx = dt

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\tan\sqrt{x}\,\sec^2\sqrt{x}}{\sqrt{x}}\,dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\tan\sqrt{x}$= t

$$sec^2\sqrt{x} \times (\frac{1}{2\sqrt{x}})\; dx = dt$$

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan\sqrt{x})^2}{2} + c$$

38. Question

Evaluate the following integrals:

$$\int \frac{x \tan^{-1} x^2}{\left(1+x^4\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\tan^{-1} x^2$ = t ⇒ $\frac{1}{1+(x^2)^2} \times 2x \times dx = dt$ � $\frac{2x}{1+x^4} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

39. Question

Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^2$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt$ � $\frac{2x}{\sqrt{1-x^4}} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

40. Question

Evaluate the following integrals:

$$\int \frac{1}{\left(\sqrt{1-x^2}\right)\sin^{-1} x} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^1$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^1}} \times dx = dt$ � $\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{1}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

41. Question

Evaluate the following integrals:

$$\int \frac{\sqrt{(2+\log x)}}{x} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put 2 + log x = t⇒ $\frac{1}{x} \times dx = dt$

$$\int \sqrt{t}\left(\frac{dt}{1}\right) = \int \sqrt{t} dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2+\log x)^{\frac{3}{2}}}{3} + c$$

42. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{(1+\tan x)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put 1 + tan x = t⇒ $sec^2 x \times dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(1+\tan x) + c$$

43. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1+\cos x)} dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 1 + cos x = t⇒ $-\sin x \times dx = dt$

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$$= -\log(1+\cos x) + c$$

44. Question

Evaluate the following integrals:

$$\int\left(\frac{1+\tan x}{1-\tan x}\right)dx$$

Answer

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

$$\int\left(\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}\right)dx = \int\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)dx$$

Put cos x - sin x = t ⇒ (- cos x - sin x) dx = dt

$$\int\left(\frac{-dt}{t}\right) = -\int\frac{1}{t}dt = -\log t + c$$

$$= -\log(\cos x - \sin x) + c$$

45. Question

Evaluate the following integrals:

i. $\int\frac{(1+\tan x)}{(x+\log\sec x)}dx$

ii. $\int\frac{(1-\sin 2x)}{(x+\cos^2 x)}dx$

Answer

(i)

Formula = $\int\frac{1}{x}dx = \log x + c$

Therefore ,

Put x + log (sec x) = t⇒$1 + \frac{1}{\sec x} \times \sec x\ \tan x\, dx = dt$

$(1+\tan x)dx = dt$

$$\int\left(\frac{dt}{t}\right) = \int\frac{1}{t}dt = \log t + c$$

$$= \log(x + \log(\sec x)) + c$$

(ii)

Formula = $\int\frac{1}{x}dx = \log x + c$

Therefore ,

Put $x + cos^2 x$ = t ⇒$1 + 2\cos x \times (-\sin x)dx = dt$

$(1-\sin 2x)dx = dt$

$$\int\left(\frac{dt}{t}\right) = \int\frac{1}{t}dt = \log t + c$$

$$= \log(x + cos^2 x) + c$$

46. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{\left(a^2 + b^2 \sin^2 x\right)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 + b^2 sin^2 x = t$ � $b^2 \times 2 \sin x \times \cos x \, dx = dt$

$(b^2 \sin 2x) dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2}\right) = \frac{1}{b^2}\int \frac{1}{t} dt = \frac{1}{b^2}\log t + c$$

$$= \frac{1}{b^2}\log|a^2 + b^2 sin^2 x| + c$$

47. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)} dx$$

Answer

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 cos^2 x + b^2 sin^2 x = t$

$(a^2 \times 2 \cos x \times (-\sin x) + b^2 \times 2 \sin x \times \cos x) dx = dt$

$(b^2 - a^2) \sin 2x \ dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2 - a^2}\right) = \frac{1}{b^2 - a^2}\int \frac{1}{t} dt = \frac{1}{b^2 - a^2}\log t + c$$

$$= \frac{1}{b^2 - a^2}\log|a^2 cos^2 x + b^2 sin^2 x| + c$$

48. Question

Evaluate the following integrals:

$$\int \left(\frac{2\cos x - 3\sin x}{3\cos x + 2\sin x}\right) dx$$

Answer

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 3cos x + 2sin x = t⇒ (2cos x - 3sin x) dx = dt

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(3\cos x + 2\sin x) + c$$

49. Question

Evaluate the following integrals:

$$\int \frac{4x}{\left(2x^2+3\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $2x^2 + 3 = t \Rightarrow (4x)\,dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(2x^2 + 3) + c$$

50. Question

Evaluate the following integrals:

$$\int \frac{(x+1)}{\left(x^2+2x-3\right)} dx$$

Answer

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $x^2+2x+3= t \Rightarrow (2x+2)\,dx = dt$ � $2(x+1)dx=dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(x^2 + 2x + 3) + c$$

51. Question

Evaluate the following integrals:

$$\int \frac{(4x-5)}{\left(2x^2-5x+1\right)} dx$$

Answer

To find: Value of $\int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$ ... (i)

Let $2x^2 - 5x + 1 = t$

$$\Rightarrow \frac{d(2x^2 - 5x + 1)}{dx} = \frac{dt}{dx}$$

$\Rightarrow 4x - 5 = \frac{dt}{dx}$

$\Rightarrow (4x - 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$2x^2 - 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|2x^2 - 5x + 1| + c$

Ans) $\log|2x^2 - 5x + 1| + c$

52. Question

Evaluate the following integrals:

$$\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$$

Answer

To find: Value of $\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$ ... (i)

Let $3x^3 - 2x^2 + 5x + 1 = t$

$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$

$\Rightarrow (9x^2 - 4x + 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$3x^3 - 2x^2 + 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|3x^3 - 2x^2 + 5x + 1| + c$

Ans) $\log|3x^3 - 2x^2 + 5x + 1| + c$

53. Question

Evaluate the following integrals:

$$\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$$

Answer

To find: Value of $\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ ... (i)

Let $\log(\tan x)$ = t

$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(\log(\tan x))}{d\tan x} \frac{d\tan x}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{\tan x} \sec^2 x = \frac{dt}{dx}$

$\Rightarrow \sec x \operatorname{cosec} x = \frac{dt}{dx}$

$\Rightarrow (\sec x \operatorname{cosec} x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}\ [\log(\tan x) = t]$

$I = \log|t| + c$

$I = \log|\log(\tan x)| + c$

Ans) $\log|\log(\tan x)| + c$

54. Question

Evaluate the following integrals:

$$\int \frac{(1+\cos x)}{(x+\sin x)^3} dx$$

Answer

To find: Value of $\int \frac{(1+\cos x)}{(x+\sin x)^3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(1+\cos x)}{(x+\sin x)^3} dx$ ... (i)

Let $x + \sin x = t$

$\Rightarrow \frac{d(x+\sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (1 + \cos x) = \frac{dt}{dx}$

$\Rightarrow (1 + \cos x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t^3}$ [$x + \sin x = t$]

$\Rightarrow I = -\frac{1}{2t^2} + c$

$I = -\frac{1}{2(x + \sin x)^2} + c$

Ans) $-\frac{1}{2(x + \sin x)^2} + c$

55. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(1 + \cos x)^2} dx$$

Answer

To find: Value of $\int \frac{\sin x}{(1 + \cos x)^2} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $1 + \cos x = t$

$\Rightarrow \frac{d(1 + \cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$

$\Rightarrow (-\sin x)dx = dt$

Putting this value in equation (i)

$I = \int -\frac{dt}{t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{t} + c$

$I = \frac{1}{1 + \cos x} + c$

Ans) $\frac{1}{1 + \cos x} + c$

56. Question

Evaluate the following integrals:

$$\int \frac{(2x + 3)}{\sqrt{x^2 + 3x - 2}} dx$$

Answer

To find: Value of $\int \frac{(2x+3)}{\sqrt{x^2+3x-2}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ … (i)

Let $x^2 + 3x - 2 = t$

$\Rightarrow (2x+3) = \frac{dt}{dx}$

$\Rightarrow (2x+3)\ dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{t}}\ [x^2 + 3x - 2 = t]$

$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$I = 2t^{\frac{1}{2}} + c$

$I = 2\sqrt{x^2 + 3x - 2} + c$

Ans) $2\sqrt{x^2 + 3x - 2} + c$

57. Question

Evaluate the following integrals:

$$\int \frac{(2x-1)}{\sqrt{x^2 - x - 1}} dx$$

Answer

To find: Value of $\int \frac{(2x-1)}{\sqrt{x^2-x-1}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ … (i)

Let $x^2 - x - 1 = t$

$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x - 1) = \frac{dt}{dx}$

$\Rightarrow (2x - 1)\ dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^{\frac{1}{2}}}\ [x^2 - x - 1 = t]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

Ans) $2\sqrt{x^2 - x - 1} + c$

58. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{x+a} + \sqrt{x+b}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\left(\sqrt{x+a}\right)^2 - \left(\sqrt{x+b}\right)^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x + a - x - b} dx$$

$$I = \frac{1}{a-b}\left[\int \sqrt{x+a}\, dx - \int \sqrt{x+b}\, dx\right]$$

$$I = \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}}\, dx - \int (x+b)^{\frac{1}{2}} dx\right]$$

$$I = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$

59. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{1-3x}-\sqrt{5-3x}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}} \times \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\sqrt{1-3x}+\sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2-\left(\sqrt{5-3x}\right)^2}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{(1-3x)-(5-3x)}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{1-3x-5+3x}dx$$

$$I = -\frac{1}{4}\left[\int \sqrt{1-3x}\,dx + \int \sqrt{5-3x}\,dx\right]$$

$$I = -\frac{1}{4}\left[\int (1-3x)^{\frac{1}{2}}\,dx + \int (5-3x)^{\frac{1}{2}}dx\right]$$

$$I = -\frac{1}{4}\left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)}\right]$$

$$I = -\frac{2}{-9\times 4}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

$$I = \frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$

60. Question

Evaluate the following integrals:

$$\int \frac{x^2}{\left(1+x^6\right)}dx$$

Answer

To find: Value of $\int \frac{x^2}{(1+x^6)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

We have, $I = \int \frac{x^2}{(1+x^6)} dx$ ... (i)

$$I = \int \frac{x^2}{1+(x^3)^2} dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \frac{1}{3}\int \frac{dt}{1+t^2} \; [\, 1+\cos x = t \,]$$

$$\Rightarrow I = \frac{1}{3}\tan^{-1}(t) + c$$

$$I = \frac{1}{3}\tan^{-1}(x^3) + c$$

Ans) $\frac{1}{3}\tan^{-1}(x^3) + c$

61. Question

Evaluate the following integrals:

$$\int \frac{x^3}{\left(1+x^8\right)} dx$$

Answer

To find: Value of $\int \frac{x^3}{(1+x^8)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^3}{(1+x^8)} dx$ ... (i)

$$I = \int \frac{x^3}{1+(x^4)^2} dx$$

Let $x^4 = t$

$$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (4x^3) = \frac{dt}{dx}$$

$$\Rightarrow (x^3)dx = \frac{dt}{4}$$

Putting this value in equation (i)

$I = \frac{1}{4}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{4}\tan^{-1}(t) + c$

$I = \frac{1}{4}\tan^{-1}(x^4) + c$

Ans) $\frac{1}{4}\tan^{-1}(x^4) + c$

62. Question

Evaluate the following integrals:

$$\int \frac{x}{\left(1+x^4\right)} dx$$

Answer

To find: Value of $\int \frac{x}{(1+x^4)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x}{(1+x^4)} dx$ ... (i)

$I = \int \frac{x}{1+(x^2)^2} dx$

Let $x^2 = t$

$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x) = \frac{dt}{dx}$

$\Rightarrow (x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{2}\tan^{-1}(t) + c$

$I = \frac{1}{2}\tan^{-1}(x^2) + c$

Ans) $\frac{1}{2}\tan^{-1}(x^2) + c$

63. Question

Evaluate the following integrals:

$$\int \frac{x^5}{\sqrt{1+x^3}} dx$$

Answer

To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ ... (i)

Let $1 + x^3 = t$

$\Rightarrow x^3 = t - 1$

$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$

$\Rightarrow (3x^2) = \frac{dt}{dx}$

$\Rightarrow x^2\, dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \frac{x^3 x^2}{\sqrt{1 + x^3}} dx$

$I = \int \frac{(t-1)}{t^{\frac{1}{2}}} \frac{dt}{3}$ [$1 + x^3 = t$]

$\Rightarrow I = \frac{1}{3} \int \frac{t}{t^{\frac{1}{2}}} dt - \frac{1}{3} \int \frac{1}{t^{\frac{1}{2}}} dt$

$\Rightarrow I = \frac{1}{3} \left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt \right]$

$\Rightarrow I = \frac{1}{3} \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right]$

$\Rightarrow I = \frac{2}{3} \left[\frac{(1 + x^3)^{\frac{3}{2}}}{3} - \frac{(1 + x^3)^{\frac{1}{2}}}{1} \right]$

$\Rightarrow I = \frac{2(1 + x^3)^{\frac{3}{2}}}{9} - \frac{2(1 + x^3)^{\frac{1}{2}}}{3} + c$

Ans) $\frac{2(1 + x^3)^{\frac{3}{2}}}{9} - \frac{2(1 + x^3)^{\frac{1}{2}}}{3} + c$

64. Question

Evaluate the following integrals:

$\int \frac{x}{\sqrt{1+x}} dx$

Answer

To find: Value of $\int \frac{x}{\sqrt{1+x}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}} dx$ ... (i)

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \frac{t - 1}{\sqrt{t}} dx\ [1 + x = t]$$

$$\Rightarrow I = \int \sqrt{t} dt - \int \frac{1}{\sqrt{t}} dt$$

$$\Rightarrow I = \left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt\right]$$

$$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$$

$$\Rightarrow I = 2\left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1}\right] + c$$

$$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$$

Ans) $\frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

65. Question

Evaluate the following integrals:

$$\int \frac{1}{x\sqrt{x^4 - 1}} dx$$

Answer

To find: Value of $\int \frac{1}{x\sqrt{x^4-1}} dx$

Formula used: $\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx$ ... (i)

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2 - 1}} dx$$

Let $x^2 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2 - 1}}\ [x^2 = t]$

$\Rightarrow I = \frac{1}{2}\sec^{-1} t + c$

$\Rightarrow I = \frac{1}{2}\sec^{-1}(x^2) + c$

Ans) $\frac{1}{2}\sec^{-1}(x^2) + c$

66. Question

Evaluate the following integrals:

$\int x\sqrt{-1}\,dx$

Answer

To find: Value of $\int x\sqrt{x-1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x-1}\,dx$ ... (i)

Let $x - 1 = t$

x = t + 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int (t+1)\sqrt{t}\,dt\ [x = t + 1]$

$\Rightarrow I = \int t\sqrt{t}dx + \int \sqrt{t}\,dx$

$\Rightarrow I = \int t^{\frac{3}{2}}dx + \int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Ans) $\frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

67. Question

Evaluate the following integrals:

$\int(1-x)\sqrt{1+x}\,dx$

Answer

To find: Value of $\int (1 - x)\sqrt{1+x}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (1 - x)\sqrt{1+x}\,dx$ ... (i)

Let $1 + x = t$

x = t - 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \{1 - (t - 1)\}\sqrt{t}\,dt\ [\,x = t - 1\,]$

$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t}\,dt$

$\Rightarrow I = \int \{2 - t\}\sqrt{t}\,dt$

$\Rightarrow I = \int 2\sqrt{t}\,dt - \int t\sqrt{t}\,dt$

$\Rightarrow I = 2\int t^{\frac{1}{2}}dx - \int t^{\frac{3}{2}}\,dx$

$\Rightarrow I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$

$\Rightarrow I = \frac{4}{3}(1+x)^{\frac{3}{2}} - \frac{2}{5}(1+x)^{\frac{5}{2}} + c$

Ans) $\frac{4}{3}(1+x)^{\frac{3}{2}} - \frac{2}{5}(1+x)^{\frac{5}{2}} + c$

68. Question

Evaluate the following integrals:

$\int x\sqrt{x^2-1}\,dx$

Answer

To find: Value of $\int x\sqrt{x^2 - 1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x^2 - 1}\,dx$ ... (i)

Let $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{2}\sqrt{t}\,dt\ [\ x = x^2 - 1\ ]$

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\,t^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$

Ans) $\frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$

69. Question

Evaluate the following integrals:

$\int x\sqrt{3x-2}\,dx$

Answer

To find: Value of $\int x\sqrt{3x - 2}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{3x - 2}\,dx$ ... (i)

Let $3x - 2 = t$

$\Rightarrow 3x = t + 2$

$\Rightarrow x = \frac{t+2}{3}$

$\Rightarrow 3 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \left(\frac{t+2}{3}\right)\sqrt{t}\,\frac{dt}{3}\ [\ t = 3x - 2\ ]$

$\Rightarrow I = \frac{1}{9}\left[\int t^{\frac{3}{2}}\,dx\ + 2\int t^{\frac{1}{2}}\,dx\right]$

$\Rightarrow I = \frac{1}{9}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$

$$\Rightarrow I = \frac{1}{9}\left[\frac{2}{5}(3x-2)^{\frac{5}{2}} + \frac{4}{3}(3x-2)^{\frac{3}{2}}\right] + c$$

$$\Rightarrow I = \frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$$

Ans) $\frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$

70. Question

Evaluate the following integrals:

$$\int \frac{dx}{x\cos^2(1+\log x)}$$

Answer

To find: Value of $\int \frac{dx}{x\cos^2(1+\log x)}$

Formula used: $\int \sec^2 x\, dx = \tan x + c$

We have, $I = \int \frac{dx}{x\cos^2(1+\log x)}$ ... (i)

Let $1 + \log x = t$

$$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{x}dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)} \; [\, t = 1 + \log x \,]$$

$$\Rightarrow I = \int \sec^2 t\, dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(1 + \log x) + c$$

Ans) $\tan(1 + \log x) + c$

71. Question

Evaluate the following integrals:

$$\int x^2 \sin x^3 dx$$

Answer

To find: Value of $\int x^2 \sin x^3\, dx$

Formula used: $\int \sin x\, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3\, dx$ ... (i)

Let $x^3 = t$

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow x^2dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \sin t \frac{dt}{3}$ [$t = x^3$]

$\Rightarrow I = \frac{1}{3}\left[\int \sin t\, dt\right]$

$\Rightarrow I = \frac{1}{3}(-\cos t) + c$

$\Rightarrow I = \frac{1}{3}(-\cos x^3) + c$

Ans) $\frac{-\cos x^3}{3} + c$

72. Question

Evaluate the following integrals:

$$\int (2x+4)\sqrt{x^2+4x+3}\, dx$$

Answer

To find: Value of $\int (2x+4)\sqrt{x^2+4x+3}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (2x+4)\sqrt{x^2+4x+3}\, dx$ ... (i)

Let $x^2 + 4x + 3 = t$

$\Rightarrow (2x+4) = \frac{dt}{dx}$

$\Rightarrow (2x+4)dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\, dt$ [$t = (2x+4)$]

$\Rightarrow I = \int t^{\frac{1}{2}}\, dx$

$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right] + c$

Ans) $\frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right]+c$

73. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(\sin x-\cos x)}dx$$

Answer

To find: Value of $\int \frac{\sin x}{(\sin x - \cos x)}$ dx

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\sin x}{(\sin x - \cos x)}$ dx ... (i)

$$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x}{(\sin x - \cos x)} dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx$$

Let $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

74. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1-\tan x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\cos x dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\cos x - \sin x)}dx + \frac{1}{2}\int \frac{(\cos x - \sin x)}{(\cos x - \sin x)}dx$$

Let $(\cos x - \sin x) = t$

$$\Rightarrow (-\sin x - \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x + \cos x)dx = -dt$$

Putting this value in equation (i)

$$I = -\frac{1}{2}\int \frac{dt}{(t)}\,dx + \frac{1}{2}\int dx$$

$$\Rightarrow I = -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$

75. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1 - \cot x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$$

Let $(\sin x - \cos x) = t$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{(t)} dx + \frac{1}{2}\int dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

Ans) $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

76. Question

Evaluate the following integrals:

$$\int \frac{\cos 2x}{(\sin x + \cos x)} dx$$

Answer

To find: Value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

Let $(\cos x + \sin x) = t$

$$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$\Rightarrow I = \log|t| + c$

$\Rightarrow I = \log|\cos x + \sin x| + c$

Ans) $\log|\cos x + \sin x| + c$

77. Question

Evaluate the following integrals:

$$\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$$

Answer

To find: Value of $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$ ... (i)

$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} dx$

$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$

Let $(\sin x + \cos x) = t$

$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x - \sin x) dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t^2}$

$\Rightarrow I = -\frac{1}{t} + c$

$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$

Ans) $\frac{-1}{\sin x + \cos x} + c$

78. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x + \log x)^2}{x} dx$$

Answer

To find: Value of $\int \frac{(x+1)(x + \log x)^2}{x} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$ ... (i)

Let $(x + \log x) = t$

$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int t^2 dt$

$\Rightarrow I = \frac{t^3}{3} + c$

$\Rightarrow I = \frac{(x+\log x)^3}{3} + c$

Ans) $\frac{(x+\log x)^3}{3} + c$

79. Question

Evaluate the following integrals:

$\int x \sin^3 x^2 \cos x^2 dx$

Answer

To find: Value of $\int x\sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sin^3 x^2 \cos x^2 dx$ ... (i)

Let $(\sin x^2) = t$

$\Rightarrow (\sin x^2 . 2x) = \frac{dt}{dx}$

$\Rightarrow (\sin x^2 . x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int t^3 \frac{dt}{2}$

$I = \frac{1}{2}\int t^3 dt$

$\Rightarrow I = \frac{1}{2}\frac{t^4}{4} + c$

$\Rightarrow I = \frac{t^4}{8} + c$

$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$

Ans) $\frac{\sin^4 x^2}{8} + c$

80. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$$

Answer

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$ ... (i)

Let $(\tan x) = t$

$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$

$\Rightarrow (\sec^2 x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{1-t^2}}$

$\Rightarrow I = \sin^{-1}(t) + c$

$\Rightarrow I = \sin^{-1}(\tan x) + c$

Ans) $\sin^{-1}(\tan x) + c$

81. Question

Evaluate the following integrals:

$$\int e^{-x} \operatorname{cosec}^2 (2e^{-x} + 5) dx$$

Answer

To find: Value of $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx$

Formula used: $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

We have, $I = \int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx$ ... (i)

Let $(2e^{-x} + 5) = t$

$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$

$\Rightarrow (e^{-x})dx = \frac{dt}{-2}$

Putting this value in equation (i)

$$I = \int \operatorname{cosec}^2(t)\ \frac{dt}{-2}$$

$$I = \frac{1}{-2}\int \operatorname{cosec}^2(t)\ dt$$

$$\Rightarrow I = \frac{1}{-2}(-\cot t) + c$$

$$\Rightarrow I = \frac{1}{2}\cot(2e^{-x} + 5) + c$$

Ans) $\frac{1}{2}\cot(2e^{-x} + 5) + c$

82. Question

Evaluate the following integrals:

$$\int 2x\sec^3(x^2+3)\tan(x^2+3)dx$$

Answer

To find: Value of $\int 2x\sec^3(x^2+3)\ \tan(x^2+3)dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int 2x\sec^2(x^2+3)\sec(x^2+3)\ \tan(x^2+3)dx$ ... (i)

Let $\sec(x^2+3) = t$

$$\Rightarrow \sec(x^2+3) = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2\ dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{\sec^3(x^2+3)}{3} + c$$

Ans) $\frac{\sec^3(x^2+3)}{3} + c$

83. Question

Evaluate the following integrals:

$$\int \frac{\sin 2x}{(a+b\cos x)^2}dx$$

Answer

To find: Value of $\int \frac{\sin 2x}{(a+b\cos x)^2} dx$

Formula used: (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin 2x}{(a+b\cos x)^2} dx$ ... (i)

$$I = \int \frac{2\sin x\cos x}{(a+b\cos x)^2} dx$$

Let $(a + b\cos x) = t$

$$\Rightarrow (\cos x) = \frac{t-a}{b}$$

$$\Rightarrow (\sin x)dx = \frac{dt}{-b}$$

Putting this value in equation (i)

$$I = \frac{2}{-b^2} \int \frac{t-a}{t^2} dt$$

$$I = \frac{2}{-b^2} \left[\int \frac{t}{t^2} dt - \int \frac{a}{t^2} dt\right]$$

$$I = \frac{2}{-b^2} \left[\int \frac{1}{t} dt - a\int \frac{1}{t^2} dt\right]$$

$$I = \frac{2}{-b^2} \left[\log|t| - a\left(-\frac{1}{t}\right) + c\right]$$

$$I = -\frac{2}{b^2} \left[\log|a+b\cos x| + \left(\frac{a}{a+b\cos x}\right)\right] + c$$

Ans) $-\frac{2}{b^2} \left[\log|a+b\cos x| + \left(\frac{a}{a+b\cos x}\right)\right] + c$

84. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3-5x)}$$

Answer

To find: Value of $\int \frac{dx}{(3-5x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3-5x)}$ ... (i)

Let $(3 - 5x) = t$

$$\Rightarrow (-5) = \frac{dt}{dx}$$

$\Rightarrow dx = \frac{dt}{-5}$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{-5}$

$I = \frac{1}{-5} \int \frac{dt}{t}$

$\Rightarrow I = \frac{1}{-5} \log |t| + c$

$\Rightarrow I = -\frac{1}{5} \log |3 - 5x| + c$

Ans) $-\frac{1}{5} \log |3 - 5x| + c$

85. Question

Evaluate the following integrals:

$\int \sqrt{1 + x}\, dx$

Answer

To find: Value of $\int \sqrt{1 + x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \sqrt{1 + x}\, dx$ ... (i)

Let $(1 + x) = t$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\, dt$

$I = \int t^{\frac{1}{2}} dt$

$\Rightarrow I = \frac{2}{3}(1 + x)^{\frac{3}{2}} + c$

Ans) $\frac{2}{3}(1 + x)^{\frac{3}{2}} + c$

86. Question

Evaluate the following integrals:

$\int x^2 e^{x^3} \cos\left(e^{x^3}\right) dx$

Answer

To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3}) dx$

Formula used: $\int \cos x\, dx = \sin x + c$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3})dx$ ... (i)

Let $e^{x^3} = t$

$$\Rightarrow e^{x^3}.\,3x^2 = \frac{dt}{dx}$$

$$\Rightarrow e^{x^3}.\,x^2.\,dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \cos(t)\,\frac{dt}{3}$$

$$I = \frac{\sin(t)}{3} + c$$

$$I = \frac{\sin\left(e^{x^3}\right)}{3} + c$$

Ans) $\dfrac{\sin\left(e^{x^3}\right)}{3} + c$

87. Question

Evaluate the following integrals:

$$\int \frac{e^{m\,\tan^{-1}x}}{\left(1+x^2\right)}dx$$

Answer

To find: Value of $\int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$ ... (i)

Let $\left(m\tan^{-1}x\right) = t$

$$\Rightarrow m\left(\frac{1}{1+x^2}\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{1}{1+x^2}\right)dx = \frac{dt}{m}$$

Putting this value in equation (i)

$$I = \int e^t \frac{dt}{m}$$

$$\Rightarrow I = \frac{e^t}{m} + c$$

$$\Rightarrow I = \frac{e^{m\tan^{-1}x}}{m} + c$$

Ans) $\frac{e^{m\tan^{-1}x}}{m} + c$

88. Question

Evaluate the following integrals:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$$

Answer

To find: Value of $\int \frac{(x+1)e^x dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x dx}{\cos^2(xe^x)}$ ... (i)

Let $(xe^x) = t$

$$\Rightarrow xe^x + e^x.1 = \frac{dt}{dx}$$

$$\Rightarrow e^x(x+1) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)}$$

$$\Rightarrow I = \int \sec^2(t) dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(xe^x) + c$$

Ans) $\tan(xe^x) + c$

89. Question

Evaluate the following integrals:

$$\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx$$

Answer

To find: Value of $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$

Formula used: $\int \cos x \, dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$ ... (i)

Let $\left(e^{\sqrt{x}}\right) = t$

$$\Rightarrow e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting this value in equation (i)

$I = \int \cos(t)\, 2dt$

$I = 2 \sin\left(e^{\sqrt{x}}\right) + c$

Ans) $2 \sin\left(e^{\sqrt{x}}\right) + c$

90. Question

Evaluate the following integrals:

$\int \sqrt{e^x - 1}\, dx$

Answer

To find: Value of $\int \sqrt{e^x - 1}\ dx$

Formula used: $\int \frac{1}{x^2+1} dx = \tan^{-1} x + c$

We have, $I = \int \sqrt{e^x - 1}\ dx \ldots$ (i)

Let $(e^x - 1) = t^2$

$\Rightarrow e^x - 1 = t^2$

$\Rightarrow e^x = t^2 + 1$

$\Rightarrow e^x = \frac{2tdt}{dx}$

$\Rightarrow dx = \frac{2tdt}{e^x}$

$\Rightarrow dx = \frac{2t}{t^2+1} dt$

Putting this value in equation (i)

$I = \int \sqrt{t^2}\, \frac{2t}{t^2+1} dt$

$\Rightarrow I = \int \frac{2t^2}{t^2+1} dt$

$\Rightarrow I = 2 \int \frac{t^2+1-1}{t^2+1} dt$

$\Rightarrow I = 2 \int \left(1 - \frac{1}{t^2+1}\right) dt$

$\Rightarrow I = 2\,[\,t - \tan^{-1} t\,] + c$

$\Rightarrow I = 2\,[\,\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1}\,] + c$

Ans) $2\,[\,\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1}\,] + c$

91. Question

Evaluate the following integrals:

Answer

To find: Value of $\int \frac{dx}{(x - \sqrt{x})}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x - \sqrt{x})}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x} - 1)}$$

Let $(\sqrt{x} - 1) = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{2}$$

$$I = \frac{1}{2} \log|t| + c$$

$$I = \frac{1}{2} \log|\sqrt{x} - 1| + c$$

Ans) $\frac{1}{2} \log|\sqrt{x} - 1| + c$

92. Question

Evaluate the following integrals:

$$\int \frac{\sec^2\left(2\tan^{-1}x\right)}{\left(1+x^2\right)} dx$$

Answer

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$

Formula used: $\int \sec^2 x \, dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)} dx$ ... (i)

Let $2\tan^{-1}x = t$

$$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{1+x^2} dx = \frac{dt}{2}$$

Putting this value in equation (i)

$I = \int \sec^2(t)\frac{dt}{2}$

$I = \frac{1}{2}\tan(t) + c$

$I = \frac{1}{2}\tan(2\tan^{-1}x) + c$

Ans) $\frac{1}{2}\tan(2\tan^{-1}x) + c$

93. Question

Evaluate the following integrals:

$$\left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$$

Answer

To find: Value of $\int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$ ... (i)

Let $x + \sin^2 x = t$

$\Rightarrow 1 + 2\sin x.\cos x = \frac{dt}{dx}$

$\Rightarrow (1 + \sin 2x)\,dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log\left|x + \sin^2 x\right| + c$

Ans) $\log\left|x + \sin^2 x\right| + c$

94. Question

Evaluate the following integrals:

$$\int\left(\frac{1-\tan x}{x+\log\cos x}\right)dx$$

Answer

To find: Value of $\int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$ ... (i)

Let $x + \log(\cos x) = t$

$\Rightarrow 1 + \frac{1.(-\sin x)}{\cos x} = \frac{dt}{dx}$

$\Rightarrow 1 - \tan x = \frac{dt}{dx}$

$\Rightarrow (1 - \tan x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log|x + \log(\cos x)| + c$

Ans) $\log|x + \log(\cos x)| + c$

95. Question

Evaluate the following integrals:

$\int \frac{(1 + \cot x)}{(x + \log \sin x)} dx$

Answer

To find: Value of $\int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$ ... (i)

Let $x + \log(\sin x) = t$

$\Rightarrow 1 + \frac{1.(\cos x)}{\sin x} = \frac{dt}{dx}$

$\Rightarrow 1 + \cot x = \frac{dt}{dx}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|x + \log(\sin x)| + c$

$I = \log|x + \log(\sin x)| + c$

Ans) $\log|x + \log(\sin x)| + c$

96. Question

Evaluate the following integrals:

$\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$

Answer

To find: Value of $\int \frac{\tan x \sec^2 x}{\left(1 - \tan^2 x\right)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{\left(1 - \tan^2 x\right)} dx$ ... (i)

Let $1 - \tan^2 x = t$

$\Rightarrow 0 - 2.\tan x.\sec^2 x = \frac{dt}{dx}$

$\Rightarrow \left(\tan x.\sec^2 x\right)dx = \frac{dt}{-2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{(-2)}$

$I = \frac{1}{2} \log |t| + c$

$I = \frac{1}{2} \log \left|1 - \tan^2 x\right| + c$

Ans) $\frac{1}{2} \log \left|1 - \tan^2 x\right| + c$

97. Question

Evaluate the following integrals:

$$\int \frac{\sin\left(2\tan^{-1} x\right)}{\left(1 + x^2\right)} dx$$

Answer

To find: Value of $\int \frac{\sin(2\tan^{-1} x)}{(1 + x^2)} dx$

Formula used: $\int \sin x \, dx = \cos x + c$

We have, $I = \int \frac{\sin(2\tan^{-1} x)}{(1 + x^2)} dx$ ... (i)

Let $2\tan^{-1} x = t$

$\Rightarrow 2\frac{1}{1 + x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{1 + x^2} = \frac{dt}{2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$$I = \int \sin(t) \frac{dt}{(2)}$$

$$I = -\frac{1}{2}\cos(t) + c$$

$$I = -\frac{1}{2}\cos(2\tan^{-1}x) + c$$

Ans) $-\frac{1}{2}\cos(2\tan^{-1}x) + c$

98. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(x^{1/2} + x^{1/3}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$\Rightarrow x^{\frac{1}{6}} = t$

$\Rightarrow 6t^5\,dt = dx$

Putting this value in equation (i)

$$I = \int \frac{6t^5\,dt}{(t^3 + t^2)}$$

$$I = \int \frac{6t^5\,dt}{t^2(t+1)}$$

$$I = 6\int \frac{t^3\,dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)}dt$$

$$I = 6\int \frac{(t+1)(t^2 - t + 1)}{(t+1)}dt - \int \frac{1}{(t+1)}dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

Ans) $\left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$

99. Question

Evaluate the following integrals:

$$\int\left(\sin^{-1} x\right)^2 dx$$

Answer

To find: Value of $\int\left(\sin^{-1} x\right)^2 dx$

Formula used: $\int \sin x\, dx = \cos x + c$

We have, $I = \int\left(\sin^{-1} x\right)^2 dx$ ... (i)

Let $\sin^{-1} x = t$, x = sint ,

$$\Rightarrow \cos t = \sqrt{1 - x^2}$$

$$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$$

$$\Rightarrow \sqrt{1 - x^2}\, dt = dx$$

$$\Rightarrow \sqrt{1 - (\sin t)^2}\, dt = dx$$

$$\Rightarrow \sqrt{1 - \sin^2 t}\, dt = dx$$

$$\Rightarrow \cos t\, dt = dx$$

Putting this value in equation (i)

$$I = \int t^2 \cos t\, dt$$

$$I = \int t^2 \cos t\, dt - \int\left[\frac{d(t^2)}{dt}\int \cos t\, dt\right] dt$$

$$I = t^2 \sin t - \int [2t.\sin t]dt$$

$$I = t^2 \sin t - 2\left\{\int t[\sin t]dt - \int\left[\frac{dt}{dt}\int \sin t\, dt\right] dt\right\}$$

$$I = t^2 \sin t - 2\left[-t\cos t + \int 1.\cos t\, dt\right]$$

$$I = t^2 \sin t + 2t\cos t - 2\sin t + c$$

$$I = \left(\sin^{-1} x\right)^2 x + 2\left(\sin^{-1} x\right)\sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1}x)^2 x + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + c$

100. Question

Evaluate the following integrals:

$$\int \frac{2x\tan^{-1}x^2}{\left(1+x^4\right)}dx$$

Answer

To find: Value of $\int \frac{2x\tan^{-1}(x^2)}{(1+x^4)}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{2x\tan^{-1}(x^2)}{(1+x^4)}dx$ ... (i)

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1+(x^2)^2} \cdot 2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1+x^4}dx = dt$$

Putting this value in equation (i)

$$I = \int t.\,dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\frac{\{\tan^{-1}(x^2)\}^2}{2} + c$

101. Question

Evaluate the following integrals:

$$\int \frac{\left(x^2+1\right)}{\left(x^4+1\right)}dx$$

Answer

To find: Value of $\int \frac{(x^2+1)}{(x^4+1)}dx$

Formula used: $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)}dx$ ... (i)

Dividing Numerator and Denominator by x^2,

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+2-2\right)}dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2-2.x.\frac{1}{x}+\left(\frac{1}{x}\right)^2+2\right)}dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(\left(x-\frac{1}{x}\right)^2+(\sqrt{2})^2\right)}dx$$

Let $x-\frac{1}{x}=t$

$$\Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2+(\sqrt{2})^2}dt$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+c$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+c$$

$$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$$

Ans) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

102. Question

Evaluate the following integrals:

$$\int \frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$$

Answer

To find: Value of $\int \frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$

Formula used: $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

We have, $I = \int \frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$ ... (i)

Let (sinx – cosx) = t

$$\Rightarrow (\cos x+\sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x+\sin x)\,dx = dt$$

$\Rightarrow t^2 = \sin^2 x - 2\sin x.\cos x + \cos^2 x$

$\Rightarrow t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow 2\sin x.\cos x = 1 - t^2$

$\Rightarrow \sin 2x = 1 - t^2$

Putting this value in equation (i)

$$\Rightarrow I = \int \frac{dt}{\sqrt{1 - t^2}}$$

$I = \sin^{-1} t$

$I = \sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x) = \theta$

$\Rightarrow I = \sin^{-1}(\sin x - \cos x) = \theta \cdots$ (ii)

$\Rightarrow \sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1 - (\sin x - \cos x)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - (\sin^2 x - 2\sin x.\cos x + \cos^2 x)}$

$\Rightarrow \cos\theta = \sqrt{1 - (1 - 2\sin x.\cos x)}$

$\Rightarrow \cos\theta = \sqrt{2\sin x.\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now $\tan\theta = \frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Comparing the value θ from eqn. (ii)

$$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Dividing Numerator and denominator from cosx

$$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$$

Ans.) $\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Objective Questions I

1. Question

Mark (√) against the correct answer in each of the following:

$$\int (2x+3)^5 \, dx = ?$$

A. $\frac{(2x+3)^6}{6}+C$

B. $\frac{(2x+3)^4}{8}+C$

C. $\frac{(2x+3)^6}{12}+C$

D. none of these

Answer

Given = $\int(2x+3)^5$

Let, 2x + 3 = z

⇒ 2dx = dz

So,

$\int(2x+3)^5 dx$

$=\int\frac{z^5}{2}dz$

$=\frac{1}{2}\frac{z^6}{6}+c$ where c is the integrating constant.

$=\frac{z^6}{12}+c$

$=\frac{(2x+3)^6}{12}+c$

2. Question

Mark (√) against the correct answer in each of the following:

$\int(3-5x)^7 dx = ?$

A. $-5(3 - 5x)^6 + C$

B. $\frac{(3-5x)^8}{-40}+C$

C. $\frac{-5(3-5x)^8}{8}+C$

D. none of these

Answer

Given = $\int(3-5x)^7$

Let, 3 – 5x = z

⇒ -5dx = dz

So,

$$\int (3-5x)^7 dx$$

$$= -\int \frac{z^7}{5} dz$$

$$= -\frac{1}{5}\frac{z^8}{8} + c$$ where c is the integrating constant.

$$= -\frac{z^8}{40} + c$$

$$= -\frac{(3-5x)^8}{40} + c$$

3. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{(2-3x)^4} dx = ?$$

A. $\frac{1}{15(2-3x)^5} + C$

B. $\frac{1}{-12(2-3x)^3} + C$

C. $\frac{1}{9(2-3x)^3} + C$

D. none of these

Answer

Given = $\int \frac{1}{(2-3x)^4}$

Let, 2 – 3x = z

⇒ -3dx = dz

So,

$$\int \frac{1}{(2-3x)^4}dx$$

$$= \int \frac{1}{z^4}\left(\frac{dz}{-3}\right)$$

$$= -\frac{1}{3}\int \frac{dz}{z^4}$$

where c is the integrating constant.

$$= -\frac{1}{3}\int z^{-4}dz$$

$$= -\frac{1}{3}\frac{z^{-3}}{-3}+c$$

$$= \frac{1}{9(2-3x)^3}+c$$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{ax+b}\,dx = ?$$

A. $\frac{2(ax+b)^{3/2}}{3a}+C$ B. $\frac{3(ax+b)^{3/2}}{2a}+C$

C. $\frac{1}{2\sqrt{ax+b}}+C$

D. none of these

Answer

Given = $\int \sqrt{ax+b}$

Let, ax + b = z^2

⇒ adx = 2zdz

So,

$$\int \sqrt{ax+b}dx$$

$$= \int z\frac{2zdz}{a}$$

$$= \frac{2}{a}\int z^2dz$$

where c is the integrating constant.

$$= \frac{2}{a}\frac{z^3}{3}+c$$

$$= \frac{2}{3a}z^3+c$$

$$= \frac{2(ax+b)^{3/2}}{3a}+c$$

5. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^2(7-4x)dx = ?$

A. $\frac{1}{4}\tan(7-4x)+C$

B. $\frac{-1}{4}\tan(7-4x)+C$

C. 4 tan (7 – 4x) + C

D. - 4 tan (7 – 4x) + C

Answer

Given = $\int \sec^2(7-4x)$

Let, 7 – 4x = z

⇒ -4dx = dz

So,

$\int \sec^2(7-4x)dx$

$= \int \sec^2 z\frac{dz}{-4}$

$= -\frac{1}{4}\int \sec^2 zdz$ where c is the integrating constant.

$= -\frac{1}{4}\tan z + c$

$= -\frac{1}{4}\tan(7-4x)+c$

6. Question

Mark (√) against the correct answer in each of the following:

$\int \cos 3x\ dx = ?$

A. $-\frac{1}{3}\sin 3x + C$

B. $\frac{1}{3}\sin 3x + C$

C. 3 sin 3x + C

D. -3 sin 3x + C

Answer

Given = $\int \cos 3x$

So, $\int \cos 3xdx = \frac{\sin 3x}{3} + c$ where c is the integrating constant.

7. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(5-3x)}dx = ?$

A. $-3e^{(5-3x)} + C$

B. $\frac{1}{3}e^{(5-3x)} + C$

C. $\frac{e^{(5-3x)}}{-3} + C$

D. none of these

Answer

Given = $\int e^{(5-3x)}$

Let, 5 – 3x = z

⇒ -3dx = dz

So,

$\int e^{(5-3x)}dx$

$= \int e^z \frac{dz}{-3}$

$= -\frac{1}{3}\int e^z dz$ where c is the integrating constant.

$= -\frac{1}{3}e^z + c$

$= -\frac{1}{3}e^{(5-3x)} + c$

8. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(3x+4)}dx = ?$

A. $\frac{3}{(\log 2)}.2^{(3x+4)} + C$

B. $\frac{2^{(3x+4)}}{3(\log 2) + C}$

C. $\frac{2^{(3x+4)}}{2(\log 3)} + C$

D. none of these

Answer

Given = $\int e^{(3x+4)}$

Let, 3x + 4 = z

⇒ 3dx = dz

So,

$$\int e^{(3x+4)}dx$$

$$=\int e^z \frac{dz}{3}$$

$$=\frac{1}{3}\int e^z dz$$

$$=\frac{1}{3}e^z + c$$

$$=\frac{1}{3}e^{(3x+4)} + c$$

where c is the integrating constant.

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^2 \frac{x}{2} dx = ?$$

A. $\tan\frac{x}{2} - x + C$

B. $\tan\frac{x}{2} + x + C$

C. $2\tan\frac{x}{2} + x + C$

D. $2\tan\frac{x}{2} - x + C$

Answer

Given = $\int \tan^2 \frac{x}{2}$

Let, $\frac{x}{2} = z$

⇒ dx = 2dz

So,

$\int \tan^2 \frac{x}{2} dx$

$= 2\int \tan^2 z dz$

$= 2\int \frac{\sin^2 z}{\cos^2 z} dz$

$= 2\int \frac{1-\cos^2 z}{\cos^2 z} dz$

$= 2\int (\sec^2 z - 1) dz$

$= 2\left[\tan z - z\right] + c$

$= 2[\tan \frac{x}{2} - \frac{x}{2}] + c$ where c is the integrating constant.

10. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{1-\cos x}\, dx = ?$

A. $-\sqrt{2} \cos \frac{x}{2} + C$

B. $-2\sqrt{2} \cos \frac{x}{2} + C$

C. $\frac{-1}{2} \cos \frac{x}{2} + C$

D. $\frac{-1}{\sqrt{2}} \cos \frac{x}{2} + C$

Answer

Given = $\int \sqrt{1-\cos x}$

So,

$\int \sqrt{1-\cos x} dx$

$= \int \sqrt{1-\cos x} \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} dx$

$= \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}} dx$

$= \int \frac{\sin x}{\sqrt{1+\cos x}} dx$

Let 1 + cosx = u^2

So, -sinxdx = 2udu

$-\int \frac{2u}{u} du = -2\int du = -2u + c = -2\sqrt{1+\cos x} + c$

where c is the integrating constant.

11. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{1+\sin x}\, dx = ?$

A. $-\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

B. $\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

C. $-2\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

D. none of these

Answer

Given = $\int \sqrt{1+\sin x}$

So,

$$\int \sqrt{1+\sin x}\,dx$$

$$=\int \sqrt{1+\sin x}\,\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}dx$$

$$=\int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}dx$$

$$=\int \frac{\cos x}{\sqrt{1-\sin x}}dx$$

Let 1 - sinx = u^2

So, -cosxdx = 2udu

$$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1-\sin x} + c$$

where c is the integrating constant.

12. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3 x\, dx = ?$

A. $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

B. $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

C. $-\frac{3}{4}\cos x - \frac{\cos 3x}{12} + C$

D. none of these

Answer

Given = $\int \sin^3 x dx$

So,

$$\int \sin^3 x dx$$
$$= \int \sin^2 x \sin x dx$$
$$= \int (1 - \cos^2 x) \sin x dx$$

Let cosx = u

So, -sinxdx = du

$$-\int (1 - u^2) du$$
$$= -\int du + \int u^2 du$$
$$= -u + \frac{u^3}{3} + c$$
$$= -\cos x + \frac{\cos^3 x}{3} + c$$

where c is the integrating constant.

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\log x}{x} dx = ?$$

A. $\frac{1}{2}(\log x)^2 + C$

B. $-\frac{1}{2}(\log x)^2 + C$

C. $\frac{2}{x^2} + C$

D. $\frac{-2}{x^2} + C$

Answer

Given = $\int \frac{\log x}{x}$

Let, logx = u

So, $\frac{1}{x} dx = du$

So,

$$\int \frac{\log x}{x} dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

where c is the integrating constant.

14. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2(\log x)}{x} dx = ?$$

A. log (tan x) + C

B. - log (tan x) + C

C. tan (tan x) + C

D. - tan (log x) + C

Answer

Given = $\int \frac{\sec^2(\log x)}{x}$

Let, logx = z

$\Rightarrow \frac{dx}{x} = dz$

So,

$$\int \frac{\sec^2(\log x)}{x} dx$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(\log x) + c$$

where c is the integrating constant.

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x(\log x)} dx = ?$$

A. $\log |x| + C$

B. $\frac{-2}{x^2} + C$

C. $(\log x)^2 + C$

D. $\log |\log x| + C$

Answer

Given = $\int \frac{1}{x(\log x)}$

Let, $\log x = z$

$\Rightarrow \frac{dx}{x} = dz$

So,

$$\int \frac{1}{x(\log x)} dx$$

$$= \int \frac{1}{z} dz$$

$$= \log z + c$$

$$= \log(\log x) + c$$

where c is the integrating constant.

16. Question

Mark (√) against the correct answer in each of the following:

$$\int e^{x^3} x^2 dx = ?$$

A. $e^{x^3} + C$

B. $\frac{1}{3} e^{x^3} + C$

C. $\frac{1}{6} e^{x^3} + C$

D. none of these

Answer

Given = $\int e^{x^3} x^2$

Let, $x^3 = z$

$\Rightarrow 3x^2 dx = dz$

$\Rightarrow x^2 dx = \frac{dz}{3}$

So,

$$\int e^{x^3} x^2 dx$$

$$= \frac{1}{3}\int e^z dz$$

$$= \frac{1}{3}e^z + c$$

$$= \frac{1}{3}e^{x^3} + c$$

where c is the integrating constant.

17. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$$

A. $e^{\sqrt{x}} + C$

B. $\frac{1}{2}e^{\sqrt{x}} + C$

C. $2e^{\sqrt{x}} + C$

D. none of these

Answer

Given = $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Let, x = z^2

⇒ dx = 2zdz

So,

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$= \int \frac{e^z}{z} 2zdz$$

$$= 2\int e^z dz$$

$$= 2e^z + c$$

$$= 2e^{\sqrt{x}} + c$$

where c is the integrating constant.

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx = ?$$

A. $\frac{e^{\tan^{-1}x}}{x}+C$

B. $e^{\tan^{-1}x}+C$

C. $e^x\tan^{-1}x+C$

D. none of these

Answer

Given = $\int\frac{e^{\tan^{-1}x}}{\left(1+x^2\right)}$

Let, $\tan^{-1}x = z$

$\Rightarrow \frac{1}{1+x^2}dx=dz$

So,

$$\int\frac{e^{\tan^{-1}x}}{\left(1+x^2\right)}dx$$

$$=\int e^z dz$$

$$=e^z+c$$

$$=e^{\tan^{-1}x}+c$$

where c is the integrating constant.

19. Question

Mark (√) against the correct answer in each of the following:

$$\int\frac{\sin\sqrt{x}}{\sqrt{x}}dx=?$$

A. $2\cos\sqrt{x}+C$

B. $-2\cos\sqrt{x}+C$

C. $-\frac{\cos\sqrt{x}}{2}+C$

D. $\frac{\cos\sqrt{x}}{2}+C$

Answer

Given = $\int\frac{\sin\sqrt{x}}{\sqrt{x}}$

Let, $x = z^2$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$$

$$= \int \frac{\sin z}{z} 2zdz$$

$$= 2\int \sin z dz$$

$$= -2\cos z + c$$

$$= -2\cos\sqrt{x} + c$$

where c is the integrating constant.

20. Question

Mark (√) against the correct answer in each of the following:

$$\int \left(\sqrt{\sin x}\right)\cos x\, dx = ?$$

A. $\frac{2}{3}(\cos x)^{3/2} + C$

B. $\frac{3}{2}(\cos x)^{3/2} + C$

C. $\frac{2}{3}(\sin x)^{3/2} + C$

D. $\frac{3}{2}(\sin x)^{3/2} + C$

Answer

Given = $\int \left(\sqrt{\sin x}\right)\cos x$

Let, sinx = z^2

⇒ cosxdx = 2zdz

So,

$$\int \left(\sqrt{\sin x}\right)\cos x dx$$

$$= 2\int z^2 dz$$

$$= 2\frac{z^3}{3} + c$$

$$= \frac{2}{3}\sin^{3/2} x + c$$

where c is the integrating constant.

21. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$$

A. $\frac{1}{2}\log\left|\tan^{-1} x\right| + C$

B. $2\sqrt{\tan^{-1} x} + C$

C. $\frac{1}{2\sqrt{\tan^{-1} x}} + C$

D. none of these

Answer

Given = $\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$

Let, $\tan^{-1}x = z^2$

$\Rightarrow \frac{1}{1+x^2}dx = 2zdz$

So,

$$\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}dx$$

$= \int \frac{2z}{z}dz$

$= 2\int dz$

$= 2z + c$

$= 2\sqrt{\tan^{-1} x} + c$

where c is the integrating constant.

22. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cot x}{\log(\sin x)}dx = ?$$

A. log |cot x| + C

B. log |cot x cosec x| + C

C. log |log sin x| + C

D. none of these

Answer

Given = $\int \frac{\cot x}{\log(\sin x)}$

Let, sinx = z

⇒ cosxdx = dz

So,

$$\int \frac{\cot x}{\log(\sin x)} dx$$

$$= \int \frac{\cos x}{\sin x \log(\sin x)} dx$$

$$= \int \frac{dz}{z \log z}$$

Let, logz = u

⇒ $\frac{1}{z} dz = du$

So,

$$\int \frac{dz}{z \log z}$$

$$= \int \frac{du}{u}$$

$$= \log u + c$$

$$= \log |\log z| + c$$

where c is the integrating constant.

23. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x \cos^2(1 + \log x)} dx = ?$$

A. tan (1 + log x) + C

B. cot (1 + log x) + C

C. sec (1 + log x) + C

D. none of these

Answer

Given = $\int \frac{1}{x \cos^2(1 + \log x)}$

Let, 1 + logx = z

⇒ $\frac{1}{x} dx = dz$

So,

$$\int \frac{1}{x\cos^2(1+\log x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(1+\log x) + c$$

where c is the integrating constant.

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx = ?$$

A. $\frac{1}{3}(\tan^{-1} x^3) + C$

B. log |tan^{-1} x^3| + C

C. $\frac{1}{6}(\tan^{-1} x^3)^2 + C$

D. none of these

Answer

Given = $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx$

Let, tan^{-1}x^3 = z

⇒ $\frac{1}{1+x^6} \times 3x^2 dx = dz$

⇒ $\frac{x^2}{1+x^6} dx = \frac{dz}{3}$

So,

$$\frac{1}{3}\int z dz$$

$$= \frac{1}{3}\frac{z^2}{2} + c$$

$$= \frac{z^2}{6} + c$$

$$= \frac{(\tan^{-1} x^3)^2}{6} + c$$

where c is the integrating constant.

25. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^5 x \tan x \, dx = ?$

A. 5 $\tan^5 x + C$

B. $\frac{1}{5}\tan^5 x + C$

C. 5 log |cos x| + C

D. none of these

Answer

Given = $\int \sec^5 x \tan x$

So, $\int \sec^5 \tan x dx = \int \sec^4 x (\sec x \tan x) dx$

Let, secx = z

⇒ secxtanxdx = dz

$\int \sec^4 x (\sec x \tan x) dx$

$= \int z^4 dz$

$= \frac{z^5}{5} + c$

$= \frac{\sec^5 x}{5} + c$

where c is the integrating constant.

26. Question

Mark (√) against the correct answer in each of the following:

$\int \operatorname{cosec}^3 (2x+1) \cot (2x+1) dx = ?$

A. $\frac{1}{4}\operatorname{cosec}^4 (2x+1) + C$

B. $-\frac{1}{3}\operatorname{cosec}^3 (2x+1) + C$

C. $-\frac{1}{6}\operatorname{cosec}^3 (2x+1) + C$

D. $\frac{1}{2}\operatorname{cosec}(2x+1)\cot(2x+1) + C$

Answer

Given = $\int \cos ec^3 (2x+1) \cot (2x+1)$

So,

$$\int \operatorname{cosec}^3(2x+1)\cot(2x+1)dx$$

$$= \int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

Let, cosec(2x + 1) = z

⇒ -2cosec(2x + 1)cot(2x + 1)dx = dz

$$\int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

$$= \int z^2 \frac{dz}{-2} =$$

$$= -\frac{1}{2}\frac{z^3}{3} + c$$

$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx = ?$$

A. log |sec ($\sin^{-1}$ x)| + C

B. log |cos ($\sin^{-1}$ x)| + C

C. tan ($\sin^{1}$ x) + C

D. none of these

Answer

Given = $\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}}$

Let, $\sin^{-1}$x = z

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx$$

$$= \int \tan z dz$$

$$= \log\left|\sec z\right| + c$$

$$= \log\left|\sec\left(\sin^{-1} x\right)\right| + c$$

where c is the integrating constant.

28. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan(\log x)}{x} dx = ?$$

A. x tan (log x) + C

B. log |tan x| + C

C. log |cos (log x)| + C

D. - log |cos (log x)| + C

Answer

Given = $\int \frac{\tan(\log x)}{x}$

Let, logx = z

⇒ $\frac{1}{x} dx = dz$

So,

$$\int \frac{\tan(\log x)}{x} dx$$

$$= \int \tan z dz$$

$$= \log |\sec z| + c$$

$$= \log |\sec(\log x)| + c$$

$$= -\log |\cos(\log x)| + c$$

where c is the integrating constant.

29. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x \cot(e^x) dx = ?$$

A. cot (e^x) + C

B. log |sin e^x| + C

C. log |cosec e^x| + C

D. none of these

Answer

Given = $\int e^x \cot(e^x) dx$

Let, $e^x = z$

⇒ $e^x dx = dz$

So,

$$\int e^x \cot\left(e^x\right)dx$$
$$= \int \cot z dz$$
$$= \log\left|\sin z\right| + c$$
$$= \log\left|\sin\left(e^x\right)\right| + c$$

where c is the integrating constant.

30. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x}{\sqrt{1+e^x}} dx = ?$$

A. $2\sqrt{1+e^x} + C$

B. $\frac{1}{2}\sqrt{1+e^x} + C$

C. $\frac{1}{\sqrt{1+e^x}} + C$

D. none of these

Answer

Given = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, $1 + e^x = z^2$

$\Rightarrow e^x dx = 2zdz$

So,

$$\int \frac{e^x}{\sqrt{1+e^x}} dx$$
$$= \int \frac{2zdz}{z}$$
$$= 2\int dz$$
$$= 2z + c$$
$$= 2\sqrt{1+e^x} + c$$

where c is the integrating constant.

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x}{\sqrt{1-x^2}} dx = ?$$

A. $\sin^{-1} x + C$

B. $\sin^{-1}\sqrt{x} + C$

C. $\sqrt{1-x^2} + C$

D. $-\sqrt{1-x^2} + C$

Answer

Given = $\int \frac{x}{\sqrt{1-x^2}} dx$

Let, $1 - x^2 = z^2$

$\Rightarrow -2xdx = 2zdz$

So,

$$\int \frac{x}{\sqrt{1-x^2}} dx$$

$$= -\int \frac{zdz}{z}$$

$$= -\int dz$$

$$= -z + c$$

$$= -\sqrt{1-x^2} + c$$

where c is the integrating constant.

32. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x (1+x)}{\cos^2 (xe^x)} dx = ?$$

A. $\tan (xe^x) + C$

B. $\cot (xe^x) + C$

C. $ex^x \tan x + C$

D. none of these

Answer

Given = $\int \frac{e^x (1+x)}{\cos^2 (xe^x)} dx$

Let, $xe^x = z$

$\Rightarrow e^x(1 + x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

33. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x + e^{-x})} = ?$$

A. cot-1 (e^x) + C

B. tan-1 (e^x) + C

C. log |e^x + 1| + C

D. none of these

Answer

Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x + 1)}dx$$

Let, e^x + 1 = z

⇒ e^xdx = dz

So,

$$\int \frac{e^x dx}{(e^x + 1)}$$

$$= \int \frac{dz}{z}$$

$$= \log|z| + c$$

$$= \tan|e^x + 1| + c$$

where c is the integrating constant.

34. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{1-4^x} dx = ?$$

A. $\sin^{-1}(2^x) + C$

B. $(\log e^2) \sin^{-1}(2^x) + C$

C. $(\log e^2) \cos^{-1}(2^x) + C$

D. $\log_2 e) \sin^{-1}(2^x) + C$

Answer

Given =

$$\int \frac{2^x dx}{1-4^x}$$

$$= \int \frac{2^x}{1-\left(2^x\right)^2} dx$$

Let, $2^x = z$

$\Rightarrow 2^x(\log 2)dx = dz$

So,

$$\int \frac{2^x dx}{1-\left(2^x\right)^2}$$

$$= \frac{1}{\log 2} \int \frac{dz}{1-z^2}$$

$$= \frac{1}{\log 2} \sin^{-1} z + c$$

$$= \frac{\sin^{-1} 2x}{\log 2} + c$$

where c is the integrating constant.

35. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(e^x - 1\right)} = ?$$

A. $\log |e^x - 1| + C$

B. $\log |1 - e^{-x}| + C$

C. $\log |e^x - 1| + C$

D. none of these

Answer

Given =

$$\int \frac{dx}{e^x - 1}$$

$$= -\int \frac{-1 + e^x - e^x}{e^x - 1} dx$$

$$= -\int \frac{e^x - 1}{e^x - 1} dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -\int dx + \int \frac{e^x}{e^x - 1} dx$$

Let, $e^x - 1 = z$

⇒ $e^x dx = dz$

So,

$$-\int dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -x + \int \frac{dz}{z}$$

$$= -x + \log z + c$$

$$= -x + \log\left|e^x - 1\right| + c$$

where c is the integrating constant.

36. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x} + x\right)} dx = ?$$

A. $\log\left|1 + \sqrt{x}\right| + C$

B. $2\log\left|1 + \sqrt{x}\right| + C$

C. $\frac{1}{\sqrt{x}} \tan^{-1}\sqrt{x} + C$

D. none of these

Answer

Given =

$$\int \frac{dx}{\left(\sqrt{x} + x\right)}$$

$$= \int \frac{1}{\sqrt{x}} \frac{1}{\left(1 + \sqrt{x}\right)} dx$$

Let, $1 + \sqrt{x} = z$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dz$

So,

$$\int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

$$= 2\int \frac{dz}{z}$$

$$= 2\log|z| + c$$

$$= 2\tan\left|1+\sqrt{x}\right| + c$$

where c is the integrating constant.

37. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1+\sin x)} = ?$$

A. tan x + sec x + C

B. tan x – sec x + C

C. $\frac{1}{2}\tan\frac{x}{2} + C$

D. none of these

Answer

Given

$$\int \frac{dx}{1+\sin x}$$

$$= \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

38. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1+\sin x)} dx = ?$$

A. x + tan x – sec x + C

B. x – tan x – sec x + C

C. x – tan x + sec x + C

D. none of these

Answer

Given

$$\int \frac{\sin x}{1+\sin x} dx$$

$$= \int dx - \int \frac{dx}{1+\sin x}$$

$$= x - \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= x - \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$

So,

$$x-\int \frac{2dz}{z^2}$$

$$=x+\frac{2}{z}+c$$

$$=x+\frac{2}{\tan\frac{x}{2}+1}+c$$

where c is the integrating constant.

39. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin\ x}{(1-\sin\ x)}dx=?$$

A. - x + sec x – tan x + C

B. x + cos x – sin x + C

C. - log |1 – sin x| + C

D. none of these

Answer

Given

$$\int \frac{\sin x}{1-\sin x}dx$$

$$=-\int dx+\int \frac{dx}{1-\sin x}$$

$$=-x+\int \frac{dx}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=-x+\int \frac{dx}{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}$$

$$=-x+\int \frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2}-1\right)^2}$$

Let, $\tan\frac{x}{2}-1=z$

⇒ $\frac{1}{2}\sec^2\frac{x}{2}dx=dz$

So,

$$-x+\int \frac{2dz}{z^2}$$

$$=-x-\frac{2}{z}+c$$

$$=-x-\frac{2}{\tan\frac{x}{2}+1}+c$$

where c is the integrating constant.

40. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1+\cos x)}=?$$

A. $\frac{1}{2}\tan\frac{x}{2}+C$

B. $-\cot\frac{x}{2}+C$

C. $\tan\frac{x}{2}+C$

D. none of these

Answer

Given

$$\int \frac{dx}{1+\cos x}$$

$$=\int \frac{dx}{1+2\cos^2\frac{x}{2}-1}$$

$$=\frac{1}{2}\int \frac{dx}{\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}\int \sec^2\frac{x}{2}dx$$

$$=\frac{1}{2}2\tan\frac{x}{2}+c$$

$$=\tan\frac{x}{2}+c$$

where c is the integrating constant.

41. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\cos x)}=?$$

A. $\frac{1}{(x-\sin x)}+C$

B. log |x – sin x| + C

C. $\log\left|\tan\frac{x}{2}\right|+C$

D. $-\cot\frac{x}{2}+C$

Answer

Given

$$\int\frac{dx}{1-\cos x}$$

$$=\int\frac{dx}{1-1+2\sin^2\frac{x}{2}}$$

$$=\frac{1}{2}\int\frac{dx}{\sin^2\frac{x}{2}}$$

$$=\frac{1}{2}\int\operatorname{cosec}^2\frac{x}{2}dx$$

$$=-\frac{1}{2}2\cot\frac{x}{2}+c$$

$$=-\cot\frac{x}{2}+c$$

where c is the integrating constant.

42. Question

Mark (√) against the correct answer in each of the following:

$$\int\left\{\frac{1-\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right\}dx=?$$

A. $2\log\left|\sec\frac{x}{2}\right|+C$

B. $2\log\left|\cos\frac{x}{2}\right|+C$

C. $2\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|+C$

D. $2\log\left|\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|+C$

Answer

Given

$$\int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}dx$$

$$=\int \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}dx$$

$$=\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

Let, $\cos\frac{x}{2}+\sin\frac{x}{2}=z$

⇒ $\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx=dz$

So,

$$\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

$$=\int \frac{dz}{z}$$

$$=\log z+c$$

$$=\log\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+c$$

where c is the integrating constant.

43. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x}\,dx=?$$

A. $\sqrt{e^x}+C$

B. $2\sqrt{e^x}+C$

C. $\frac{1}{2}\sqrt{e^x}+C$

D. none of these

Answer

Given

$$\int \sqrt{e^x}\,dx$$

$$= \int \left(e^x\right)^{\frac{1}{2}} dx$$

$$= \int e^{\frac{1}{2}x} dx$$

$$= 2e^{\frac{1}{2}x} + c$$

$$= 2\sqrt{e^x} + c$$

where c is the integrating constant.

44. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1+\cos x)} dx = ?$$

A. $x + \tan\frac{x}{2} + C$

B. $-x + \tan\frac{x}{2} + C$

C. $x - \tan\frac{x}{2} + C$

D. none of these

Answer

Given

$$\int \frac{\cos x dx}{1+\cos x}$$

$$= \int \frac{1+\cos x - 1}{1+\cos x} dx$$

$$= \int dx - \int \frac{dx}{1+\cos x}$$

$$= x - \tan\frac{x}{2} + c$$

[From Question no. 40] where c is the integrating constant.

45. Question

Mark (√) against the correct answer in each of the following:

$$\int \sec^2 x \ \mathrm{cosec}^2 x \, dx = ?$$

A. tan x – cot x + C

B. tan x + cot x + C

C. - tan x + cot x + C

D. none of these

Answer

Given

$$\int \sec^2 x \operatorname{cosec}^2 x dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

46. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

A. tan x + x + C

B. tan x – x + C

C. - tan x + x + C

D. none of these

Answer

Given

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx$$

$$= \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= 2\tan \frac{x}{2} - x + c$$

where c is the integrating constant.

47. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(1+\cos x)}{(1-\cos x)} dx = ?$

A. $-2\cot\frac{x}{2} - x + C$

B. $-2\cot\frac{x}{2} + x + C$

C. $2\cot\frac{x}{2} + x + C$

D. none of these

Answer

Given

$$\int \frac{(1+\cos 2x)}{(1-\cos 2x)} dx$$

$$= \int \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} dx$$

$$= \int \cot^2\frac{x}{2} dx$$

$$= \int \left(\operatorname{cosec}^2\frac{x}{2} - 1\right) dx$$

$$= -2\cot\frac{x}{2} - x + c$$

where c is the integrating constant.

48. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$

A. tan x + cot x + C

B. tan x – cot x + C

C. - tan x + cot x + C

D. none of these

Answer

Given

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

49. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \ \sin^2 x} dx = ?$$

A. cot x + tan x + C

B. - cot x + tan x + C

C. cot x – tan x + C

D. - cot x – tan x + C

Answer

Given

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\tan x - \cot x + c$$

where c is the integrating constant.

50. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

A. sin x + x cos α + C

B. 2sin x + x cos α + C

C. 2 sin x + 2x cos α + C

D. none of these

Answer

Given

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$$

$$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\int \cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\left(\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right)$$

$$= 2\int (\cos x + \cos \alpha) dx$$

$$= 2[\sin x + x\cos\alpha] + c$$

where c is the integrating constant.

51. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left\{\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right\} dx = ?$$

A. $2x^2 + C$

B. $\frac{x^2}{2} + C$

C. $\frac{2}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \cos 2x = 2\cos^2 x$; $1 - \cos 2x = 2\sin^2 x$

Therefore ,

$$\Rightarrow \int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1}\sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1}\tan x \, dx$$

$\Rightarrow \int x\,dx = \frac{x^2}{2} + c$

52. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}(\sec x + \tan x)dx = ?$$

A. $\frac{\pi x}{4} + \frac{x^2}{4} + C$

B. $\frac{\pi x}{4} - \frac{x^2}{4} + C$

C. $\frac{1}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos\frac{x}{2} + \sin\frac{x}{2})^2$

$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$; $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Therefore ,

$\Rightarrow \int \tan^{-1}(\sec x + \tan x)\,dx = \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)dx$

$\Rightarrow \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}dx = \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}dx$

$\Rightarrow \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^1}{(\cos\frac{x}{2}-\sin\frac{x}{2})}dx = \int \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}dx$

(Multiply by $\sec\frac{x}{2}$ in numerator and denominator)

$\Rightarrow \int \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}dx = \int \tan^{-1}\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{\tan\frac{\pi}{4}-\tan\frac{\pi}{4}\tan\frac{x}{2}}dx = \int \tan^{-1}\tan(\frac{\pi}{4}+\frac{x}{2})\,dx$

$\Rightarrow \int (\frac{\pi}{4}+\frac{x}{2})dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$

53. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\sin x)}{(1-\sin x)}dx = ?$$

A. 2 tan x + x – 2sec x + C

B. 2 tan x – x + 2 sec x + C

C. 2 tan x – x – 2sec x + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$

$\Rightarrow \int \frac{(1+\sin x)^2}{1-sin^2 x} dx = \int \frac{1+sin^2 x+2\sin x}{cos^2 x} dx$

$\Rightarrow \int \frac{1}{cos^2 x} dx + 2\int \frac{\sin x}{cos^2 x} dx + \int \frac{sin^2 x}{cos^2 x} dx$

$\Rightarrow \int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx + \int tan^2 x\, dx$

$\Rightarrow \int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx + \int (-1 + sec^2 x)\, dx$

$\Rightarrow 2\int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx - \int 1\, dx$

Put cos x = t

Therefore -> sin x dx = - dt

$\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$

$\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$

$\Rightarrow 2\tan x + 2\sec x - x + c$

54. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^4}{(1+x^2)} dx = ?$$

A. $\frac{x^3}{3} + x + \tan^{-1} x + C$

B. $\frac{-x^3}{3} + x - \tan^{-1} x + C$

C. $\frac{x^3}{3} - x + \tan^{-1} x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\, x$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{x^4+1-1}{1+x^2} dx$

$\Rightarrow \int \frac{x^4-1}{1+x^2} dx + \int \frac{1}{1+x^2} dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \int x^2 - 1\, dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \frac{x^3}{3} - x + \tan^{-1} x + c$

55. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx = ?$$

A. x cos 2α - sin 2α . log |sin (x + α)| + C

B. x cos 2α + sin 2α . log|sin (x + α)| + C

C. x cos 2α + sin α . log |sin (x + α)| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin(x+\propto-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \frac{\sin(x+\propto)\cos(-2\propto)+\cos(x+\propto)\sin(-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \cos(2\propto)\, dx - \sin 2\propto \int \cot(x+\propto)\, dx$

$\Rightarrow \cos(2\propto)\, x - \sin 2\propto \log|\sin(x+\propto)| + c$

56. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x+3}-\sqrt{x+2}\right)} dx = ?$$

A. $\frac{2}{3}(x+3)^{3/2} - \frac{2}{3}(x+3)^{3/2} + C$

B. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+3)^{3/2} + C$

C. $\frac{3}{2}(x+3)^{3/2} - \frac{3}{2}(x+3)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{x+3}+\sqrt{x+2})}{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})} dx$ (Rationalizing the denominator)

$\Rightarrow \int (\sqrt{x+3}+\sqrt{x+2}) dx$

$\Rightarrow \int \sqrt{x+3}\, dx + \int \sqrt{x+2}\, dx$

$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{3} + \frac{2(x+2)^{\frac{3}{2}}}{3} + c$

57. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx = ?$$

A. - log |cos x – sin x| + C

B. log |cos x – sin x| + C

C. log |cos x + sin x | + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\Rightarrow \int \frac{-dt}{t} = -\log t + c$

$\Rightarrow -\log|\cos x - \sin x| + c$

59. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^2}{(1+x^6)} dx = ?$$

A. $\sin^{-1} x^3 + C$

B. $\cos^{-1} x^3 + C$

C. $\tan^{-1} x^3 + C$

D. $\cot^{-1} x^3 + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t \; 3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{1+t^2}$

$\Rightarrow \tan^{-1} t + c$

$\Rightarrow \tan^{-1} x^3 + c$

59. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x\sqrt{x^6 - 1}} = ?$$

A. $\frac{1}{3}\sec^{-1} x^3 + C$

B. $\frac{1}{3}\text{cosec}^{-1} x^3 + C$

C. $\frac{1}{3}\cot^{-1} x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \;\; ; \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t \,, 3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{x \times 3x^2 \sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \sec^{-1} t + c$

$\Rightarrow \frac{1}{3} \sec^{-1} x^3 + c$

60. Question

Mark (√) against the correct answer in each of the following:

$$\int \left\{ (2x+1)\sqrt{x^2 + x + 1} \right\} dx = ?$$

A. $\frac{3}{2}\left(x^2 + x + 1\right)^{3/2} + C$

B. $\frac{2}{3}\left(x^2 + x + 1\right)^{3/2} + C$

C. $\frac{3}{2}(2x+1)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x + 1)dx = dt$

$\Rightarrow \int \sqrt{t} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3} t^{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}(x^2 + x + 1)^{\frac{3}{2}} + c$

61. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left\{\sqrt{2x+3} + \sqrt{2x+3}\right\}} = ?$$

A. $\frac{1}{18}(2x+3)^{3/2} + \frac{1}{18}(2x-3)^{3/2} + C$

B. $\frac{1}{18}(2x+3)^{3/2} - \frac{1}{18}(2x-3)^{3/2} + C$

C. $\frac{1}{12}(2x+3)^{3/2} - \frac{1}{12}(2x-3)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{(\sqrt{2x+3} + \sqrt{2x-3})(\sqrt{2x+3} - \sqrt{2x-3})} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\sqrt{2x+3} - \sqrt{2x-3}}{6} dx$

$\Rightarrow \frac{1}{6} \int \sqrt{2x+3}\, dx - \frac{1}{6} \int \sqrt{2x-3}\, dx$

$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3 \times 6 \times 2} - \frac{2(2x-3)^{\frac{3}{2}}}{3 \times 6 \times 2} + c$

$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$

62. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan x\, dx = ?$$

A. log |cos x| + C

B. - log |cos x| + C

C. log |sin x| + C

D. - log |sin x| + C

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put cos x = t -sin x dx = dt

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow - \log t + c$

$\Rightarrow - \log|\cos x| + c$

63. Question

Mark (√) against the correct answer in each of the following:

$\int \sec x \, dx = ?$

A. log |sec x - tan x| + C

B. - log |sec x + tan x| + C

C. log |sec x + tan x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(sec^2 x + \sec x \tan x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\sec x + \tan x| + c$

64. Question

Mark (√) against the correct answer in each of the following:

$\int \operatorname{cosec} x \, dx = ?$

A. log |cosec x - cot x| + C

B. - log |cosec x - cot x| + C

C. log |cosec x + cot x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$

$\Rightarrow \int \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$

Put $\operatorname{cosec} x - \cot x = t$, $(\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\operatorname{cosec} x - \cot x| + c$

65. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(1+\sin x)}{(1+\cos x)} dx = ?$

A. $\tan \frac{x}{2} + 2\log\left|\cos \frac{x}{2}\right| + C$

B. $-\tan \frac{x}{2} + 2\log\left|\cos \frac{x}{2}\right| + C$

C. $\tan \frac{x}{2} - 2\log\left|\cos \frac{x}{2}\right| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2\cos^2 \frac{x}{2}} dx$

$$\Rightarrow \int \frac{1}{2cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}} dx = \frac{1}{2}\int sec^2\frac{x}{2}dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}dx$$

$$\Rightarrow \frac{1}{2}\tan\frac{x}{2} \times 2 + \int tan\ \frac{x}{2}dx$$

$$\Rightarrow \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$$

$$\Rightarrow \tan\frac{x}{2} - 2\log|\cos\frac{x}{2}| + c$$

66. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan x}{(\sec x + \cos x)} dx = ?$$

A. $\tan^{-1}(\cos x) + C$

B. $-\tan^{-1}(\cos x) + C$

C. $\cot^{-1}(\cos x) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$$\Rightarrow \int \frac{\sec x \tan x}{sec^2 x + 1} dx$$

Put sec x = t (sec x tan x) dx = dt

$$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$$

$$\Rightarrow \tan^{-1} \sec x + c$$

$$\Rightarrow -\tan^{-1}(\cos x) + c$$

67. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{\frac{1+x}{1-x}} dx = ?$$

A. $\sin^{-1} x + \sqrt{1-x^2} + C$

B. $\sin^{-1} x + (1+x^2) + C$

C. $\sin^{-1} x - \sqrt{1-x^2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}} dx$

$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$

Put $1 - x^2 = t$ -2x dx = dt

$\Rightarrow \sin^{-1} x - \frac{1}{2} \int \frac{1}{\sqrt{t}} dt + c$

$\Rightarrow \sin^{-1} x - \frac{1}{2} \frac{\sqrt{t}}{\frac{1}{2}} + c$

$\Rightarrow \sin^{-1} x - \sqrt{t} + c \;\; = \;\; \sin^{-1} x - \sqrt{1 - x^2} + c$

68. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x^2} e^{-1/x} dx = ?$$

A. $e^{-1/x} + C$

B. $-e^{-1/x} + C$

C. $\frac{e^{-1/x}}{x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int sec^2 x dx = tan\, x$

Therefore ,

Put $-\frac{1}{x} = t \; \frac{1}{x^2} dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c$

$\Rightarrow e^{-\frac{1}{x}} + c$

69. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^3}{(1 + x^8)} dx = ?$$

A. $\tan^{-1} x^4$ + C

B. 4 $\tan^{-1} x^4$ + C

C. $\frac{1}{4} \tan^{-1} x^4 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^4 = t$ $4x^3 dx = dt$

$\Rightarrow \frac{1}{4} \int \frac{1}{1+t^2} dt$

$\Rightarrow \frac{1}{4} \tan^{-1} t + c$

$\Rightarrow \frac{1}{4} \tan^{-1} x^4 + c$

70. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = ?$$

A. $\frac{1}{3}(x+\log x)^3 + C$

B. $\frac{x^2}{2} + x + C$

C. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^1 + \log x = t$ $(1 + \frac{1}{x})dx = dt \Rightarrow (\frac{x+1}{x})dx = dt$

$\Rightarrow \int t^2 dt$

$\Rightarrow \frac{t^3}{3} + c$

$\Rightarrow \frac{(x+\log x)^3}{3} + c$

71. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2x \tan^{-1} x^2}{(1+x^4)} dx = ?$$

A. $(\tan^{-1} x^2)^2 + C$

B. $2 \tan^{-1} x^2 + C$

C. $\frac{1}{2}(\tan^{-1} x^2)^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan^{-1} x^2 = t$ $(\frac{1}{1+(x^2)^2} \times 2x)dx = dt \Rightarrow (\frac{2x}{1+x^4})dx = dt$

$\Rightarrow \int t^1 dt$

$\Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\tan^{-1} x^2)^2}{2} + c$

72. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(2-3x)} = ?$$

A. - 3 log |2 – 3x| + C

B. $-\frac{1}{3}\log|2-3x| + C$

C. - log |2 – 3x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $2 - 3x = t$ $-3dx = dt$

$\Rightarrow -\frac{1}{3}\int \frac{1}{t} dt$

$\Rightarrow -\frac{1}{3}\log t + c$

$\Rightarrow -\frac{1}{3}\log|2-3x| + c$

73. Question

Mark (√) against the correct answer in each of the following:

$$\int x\sqrt{x^2-1}\,dx = ?$$

A. $\frac{2}{3}(x^2-1)^{3/2} + C$

B. $\frac{1}{3}(x^2-1)^{3/2} + C$

C. $\frac{1}{\sqrt{x^2-1}} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $x^2 - 1 = t\ 2xdx = dt$

$\Rightarrow \int \sqrt{t}dt$

$\Rightarrow \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

74. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(5-3x)} dx = ?$

A. $\frac{3^{(5-3x)}}{3(\log 3)} + C$

B. $\frac{3^{(4-3x)}}{(\log 3)} + C$

C. $-3^{(5 - 3x)}$ log 3 + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put $5 - 3x = t\ -3dx = dt$

$\Rightarrow -\frac{1}{3}\int 3^t dt$

$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$

$\Rightarrow -\frac{3^{(5-3x)}}{3\log 3} + c$

75. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\tan x} \sec^2 x\, dx = ?$

A. $e^{\tan x}$ + tan x + C

B. $e^{\tan x}$. tan x + C

C. $e^{\tan x}$ + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t$ $sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

76. Question

Mark (√) against the correct answer in each of the following:

$$\int e^{\cos^2 x} \sin 2x \, dx = ?$$

A. $e^{\cos^2 x} + C$

B. $-e^{\cos^2 x} + C$

C. $e^{\sin^2 x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $cos^2 x = t \Rightarrow 2\cos x\,(-\sin x) dx = dt \Rightarrow -\sin 2x \ dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{cos^2 x} + c$

77. Question

Mark (√) against the correct answer in each of the following:

$$\int x \sin^3 x^2 \cos x^2 dx = ?$$

A. $\frac{1}{4} \sin^4 x^2 + C$

B. $\frac{1}{8} \sin^4 x^2 + C$

C. $\frac{1}{2} \sin^4 x^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x \cos x^2 \, dx = dt$

$\Rightarrow \frac{1}{2} \int t^3 dt$

$\Rightarrow \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$\Rightarrow \frac{(\sin x^2)^4}{8} + c$

78. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx = ?$$

A. $\sin\left(e^{\sqrt{x}}\right) + C$

B. $\frac{1}{2}\sin\left(e^{\sqrt{x}}\right) + C$

C. $2\sin\left(e^{\sqrt{x}}\right) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}})dx = dt$

$\Rightarrow \int 2dt$

$\Rightarrow 2t + c \Rightarrow 2\sin e^{\sqrt{x}} + c$

79. Question

Mark (√) against the correct answer in each of the following:

$$\int x^2 \sin x^3 dx = ?$$

A. $\cos x^3 + C$

B. $-\cos x^3 + C$

C. $-\frac{1}{3}\cos x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{1}{3}\int \sin t\, dt$

$\Rightarrow -\frac{1}{3}\cos t + c \Rightarrow -\frac{1}{3}\cos x^3 + c$

80. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx = ?$$

A. tan (xe^x) + C

B. - tan (xe^x) + C

C. cot (xe^x) + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$\Rightarrow \int \frac{dt}{cos^2 t} \Rightarrow \int sec^2 t\, dt = \tan t + c$

$\Rightarrow \tan(xe^x) + c$

81. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x\sqrt{x^4 - 1}} dx = ?$$

A. $\sec^{-1} x^2$ + C

B. $\frac{1}{2}\sec^{-1} x^2 + C$

C. $\operatorname{cosec}^{-1} x^2$ + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}} dt$

$\Rightarrow \frac{1}{2}\sec^{-1} t + c \Rightarrow \frac{1}{2}\sec^{-1} x^2 + c$

82. Question

Mark (√) against the correct answer in each of the following:

$$\int x\sqrt{x-1}\, dx = ?$$

A. $\frac{2}{3}(x-1)^{3/2}+C$

B. $\frac{2}{5}(x-1)^{5/2}+C$

C. $\frac{2}{5}(x-1)^{5/2}+\frac{3}{2}(x-1)^{3/2}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\,;\ \int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1}t+c$

Therefore ,

Put $x-1=t \Rightarrow x = t+1 \Rightarrow dx = dt$

$\Rightarrow \int (t+1)\times\sqrt{t}dt \Rightarrow \int t^{\frac{3}{2}}dt+\int t^{\frac{1}{2}}dt$

$\Rightarrow \frac{t^{\frac{5}{2}}}{\frac{5}{2}}+\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c \Rightarrow \frac{2t^{\frac{5}{2}}}{5}+\frac{2t^{\frac{3}{2}}}{3}+c$

$\Rightarrow \frac{2(x-1)^{\frac{5}{2}}}{5}+\frac{2(x-1)^{\frac{3}{2}}}{3}+c$

83. Question

Mark (√) against the correct answer in each of the following:

$\int x\sqrt{x^2-x}\,dx = ?$

A. $\frac{1}{3}(x^2-1)^{3/2}+C$

B. $\frac{2}{3}(x^2-1)^{3/2}+C$

C. $\frac{1}{\sqrt{x^2-1}}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\,;\ \int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1}t+c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2-1}dx$

Put $x^2-1=t \Rightarrow 2xdx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{2} \Rightarrow \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt$

$\Rightarrow \frac{t^{\frac{3}{2}}}{3}+c \Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3}+c$

$\Rightarrow \frac{1}{3}(x^2-1)^{\frac{3}{2}}+c$

84. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1+\sqrt{x}\right)} = ?$$

A. $\sqrt{x} - \log\left|1+\sqrt{x}\right| + C$

B. $\sqrt{x} + \log\left|1+\sqrt{x}\right| + C$

C. $2\sqrt{x} - 2\log\left|1+\sqrt{x}\right| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int \frac{1}{1+\sqrt{x}} dx$

Put $x = t^2 \Rightarrow dx = 2tdt$

$\Rightarrow \int \frac{2t}{1+t} dt \Rightarrow 2\int \frac{t}{1+t} dt \Rightarrow 2\int \frac{t+1-1}{1+t} dt \Rightarrow 2\int dt - 2\int \frac{1}{1+t} dt$

$\Rightarrow 2t - 2\log(1+t) + c \Rightarrow 2\sqrt{x} - 2\log(1+\sqrt{x}) + c$

85. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1}\, dx$$

A. $\frac{3}{2}\left(e^x - 1\right)^{3/2} + C$

B. $\frac{1}{2}\left(e^x - 1\right)^{1/2} + C$

C. $\frac{2}{3}\left(e^x - 1\right)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t}dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2}dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2}dz \Rightarrow 2\int \frac{1+z^2}{1+z^2}dz - 2\int \frac{1}{1+z^2}dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2}dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

86. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(\sin x - \cos x)}dx = ?$$

A. $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. log |sin x – cos x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx$

Put $(\sin x - \cos x) = t\ (\sin x + \cos x)\ dx = dt$

$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t}dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

87. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\tan x)} = ?$$

A. $\frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. $\frac{1}{2}x-\frac{1}{2}\log|\sin x-\cos x|+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\sin x}{\cos x}} dx \Rightarrow \int \frac{\cos x}{\cos x - \sin x} dx$

We can write $\cos x = \frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\cos x - \sin x)+(\sin x + \cos x)]}{(\cos x - \sin x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\cos x - \sin x)}{\cos x - \sin x} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

Put $(\cos x - \sin x) = t\ (\sin x + \cos x)\ dx = -dt$

$\Rightarrow \frac{x}{2} - \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} - \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

88. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\cot x)} = ?$$

A. log |sin x – cos x| + C

B. $\frac{1}{2}\log|\sin x-\cos x|+C$

C. $\frac{1}{2}x-\frac{1}{2}\log|\sin x-\cos x|+C$

D. $\frac{1}{2}x+\frac{1}{2}\log|\sin x-\cos x|+C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\cos x}{\sin x}} dx \Rightarrow \int \frac{\sin x}{\sin x - \cos x} dx$

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x)+(\sin x + \cos x)]}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx \Rightarrow \frac{x}{2}+\frac{1}{2}\int \frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

Put $(\sin x - \cos x) = t\ (\sin x + \cos x)\ dx = dt$

$\Rightarrow \frac{x}{2}+\frac{1}{2}\int \frac{1}{t}dt \Rightarrow \frac{x}{2}+\frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

89. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx = ?$$

A. $\sin^{-1}$ (tan x) + C

B. $\cos^{-1}$ (sin x) + C

C. tan-1 (cos x) + C

D. $\tan^{-1}$ (sin x) + C

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\ ;\ \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

Put $\tan x = t \Rightarrow sec^2 x\ dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1-t^2}}dt \Rightarrow \sin^{-1}t + c$

$\Rightarrow \sin^{-1}(\tan x) + c$

90. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x^2+1)}{(x^4+1)}dx = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(x-\frac{1}{x}\right)+C$

B. $\frac{1}{\sqrt{2}}\cot^{-1}\left\{\left(x-\frac{1}{x}\right)\right\}+C$

C. $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x-\frac{1}{x}\right)\right\}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\ ;\ \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2}dx$

Put $x-\frac{1}{x}=t \Rightarrow (1+\frac{1}{x^2})\,dx = dt$

$\Rightarrow \int \frac{1}{t^2+2}dt \Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$

$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}[\frac{1}{\sqrt{2}}(x-\frac{1}{x})]+c$

91. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin^6 x}{\cos^8} dx = ?$$

A. $\frac{1}{7}\tan^7 x + C$

B. $\frac{1}{7}\sec^7 x + C$

C. $\log|\cos^6 x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\,;\ \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

$\Rightarrow \int \frac{\sin^6 x}{\cos^6 x \cos^2 x}dx \Rightarrow \int \frac{\tan^6 x}{\cos^2 x}dx \Rightarrow \int \tan^6 x \sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7}+c$

$\Rightarrow \frac{(\tan x)^7}{7}+c$

92. Question

Mark (√) against the correct answer in each of the following:

$$\int \sec^5 x \tan x \, dx = ?$$

A. $\frac{1}{5}\tan^5 x + C$

B. $\frac{1}{5}\sec^5 x + C$

C. 5 log |cos x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c\,;\ \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

$\Rightarrow \int \sec^4 x \sec x \tan x \, dx$

Put $\sec x = t \Rightarrow \sec x \tan x \, dx = dt$

$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$

$\Rightarrow \frac{(\sec x)^5}{5} + c$

93. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^5 x \, dx = ?$

A. $\frac{1}{6}\tan^6 x + C$

B. $\frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x + \log|\sec x| + C$

C. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log|\sec x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \, ; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int tan^3 x \, tan^2 x dx \Rightarrow \int tan^3 x(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int tan^3 x dx \Rightarrow \int tan^3 x sec^2 x dx - \int tan^1 x \, tan^2 x dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x \, (sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x \, sec^2 x dx + \int \tan x \, dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^3 dt - \int t^1 dt + \log|\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log|\sec x| + c$

94. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3 x \cos^3 x \, dx = ?$

A. $-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

B. $\frac{1}{4}\cos^4 x - \frac{1}{6}\cos^6 x + C$

C. $\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \cos x\,(cos^2 x\, sin^3 x)dx \Rightarrow \int \cos x\,((1 - sin^2 x)\, sin^3 x)dx$

$\Rightarrow \int \cos x\,(sin^3 x - sin^5 x)dx \Rightarrow \int sin^3 x cos\, x dx - \int sin^5 x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$

$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

95. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^4 x \tan x\, dx = ?$

A. $\frac{1}{2}\sec^2 x + \frac{1}{4}\sec^4 x + C$

B. $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

C. $\frac{1}{2}\sec x + \log|\sec x + \tan x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x\, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x\, dx$

$\Rightarrow \int sec^2 x \tan x\, dx + \int tan^3 x sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$

96. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\log\, \tan x}{\sin x \cos x} dx = ?$

A. log {log (tan x)| + C

B. $\frac{1}{2}(\log\, \tan x)^2 + C$

C. log (sin x cos x) + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x \, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x \, dx$

$\Rightarrow \int sec^2 x \tan x \, dx + \int tan^3 x sec^2 x dx$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\log|\tan x|)^2}{2} + c$

97. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3 (2x + 1) dx = ?$

A. $\frac{1}{8} \sin^4 (2x + 1) + C$

B. $\frac{1}{2} \cos(2x + 1) + \frac{1}{3} \cos^3 (2x + 1) + C$

C. $-\frac{1}{2} \cos(2x + 1) + \frac{1}{6} \cos^3 (2x + 1) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sin^2 (2x + 1) \sin(2x + 1) \, dx \Rightarrow \int (1 - cos^2 (2x + 1)) \sin(2x + 1) \, dx$

$\Rightarrow \int \sin(2x + 1) \, dx - \int cos^2 (2x + 1) \sin(2x + 1) \, dx$

Put $\cos(2x + 1) = t \Rightarrow -2 \sin(2x + 1) \, dx = dt$

$\Rightarrow -\int \frac{dt}{2} - (-\frac{1}{2}) \int t^2 dt \Rightarrow -\frac{1}{2} \int dt + \frac{1}{2} \int t^2 dt$

$\Rightarrow -\frac{1}{2} t + \frac{1}{2} \frac{t^3}{3} + c \Rightarrow -\frac{1}{2} t + \frac{t^3}{6} + c$

$\Rightarrow -\frac{1}{2} \cos(2x + 1) + \frac{[\cos(2x+1)]^3}{6} + c$

98. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sqrt{\tan x}}{\sin x + \cos x} dx = ?$

A. $2\sqrt{\tan x}+C$

B. $2\sqrt{\cot x}+C$

C. $2\sqrt{\sec x}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x \times \cos x} dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x} \times \frac{1}{\sec x}} dx \Rightarrow \int \frac{sec^2 x}{\sqrt{\tan x}} dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}} + c \Rightarrow 2\sqrt{t} + c$

$\Rightarrow 2\sqrt{\tan x} + c$

99. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos + \sin x)}{(1-\sin 2x)} dx = ?$$

A. log |sin x – cos x| + C

B. $\frac{1}{(\cos x - \sin x)} + C$

C. log |cos x + sin x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\cos x + \sin x}{cos^2 x + sin^2 x - \sin 2x} dx \Rightarrow \int \frac{\cos x + \sin x}{(\cos x - \sin x)^2} dx$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x) dx = -dt$

$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x - \sin x} + c$

100. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1} dx = ?$$

A. $\frac{2}{3}(e^x - 1)^{3/2} + C$

B. $\frac{1}{2} \cdot \frac{e^x}{\sqrt{e^x - 1}} + C$ e

C. $2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2} dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

101. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{\sin^3 x \cos x}} = ?$$

A. $2\sqrt{\tan x} + C$

B. $2\sqrt{\cot x} + C$

C. $-2\sqrt{\tan x} + C$

D. $\frac{-2}{\sqrt{\tan x}} + C$

Answer

Let $I = \int \frac{dx}{\sqrt{\sin^3 x \cos x}}$

Now multiplying and dividing by $\cos^2$x, we get,

$$I = \int \frac{dx}{\sqrt{\sin^3 x \times \cos x}} \times \frac{1}{\cos^2 x} \times \cos^2 x$$

$$I = \int \frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}} dx$$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$$

Let tan x = t

Differentiating both sides, we get,

$\sec^2 x \, dx = dt$

Therefore,

$I = \int \frac{dt}{t^{3/2}}$

Integrating, we get,

$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$

$I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C$

$I = -\frac{2}{\sqrt{t}} + C$

$I = -\frac{2}{\sqrt{\tan x}} + C$

Exercise 13B

1. Question

Evaluate the following integrals:

(i) $\int \sin^2 x \, dx$

(ii) $\int \cos^2 x \, dx$

Answer

i) $\int sin^2 x dx$

$\Rightarrow \int sin^2 x dx$

Now, we know that $1-\cos 2x = 2\sin^2 x$

So, applying this identity in the given integral, we get,

$$\int sin^2 x dx = \int \frac{(1 - cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2x dx)$

$\Rightarrow \frac{x}{2} - \frac{sin2x}{2 \times 2} + c$

$\Rightarrow \frac{x}{2} - \frac{sin2x}{4} + c$

Ans: $\int sin^2 x dx = \frac{x}{2} - \frac{sin2x}{4} + c$

ii) $\int cos^2 x \, dx$

$\Rightarrow \int cos^2 x \, dx$

Now, we know that $1+\cos 2x = 2\cos^2 x$

So, applying this identity in the given integral, we get,

$$\int cos^2x\,dx = \int \frac{(1+cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cos2xdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{4} + c$$

Ans: $\int cos^2x\,dx = \frac{x}{2} + \frac{sin2x}{4} + c$

2. Question

Evaluate the following integrals:

(i) $\int \cos^2(x/2)dx$

(ii) $\int \cot^2(x/2)dx$

Answer

(i) $\int \cos^2(x/2)dx$

$\Rightarrow \int cos^2\left(\frac{x}{2}\right)dx$

Now, we know that 1+cosx=2cos^2 (x/2)

So, applying this identity in the given integral, we get,

$$\int cos^2(\frac{x}{2})\,dx = \int \frac{(1+cosx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cosxdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

Ans: $\frac{x}{2} + \frac{sin2x}{2} + c$

ii) $\int \cot^2(x/2)dx$

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$

Now, we know that cosec^{2}x-cot^2 x=1

So, applying this identity in the given integral we get,

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx = \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx$

$\Rightarrow \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx = \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx$

$\Rightarrow \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx = \frac{-cotx}{\frac{1}{2}} - x + c$

$\Rightarrow$-2cotx-x+c

$\Rightarrow\int cot^2\left(\frac{x}{2}\right)dx$ =-2cotx-x+c

Ans: -2cotx-x+c

3. Question

Evaluate the following integrals:

(i) $\int \sin^2 nx\, dx$

(ii) $\int \sin^5 x\, dx$

Answer

i)$\int sin^2 nxdx$

$\Rightarrow\int sin^2 nxdx$

Now, we know that 1-cos2nx=2sin^{2}nx

So, applying this identity in the given integral, we get,

$$\int sin^2 nxdx = \int \frac{(1-cos2nx)dx}{2}$$

$\Rightarrow\frac{1}{2}(\int dx - \int cos2nxdx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2nx}{2n\times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4n} + c$$

Ans: $\int sin^2 nxdx = \frac{x}{2} - \frac{sin2nx}{4n} + c$

(ii) $\int \sin^5 x\, dx$

We know that 1-cos^{2}x=sin^{2}x

$\Rightarrow\int sin^5 xdx = \int(1-cos^2 x)^2 sinxdx$

⇒Put cosx=t

⇒-sinxdx=dt

$\Rightarrow\int(1-cos^2 x)^2 sinxdx = -\int(1-t^2)^2 dt$

$\Rightarrow-\int(1-t^2)^2 dt = -\int(1+t^4-2t^2)dt$

$\Rightarrow-\int dt + \int 2t^2 dt - \int t^4 dt$

$\Rightarrow-t+\frac{2t^3}{3}-\frac{t^5}{5}+c$

Resubstituting the value of t=cosx we get,

$\Rightarrow-cosx+\frac{2cos^3 x}{3}-\frac{cos^5 x}{5}+c$

Ans: $-cosx+\frac{2cos^3 x}{3}-\frac{cos^5 x}{5}+c$

4. Question

Evaluate the following integrals:

$$\int \cos^3(3x+5)\,dx$$

Answer

Substitute 3x+5=u

⇒3dx=du

⇒dx=du/3

⇒$\int cos^3(3x+5)dx = \frac{1}{3}\int cos^3(u)du$

Now We know that 1-cos^{2}x=sin^{2}x ,

⇒$\frac{1}{3}\int cos^3(u)du = \frac{1}{3}\int(1-sin^2(u))cosu\,du$

⇒Substitute sinu=t

⇒cosu du=dt

⇒$\frac{1}{3}\int(1-sin^2(u))cosu\,du = \frac{1}{3}\int(1-t^2)\,dt$

⇒$\frac{1}{3}\int dt - \frac{1}{3}\int t^2 dt$

⇒$\frac{t}{3} - \frac{t^3}{3\times3} + c$

⇒$\frac{t}{3} - \frac{t^3}{9} + c$

Resubstituting the value of t=sinu and u=3x+5 we get,

⇒$\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Ans: $\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

5. Question

Evaluate the following integrals:

$$\int \sin^7(3-2x)\,dx$$

Answer

⇒$-\int sin^7(2x-3)dx$

Substitute 2x-3=u

⇒ 2dx=du

⇒dx=du/2

⇒$-\left(\frac{1}{2}\right)\int sin^7(u)du$

⇒ We know that 1-cos^{2}x=sin^{2}x

⇒$-\left(\frac{1}{2}\right)\int\left(1-cos^2(u)\right)^3 sinu\,du$

⇒Put cosu=t

⇒-sinxdu=dt

⇒$\left(\frac{1}{2}\right)\int(1-t^2)^3 dt$

⇒$\left(\frac{1}{2}\right)\int(1-t^6-3t^2+3t^4)dt$

$\Rightarrow\left(\frac{1}{2}\right)[\int dt - \int t^6 dt - \int 3t^2 dt + \int 3t^4 dt]$

$\Rightarrow\left(\frac{1}{2}\right)\left[t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5}\right] + c$

$\Rightarrow\left(\frac{1}{2}\right)\left[t - \frac{t^7}{7} - t^3 + \frac{3t^5}{5}\right] + c$

Resubstituting the value of t=cosu and u=2x-3 we get

$\Rightarrow\left(\frac{1}{2}\right)\left[\cos(2x-3) - \frac{cos^7(2x-3)}{7} - cos^3(2x-3) + \frac{3cos^5(2x-3)}{5}\right] + c$

$\Rightarrow\frac{\cos(2x-3)}{2} - \frac{cos^7(2x-3)}{14} - \frac{cos^3(2x-3)}{2} + \frac{3cos^5(2x-3)}{10} + c$

Now as we know cos(-x)=cosx

$\Rightarrow\frac{\cos(2x-3)}{2} - \frac{cos^7(2x-3)}{14} - \frac{cos^3(2x-3)}{2} + \frac{3cos^5(2x-3)}{10} + c$

$=\frac{\cos(3-2x)}{2} - \frac{cos^7(3-2x)}{14} - \frac{cos^3(3-2x)}{2} + \frac{3cos^5(3-2x)}{10} + c$

Ans: $\frac{\cos(3-2x)}{2} - \frac{cos^7(3-2x)}{14} - \frac{cos^3(3-2x)}{2} + \frac{3cos^5(3-2x)}{10} + c$

6. Question

Evaluate the following integrals:

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

Answer

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

$\Rightarrow\int \frac{1-cos2x}{1+cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int \frac{1-cos2x}{1+cos2x}dx = \int \frac{2sin^2 x}{2cos^2 x}dx$

$\Rightarrow\int tan^2 x\, dx$

Now sec^{2}x-1=tan^{2}x

$\Rightarrow\int (sec^2 x - 1)dx$

$\Rightarrow\int sec^2 x dx - \int dx$

⇒tanx-x+c

Ans: tanx-x+c

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx = \int \frac{2cos^2x}{2sin^2x}dx$

$\Rightarrow\int cot^2x\, dx$

Now cosec^{2}x-1=cot^{2}x

$\Rightarrow\int (cosec^2x - 1)dx$

$\Rightarrow\int cosec^2x dx - \int dx$

⇒-cotx-x+c

Ans: -cotx-x+c

7. Question

Evaluate the following integrals:

(i) $\int \frac{1-cosx}{1+cosx}dx$

(ii) $\int \frac{1+cosx}{1-cosx}dx$

Answer

i) $\int \frac{1-cosx}{1+cosx}dx$

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx = \int \frac{2sin^2(\frac{x}{2})}{2cos^2(\frac{x}{2})}dx$

$\Rightarrow\int tan^2(\frac{x}{2})dx$

Now sec^2(x/2)-1=tan2(x/2)

$\Rightarrow\int \left(sec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int sec^2(\frac{x}{2})dx - \int dx$

⇒2tan(x/2)-x+c

Ans: 2tan(x/2)-x+c

(ii) $\int \frac{1+cosx}{1-cosx}dx$

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx = \int \frac{2cos^2(\frac{x}{2})}{2sin^2(\frac{x}{2})}dx$

$\Rightarrow\int cot^2(\frac{x}{2})dx$

Now cosec^2 (x/2)-1=cot^2 (x/2)

$\Rightarrow\int \left(cosec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int cosec^2(\frac{x}{2})dx - \int dx$

⇒-2cot(x/2)-x+c

Ans: ⇒-2cot(x/2)-x+c

8. Question

Evaluate the following integrals:

$$\int \sin 3x \cos 4x \, dx$$

Answer

⇒$\int sin3x\ cos4x\ dx$

Applying the formula: sinx×cosy=1/2(sin(x+y)-sin(y-x))

⇒$\frac{1}{2}\int (sin7x - sinx)dx$

⇒$\frac{1}{2}\int sin7x\ dx - \frac{1}{2}\int sinx\ dx$

⇒$\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Ans: $\frac{-cos7x}{14} + \frac{cosx}{2} + c$

9. Question

Evaluate the following integrals:

$$\int \cos 4x \cos 3x \, dx$$

Answer

⇒$\int cos4x\ cos3x\ dx$

Applying the formula: cosx×cosy=1/2(cos(x+y)+cos(x-y))

⇒$\frac{1}{2}\int (cos7x + cosx)dx$

⇒$\frac{1}{2}\int cos7x\ dx + \frac{1}{2}\int cosx\ dx$

⇒$\frac{sin7x}{14} + \frac{sinx}{2} + c$

Ans: $\frac{sin7x}{14} + \frac{sinx}{2} + c$

10. Question

Evaluate the following integrals:

$$\int \sin 4x \sin 8x \, dx$$

Answer

⇒$\int sin4x\ sin8x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

⇒$\frac{1}{2}\int (cos4x - cos12x)dx$

⇒$\frac{1}{2}\int cos4x\ dx - \frac{1}{2}\int cos12x\ dx$

⇒$\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Ans: $\frac{sin4x}{8} - \frac{sin12x}{24} + c$

11. Question

Evaluate the following integrals:

$$\int \sin\ 6x\ \cos\ x\ dx$$

Answer

$\Rightarrow \int sin6x\ cosx\ dx$

Applying the formula: sinx×cosy=1/2(sin(y+x)-sin(y-x))

$\Rightarrow \frac{1}{2}\int (sin7x - \sin(-5x))dx$

$\Rightarrow \frac{1}{2}\int sin7x\ dx + \frac{1}{2}\int sin5x\ dx$

$\Rightarrow \frac{-cos7x}{14} - \frac{cosx}{10} + c$

Ans: $\frac{-cos7x}{14} - \frac{cosx}{10} + c$

12. Question

Evaluate the following integrals:

$$\int \sin\ x\sqrt{1+\cos\ 2x}\ dx$$

Answer

we know that $1+cos2x=2cos^2x$

So, applying this identity in the given integral we get,

$\Rightarrow \int sinx\sqrt{1+cos2x}dx$

$\Rightarrow \int sinx\sqrt{(2cos^2x)}dx$

$\Rightarrow \sqrt{2}\int sinxcosxdx$

Let sinx =t

⇒ cosx dx=dt

$\Rightarrow \sqrt{2}\int tdt$

$\Rightarrow \sqrt{2}\frac{t^2}{2} + c = \frac{t^2}{\sqrt{2}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^2x}{\sqrt{2}} + c$

Ans: $\frac{sin^2x}{\sqrt{2}} + c$

13. Question

Evaluate the following integrals:

$$\int \cos^4 x\, dx$$

Answer

$\Rightarrow \int cos^2xcos^2xdx$

$\Rightarrow \int (\frac{1+cos2x}{2})(\frac{1+cos2x}{2})dx \ldots (\frac{1+cos2x}{2} = cos^2x)$

$\Rightarrow \frac{1}{4}\int (1+cos2x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1+cos^2 2x+2cos2x)dx$

$\Rightarrow \frac{1}{4}[\int 1dx \ +\int cos^2 2xdx+\int 2cos2x\ dx$

$\Rightarrow \frac{1}{4}[x+\int \frac{(1+cos4x)dx}{2}+2\frac{sin2x}{2}]$ …(1+cos4x=2cos^{2}x)

$\Rightarrow \frac{1}{4}[x+\frac{1}{2}(\int dx+\int cos4xdx)+sin2x]+c$

$\Rightarrow [\frac{x}{4}+\frac{1}{2}\times\frac{1}{4}\left(\int dx+\int cos4xdx\right)+\frac{sin2x}{4}\right]+c$

$\Rightarrow [\frac{x}{4}+\left(\frac{x}{8}+\frac{sin4x}{32}\right)+\frac{sin2x}{4}\right]+c$

$\Rightarrow \frac{3x}{8}+\frac{sin4x}{32}+\frac{sin2x}{4}+c$

Ans: $\frac{3x}{8}+\frac{sin4x}{32}+\frac{sin2x}{4}+c$

14. Question

Evaluate the following integrals:

$$\int \cos 2x \cos 4x \cos 6x \, dx$$

Answer

$\Rightarrow \int cos2x\ cos4x\ cos\ 6xdx$

$\Rightarrow \frac{1}{2}\int (cos6x+cos2x)cos6xdx$

$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{2}\int cos2xcos6xdx$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int (cos8x+cos4x)dx$$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int cos8x\,dx+\frac{1}{4}\int cos4x\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(1+cos12x)dx}{2}+\frac{1}{4}\frac{sin8x}{8}+\frac{1}{4}\frac{sin4x}{4}+c$$

$$\Rightarrow \frac{1}{4}\left(x+\frac{sin12x}{12}\right)+\frac{sin8x}{32}+\frac{sin4x}{16}+c$$

$$\Rightarrow \frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$$

Ans: $\frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x \, dx$$

Answer

Let sinx =t

⇒ cosx dx =dt

$\Rightarrow \int sin^3 x\ cosx\ dx=\int t^3 dt$

$\Rightarrow \frac{t^4}{4} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^4 x}{4} + c$

Ans: $\frac{sin^4 x}{4} + c$

16. Question

Evaluate the following integrals:

$$\int \sec^4 x \, dx$$

Answer

$\Rightarrow \int sec^4 dx = \int sec^2 x sec^2 x dx$

$\Rightarrow \int sec^2 x(1 + tan^2 x)dx$

$\Rightarrow$Put tanx=t $\Rightarrow$sec^{2}dx=dt

$\Rightarrow \int (1 + t^2)dt$

$\Rightarrow t + \frac{t^3}{3} + c$

Resubstituting the value of t=tanx we get

$\Rightarrow tanx + \frac{tan^3 x}{3} + c$

Ans: $tanx + \frac{tan^3 x}{3} + c$

17. Question

Evaluate the following integrals:

$$\int \cos^3 x \sin^4 x \, dx$$

Answer

$\Rightarrow \int cos^3 x sin^4 x \, dx$

$\Rightarrow \int cosx \, sin^4 x cos^2 x dx$

$\Rightarrow \int cosx \, sin^4 x(1 - sin^2 x)dx$

Put sinx=t

$\Rightarrow$cosxdx=dt

$\Rightarrow \int t^4(1 - t^2)dt$

$\Rightarrow \int t^4 dt - \int t^6 \, dt$

$\Rightarrow \frac{t^5}{5} - \frac{t^7}{7} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Ans: $\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

18. Question

Evaluate the following integrals:

$$\int \cos^4 x \sin^3 x \, dx$$

Answer

$\Rightarrow \int cos^4 x sin^3 x\, dx$

$\Rightarrow \int sinx\, sin^2 x cos^4 x dx$

$\Rightarrow \int sinx\, cos^4 x(1 - cos^2 x)dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^4(t^2 - 1)dt$

$\Rightarrow \int t^6 dt - \int t^4\, dt$

$\Rightarrow \frac{t^7}{7} - \frac{t^5}{5} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Ans: $\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

19. Question

Evaluate the following integrals:

$$\int \sin^{2/3} x \cos^3 x \, dx$$

Answer

$\Rightarrow \int cos^3 x sin^{\frac{2}{3}} x\, dx$

$\Rightarrow \int cosx\, cos^2 x sin^{\frac{2}{3}} x dx$

$\Rightarrow \int cosx\, (1 - sin^2 x) sin^{\frac{2}{3}} x dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^{\frac{2}{3}}(1 - t^2)dt$

$\Rightarrow \int t^{\frac{2}{3}} dt - \int t^{\frac{8}{3}}\, dt$

$\Rightarrow \frac{t^{\frac{5}{3}}}{\frac{5}{3}} - \frac{t^{\frac{11}{3}}}{\frac{11}{3}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$

Ans: $\frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$

20. Question

Evaluate the following integrals:

$$\int \cos^{3/5} x \sin^3 x \, dx$$

Answer

$\Rightarrow \int sin^3 x cos^{\frac{3}{5}} x\, dx$

$\Rightarrow \int sinx\, sin^2 x cos^{\frac{3}{5}} xdx$

$\Rightarrow \int sinx\,(1 - cos^2 x) cos^{\frac{3}{5}} xdx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^{\frac{3}{5}}(t^2 - 1)dt$

$\Rightarrow \int t^{\frac{13}{5}} dt - \int t^{\frac{3}{5}} dt$

$$\Rightarrow \frac{t^{\frac{18}{5}}}{\frac{18}{5}} - \frac{t^{\frac{8}{5}}}{\frac{8}{5}} + c$$

Resubstituting the value of t=cosx we get

$$\Rightarrow \frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$$

Ans: $\frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$

21. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^4 2x \, dx$$

Answer

$$\Rightarrow \int cosec^4 2xdx$$

$$\Rightarrow \int cosec^2 2xcosec^2 2xdx$$

$$\Rightarrow \int cosec^2 2x\,(1 + cot^2 2x)dx$$

⇒cot2x=t ⇒-2cosec^2 2xdx=dt

$$\Rightarrow -1/2 \int (1 + t^2)dt$$

$$\Rightarrow -1/2 \int dt - 1/2 \int t^2 dt$$

$$\Rightarrow -(\frac{1}{2})\text{t} - \frac{t^3}{6} + c$$

Resubstituting the value of t=cotx we get

$$\Rightarrow -\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$$

Ans: $-\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$

22. Question

Evaluate the following integrals:

$$\int \frac{\cos 2x}{\cos x} dx$$

Answer

$$\Rightarrow \int \frac{\cos 2x}{\cos x} dx = \int \frac{2\cos^2 x - 1}{\cos x} dx$$

$$\Rightarrow \int \frac{2\cos^2 x}{\cos x} dx - \int \frac{1}{\cos x} dx$$

$$\Rightarrow \int 2\cos x dx - \int \sec x dx$$

$$\Rightarrow 2\sin x - \log|\sec x + \tan x| + c$$

Ans: 2sinx-log|secx+tanx|+c

23. Question

Evaluate the following integrals:

$$\int \frac{\cos x}{\cos(x+\alpha)} dx$$

Answer

$$\Rightarrow \int \frac{\cos x}{\cos(x+\alpha)} dx = \int \frac{\cos\big((x+\alpha)-\alpha\big)}{\cos(x+\alpha)} dx$$

$$\Rightarrow \int \frac{\cos(x+\alpha)\cos\alpha + \sin(x+\alpha)\sin\alpha}{\cos(x+\alpha)} dx$$

$$\Rightarrow \int \cos\alpha dx + \int \tan(x+\alpha)\sin\alpha\, dx$$

Now α is a constant

$$\Rightarrow x\cos\alpha - \sin\alpha\log|\cos(x+\alpha)| + c$$

Ans:xcos α-sin αlog|cos(x+ α)|+c

24. Question

Evaluate the following integrals:

$$\int \cos^3 x \sin 2x\, dx$$

Answer

$$\Rightarrow \int \sin 2x \cos^3 x dx$$

$$\Rightarrow \int 2\sin x\cos x\cos^3 x dx$$

$$\Rightarrow \int 2sinx\,cos^4 x dx$$

Now put cosx=t

⇒-sinxdx=dt

$$\Rightarrow -2\int t^4 dt$$

$$\Rightarrow -2 \times \frac{t^5}{5} + c$$

Resubstituting the value of t= cosx we get,

$$\Rightarrow \frac{-2cos^5 x}{5} + c$$

Ans: $\frac{-2cos^5 x}{5} + c$

25. Question

Evaluate the following integrals:

$$\int \frac{\cos^9 x}{\sin x} dx$$

Answer

$$\Rightarrow \int \frac{cos^9 x}{sinx} dx$$

$$\Rightarrow \int \frac{cos^9 x}{sin^2 x} sinxdx$$

$$\Rightarrow \int \frac{cos^9 x}{1 - cos^2 x} sinxdx$$

Put cosx =t

⇒ -sinxdx=dt

$$\Rightarrow \int \frac{t^9}{t^2 - 1} dt$$

Now put t^2-1=a

⇒2tdt=da

And $t^8=(a+1)^4$

$$\Rightarrow \frac{1}{2}\int \frac{(a+1)^4}{a} da$$

$$\Rightarrow \frac{1}{2}\int (a^3 + 4a^2 + 6a + \frac{1}{a} + 4) da$$

$$\Rightarrow \frac{1}{2}\left(\frac{a^4}{4} + \frac{4a^3}{3} + \frac{6a^2}{2} + lna + 4a\right) + c$$

$$\Rightarrow \left(\frac{a^4}{8} + \frac{2a^3}{3} + \frac{3a^2}{2} + \frac{lna}{2} + 2a\right) + c$$

Resubstituting the value of $a=t^2-1$ and t=cosx ⇒$a=cos^2x-1=-sin^2x$ we get

$$\Rightarrow \left(\frac{(-sin^2x)^4}{8} + \frac{2(-sin^2x)^3}{3} + \frac{3(-sin^2x)^2}{2} + \frac{\ln|(-sin^2x)|}{2} + 2(-sin^2x)\right) + c$$

$$\Rightarrow \left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \frac{2\ln|(-sinx)|}{2} - 2sin^2x\right) + c$$

$$\Rightarrow \left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \ln(sinx) - 2sin^2x\right) + c$$

Ans: $\left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \ln(sinx) - 2sin^2x\right) + c$

26. Question

Evaluate the following integrals:

$$\int \cos^4 2x\,dx$$

Answer

$\Rightarrow \int cos^2 2xcos^2 2xdx$

$\Rightarrow \int (\frac{1+cos4x}{2})(\frac{1+cos4x}{2})dx \ldots (\frac{1+cos4x}{2} = cos^2 2x)$

$\Rightarrow \frac{1}{4}\int (1+cos4x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1+cos^2 4x + 2cos4x)dx$

$\Rightarrow \frac{1}{4}[\int 1dx + \int cos^2 4xdx + \int 2cos4x\,dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos8x)dx}{2} + 2\frac{sin4x}{4}]$ ...(1+cos8x=2cos^2 4x)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}\left(\int dx + \int cos8xdx\right) + (\frac{sin4x}{2})\Big] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2} \times \frac{1}{4}\left(\int dx + \int cos8xdx\right) + \frac{sin4x}{8}\Big] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin8x}{64}\right) + \frac{sin4x}{8}\Big] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Ans: $\frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

27. Question

Evaluate the following integrals:

$$\int \frac{\sin^2 x}{(1+\cos x)^2}dx$$

Answer

Doing tangent half angle substitution we get,

$$\Rightarrow \int \frac{sin^2x}{(1+cos^2x)}dx = \int \frac{\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)^2}{[1+(\frac{1-tan^2\frac{x}{2}}{1-tan^2\frac{x}{2}})]^2}$$

Substitute u=tan(x/2)

⇒2du=sec^2(x/2)dx

⇒dx=$\frac{2du}{u^2+1}$

⇒$2\int \frac{u^2}{1+u^2}du$

⇒$2\int \frac{1+u^2}{1+u^2}du - 2\int \frac{1}{1+u^2}du$

⇒$2\int du - \tan^{-1}u + c$

⇒$2u - \tan^{-1}u + c$

Resubstituting the values we get,

$$\Rightarrow 2\tan\frac{x}{2} - \tan^{-1}\tan\frac{x}{2} + c$$

$$\Rightarrow 2\tan\frac{x}{2} - \frac{x}{2} + c$$

Ans: $2\tan\frac{x}{2} - \frac{x}{2} + c$

28. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3\cos x + 4\sin x)}$$

Answer

$$\int \frac{dx}{3cosx + 4sinx} = \int \frac{dx}{3\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 4\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 8tan\frac{x}{2} - 3tan^2\frac{x}{2}}$$

Let tan$\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{3 + 8t - 3t^2} = \frac{2}{3}\int \frac{dt}{1 + \frac{8}{3}t - t^2} = \frac{2}{3}\int \frac{dt}{1 - \left(t - \frac{4}{3}\right)^2 + \frac{16}{9}}$$

$$\Rightarrow \frac{2}{3}\int \frac{dt}{\frac{25}{9} - \left(t - \frac{4}{3}\right)^2} = \frac{2}{3}\int \frac{dt}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{2 \times \frac{5}{3}} \ln\left|\frac{\frac{5}{3} + \left(t - \frac{4}{3}\right)}{\frac{5}{3} - \left(t - \frac{4}{3}\right)}\right| + c = \frac{1}{5}\ln\left|\frac{1 + 3t}{9 - 3t}\right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5}\ln\left|\frac{1 + 3tan\frac{x}{2}}{9 - 3tan\frac{x}{2}}\right| + c$$

Ans: $\frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right|+c$

29. Question

Evaluate the following integrals:

$$\int\frac{dx}{(a\cos x+b\sin x)^2},\ a>0 \text{ and } b>0$$

Answer

$$\int\frac{dx}{(acosx+bsinx)^2}$$

Taking bcosx common from the denominator we get,

$$\int\frac{dx}{b^2cos^2x(\frac{a}{b}+tanx)^2}$$

$$\Rightarrow\frac{1}{b^2}\int\frac{sec^2xdx}{(\frac{a}{b}+tanx)^2}$$

Let (a/b)+tanx=t

$\therefore sec^2xdx=dt$

$$\Rightarrow\frac{1}{b^2}\int\frac{dt}{t^2}=\frac{-1}{b^2}\times\frac{1}{t}=\frac{-1}{b^2t}+c$$

Resubstituting the value of t = (a/b)+tanx we get

$$\Rightarrow\frac{-1}{b^2(\frac{a}{b}+tanx)}+c=\frac{-1}{ab+b^2tanx}+c$$

Ans: $\frac{-1}{ab+b^2tanx}+c$

30. Question

Evaluate the following integrals:

$$\int\frac{dx}{(\cos x-\sin x)}$$

Answer

$$\int\frac{dx}{cosx-sinx}=\int\frac{dx}{\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)-\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow\int\frac{sec^2\frac{x}{2}dx}{1-2tan\frac{x}{2}-tan^2\frac{x}{2}}$$

Let $tan\frac{x}{2}$=t

$\therefore\frac{1}{2}sec^2\frac{x}{2}dx=dt$

$\Rightarrow \int \frac{2dt}{1-2t-t^2} = -2\int \frac{dt}{t^2+2t-1} = -2\int \frac{dt}{(t+1)^2-2}$

$= -2\int \frac{dt}{(t+1)^2-(\sqrt{2})^2}$

$\Rightarrow -2 \times \frac{1}{2\times\sqrt{2}} \ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c$ resubstituting the value of t we get

$\Rightarrow \frac{-1}{\sqrt{2}} \ln\left|\frac{tan\frac{x}{2}+1-\sqrt{2}}{tan\frac{x}{2}+1+\sqrt{2}}\right| + c = \frac{-1}{\sqrt{2}} \ln\left|\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)\right| + c$

Ans: $\frac{-1}{\sqrt{2}} \ln\left|\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)\right| + c$

31. Question

Evaluate the following integrals:

$\int (2\tan x - 3\cot x)^2 dx$

Answer

$\int (2tanx - 3cotx)^2 dx$

$\Rightarrow \int (4tan^2x + 9cot^2x - 12tanxcotx)dx$

$\Rightarrow \int (4(sec^2x - 1) + 9(cosec^2x - 1) - 12)dx$

$\Rightarrow \int 4sec^2x\, dx + \int 9cosec^2x dx - \int 25dx$

$\Rightarrow 4\tan x - 9\cot x - 25x + c$

Ans: 4tanx-9cotx-25x+c

32. Question

Evaluate the following integrals:

$\int \sin x \sin 2x \sin 3x\, dx$

Answer

$\Rightarrow \int sinx\ sin2xsin3x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2}\int (cos2x - cos4x)sin2xdx$

$\Rightarrow \frac{1}{2}\int sin2xcos2x\ dx - \frac{1}{2}\int sin2xcos4x\ dx$

$\Rightarrow \frac{1}{4}\int sin4x\ dx - \frac{1}{4}\int (sin6x - sin2x)dx$

$\Rightarrow \frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Ans: $\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

33. Question

Evaluate the following integrals:

$$\int\left(\frac{1-\cot x}{1+\cot x}\right)dx$$

Answer

$$\Rightarrow \int \frac{1-cotx}{1+cotx}dx = \int \frac{1-\frac{cosx}{sinx}}{1+\frac{cosx}{sinx}}dx$$

$$\Rightarrow \int \frac{sinx - cosx}{sinx + cosx}dx = -\int \frac{cosx - sinx}{sinx + cosx}dx$$

$$\Rightarrow -\int \frac{d(sinx + cosx)}{sinx + cosx}$$

$$\Rightarrow -\log|\text{sinx} + \text{cosx}| + \text{c}$$

Ans: -log(sinx+cosx)+c

34. Question

Evaluate the following integrals:

$$\int\frac{dx}{(2\sin x + \cos x + 3)}$$

Answer

$$\int \frac{dx}{cosx + 2sinx + 3} = \int \frac{dx}{\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right) + 2\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right) + 3}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 1 + 3tan^2\frac{x}{2} + 4tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let tan$\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{4 + 4t + 2t^2} = \int \frac{dt}{2 + 2t + t^2} = \frac{2}{3}\int \frac{dt}{(t+1)^2 + 2 - 1}$$

$$\Rightarrow \int \frac{dt}{(t+1)^2 + 1} = \int \frac{dt}{(1)^2 + (t+1)^2}$$

$$\Rightarrow \tan^{-1}(t+1) + c$$

Resubstituting the value of t we get

$$\Rightarrow \tan^{-1}(tan\frac{x}{2} + 1) + c$$

Ans: $\tan^{-1}(tan\frac{x}{2} + 1) + c$

Exercise 13C

1. Question

Evaluate the following integrals:

$$\int x e^x dx$$

Answer

Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x.e^x dx = x\int e^x - \int\frac{dx}{dx}.\int e^x dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1) + c$$

2. Question

Evaluate the following integrals:

$$\int x \cos x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos xdx = x\int \cos x - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx$$

$$= x\sin x - \int 1.\sin xdx$$

$$= x\sin x + \cos x + c$$

3. Question

Evaluate the following integrals:

$$\int x e^{2x} dx$$

Answer

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{dx}{dx}.\int e^{2x}dx\right]dx$$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{2\times 2} + c$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

4. Question

Evaluate the following integrals:

$$\int x \sin 3x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\sin 3xdx = x\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{3\times 3}\right) + c$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{9}\right) + c$$

5. Question

Evaluate the following integrals:

$$\int x\cos 2x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos 2x dx = x\int \cos 2x dx - \int\left[\frac{dx}{dx}.\int \cos 2x dx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1.\left(\frac{\sin 2x}{2}\right)dx$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{2\times 2}\right) + c$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{4}\right) + c$$

6. Question

Evaluate the following integrals:

$$\int x \log 2x\, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log 2x is the first function, and x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\log 2x dx = \log 2x\int x dx - \int\left[\frac{d\log 2x}{dx}.\int x dx\right]dx$$

$$= \log 2x.\frac{x^2}{2} - \int\left[\frac{1\times 2}{2x}\frac{x^2}{2}\right]dx$$

$$= \frac{x^2}{2}\log 2x - \int \frac{x}{2}dx$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{2\times 2} + c$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$$

7. Question

Evaluate the following integrals:

$$\int x\, \mathrm{cosec}^2 x\, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\mathrm{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\,\mathrm{cosec}^2 x dx = x\int \mathrm{cosec}^2 x - \int\left[\frac{dx}{dx}.\int \mathrm{cosec}^2 x dx\right]dx$$

$$= x(-\cot x) - \int 1.(-\cot x)dx$$

$$= -x\cot x + \int \cot x dx$$

$$= -x\cot x + \ln|\sin x| + c$$

8. Question

Evaluate the following integrals:

$$\int x^2 \cos x \; dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^2 \cos x dx = x^2\int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx$$

$$= x^2 \sin x - \int[2x \times \sin x]dx$$

$$= x^2 \sin x - 2\left[\int x \sin x dx\right]$$

Again applying by the part method in the second half, we get

$$x^2 \sin x - 2\int x \sin x dx$$

$$= x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x)dx\right]$$

$$= x^2 \sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

9. Question

Evaluate the following integrals:

$$\int x \sin^2 x \; dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\sin^2 x = \frac{1+\cos 2x}{2}$

We have

$$\int x\sin^2 x dx = \int x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$=\int\left(\frac{x}{2}-\frac{x\cos 2x}{2}\right)dx$$

$$=\int\frac{x}{2}dx-\int\frac{x\cos 2x}{2}dx$$

$$=\frac{x^2}{2\times 2}-\frac{1}{2}\int x\cos 2x dx$$

Taking X as first function and Cos 2x as the second function.

$$=\frac{x^2}{4}-\frac{1}{2}\left\{x\int\cos 2x dx-\int\left(\frac{dx}{dx}.\int\cos 2x dx\right)dx\right\}$$

$$=\frac{x^2}{4}-\frac{1}{2}\left\{x.\frac{\sin 2x}{2}-\int\left(1.\frac{\sin 2x}{2}\right)dx\right\}$$

$$=\frac{x^2}{4}-\frac{1}{2}\left\{\frac{x\sin 2x}{2}-\left(\frac{-\cos 2x}{2\times 2}\right)\right\}+c$$

$$=\frac{x^2}{4}-\frac{1}{2}\left\{\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right\}+c$$

$$=\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}+c$$

10. Question

Evaluate the following integrals:

$$\int x\ \tan^2 x\ dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\int x\tan^2 x dx = \int x\left(\sec^2 x-1\right)dx$$

$$=\int x\sec^2 x dx-\int x dx$$

Using x as the first function and $\sec^2 x$ as the second function

$$\int x\sec^2 x dx - \int x dx$$

$$= \left\{ x\int \sec^2 x dx - \int\left(\frac{dx}{dx}.\int \sec^2 x dx\right)dx \right\} - \frac{x^2}{2}$$

$$= \left\{ x.\tan x - \int 1.\tan x dx \right\} - \frac{x^2}{2}$$

$$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$$

$$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$$

$$x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$$

11. Question

Evaluate the following integrals:

$$\int x^2 e^x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^2 e^x dx = \left[x^2\int e^x dx - \int\left(\frac{dx^2}{dx}.\int e^x dx\right)dx \right]$$

$$= x^2 e^x - \int 2x.e^x dx$$

$$= x^2 e^x - 2\int x e^x dx$$

$$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\frac{dx}{dx}.\int e^x dx\right)dx \right]$$

$$= x^2 e^x - 2\left[x e^x - \int 1.e^x dx \right]$$

$$= x^2 e^x - 2\left[x e^x - e^x \right] + c$$

$$= x^2 e^x - 2x e^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

12. Question

Evaluate the following integrals:

$$\int x^2 \cos^3 x\, dx$$

Answer

We know that Cos3x = 4Cos^{3}x - 3Cosx

Cos^{3}x = $\frac{\cos 3x + 3\cos x}{4}$

$$\int x^2 \cos^3 x dx = \int x^2 \left(\frac{\cos 3x + 3\cos x}{4} \right) dx$$
$$= \frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx \right)$$

Taking X^2 as the first function and cos 3x and cos x as the second function and applying By part method.

$$\frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx \right)$$
$$= \frac{1}{4}\left\{ \left(x^2 \int \cos 3x dx - \int \left[\frac{dx^2}{dx} . \int \cos 3x dx \right] dx \right) + 3\left(x^2 \int \cos x dx - \int \left[\frac{dx^2}{dx} . \int \cos x dx \right] dx \right) \right\}$$
$$= \frac{1}{4}\left\{ \left(\frac{x^2 . \sin 3x}{3} - \int 2x . \frac{\sin 3x}{3} dx \right) + 3\left(x^2 \sin x - \int 2x . \sin x dx \right) \right\}$$
$$= \frac{1}{4}\left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\int x \sin 3x dx \right) + 3\left(x^2 \sin x - 2\int x \sin x dx \right) \right\}$$
$$= \frac{1}{4}\left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\int \sin 3x dx - \int \left(\frac{dx}{dx} . \int \sin 3x dx \right) dx \right] \right) + 3\left(x^2 \sin x - 2\left[x\int \sin x dx - \int \left(\frac{dx}{dx} . \int \sin x dx \right) dx \right] \right) \right\}$$
$$= \frac{1}{4}\left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x \frac{-\cos 3x}{3} - \int 1 . \frac{-\cos 3x}{3} dx \right] \right) + 3\left(x^2 \sin x - 2\left[-x \cos x - \int -\cos x dx \right] \right) \right\}$$
$$= \frac{1}{4}\left\{ \left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[\frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} \right] \right) + 3\left(x^2 \sin x + 2x \cos x - 2 \sin x \right) \right\} + c$$
$$= \frac{1}{4}\left\{ \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + 3x^2 \sin x + 6x \cos x - 6 \sin x \right\} + c$$
$$= \frac{x^2 \sin 3x}{12} + \frac{x \cos 3x}{18} - \frac{\sin 3x}{54} + \frac{3x^2 \sin x}{4} + \frac{3x \cos x}{2} - \frac{3}{2} \sin x + c$$

13. Question

Evaluate the following integrals:

$$\int x^2 e^{3x} dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\int x^2 e^{3x} dx = x^2 \int e^{3x} dx - \int \left(\frac{dx^2}{dx} . \int e^{3x} dx \right) dx$$

$$= x^2 \frac{e^{3x}}{3} - \int 2x . \frac{e^{3x}}{3} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \int x e^{3x} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \int e^{3x} dx - \int \left[\frac{dx}{dx} . \int e^{3x} dx \right] dx \right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx \right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \frac{e^{3x}}{9} \right) + c$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$$

$$= e^{3x} \left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} \right) + c$$

14. Question

Evaluate the following integrals:

$$\int x^2 \sin^2 x \, dx$$

Answer

We can write $\sin^2 x = \frac{1 - \cos 2x}{2}$

We have

$$\int x^2 \left(\frac{1 - \cos 2x}{2} \right) dx = \int \frac{x^2}{2} - \frac{x^2 \cos 2x}{2} dx$$

$$= \int \frac{x^2}{2} dx - \int \frac{x^2 \cos 2x}{2} dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$=\frac{x^3}{3\times 2}-\frac{1}{2}\int x^2\cos 2x dx$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2\int\cos 2x dx-\int\left[\frac{dx^2}{dx}.\int\cos 2x dx\right]dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int 2x.\frac{\sin 2x}{2}dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int x.\sin 2x dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\int\sin 2x dx-\int\left(\frac{dx}{dx}.\int\sin 2x dx\right)dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\frac{-\cos 2x}{2}-\int 1.\frac{-\cos 2x}{2}dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}+\frac{x\cos 2x}{2}-\frac{\sin 2x}{4}\right)+c$$

$$=\frac{x^3}{6}-\frac{x^2\sin 2x}{4}-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}+c$$

15. Question

Evaluate the following integrals:

$$\int x^3\log\ 2x\ dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log2x is the first function, and x^3 is the second function.

Using Integration by part

$$\int a.b.dx=a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^3\log 2x dx=\log 2x\int x^3dx-\int\left(\frac{d\log 2x}{dx}.\int x^3dx\right)dx$$

$$=\log 2x\frac{x^4}{4}-\int\frac{1.2}{2x}.\frac{x^4}{4}dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}\int x^3dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}.\frac{x^4}{4}+c$$

$$=\log 2x\frac{x^4}{4}-\frac{x^4}{16}+c$$

16. Question

Evaluate the following integrals:

$$\int x \cdot \log(x+1)dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 1) is first function and x is second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x\log(x+1) = \log(x+1)\int xdx - \int\left(\frac{d\log(x+1)}{dx}.\int xdx\right)dx$$

$$= \log(x+1)\frac{x^2}{2} - \int\frac{1}{x+1}\times\frac{x^2}{2}dx$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2-1+1}{x+1}dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{x^2-1}{x+1}+\frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{(x+1)(x-1)}{x+1}+\frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int(x-1)+\frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

17. Question

Evaluate the following integrals:

$$\int\frac{\log x}{x^n}dx$$

Answer

We can write it as $\int x^{-n}.\log x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logx is the first function, and x^{-n} is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^{-n}\log x dx = \log x\int x^{-n}dx - \int\left(\frac{d\log x}{dx}.\int x^{-n}dx\right)dx$$

$$= \log x\left(\frac{x^{-n+1}}{-n+1}\right) - \int\frac{1}{x}.\frac{x^{-n+1}}{-n+1}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\int\frac{x^{-n}.x}{x}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\times\frac{x^{-n+1}}{-n+1} + c$$

$$= \frac{x^{-n+1}\log x}{1-n} - \frac{x^{-n+1}}{(1-n)^2} + c$$

18. Question

Evaluate the following integrals:

$$\int 2x^3 e^{x^2}\, dx$$

Answer

We can write it as $\int 2.x.x^2.e^{x^2}dx$

Let $x^2 = t$

$2xdx = dt$

Using the relation in the above condition, we get

$$\int 2x.x^2.e^{x^2}dx = \int t.e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int te^t dt = t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt$$

$$= te^t - \int 1.e^t dt$$

$$= te^t - e^t + c$$

Replacing t with x^2,we get

$$x^2e^{x^2} - e^{x^2} + c$$

$$= e^{x^2}\left(x^2 - 1\right) + c$$

19. Question

Evaluate the following integrals:

$$\int x \sin^3 x \, dx$$

Answer

We know that Sin3x = 3Sinx - 4Sin^{3}x

Sin^{3}x = (3Sinx - Sin3x)/4

$$\int x \sin^3 x dx = \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$= \frac{1}{4}\int 3x \sin x - x \sin 3x dx$$

$$= \frac{3}{4}\int x \sin x dx - \frac{1}{4}\int x \sin 3x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sinx and sin3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{3}{4}\int x \sin x dx - \frac{1}{4}\int x \sin 3x dx$$

$$= \frac{3}{4}\left(x\int \sin x dx - \int\left[\frac{dx}{dx}.\int \sin x dx\right]dx\right) - \frac{1}{4}\left(s\int \sin 3x dx - \int\left[\frac{dx}{dx}.\int \sin 3x dx\right]dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \int \cos x dx\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \int \frac{\cos 3x}{3}dx\right)$$

$$= \frac{3}{4}(-x\cos x + \sin x) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right) + c$$

$$= \frac{-3x\cos x}{4} + \frac{3\sin x}{4} + \frac{x\cos 3x}{12} - \frac{\sin 3x}{36} + c$$

20. Question

Evaluate the following integrals:

$$\int x \cos^3 x \, dx$$

Answer

We can write cos^{3}x = (cos3x + 3cosx)/4, we have

$$\int x\cos^3 x dx = \int x\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\int x\cos 3x dx + \frac{3}{4}\int x\cos x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and cosx and cos3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{4}\left(x\int \cos 3xdx - \int\left[\frac{dx}{dx}.\int \cos 3xdx\right]dx\right) + \frac{3}{4}\left(x\int \cos xdx - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx\right)$$

$$= \frac{1}{4}\left(x\frac{\sin 3x}{3} - \int\frac{\sin 3x}{3}dx\right) + \frac{3}{4}\left(x\sin x - \int \sin xdx\right)$$

$$= \frac{1}{4}\left(\frac{x\sin 3x}{3} + \frac{\cos 3x}{9}\right) + \frac{3}{4}(x\sin x + \cos x) + c$$

$$= \frac{x\sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3x\sin x}{4} + \frac{3\cos x}{4} + c$$

21. Question

Evaluate the following integrals:

$$\int x^3 \cos x^2 dx$$

Answer

We can write it as

$$\int x.x^2 \cos x^2 dx$$

Now let $x^2 = t$

2xdx = dt

Xdx = dt/2

Now

$$\frac{1}{2}\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and cost as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\cos tdt = \frac{1}{2}\left(t\int \cos tdt - \int\left[\frac{dt}{dt}.\int \cos tdt\right]dt\right)$$

$$= \frac{1}{2}\left(t\sin t - \int \sin tdt\right)$$

$$= \frac{1}{2}(t\sin t + \cos t) + c$$

Replacing t with x^2

$$= \frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + c$$

22. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x)dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(cosx) is the first function and sinx as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \sin x \log(\cos x)dx = \log(\cos x)\int \sin xdx - \int\left(\frac{d\log(\cos x)}{dx}.\int \sin xdx\right)dx$$

$$= -\cos x\log(\cos x) + \int\frac{-\sin x}{\cos x}.\cos xdx$$

$$= -\cos x\log(\cos x) - \int \sin xdx$$

$$= -\cos x\log(\cos x) + \cos x + c$$

23. Question

Evaluate the following integrals:

$$\int x \sin x \cos x \; dx$$

Answer

We know that Sin2x = 2Sinxcosx

$$\int x\sin x\cos xdx = \frac{1}{2}\int x\sin 2xdx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sin2x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int x\sin 2xdx = \frac{1}{2}\left(x\int \sin 2xdx - \int\left[\frac{dx}{dx}.\int \sin 2xdx\right]dx\right)$$

$$= \frac{1}{2}\left(x\frac{-\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right)$$

$$= \frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right) + c$$

$$= \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

24. Question

Evaluate the following integrals:

$$\int \cos\sqrt{x}\ dx$$

Answer

Let √x = t

$$\frac{1}{2\sqrt{x}}dx = dt$$
$$\Rightarrow dx = 2\sqrt{x}dt$$
$$\Rightarrow dx = 2tdt$$

We can write it as

$$\int \cos\sqrt{x}dx = 2\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and cos t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow 2\int t\cos tdt = 2\left(t\int \cos tdt - \int\left[\frac{dt}{dt}\right]\int \cos tdt\right)dt$$
$$= 2\left(t\sin t - \int \sin tdt\right)$$
$$= 2t\sin t + 2\cos t + c$$

Replacing t with √x

= 2√xsin√x + 2cos√x + c

= 2(cos√x + √xsin√x) + c

25. Question

Evaluate the following integrals:

$$\int \operatorname{cosec}^3 x\, dx$$

Answer

We can write it as $\int \operatorname{cosec}^3 dx = \int \operatorname{cosec}x.\operatorname{cosec}^2 xdx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cosecx is first function and $\operatorname{cosec}^2 x$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \operatorname{cosec}x.\operatorname{cosec}^2 x dx = \operatorname{cosec}x \int .\operatorname{cosec}^2 x dx - \int \left(\frac{d \operatorname{cosec}x}{dx} . \int .\operatorname{cosec}^2 x dx \right) dx$$

$$= \operatorname{cosec}x\left(-\cot x\right) - \int \left(-\operatorname{cosec}x.\cot x\right)\left(-\cot x\right) dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x.\cot^2 x dx$$

We know that $Cot^2x = Cosec^2x - 1$

$$-\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x \left(\operatorname{cosec}^2 x - 1\right) dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec}x dx$$

We can write $\int \operatorname{cosec}^3 x dx = I$

$$\Rightarrow \int \operatorname{cosec}^3 x dx - \operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec}x dx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 x dx = -\operatorname{cosec}x.\cot x + \int \operatorname{cosec}x dx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 x dx = -\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x| + c_1$$

$$\Rightarrow \int \operatorname{cosec}^3 x dx = \frac{-\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x|}{2} + c$$

26. Question

Evaluate the following integrals:

$$\int x \sin^3 x \cos x \, dx$$

Answer

We can write it as $\int x \sin^2 x \sin x \cos x dx$

We also know that 2sinx.cosx = sin2x

$$\int x \sin^2 x \sin x \cos x dx = \frac{1}{2}\int x \sin^2 x \sin 2x dx$$

We also know that $\sin^2 x = \frac{1-\cos 2x}{2}$

$$\frac{1}{2}\int x \sin^2 x \sin 2x dx = \frac{1}{2}\int x.\left(\frac{1-\cos 2x}{2}\right)\sin 2x dx$$

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \int \frac{x \cos 2x \sin 2x}{2} dx\right)\right]$$

Here Sin4x = 2sin2x.cos2x

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \frac{1}{4}\int x \sin 4x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int \sin 2xdx - \int\left(\frac{dx}{dx}.\int \sin 2xdx\right)dx\right\}\right) - \left(\frac{1}{4}\left\{x\int \sin 4x - \int\left(\frac{dx}{dx}.\int \sin 4xdx\right)dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \int\frac{\cos 4x}{4}dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \frac{\sin 2x}{4}\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \frac{\sin 4x}{16}\right\}\right)\right] + c$$

$$= \frac{-x\cos 2x}{8} + \frac{\sin 2x}{16} + \frac{x\cos 4x}{32} - \frac{\sin 4x}{128} + c$$

27. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x)dx$$

Answer

Let cosx = t

- sinxdx = dt

Now the integral we have is

$$\int \sin x \log(\cos x)dx = -\int \log tdt$$

$$= -\int 1.\log tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$-\int 1.\log tdt = \log t\int 1dt - \int\left(\frac{d\log t}{dt}.\int 1.dt\right)dt$$

$$= -\log t.t + \int\frac{1}{t}.tdt$$

$$= -t\log t + t + c$$

Replacing t with cosx

$$t(-\log t + 1) + c$$

$$= \cos x(1 - \log(\cos x)) + c$$

28. Question

Evaluate the following integrals:

$$\int \frac{\log(\log x)}{x} dx$$

Answer

Let logx = t

1/x dx = dt

$$\int \frac{\log(\log x)}{x} dx = \int \log t dt = \int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int b dx - \int \left[\frac{da}{dx}.\int b dx\right] dx$$

$$\int 1.\log t dt = \log t \int 1 dt - \int \left(\frac{d \log t}{dt}.\int 1.dt\right) dt$$

$$= t.\log t - \int \frac{1}{t} t dt$$

$$= t \log t - t + c$$

Now replacing t with logx

$$\log x.\log(\log x) - \log x + c$$

$$= \log x(\log(\log x) - 1) + c$$

29. Question

Evaluate the following integrals:

$$\int \log(2 + x^2) dx$$

Answer

$$= \int 1.\log(2 + x^2) dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(2 + x^2)$ is the first function and 1 as the second function.

$$\int a.b.dx = a\int b dx - \int \left[\frac{da}{dx}.\int b dx\right] dx$$

$$\int 1.\log\left(2+x^2\right)dx = \log\left(2+x^2\right)\int 1dx - \int\left(\frac{d\log\left(2+x^2\right)}{dx}.\int 1dx\right)dx$$

$$= \log\left(2+x^2\right).x - \int \frac{1.2x}{2+x^2}.xdx$$

$$= x\log\left(2+x^2\right) - \int \frac{2x^2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\int \frac{x^2+2-2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\left[\left(\int 1dx\right) - \int \frac{2}{2+x^2}dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - \left(2\int \frac{1}{2+x}\right)dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - 2\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right)\right] + c$$

$$= x\log\left(2+x^2\right) - 2x + 2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

30. Question

Evaluate the following integrals:

$$\int \frac{x}{(1+\sin\ x)}dx$$

Answer

$$\int \frac{x}{1+\sin x}dx = \int \frac{x(1-\sin x)}{(1+\sin x).(1-\sin x)}dx$$

We can write it as $= \int \frac{x(1-\sin x)}{1-\sin^2 x}dx$

$$= \int \frac{x(1-\sin x)}{\cos^2 x}dx$$

$$= \int x\sec^2 xdx - \int x\tan x\sec xdx$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and secxtanx as the second function, we have

$$\int x\sec^2 xdx - \int x\sec x\tan xdx = \left(x\int \sec^2 xdx - \int\left(\frac{dx}{dx}.\int \sec^2 xdx\right)dx\right)$$
$$-\left(x\int \sec x\tan xdx - \int\left(\frac{dx}{dx}.\int \sec x\tan xdx\right)dx\right)$$

$$= \left(x\tan x - \int 1.\tan xdx\right) - \left(x.\sec x - \int 1.\sec xdx\right)$$

$$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + c$$

$$= x(\tan x - \sec x) + \ln|1+\sin x| + c$$

31. Question

Evaluate the following integrals:

$$\int\left\{\frac{1}{\log x}-\frac{1}{(\log\ x)^2}\right\}dx$$

Answer

Let us assume logx = t

X = e^t

dx = e^tdt

Now we have

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Considering f(x) = 1/t ; f'(x) = - $1/t^2$

$$\frac{d}{dt}\left(\frac{1}{t}\right)=-\frac{1}{t^2}$$

By the integral property of $\int\{f(x)+f'(x)\}e^x dx=e^x.f(x)+c$

So the solution of the integral is

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=e^t\times\frac{1}{t}+c$$

Substituting the value of t as logx

$$=e^{\log x}\times\frac{1}{\log x}+c$$

$$=\frac{x}{\log x}+c$$

32. Question

Evaluate the following integrals:

$$\int e^{-x}\cos\ 2x\ \cos\ 4x\ dx$$

Answer

$$\cos A.\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

We know that $\Rightarrow \cos 4x.\cos 2x=\frac{1}{2}\left[\cos(4x+2x)+\cos(4x-2x)\right]$

$$=\frac{1}{2}\left[\cos 6x+\cos 2x\right]$$

Putting in the original equation

$$\int e^{-x}\cos 2x.\cos 4x dx = \int e^{-x}\left(\frac{1}{2}[\cos 6x + \cos 2x]\right)$$

$$= \frac{1}{2}\left[\left(\int e^{-x}\cos 6x dx\right) + \left(\int e^{-x}\cos 2x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cos6x and cos2x is first function and e^{-x} as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

Solving both parts individually

$$I = \int e^{-x}\cos 6x dx = \cos 6x\int e^{-x}dx - \int\left(\frac{d\cos 6x}{dx}.\int e^{-x}dx\right)dx$$

$$I = \cos 6x.\left(-e^{-x}\right) - \int\left(-6\sin 6x\right).\left(-e^{-x}\right)dt$$

$$I = -\cos 6x.e^{-x} - 6\int \sin 6x.e^{-x}dx$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\int e^{-x}dx - \int\left(\frac{d\sin 6x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\left(-e^{-x}\right) - \int\left(6\cos 6x\right).\left(-e^{-x}\right)dt\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6\int e^{-x}\cos 6x dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6I\right]$$

$$I = -e^{-x}\cos 6x + 6e^{-x}\sin 6x - 36I$$

$$37I = e^{-x}\left(6\sin 6x - \cos 6x\right)$$

$$I = \frac{e^{-x}\left(6\sin 6x - \cos 6x\right)}{37}$$

Solving the second part,

$$I=\int e^{-x}\cos 2x dx=\cos 2x\int e^{-x}dx-\int\left(\frac{d\cos 2x}{dx}.\int e^{-x}dx\right)dx$$

$$J=\cos 2x.\left(-e^{-x}\right)-\int\left(-2\sin 2x\right).\left(-e^{-x}\right)dt$$

$$J=-\cos 2x.e^{-x}-2\int\sin 2x.e^{-x}dx$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\int e^{-x}dx-\int\left(\frac{d\sin 2x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\left(-e^{-x}\right)-\int\left(2\cos 2x\right).\left(-e^{-x}\right)dt\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2\int e^{-x}\cos 2x dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2J\right]$$

$$J=-e^{-x}\cos 2x+2e^{-x}\sin 2x-4J$$

$$5J=e^{-x}\left(2\sin 2x-\cos 2x\right)$$

$$J=\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}$$

Putting in the obtained equation

$$=\frac{1}{2}\left[\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}\right]+c$$

$$=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{74}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{10}+c$$

$$=e^{-x}\left(\frac{\left(6\sin 6x-\cos 6x\right)}{74}+\frac{\left(2\sin 2x-\cos 2x\right)}{10}\right)+c$$

33. Question

Evaluate the following integrals:

$$\int e^{\sqrt{x}}dx$$

Answer

Let √x = t

$$\frac{1}{2\sqrt{x}}dx=dt$$

$$dx=2\sqrt{x}dt$$

$$\Rightarrow dx=2tdt$$

Replacing in the original equation , we get

$$\int e^{\sqrt{x}}dx=\int e^{t}.2tdt$$

$$=2\int te^{t}dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int te^t dt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[te^t - \int 1.e^t dt\right]$$

$$= 2\left[te^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with √x

$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

34. Question

Evaluate the following integrals:

$$\int e^{\sin x}\sin\ 2x\ dx$$

Answer

We can write Sin2x = 2sinx.cosx

$$\int e^{\sin x}\sin 2x dx = 2\int e^{\sin x}.\sin x\cos x dx$$

Let Sinx = t

Cosxdx = dt

$$2\int e^{\sin x}\sin x\cos x dx = 2\int e^t.t.dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int e^t.tdt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[t.e^t - \int 1.e^t dt\right]$$

$$= 2\left[t.e^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with sin x

$= 2e^{\sin x}(\sin x - 1) + c$

35. Question

Evaluate the following integrals:

$$\int\frac{x\ \sin^{-1}x}{\sqrt{1-x^2}}dx$$

Answer

Let $\sin^{-1}x = t$

$X = \sin t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

Putting this in the original equation, we get

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}dx = \int t.\sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t.\sin tdt = t\int \sin tdt - \int\left(\frac{dt}{dt}.\int \sin tdt\right)dt$$

$$= t(-\cos t) - \int 1.(-\cos t)dt$$

$$= -t\cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$= - t(\sqrt{1 - \sin^2 t}) + \sin t + c$

Now replacing $\sin^{-1}x = t$

$= - \sin^{-1}x(\sqrt{1 - x^2}) + x + c$

36. Question

Evaluate the following integrals:

$$\int \frac{x^2 \tan^{-1} x}{(1+x^2)}dx$$

Answer

Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2}dx = dt$$

Now we have

$$\int \frac{x^2 \tan^{-1} x}{(1+x^2)}dx = \int \tan^2 t.tdt$$

$$\int t.\tan^2 tdt = \int t(\sec^2 t - 1)dt$$

$$= \int t\sec^2 tdt - \int tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t\sec^2 tdt - \int tdt = t\int \sec^2 tdt - \int\left(\frac{dt}{dt}.\int \sec^2 tdt\right)dt - \frac{t^2}{2}$$

$$= t.\tan t - \int \tan tdt - \frac{t^2}{2}$$

$$= t.\tan t - \ln|\sec t| - \frac{t^2}{2} + c$$

We know that sec t = $\sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x.x - \ln|\sqrt{\tan^2 t + 1}| - \frac{\tan^2 x}{2} + c$$

$$= x\tan^{-1} x - \ln|\sqrt{x^2 + 1}| - \frac{\tan^2 x}{2} + c$$

37. Question

Evaluate the following integrals:

$$\int \frac{\log(x+2)}{(x+2)^2}dx$$

Answer

We can write it as $\int \log(x+2).\frac{1}{(x+2)^2}dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 2) is first function and $(x + 2)^{-2}$ as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \log(x+2).\frac{1}{(x+2)^2}dx = \log(x+2)$$

$$\int \frac{1}{(x+2)^2}dx - \int\left(\frac{d\log(x+2)}{dx}.\int\frac{1}{(x+2)^2}dx\right)dx$$

$$= \log(x+2).\frac{-1}{(x+2)} - \int\frac{1}{x+2}.\frac{-1}{(x+2)}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} + \int\frac{1}{(x+2)^2}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} - \frac{1}{(x+2)} + c$$

38. Question

Evaluate the following integrals:

$$\int x \sin^{-1} x\ dx$$

Answer

Let x = sin t ; t = $\sin^{-1}x$

dx = cos t dt

$$\Rightarrow \int x\sin^{-1}x dx = \int \sin t.\sin^{-1}(\sin t)\cos t dt$$

$$= \int \sin t.t.\cos t dt$$

We know that sin 2t = 2 sint×cost

We have $\int t\cos t\sin t dt = \frac{1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin 2t as the second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{1}{2}\int t\sin 2t dt = \frac{1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int\sin 2t dt\right]dt\right)$$

$$= \frac{1}{2}\left(t.\frac{-\cos 2t}{2} + \int\frac{\cos 2t}{2}dt\right)$$

$$= \frac{1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{-t\cos 2t}{4} + \frac{\sin 2t}{8} + c$$

We know that cos2t = 1 - 2$\sin^2$t , sin2t = 2sint×cost and cos t = √1 - $\sin^2$t

Replacing in above equation

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{\sqrt{1-\sin^2 t}}{4}.\sin t + c$$

$$= \frac{-\sin^{-1} x\left(1-2x^2\right)}{4} + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\sin^{-1} x - \frac{\sin^{-1} x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

39. Question

Evaluate the following integrals:

$$\int x \cos^{-1} x\, dx$$

Answer

Let x = cos t ; t = $\cos^{-1}x$

dx = - sin t dt

$$\int x\cos^{-1} x dx = -\int \cos t.\cos^{-1}(\cos t)\sin t dt$$

$$= -\int \cos t.t.\sin t.dt$$

We know that sin 2t = 2 sint×cost

We have $-\int t\cos t\sin t dt = \frac{-1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and sin 2t as second function.

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{-1}{2}\int t\sin 2t dt = \frac{-1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{-1}{2}\left(t.\frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2} dt\right)$$

$$= \frac{-1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{t\cos 2t}{4} - \frac{\sin 2t}{8} + c$$

We know that cos2t = $2\cos^2 t$ - 1 and sin2t = 2sint×cost and sint = $\sqrt{1 - \cos^2 t}$

Replacing in above equation

$$= \frac{t\left(2\cos^2 t - 1\right)}{4} - \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{t\left(2\cos^2 t - 1\right)}{4} - \frac{\sqrt{1-\cos^2 t}}{4}.\cos t + c$$

$$= \frac{\cos^{-1} x\left(2x^2 - 1\right)}{4} - \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\cos^{-1} x - \frac{\cos^{-1} x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\cos^{-1} x + \frac{\sin^{-1} x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

40. Question

Evaluate the following integrals:

$$\int \cot^{-1} x\ dx$$

Answer

We can write it as $\int \cot^{-1} x.1dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cot^{-1}x$ is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \cot^{-1} x.1dx = \cot^{-1} x\int 1dx - \int\left(\frac{d\cot^{-1} x}{dx}.\int 1dx\right)dx$$

$$= \cot^{-1} x.x - \int \frac{-1}{1+x^2}.x.dx$$

$$= x\cot^{-1} x + \int \frac{x}{1+x^2}dx$$

Let $1 + x^2 = t$

$2xdx = dt$

$Xdx = dt/2$

$$\Rightarrow \int \cot^{-1} xdx = x\cot^{-1} x + \int \frac{dt}{2t}$$

$$= x\cot^{-1} x + \frac{\log t}{2} + c$$

Now replacing t with $1 + x^2$

$= x\cot^{-1}x + \log(1 + x^2)/2 + c$

41. Question

Evaluate the following integrals:

$\int x \cot^{-1} x \, dx$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$\therefore \int x \cot^{-1} x \, dx$

$= \cot^{-1} x \int x dx - \int \left\{\frac{d}{dx}(\cot^{-1} x) \int x dx\right\} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \int \frac{1}{(1+x^2)} \times \frac{x^2}{2} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{(1+x^2)} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{1+x^2-x^2}{(1+x^2)} dx$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int 1 - \frac{1}{(1+x^2)} dx$

$= \frac{\mathbf{x^2 \cot^{-1} x}}{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}[\mathbf{x} - \mathbf{\tan^{-1} x}] + \mathbf{c}$, where c is the integrating constant

42. Question

Evaluate the following integrals:

$\int x^2 \cot^{-1} x \, dx$

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Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$\therefore \int x^2 \cot^{-1} x \, dx$

$= \cot^{-1} x \int x^2 dx - \int \left\{\frac{d}{dx}(\cot^{-1} x) \int x^2 dx\right\} dx$

$= \frac{x^3 \cot^{-1} x}{3} - \int \frac{1}{(1+x^2)} \times \frac{x^3}{3} dx$

$= \frac{x^3 \cot^{-1} x}{3} - \frac{1}{3} \int \frac{x^3}{(1+x^2)} dx$

Taking $(1+x^2)=a$,

$2xdx=da$ i.e. $xdx=da/2$

Again, x^2=a-1

$\therefore \frac{1}{3}\int \frac{x^2 \times xdx}{(1+x^2)}$

$= \frac{1}{3}\int \frac{(a-1)da}{2a}$

$= \frac{1}{6}\int \left(1-\frac{1}{a}\right)da$

$= \frac{1}{6}(a-\ln a)$

Replacing the value of a, we get,

$\therefore \frac{1}{6}(a-\ln a)$

$= \frac{1}{6}[(1+x^2)-\ln|x^2+1|+c_1$

$= \frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+\left(c_1+\frac{1}{6}\right)$

$= \frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$

The total integration yields as

$= \frac{x^3\cot^{-1}x}{3}+\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$, where c is the integrating constant

43. Question

Evaluate the following integrals:

$\int \sin^{-1}\sqrt{x}\ dx$

Answer

Tip – If f_1(x) and f_2(x) are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where f_1(x) and f_2(x) are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$\therefore \int \sin^{-1}\sqrt{x}dx$

$= \sin^{-1}\sqrt{x}\int dx-\int\left\{\frac{d}{dx}(\sin^{-1}\sqrt{x})\int dx\right\}dx$

$= x\sin^{-1}\sqrt{x}-\int\frac{1}{2\sqrt{x}\sqrt{1-x}}\times xdx$

$= x\sin^{-1}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}}{\sqrt{1-x}}dx$

Taking (1-x)=a^2,

-dx=2ada i.e. dx=-2ada

Again, x=1-a^2

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

44. Question

Evaluate the following integrals:

$$\int \cos^{-1}\sqrt{x}\ dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times xdx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$\mathbf{= x\cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

45. Question

Evaluate the following integrals:

$$\int \cos^{-1}\left(4x^3 - 3x\right)dx$$

Answer

Formula to be used – We know , cos3x = $4\cos^3x$-3cosx

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

Assuming x = cosa, $4\cos^3a$-3cosa=cos3a

And, dx = -sinada

Hence, a=$\cos^{-1}x$

Again, sina=√(1-x^2)

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

$$= \int \cos^{-1}(\cos 3a)\{-\sin a da\}$$

$$= -3\int a\sin a da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = sina,

$$\therefore -3\int a\sin a da$$

$$= -3\left[a\int \sin a da - \int\left\{\frac{d}{dx}a\int \sin a da\right\}da\right]$$

$$= 3a\cos a - \int \cos a da$$

$$= 3a\cos a - \sin a + c$$

Replacing the value of a we get,

$$\therefore 3a\cos a - \sin a + c$$

$= \mathbf{3x\cos^{-1}x - \sqrt{1-x^2} + c}$, where c is the integrating constant

46. Question

Evaluate the following integrals:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $f_2(x) = 1$,

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\int dx - \int\left[\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}\int dx\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int\left[\frac{\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int\frac{-4x^2dx}{(1+x^2)^2\times\frac{1}{1+x^2}\times 2x}$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int\frac{2xdx}{1+x^2}$$

Now,

$$\int\frac{2xdx}{1+x^2}$$

$$= \int\frac{d(1+x^2)}{1+x^2}$$

$$= \ln(1+x^2) + c$$

Again, we know,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\Rightarrow 2x = \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Replacing x by tanx, it is obtained that,

$$2\tan x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

So, the final integral yielded is

$\mathbf{2xtanx - ln(1 + x^2) + c}$, where c is the integrating constant

47. Question

Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Answer

Formula to be used – We know, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Assuming x = tana,

$$\frac{2\tan a}{1-\tan^2 a} = \tan 2a$$

And, dx = $\sec^2 a\,da$

Hence, a=$\tan^{-1}x$

Now, $\sec^2 a-\tan^2 a=1$, so,seca=√(1+x^2)

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 2a)\{\sec^2 a\,da\}$$

$$= 2\int a\sec^2 a\,da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = $\sec^2 a$,

$$\therefore 2\int a\sec^2 a\,da$$

$$= 2\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 2a\tan a - \int \tan a\,da$$

$$= 2a\tan a - \ln|\sec a| + c$$

Replacing the value of a we get,

$$\therefore 2a\tan a - \ln|\sec a| + c$$

$= \mathbf{2x\tan^{-1}x - \ln\sqrt{1+x^2} + c}$, where c is the integrating constant

48. Question

Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$$

Answer

Formula to be used – We know, $\tan 3x = \frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$$

Assuming x = tana,

$$\frac{3\tan a-\tan^3 a}{1-3\tan^2 a} = \tan 3a$$

And, dx = $\sec^2 a\,da$

Hence, a=$\tan^{-1}x$

Now, $\sec^2 a - \tan^2 a = 1$, so, seca=√$(1+x^2)$

$$\therefore \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 3a)\{\sec^2 a\,da\}$$

$$= 3\int a\sec^2 a\,da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x) = \sec^2 a$,

$$\therefore 3\int a\sec^2 a\,da$$

$$= 3\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 3a\tan a - \frac{3}{2}\int \tan a\,da$$

$$= 3a\tan a - \frac{3}{2}\ln|\sec a| + c$$

Replacing the value of a we get,

$$\therefore 3a\tan a - \frac{3}{2}\ln|\sec a| + c$$

$= \mathbf{3x\tan^{-1}x - \frac{3}{2}\ln\sqrt{1+x^2} + c}$, where c is the integrating constant

49. Question

Evaluate the following integrals:

$$\int \frac{\sin^{-1}x}{x^2}dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx} f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$$\therefore \int \frac{\sin^{-1}x}{x^2}dx$$

$$= \sin^{-1}x\int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int\frac{1}{x^2}dx\right\}dx$$

$$= \frac{-\sin^{-1}x}{x} - \int\frac{1}{\sqrt{1-x^2}}\times\left(-\frac{1}{x}\right)dx$$

$$= \frac{-\sin^{-1}x}{x} + \int\frac{1}{x\sqrt{1-x^2}}dx$$

Taking x= sina, dx = cosada

Hence, coseca=1/x

Now, $\text{cosec}^2a-\cot^2a = 1$ so $\cot a=\sqrt{(1-x^2)}/x$

$$\therefore \int\frac{1}{x\sqrt{1-x^2}}dx$$

$$= \int\frac{1}{\text{sinacosa}}(\text{cosada})$$

$$= \int \text{cosecada}$$

$$= \ln|\text{coseca} - \text{cota}| + c$$

Replacing the value of a, we get,

$$\therefore \ln|\text{coseca} - \text{cota}| + c$$

$$= \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$$

The total integration yields as

$= \frac{-\sin^{-1}x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$, where c is the integrating constant

50. Question

Evaluate the following integrals:

$$\int\frac{\tan x\sec^2 x}{(1-\tan^2 x)}dx$$

Answer

Say, tanx = a

Hence, $\sec^2xdx=da$

$$\therefore \int\frac{\tan x\sec^2x}{1-\tan^2x}dx$$

$$= \int \frac{ada}{1-a^2}$$

Now, taking $1-a^2 = k$, $-2ada=dk$ i.e. $ada=-dk/2$

$$\therefore \int \frac{ada}{1-a^2}$$

$$= \int \frac{-dk}{2k}$$

$$= -\frac{1}{2}\ln|k| + c$$

Replacing the value of k,

$$-\frac{1}{2}\ln|k| + c$$

$$= -\frac{1}{2}\ln|1-a^2| + c$$

Replacing the value of a,

$$-\frac{1}{2}\ln|1-a^2| + c$$

$= -\frac{1}{2}\mathbf{\ln|1-\tan^2 x| + c}$, where c is the integrating constant

51. Question

Evaluate the following integrals:

$$\int e^{3x} \sin 4x \ dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 4x$ and $f_2(x) = e^{3x}$,

$$\therefore \int e^{3x}\sin 4x dx$$

$$= \sin 4x \int e^{3x}dx - \int\left\{\frac{d}{dx}(\sin 4x)\int e^{3x}dx\right\}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \int 4\cos 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\int e^{3x}\cos 4x dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\left[\cos 4x\int e^{3x}dx - \int\left\{\frac{d}{dx}(\cos 4x)\int e^{3x}dx\right\}dx\right]$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{4}{3}\int 4\sin 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{16}{9}\int e^{3x}\sin 4x dx$$

$$\therefore \left(1+\frac{16}{9}\right)\int e^{3x}\sin 4x dx = \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} + c_1$$

$$\Rightarrow \frac{25}{9}\int e^{3x}\sin 4x dx = \frac{3e^{3x}\sin 4x - 4e^{3x}\cos 4x}{9} + c_1$$

$\Rightarrow \int e^{3x}\sin 4x dx = \frac{e^{3x}}{25}(\mathbf{3\sin 4x - 4\cos 4x}) + \mathbf{c}$, where c is the integrating constant

52. Question

Evaluate the following integrals:

$$\int e^{2x}\sin x\, dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin x dx$$

$$= \sin x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin x}{2} - \int \cos x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{2}\int \sin x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}\int e^{2x}\sin x dx$$

$$\therefore \left(1+\frac{1}{4}\right)\int e^{2x}\sin x dx = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} + c_1$$

$$\Rightarrow \frac{5}{4}\int e^{2x}\sin x dx = \frac{2e^{2x}\sin x - e^{2x}\cos x}{4} + c_1$$

$\Rightarrow \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(\mathbf{2\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

53. Question

Evaluate the following integrals:

$$\int e^{2x}\sin x\cos x\, dx$$

Answer

$$\int e^{2x}\sin x\cos x dx$$

$= \frac{1}{2}\int e^{2x} \times 2\sin x\cos x dx$

$= \frac{1}{2}\int e^{2x}\sin 2x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 2x$ and $f_2(x) = e^{2x}$,

$\therefore \int e^{2x}\sin 2x dx$

$= \sin 2x \int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin 2x)\int e^{2x}dx\right\}dx$

$= \frac{e^{2x}\sin 2x}{2} - \int 2\cos 2x \times \frac{e^{2x}}{2}dx$

$= \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x dx$

$= \frac{e^{2x}\sin 2x}{2} - \left[\cos 2x \int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos 2x)\int e^{2x}dx\right\}dx\right]$

$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int 2\sin 2x \times \frac{e^{2x}}{2}dx$

$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int e^{2x}\sin x dx$

$\therefore (1+1)\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} + c_1$

$\Rightarrow 2\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x - e^{2x}\cos 2x}{2} + c_1$

$\Rightarrow \int e^{2x}\sin 2x dx = \frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'$

$\therefore \frac{1}{2}\int e^{2x}\sin 2x dx$

$= \frac{1}{2} \times \left[\frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'\right]$

$= \boldsymbol{\frac{e^{2x}}{8}(\sin 2x - \cos 2x) + c}$, where c is the integrating constant

54. Question

Evaluate the following integrals:

$\int e^{2x}\cos(3x+4)dx$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos(3x+4)$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\cos(3x+4)\,dx$$

$$= \cos(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\cos(3x+4)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \int 3\sin(3x+4)\times\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\sin(3x+4)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{3}{2}\int 3\cos(3x+4)\times\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{9}{4}\int e^{2x}\cos(3x+4)dx$$

$$\therefore \left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x+4)dx = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} + c_1$$

$$\Rightarrow \frac{13}{4}\int e^{2x}\cos(3x+4)\,dx = \frac{2e^{2x}\cos(3x+4) + 3e^{2x}\sin(3x+4)}{4} + c_1$$

$\Rightarrow \int e^{2x}\cos(3x+4)\,dx = \frac{e^{2x}}{13}\left(\mathbf{2cos(3x+4) + 3sin(3x+4)}\right) + \mathbf{c}$, where c is the integrating constant

55. Question

Evaluate the following integrals:

$$\int e^{-x}\cos x\,dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos x$ and $f_2(x) = e^{-x}$,

$$\therefore \int e^{-x}\cos x\,dx$$

$$= \cos x\int e^{-x}dx - \int\left\{\frac{d}{dx}\cos x\int e^{-x}dx\right\}dx$$

$$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx$$

$$= -e^{-x}\cos x - \left[\sin x\int e^{-x}dx - \int\left\{\frac{d}{dx}\sin x\int e^{-x}dx\right\}dx\right]$$

$$= -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]$$

$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$

$\therefore (1+1)\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow 2\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$

$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\mathbf{sinx - cosx}) + \mathbf{c}$, where c is the integrating constant

56. Question

Evaluate the following integrals:

$\int e^{x}(\sin x + \cos x)dx$

Answer

$\int e^{x}(\sin x + \cos x)dx$

$= \int e^{x}\sin x dx + \int e^{x}\cos x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^{x}\sin x dx + \int e^{x}\cos x dx$

$= \sin x\int e^{x}dx - \int\left[\frac{d}{dx}(\sin x)\int e^{x}dx\right]dx + \int e^{x}\cos x dx$

$= e^{x}\sin x - \int e^{x}\cos x dx + \int e^{x}\cos x dx + c$

$= \mathbf{e^{x}sinx + c}$, where c is the integrating constant

57. Question

Evaluate the following integrals:

$\int e^{x}(\cot x - \mathrm{cosec}^{2}x)dx$

Answer

$\int e^{x}(\cot x - \mathrm{cosec}^{2}x)dx$

$= \int e^{x}\cot x dx + \int e^{x}\mathrm{cosec}^{2}x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx$

$= \cot x \int e^x dx - \int \left[\frac{d}{dx}(\cot x) \int e^x dx\right] dx + \int e^x \operatorname{cosec}^2 x dx$

$= e^x \cot x - \int e^x \operatorname{cosec}^2 x dx + \int e^x \operatorname{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

58. Question

Evaluate the following integrals:

$\int e^x \sec x (1 + \tan x) dx$

Answer

$\int e^x \sec x (1 + \tan x) dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$

$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

59. Question

Evaluate the following integrals:

$\int e^x \left(\tan^{-1} x + \frac{1}{1 + x^2}\right) dx$

Answer

$\int e^x \left(\tan^{-1} x + \frac{1}{1 + x^2}\right) dx$

$= \int e^x \tan^{-1} x dx + \int \frac{e^x}{1 + x^2} dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan^{-1} x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \tan^{-1} x\,dx + \int \frac{e^x}{1+x^2}dx$$

$$= \tan^{-1} x \int e^x dx - \int \left[\frac{d}{dx}(\tan^{-1} x) \int e^x dx\right] dx + \int \frac{e^x}{1+x^2}dx$$

$$= e^x \tan^{-1} x - \int \frac{e^x}{1+x^2}dx + \int \frac{e^x}{1+x^2}dx + c$$

$= \mathbf{e^x \tan^{-1} x + c}$, where c is the integrating constant

60. Question

Evaluate the following integrals:

$$\int e^x (\cot x + \log \sin x) dx$$

Answer

$$\int e^x(\cot x + \log \sin x)dx$$

$$= \int e^x \cot x\,dx + \int e^x \log \sin x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logsinx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \cot x\,dx + \int e^x \log \sin x dx$$

$$= \int e^x \cot x\,dx + \log \sin x \int e^x dx - \int \left[\frac{d}{dx}(\log \sin x) \int e^x dx\right]$$

$$= \int e^x \cot x\,dx + e^x \log \sin x - \int e^x \cot x\,dx + c$$

$= \mathbf{e^x \log|\sin x| + c}$, where c is the integrating constant

61. Question

Evaluate the following integrals:

$$\int e^x (\tan x - \log \cos x) dx$$

Answer

$$\int e^x(\tan x + \log \cos x)dx$$

$$= \int e^x \tan x\,dx + \int e^x \log \cos x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \tan x\,dx - \int e^x \log \cos x dx$$

$$= \int e^x \tan x\, dx - \log\cos x \int e^x dx + \int \left[\frac{d}{dx}(\log\cos x) \int e^x dx\right]$$

$$= \int e^x \tan x\, dx - e^x \log\cos x - \int e^x \tan x\, dx + c$$

$= \mathbf{e^x \log|\sec x| + c}$, where c is the integrating constant

62. Question

Evaluate the following integrals:

$$\int e^x \left[\sec x + \log(\sec x + \tan x)\right] dx$$

Answer

$$\int e^x [\sec x + \log(\sec x + \tan x)] dx$$

$$= \int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x) dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x) dx$$

$$= \int e^x \sec x dx + \log(\sec x + \tan x) \int e^x dx - \int \left[\frac{d}{dx}(\log(\sec x + \tan x)) \int e^x dx\right]$$

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int \frac{e^x \tan x \times (\sec^2 x + \sec x \tan x)\, dx}{\sec x + \tan x} + c$$

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int e^x \sec x\, dx + c$$

$= \mathbf{e^x \log|\sec x + \tan x| + c}$, where c is the integrating constant

63. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$$

Answer

$$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$$

$$= \int e^x (\sec^2 x + \tan x) dx$$

$$= \int e^x \sec^2 x dx + \int e^x \tan x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY

PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = tanx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec^2 x dx + \int e^x \tan x dx$$

$$= \int e^x \sec^2 x dx + \tan x \int e^x dx - \int\left[\frac{d}{dx}(\tan x)\int e^x dx\right]$$

$$= \int e^x \sec^2 x\, dx + e^x \tan x - \int e^x \sec^2 x\, dx + c$$

$= \mathbf{e^x \tan x + c}$, where c is the integrating constant

64. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{\sin x \cos x - 1}{\sin^2 x}\right)dx$$

Answer

$$\int e^x\left(\frac{\sin x \cos x - 1}{\sin^2 x}\right)dx$$

$$= \int e^x(\cot x - \operatorname{cosec}^2 x)dx$$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = cotx and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= \cot x \int e^x dx - \int\left\{\frac{d}{dx}(\cot x)\int e^x dx\right\}dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx + c$$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

65. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$

Answer

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$

$$= \int e^x(\sec x + \sec x\tan x)dx$$

$$= \int e^x\sec xdx + \int e^x\sec x\tan xdx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x\sec xdx + \int e^x\sec x\tan xdx$$

$$= \sec x\int e^xdx - \int\left[\frac{d}{dx}(\sec x)\int e^xdx\right]dx + \int e^x\sec x\tan xdx$$

$$= e^x\sec x - \int e^x\sec x\tan xdx + \int e^x\sec x\tan xdx + c$$

$= \mathbf{e^x\sec x + c}$, where c is the integrating constant

66. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$$

Answer

$$\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$$

$$= \int e^x\left(\frac{1-\sin x\cos x}{\sin^2 x}\right)dx$$

$$= \int e^x(\operatorname{cosec}^2 x - \cot x)dx$$

$$= \int e^x\operatorname{cosec}^2 xdx - \int e^x\cot xdx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x\operatorname{cosec}^2 xdx - \int e^x\cot xdx$$

$$= \int e^x\operatorname{cosec}^2 xdx - \cot x\int e^xdx + \int\left\{\frac{d}{dx}(\cot x)\int e^xdx\right\}dx$$

$$= \int e^x\operatorname{cosec}^2 xdx - e^x\cot x - \int e^x\operatorname{cosec}^2 xdx$$

$= \mathbf{-e^x\cot x + c}$, where c is the integrating constant

67. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

Answer

$$\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=\left(\frac{1+\frac{2\tan {}^{x}/_{2}}{1+\tan^2({}^{x}/_{2})}}{1+\frac{1-\tan^2({}^{x}/_{2})}{1+\tan^2({}^{x}/_{2})}}\right)$$

$$=\frac{\left(1+\tan {}^{x}/_{2}\right)^2}{2}$$

$$\therefore \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

$$=\int e^x\times\frac{\left(1+\tan {}^{x}/_{2}\right)^2}{2}$$

$$=\int\frac{e^x\left(1+\tan^2 {}^{x}/_{2}+2\tan {}^{x}/_{2}\right)}{2}dx$$

$$=\int\frac{e^x\left(\sec^2 {}^{x}/_{2}+2\tan {}^{x}/_{2}\right)}{2}dx$$

$$=\int\frac{e^x\sec^2 {}^{x}/_{2}\,dx}{2}+\int e^x\tan {}^{x}/_{2}\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int\frac{e^x\sec^2 {}^{x}/_{2}\,dx}{2}+\int e^x\tan {}^{x}/_{2}\,dx$$

$$=\int\frac{e^x\sec^2 {}^{x}/_{2}\,dx}{2}+\tan {}^{x}/_{2}\int e^x dx-\int\left[\frac{d}{dx}\left(\tan {}^{x}/_{2}\right)\int e^x dx\right]dx$$

$$=\int\frac{e^x\sec^2 {}^{x}/_{2}\,dx}{2}+e^x\tan {}^{x}/_{2}-\int\frac{e^x\sec^2 {}^{x}/_{2}\,dx}{2}+c$$

$\mathbf{=e^x\tan {}^{x}/_{2}+c}$, where c is the integrating constant

68. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$$

Answer

$$\int e^x\left(\frac{\sin 4x-1}{1-\cos 4x}\right)dx$$

$= \int e^x \left(\frac{2\sin 2x\cos 2x - 4}{2\sin^2 2x}\right) dx$

$= \int e^x(\cot 2x - 2\mathrm{cosec}^2 2x)dx$

$= \int e^x \cot 2x dx - \int 2e^x \mathrm{cosec}^2 2x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \cot 2x dx - \int 2e^x \mathrm{cosec}^2 2x dx$

$= \cot 2x \int e^x dx - \int \left\{\frac{d}{dx}(\cot 2x) \int e^x dx\right\} dx - \int 2e^x \mathrm{cosec}^2 2x dx$

$= e^x \cot 2x + \int 2e^x \mathrm{cosec}^2 2x dx - \int 2e^x \mathrm{cosec}^2 2x dx + c$

$= \mathbf{e^x \cot 2x + c}$, where c is the integrating constant

69. Question

Evaluate the following integrals:

$$\int \frac{e^x\left[\sqrt{1-x^2}\sin^{-1}x + 1\right]}{\sqrt{1-x^2}} dx$$

Answer

$\int \frac{e^x[\sqrt{1-x^2}\sin^{-1}x + 1]}{\sqrt{1-x^2}} dx$

$= \int e^x \left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right) dx$

$= \int e^x \sin^{-1}x\, dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sin^{-1}x\, dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$

$= \sin^{-1}x \int e^x dx - \int \left\{\frac{d}{dx}(\sin^{-1}x) \int e^x dx\right\} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$

$= e^x \sin^{-1}x - \int \frac{e^x}{\sqrt{1-x^2}} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx + c$

$= \mathbf{e^x \sin^{-1}x + c}$, where c is the integrating constant

70. Question

Evaluate the following integrals:

$$\int e^x \left(\frac{1 + x \log x}{x} \right) dx$$

Answer

$$\int e^x \left(\frac{1 + x\log x}{x} \right) dx$$

$$= \int e^x \left(\frac{1}{x} + \log x \right) dx$$

$$= \int \frac{e^x}{x} dx + \int e^x \log x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x)dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x} dx + \int e^x \log x dx$$

$$= \int \frac{e^x}{x} dx + \log x \int e^x dx - \int \left[\frac{d}{dx} (\log x) \int e^x dx \right] dx$$

$$= \int \frac{e^x}{x} dx + e^x \log x - \int \frac{e^x}{x} dx + c$$

$= \mathbf{e^x \log x + c}$, where c is the integrating constant

71. Question

Evaluate the following integrals:

$$\int e^x \cdot \frac{x}{(1+x)^2} dx$$

Answer

$$\frac{x}{(1+x)^2} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2}$$

$\Rightarrow x = A(1 + x) + B$

For x=-1, equation: -1 = B i.e. B = -1

For x=0, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x}{(1+x)^2}$$

$$= \frac{1}{(1+x)} - \frac{1}{(1+x)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right] dx$$

$$= \int e^x \times \frac{1}{(1+x)} dx - \int e^x \times \frac{1}{(1+x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(1+x)}dx - \int \frac{e^x}{(1+x)^2}dx$$

$$= \frac{1}{(1+x)}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right]dx - \int \frac{e^x}{(1+x)^2}dx$$

$$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx + c$$

$= \frac{e^x}{(1+x)} + c$, where c is the integrating constant

72. Question

Evaluate the following integrals:

$$\int e^x \frac{(x-1)}{(x+1)^3}dx$$

Answer

$$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$\Rightarrow x - 1 = A(x+1)^2 + B(x+1) + C$

For x=-1, equation: -2 = C i.e. C = -2

For x=0, equation: -1 = A+B-2 i.e. A+B = 1

For x=1, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+A = 1

⇨A = 0

And, B = 1

$$\therefore \frac{x-1}{(x+1)^3}$$

$$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]dx$$

$$= \int e^x \times \frac{1}{(x+1)^2}dx - \int e^x \times \frac{2}{(x+1)^3}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x+1)^2}dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{1}{(x+1)^2}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)\int e^x dx\right]dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{e^x}{(x+1)^2} + \int \frac{2e^x}{(x+1)^3}dx - \int \frac{2e^x}{(x+1)^3}dx + c$$

$= \frac{e^x}{(x+1)^2} + c$, where c is the integrating constant

73. Question

Evaluate the following integrals:

$$\int e^x \frac{(2-x)}{(1-x)^2}dx$$

Answer

$$\frac{2-x}{(1-x)^2} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2}$$

$\Rightarrow 2 - x = A(1-x) + B$

For x=1, equation: 1 = B i.e. B = 1

For x=2, equation: 0 = -A+1 i.e. A = 1

$$\therefore \frac{2-x}{(1-x)^2}$$

$$= \frac{1}{(1-x)} + \frac{1}{(1-x)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right]dx$$

$$= \int e^x \times \frac{1}{(1-x)^2}dx + \int e^x \times \frac{1}{1-x}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{(1-x)^2}dx + \int \frac{e^x}{1-x}dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{1}{1-x}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{1-x}\right)\int e^x dx\right]dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2}dx + c$$

$= \frac{e^x}{1-x} + c$, where c is the integrating constant

74. Question

Evaluate the following integrals:

$$\int e^{x} \cdot \frac{(x-3)}{(x-1)^{3}} dx$$

Answer

$$\frac{x-3}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$\Rightarrow x - 3 = A(x-1)^2 + B(x-1) + C$

For x=1, equation: -2 = C i.e. C = -2

For x=0, equation: -3 = A-B-2 i.e. B = A+1

For x=3, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+3A = 1

⇨A = 0

And, B = 1

$\therefore \frac{x-3}{(x-1)^3}$

$= \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$

The given equation becomes

$$\int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right] dx$$

$$= \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) \int e^x dx\right] dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{(x-1)^2}} + \mathbf{c}$, where c is the integrating constant

75. Question

Evaluate the following integrals:

$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

Answer

$$\int e^{3x}\left(\frac{3x-1}{9x^2}\right)dx$$

$$=\int \frac{e^{3x}}{3x}dx-\int \frac{e^{3x}}{9x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/3xand $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x}dx-\int \frac{e^{3x}}{9x^2}dx$$

$$=\frac{1}{3x}\int e^{3x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{3x}\right)\int e^{3x}dx\right]dx-\int \frac{e^{3x}}{9x^2}dx$$

$$=\frac{e^{3x}}{9x}+\int \frac{e^{3x}}{9x^2}dx-\int \frac{e^{3x}}{9x^2}dx+c$$

$=\mathbf{\frac{e^{3x}}{9x}+c}$, where c is the integrating constant

76. Question

Evaluate the following integrals:

$$\int \frac{(x+1)}{(x+2)^2}e^x dx$$

Answer

$$\frac{x+1}{(x+2)^2}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}$$

$\Rightarrow x+1=A(x+2)+B$

For x=-2, equation: -1 = B i.e. B = -1

For x=-1, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x+1}{(x+2)^2}$$

$$=\frac{1}{(x+2)}-\frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(x+2)}-\frac{1}{(x+2)^2}\right]dx$$

$$=\int e^x\times\frac{1}{x+2}dx-\int e^x\times\frac{1}{(x+2)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(x+2) and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x+2}dx - \int \frac{e^x}{(x+2)^2}dx$$

$$= \frac{1}{x+2}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x+2}\right)\int e^x dx\right]dx - \int \frac{e^x}{(x+2)^2}dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2}dx - \int \frac{e^x}{(x+2)^2}dx + c$$

$= \frac{e^x}{x+2} + c$, where c is the integrating constant

77. Question

Evaluate the following integrals:

$$\int \frac{x e^{2x}}{(1+2x)^2}dx$$

Answer

$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$\Rightarrow x = A(1+2x) + B$

For x=-1/2, equation: -1/2 = B i.e. B = -1/2

For x=0, equation: 0 = A-1/2 i.e. A = 1/2

$$\therefore \frac{x}{(1+2x)^2}$$

$$= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$$

The given equation becomes

$$\int e^{2x}\left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right]dx$$

$$= \int e^{2x} \times \frac{1}{2(1+2x)}dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(1+2x) and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int e^{2x} \times \frac{1}{2(1+2x)}dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}dx$$

$$= \frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}dx\right]dx - \int \frac{e^{2x}}{(1+2x)^2}dx\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{(2x+1)^2}dx - \int \frac{e^{2x}}{(2x+1)^2}dx\right]$$

$= \frac{e^{2x}}{4(2x+1)} + c$, where c is the integrating constant

78. Question

Evaluate the following integrals:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

Answer

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

$$=\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{1}{2x}\int e^{2x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{2x}\right)\int e^{2x}dx\right]dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{e^{2x}}{4x}+\int \frac{e^{2x}}{4x^2}dx-\int \frac{e^{2x}}{4x^2}dx+c$$

$=\frac{e^{2x}}{4x}+c$, where c is the integrating constant

79. Question

Evaluate the following integrals:

$$\int e^{x}\left(\log x+\frac{1}{x^2}\right)dx$$

Answer

$$\int e^{x}\left(\log x+\frac{1}{x^2}\right)dx$$

$$=\int e^{x}\log x dx-\int \frac{e^{x}}{x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^{x}$ in the first integral and keeping the second integral intact,

$$\int e^{x}\log x dx-\int \frac{e^{x}}{x^2}dx$$

$$=\log x\int e^{x}dx-\int\left[\frac{d}{dx}(\log x)\int e^{x}dx\right]dx-\int \frac{e^{x}}{x^2}dx$$

$$=e^{x}\log x-\int \frac{e^{x}}{x}dx-\int \frac{e^{x}}{x^2}dx$$

$$=e^{x}\log x-\left[\frac{1}{x}\int e^{x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{x}\right)\int e^{x}dx\right]dx\right]-\int \frac{e^{x}}{x^2}dx$$

$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$

$= \mathbf{e^x \left(\log x - \frac{1}{x}\right) + c}$, where c is the integrating constant

80. Question

Evaluate the following integrals:

$$\int \frac{\log x}{(1+\log x)^2} dx$$

Answer

$$\frac{\log x}{(1+\log x)^2} = \frac{A}{(1+\log x)} + \frac{B}{(1+\log x)^2}$$

$\Rightarrow \log x = A(1+\log x) + B$

For x=1, equation: 0 = A+B

For x=1/e, equation: -1 = B i.e. B = -1

So, A = 1

$\therefore \frac{\log x}{(1+\log x)^2}$

$= \frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}$

The given equation becomes

$$\int \left[\frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}\right] dx$$

$$= \int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(1+logx) and $f_2(x)$ = 1in the second integral and keeping the first integral intact,

$$\int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$$

$$= \frac{1}{(1+\log x)} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{(1+\log x)}\right) \int dx\right] dx - \int \frac{1}{(1+\log x)^2} dx$$

$$= \frac{x}{(1+\log x)} + \int \frac{1}{(1+\log x)^2} dx - \int \frac{1}{(1+\log x)^2} dx + c$$

$= \mathbf{\frac{x}{(1+\log x)} + c}$, where c is the integrating constant

81. Question

Evaluate the following integrals:

$$\int \{\sin(\log x) + \cos(\log x)\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \sin(\log x) dx + \int \cos(\log x) dx$$

$$= \sin(\log x) \int dx - \int \left[\frac{d}{dx}(\sin(\log x)) \int dx\right] dx + \int \cos(\log x) dx$$

$$= x \sin(\log x) - \int \cos(\log x)\, dx + \int \cos(\log x)\, dx + c$$

$= \mathbf{e^{\log x} \sin(\log x) + c}$, where c is the integrating constant

82. Question

Evaluate the following integrals:

$$\int \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \frac{1}{\log x} dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{1}{\log x} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{\log x}\right) \int dx\right] dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} dx - \int \frac{1}{(\log x)^2} dx + c$$

$= \mathbf{\frac{x}{\log x} + c}$, where c is the integrating constant

83. Question

Evaluate the following integrals:

$$\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = log(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$$

$$= \log(\log x)\int dx - \int\left[\frac{d}{dx}(\log(\log x))\int dx\right]dx + \int \frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \int \frac{1}{\log x}dx + \int \frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \left[\frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx\right] + \int \frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx + c$$

$= \mathbf{x}\left[\mathbf{\log(\log x)} - \frac{\mathbf{1}}{\mathbf{\log x}}\right] + \mathbf{c}$, where c is the integrating constant

84. Question

Evaluate the following integrals:

$$\int\left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)dx$$

Answer

It is know that $\sin^{-1}x+\cos^{-1}x = \pi/2$

$$\therefore \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)$$

$$= \frac{2}{\pi}(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x})$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}\,dx$$

$$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}(\sin^{-1}\sqrt{x})\int dx\right\}dx$$

$$= x\sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c'$, where c' is the integrating constant

For the second term,

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c''$, where c'' is the integrating constant

$$\therefore \int \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right) dx$$

$= \frac{2}{\pi}\int (\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x})dx$

$= \frac{2}{\pi}\Bigg[x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] - x\cos^{-1}\sqrt{x}$
$+ \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right]\Bigg] + c$

$= \frac{2}{\pi}\left[\sqrt{x-x^2} + x(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}) + \sin^{-1}\sqrt{1-x}\right] + c$ where c is the integrating constant

85. Question

Evaluate the following integrals:

$\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$

Answer

Tip – 5^x is to be replaced by a

$\therefore 5^x = a$

$\Rightarrow 5^x \log 5 dx = da$

$\Rightarrow 5^x dx = \frac{da}{\log 5}$

The equation becomes as follows:

$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$

Tip – 5^a is to be replaced by k

$\therefore 5^a = k$

$\Rightarrow 5^a \log 5 da = dk$

$\Rightarrow 5^a da = \frac{dk}{\log 5}$

The equation becomes as follows:

$\int 5^k \times \frac{1}{(\log 5)^2} dk$

$= \frac{1}{(\log 5)^2}\int 5^k dk$

$= \frac{5^k}{(\log 5)^3} + c$

Re-replacing the value of k,

$\frac{5^{5^a}}{(\log 5)^3} + c$

Re-replacing the value of a,

$\frac{5^{5^{5^x}}}{(\log 5)^3} + c$, where c is the integrating constant

86. Question

Evaluate the following integrals:

$$\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$$

Answer

$$\left(\frac{1+\sin 2x}{1+\cos 2x}\right)$$

$$=\left(\frac{1+\frac{2\tan x}{1+\tan^2 x}}{1+\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$$

$$=\frac{(1+\tan x)^2}{2}$$

$$\therefore \int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$$

$$=\int e^{2x}\times\frac{(1+\tan x)^2}{2}$$

$$=\int \frac{e^{2x}(1+\tan^2 x+2\tan x)}{2}dx$$

$$=\int \frac{e^{2x}(\sec^2 x+2\tan x)}{2}dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}+\int e^{2x}\tan x\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2}+\int e^{2x}\tan x\,dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}+\tan x\int e^{2x}dx-\int\left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}+\frac{1}{2}e^{2x}\tan x-\int \frac{e^{2x}\sec^2 x dx}{2}+c$$

$=\frac{1}{2}\mathbf{e^x \tan {}^{x}/_{2} + c}$, where c is the integrating constant

87. Question

Evaluate the following integrals:

$$\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

Answer

$$\left(\frac{1-\sin 2x}{1-\cos 2x}\right)$$

$$= \left(\frac{1 - \frac{2\tan x}{1 + \tan^2 x}}{1 - \frac{1 - \tan^2 x}{1 + \tan^2 x}} \right)$$

$$= \frac{(1 - \tan x)^2}{2}$$

$$\therefore \int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

$$= \int e^{2x} \times \frac{(1 - \tan x)^2}{2}$$

$$= \int \frac{e^{2x}(1 + \tan^2 x - 2\tan x)}{2} dx$$

$$= \int \frac{e^{2x}(\sec^2 x - 2\tan x)}{2} dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x} \tan x \, dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x} \tan x \, dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \tan x \int e^{2x} dx + \int \left[\frac{d}{dx}(\tan x) \int e^{2x} dx \right] dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \frac{1}{2} e^{2x} \tan x + \int \frac{e^{2x}\sec^2 x dx}{2} + c$$

$= -\frac{1}{2} e^x \tan {}^x/_2 + c$, where c is the integrating constant

Objective Questions II

1. Question

Mark (√) against the correct answer in each of the following:

$$\int x \, e^x dx = ?$$

A. $e^x (1 – x) + C$

B. $e^x (x – 1) + C$

C. $e^x (x – 1) + C$

D. none of these

Answer

To find: Value of $\int x \, e^x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int x\, e^x dx$ ... (i)

$$I = \int x\, e^x dx$$

$$\Rightarrow x\int e^x dx - \int\left[\frac{d(x)}{x}\int e^x dx\right]dx$$

$$\Rightarrow I = x\, e^x - \int 1.e^x dx$$

$$\Rightarrow I = x\, e^x - e^x + c$$

$$\therefore I = e^x (x-1) + c$$

Ans) c $e^x (x-1) + c$

2. Question

Mark (√) against the correct answer in each of the following:

$$\int x\, e^{2x}\, dx = ?$$

A. $\frac{1}{2}x\, e^{2x} + \frac{1}{4}e^{2x} + C$

B. $\frac{1}{2}x\, e^{2x} - \frac{1}{4}e^{2x} + C$

C. $2x\, e^{2x} + 4e^{2x} + C$

D. none of these

Answer

To find: Value of $\int x\, e^{2x} dx$

Formula used:

$$\textbf{(i)} \int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$$

We have, $I = \int x\, e^{2x} dx$ ... (i)

$$I = \int x\, e^{2x} dx$$

$$\Rightarrow x\int e^{2x} dx - \int\left[\frac{d(x)}{x}\int e^{2x} dx\right]dx$$

$$\Rightarrow I = x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$\Rightarrow I = x\frac{e^{2x}}{2} - \frac{1}{2}\int\frac{e^{2x}}{2}dx$$

$$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x} dx$$

$$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\frac{e^{2x}}{2} + c$$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

$\therefore I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Ans) B $\frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

3. Question

Mark (√) against the correct answer in each of the following:

$\int x\cos 2x\,dx = ?$

A. $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

B. $\frac{1}{2}x\sin 2x - \frac{1}{4}\cos 2x + C$

C. 2x sin 2x + 4 cos 2x + C

D. none of these

Answer

To find: Value of $\int x\cos 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\cos t\frac{dt}{2}$

$I = \frac{1}{4}\int t\cos t\,dt$

Taking 1st function as t and second function as $\cos t$

$\Rightarrow I = \frac{1}{4}\left[t\int \cos t\,dt - \int\left(\frac{dt}{dt}\int \cos t\,dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(\sin t) - \int(1\,(\sin t))dt\right]$

$\Rightarrow I = \frac{1}{4}[t(\sin t) - (-\cos t)] + c$

$\Rightarrow I = \frac{1}{4}[t \sin t + \cos t] + c$

$\Rightarrow I = \frac{1}{4}[2x \sin 2x + \cos 2x] + c$

$\Rightarrow I = \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

Ans) A $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

4. Question

Mark (√) against the correct answer in each of the following:

$\int x \sec^2 x \, dx = ?$

A. x tan x – log |cos x| + C

B. x tan x + log |cos x| + C

C. x tan x + log |sec x| + C

D. none of these

Answer

To find: Value of $\int x \sec^2 x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int x \sec^2 x \, dx$ … (i)

Taking 1st function as x and second function as $\sec^2 x$

$\Rightarrow I = \left[x \int \sec^2 x \, dx - \int \left(\frac{dx}{dx} \int \sec^2 x \, dx \right) dx \right]$

$\Rightarrow I = \left[x \tan x - \int (1 \tan x) dx \right]$

$\Rightarrow I = \left[x \tan x - \int \tan x dx \right]$

$\Rightarrow I = [x \tan x - (-\log|\cos x|)] + c$

$\Rightarrow I = x \tan x + \log|\cos x| + c$

Ans) B $x \tan x + \log|\cos x| + c$

5. Question

Mark (√) against the correct answer in each of the following:

$\int x \sin 2x \, dx = ?$

A. $\frac{1}{2}x \cos 2x + \frac{1}{4}\sin 2x + C$

B. $-\frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$

C. $-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+C$

D. none of these

Answer

To find: Value of $\int x\sin 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\sin 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\sin t \frac{dt}{2}$

$I = \frac{1}{4}\int t\sin t\, dt$

Taking 1st function as t and second function as $\sin t$

$\Rightarrow I = \frac{1}{4}\left[t\int \sin t\, dt - \int\left(\frac{dt}{dt}\int \sin t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(-\cos t) - \int(1(-\cos t))dt\right]$

$\Rightarrow I = \frac{1}{4}\left[-t\cos t - \int -\cos t\, dt\right]$

$\Rightarrow I = \frac{1}{4}[-t\cos t + \sin t]+c$

$\Rightarrow I = \frac{1}{4}[-2x\cos 2x + \sin 2x]+c$

$\Rightarrow I = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Ans) C $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

6. Question

Mark (√) against the correct answer in each of the following:

$\int x\log x\, dx = ?$

A. $x\log x+\frac{1}{2}x^2+C$

B. $\frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C$

C. $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$

D. none of these

Answer

To find: Value of $\int x\log x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x\log x\, dx$ ... (i)

Taking 1st function as $\log x$ and second function as x

$\Rightarrow I = \left[\log x \int x\, dx - \int \left(\frac{d\log x}{dx} \int x\, dx\right)dx\right]$

$\Rightarrow I = \left[\log x \frac{x^2}{2} - \int \left(\frac{1}{x}\frac{x^2}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x \frac{x^2}{2} - \int \left(\frac{x}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x \frac{x^2}{2} - \frac{1}{2}\int x dx\right]$

$\Rightarrow I = \left[\log x \frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}\right] + c$

$\Rightarrow I = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$

Ans) C $\frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$

7. Question

Mark (√) against the correct answer in each of the following:

$\int x\, \mathrm{cosec}^2 x\, dx = ?$

A. x cot x – log |sin x| + C

B. - cot x + log |sin x| + C

C. x tan x – log |sec x| + C

D. none of these

Answer

To find: Value of $\int x\, \mathrm{cosec}^2 x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x \operatorname{cosec}^2 x dx$ ... (i)

$$I = \int x \operatorname{cosec}^2 x dx$$

$$\Rightarrow x\int \operatorname{cosec}^2 x dx - \int \left[\frac{d(x)}{x} \int \operatorname{cosec}^2 x dx\right] dx$$

$$\Rightarrow I = x(-\cot x) - \int 1.(-\cot x) dx$$

$$\Rightarrow I = -x(\cot x) + \log|\sin x| + c$$

Ans) D None of these

8. Question

Mark (√) against the correct answer in each of the following:

$$\int x \sin x \cos x \, dx = ?$$

A. $-\frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + C$

B. $\frac{1}{4} x \cos 2x - \frac{1}{8} \sin 2x + C$

C. $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$

D. $-\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C$

Answer

To find: Value of $\int x \sin x \cos x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int x \sin x \cos x dx$ ... (i)

$$I = \frac{1}{2}\int x\, 2\sin x \cos x dx$$

$$I = \frac{1}{2}\int x \sin 2x dx$$

$$\Rightarrow \frac{1}{2}\left[x\int \sin 2x dx - \int \left[\frac{d(x)}{x} \int \sin 2x dx\right] dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-x\cos 2x}{2} - \int \left[1 \frac{-\cos 2x}{2}\right] dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right] + c$$

$$\Rightarrow \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Ans) D $\frac{-x\cos 2x}{4}+\frac{\sin 2x}{8}+c$

9. Question

Mark (√) against the correct answer in each of the following:

$\int x\cos^2 x\,dx = ?$

A. $\frac{x^2}{4}-\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+C$

B. $\frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+C$

C. $\frac{x^2}{4}+\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}+C$

D. none of these

Answer

To find: Value of $\int x\cos^2 x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos^2 x\,dx$ ... (i)

$I = \int x\,\frac{1}{2}(1+\cos 2x)dx$

$I = \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 2x\,dx$

$I = \frac{1}{2}\,\frac{x^2}{2}+\frac{1}{2}\left[x\int \cos 2x\,dx - \int\left[\frac{d(x)}{x}\int \cos 2x\,dx\right]dx\right]$

$I = \frac{1}{2}\,\frac{x^2}{2}+\frac{1}{2}\left[x\,\frac{\sin 2x}{2} - \int 1.\frac{\sin 2x}{2}dx\right]$

$I = \frac{1}{2}\,\frac{x^2}{2}+\frac{1}{2}\left[x\,\frac{\sin 2x}{2} - \frac{1}{2}\int \sin 2x\,dx\right]$

$I = \frac{1}{2}\,\frac{x^2}{2}+\frac{1}{2}\left[x\,\frac{\sin 2x}{2} - \frac{1}{2}\left(-\frac{\cos 2x}{2}\right)+c\right]$

$I = \frac{1}{2}\,\frac{x^2}{2}+\frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}+c\right]$

$I = \frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+c$

Ans) D $\frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+c$

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\log x}{x^2} dx = ?$$

A. $-\frac{1}{x}(\log x + 1) + C$

B. $\frac{1}{x}(\log x - 1) + C$

C. $\frac{1}{x}(\log x + 1) + C$

D. none of these

Answer

To find: Value of $\int \frac{logx}{x^2} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \frac{\log x}{x^2} dx$ ... (i)

$I = \int x^{-2} \log x dx$

$\Rightarrow \log x \int x^{-2}dx - \int \left[\frac{d(\log x)}{x} \int x^{-2}dx \right] dx$

$\Rightarrow \log x \frac{x^{-1}}{-1} - \int \left(\frac{1}{-x^2} \right) dx$

$\Rightarrow -\frac{\log x}{x} + \left(-\frac{1}{x} \right) + c$

$\Rightarrow -\frac{1}{x}(\log x + 1) + c$

Ans) A $-\frac{1}{x}(\log x + 1) + c$

11. Question

Mark (√) against the correct answer in each of the following:

$$\int \log x \, dx = ?$$

A. $\frac{1}{x} + C$

B. $\frac{1}{2}(\log x)^2 + C$

C. x (log x + 1) + C

D. x (log x – 1) + C

Answer

To find: Value of $\int \log x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \log x \,.\, 1.\, dx$ ... (i)

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \left[\log x \int 1\,dx - \int\left(\frac{d\log x}{dx}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\,x\right)dx\right]$$

$$\Rightarrow I = \left[\log x \,.\, x - \int 1dx\right]$$

$$\Rightarrow I = [\log x \,.\, x - x] + c$$

$$\Rightarrow I = x(\log x - 1) + c$$

Ans) D $x(\log x-1)+c$

12. Question

Mark (√) against the correct answer in each of the following:

$\int \log_{10} x\, dx = ?$

A. $\frac{1}{x}\log_e 10 + C$

B. $\frac{1}{x}\log_{10} e + C$

C. x (log x – 1) $\log_e$ 10 + C

D. x(log x – 1) $\log_{10}$ e + C

Answer

To find: Value of $\int \log_{10} x\, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \log_{10} x\, dx$ ... (i)

$$I = \int \log_{10} x\, dx = \int \frac{\log x}{\log 10}\, dx$$

$$I = \frac{1}{\log_e 10}\int \log x \,.\, 1\, dx$$

Taking 1st function as $\log x$ and second function as 1

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \int 1\,dx - \int\left(\frac{d\log x}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\,x\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int 1dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}[\log x \,.\, x - x]+c$

$\Rightarrow I = x(\log x-1)\log_{10} e + c$

Ans) D $x(\log x-1)\log_{10} e + c$

13. Question

Mark (√) against the correct answer in each of the following:

$\int(\log x)^2\,dx = ?$

A. $\frac{2\log x}{x} + C$

B. $\frac{1}{3}(\log x)^3 + C$

C. $x(\log x)^2 - 2x\log x + 2x + C$

D. $x(\log x)^2 + 2x\log x - 2x + C$

Answer

To find: Value of $\int(\log x)^2\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int(\log x)^2 \,.\, 1 \,.\, dx$ ... (i)

Taking 1st function as $(\log x)^2$ and second function as 1

$\Rightarrow I = \left[(\log x)^2\int 1\,dx - \int\left(\frac{d(\log x)^2}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \left[(\log x)^2\int 1\,dx - \int\left(\frac{2(\log x)}{x}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \left[(\log x)^2 .\, x - 2\int \log x dx\right]$

$\Rightarrow I = [(\log x)^2 .\, x - 2(x\log x - x)]+c$

$\Rightarrow I = [(\log x)^2 .\, x - 2x\log x + 2x]+c$

$\Rightarrow I = x(\log x)^2 - 2x\log x + 2x + c$

Ans) C $x(\log x)^2 - 2x\log x + 2x + c$

14. Question

Mark (√) against the correct answer in each of the following:

$$\int e^{\sqrt{x}}\,dx = ?$$

A. $e^{\sqrt{x}} + \sqrt{x} + C$

B. $\frac{1}{2}e^{\sqrt{x}}\left(\sqrt{x}+1\right)+C$

C. $2e^{\sqrt{x}}\left(\sqrt{x}-1\right)+C$

D. none of these

Answer

To find: Value of $\int e^{\sqrt{x}}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int e^{\sqrt{x}}dx$ ... (i)

Putting $\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\,dt$

$\Rightarrow$ dx = 2t dt

$\Rightarrow I = 2\int t.e^t\,dt$

$\Rightarrow I = 2\left[t\int e^t\,dt - \int\left[\frac{d(t)}{dt}\int e^t dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int[1\,e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\,2(t-1)+c$

$\therefore I = 2\,e^{\sqrt{x}}\,(\sqrt{x}-1)+c$

Ans) C $2\,e^{\sqrt{x}}\,(\sqrt{x}-1)+c$

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos\sqrt{x}\,dx = ?$$

A. $\sin\sqrt{x} + \cos\sqrt{x} + C$

B. $\frac{1}{2}\left(\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}\right) + C$

C. $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C$

D. none of these

Answer

To find: Value of $\int \cos\sqrt{x}\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos\sqrt{x}\, dx$ ... (i)

Putting $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\, dt$

⇒ dx = 2t dt

$\Rightarrow I = \int \cos t \,.\, 2t\, dt$

$\Rightarrow I = 2\int t.\, \cos t\, dt$

$\Rightarrow I = 2\left[t\int \cos t\ dt - \int\left[\frac{d(t)}{dt}\int \cos t\, dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int[1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\, 2(t-1) + c$

$\therefore I = 2\, e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$

Ans) C $2\, e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$

16. Question

Mark (√) against the correct answer in each of the following:

$\int \cos(\log x)dx = ?$

A. $\frac{x}{2}\left[\cos(\log x) - \sin(\log x)\right] + C$

B. $\frac{x}{2}\left[\cos(\log x) + \sin(\log x)\right] + C$

C. 2x [cos (log x) + sin (log x)] + C

D. 2x [cos (log x) – sin (log x)] + C

Answer

To find: Value of $\int \cos(logx)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos(\log x)\,dx$ ... (i)

$I = \int 1.\ \cos(\log x)\,dx$

Taking cos(logx) as first function and 1 as second function.

$\Rightarrow I = \left[\cos\log x\int 1\,dx - \int\left[\frac{d[\cos(\log x)]}{dx}\int 1dx\right]dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) - \int\left[-\sin(\log x)\frac{1}{x}x\right]dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int[\sin(\log x)]dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int[1.\sin(\log x)]dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{\sin(\log x)\int 1\,dx - \left(\frac{d\sin(\log x)}{dx}\int 1.dx\right)dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\sin(\log x) - \left(\cos(\log x)\frac{1}{x}x\right)dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\sin(\log x) - \left(\cos(\log x)\frac{1}{x}x\right)dx\right\}\right]$

$\Rightarrow I = [x.\cos(\log x) + \{x.\sin(\log x) - (\cos(\log x))dx\}]$

$\Rightarrow I = [x.\cos(\log x) + x.\sin(\log x) - I]$

$\Rightarrow 2I = [x.\cos(\log x) + x.\sin(\log x)]$

$\Rightarrow I = \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

Ans) B $\frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

17. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^3 x\,dx = ?$

A. $\frac{1}{2}\{\sec x\tan x - \log|\sec x + \tan x|\} + C$

B. $\frac{1}{2}\{\sec x\tan x + \log|\sec x + \tan x|\} + C$

C. 2{sec x tan x + log |sec x + tan x|}+C

D. none of these

Answer

To find: Value of $\int \sec^3 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sec^3 x\, dx$ ... (i)

$I = \int \sec x \sec^2 x\, dx$

Taking secx as first function and $\sec^2 x$ as second function.

$\Rightarrow I = \left[\sec x \int \sec^2 x\, dx - \int\left[\frac{d[\sec x]}{dx}\int \sec^2 x\, dx\right]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int[\sec x \tan x \tan x]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int[\sec x \tan^2 x]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int[\sec x(\sec^2 x - 1)]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int(\sec^3 x - \sec x)\, dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int \sec^3 x\, dx + \int \sec x dx\right]$

$\Rightarrow I = [\sec x \tan x - I + \log|\sec x + \tan x| + c]$

$\Rightarrow 2I = [\sec x \tan x + \log|\sec x + \tan x| + c]$

$\Rightarrow I = \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Ans) B $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

18. Question

Mark (√) against the correct answer in each of the following:

$$\int\left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\}dx = ?$$

A. x log x + C

B. $\frac{x}{\log x} + C$

C. $x + \frac{1}{\log x} + C$

D. none of these

Answer

To find: Value of $\int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$ ... (i)

Put $t = logx$

$e^t = e^{logx} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx$

We know $\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx = e^t \frac{1}{t}$

$\Rightarrow \frac{x}{logx} + c$

Ans) B $\frac{x}{logx} + c$

19. Question

Mark (√) against the correct answer in each of the following:

$\int 2x^3 e^{x^2} dx = ?$

A. $e^{x^2}\left(x^2 - 1\right) + C$

B. $e^{x^2}\left(x^2 + 1\right) + C$

C. $e^{x^2}\left(x + 1\right) + C$

D. none of these

Answer

To find: Value of $\int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \left\{\frac{1}{(logx)} - \frac{1}{(logx)^2}\right\} dx$ ... (i)

Put $t = logx$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx$

We know $\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx = e^t \frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

20. Question

Mark (√) against the correct answer in each of the following:

$\int \left(x2^x\right) dx = ?$

A. $\frac{2^x}{(\log 2)}(x + \log 2) + C$

B. $\frac{2^x}{(\log 2)^2}(x + \log 2 - 1) + C$

C. $\frac{x \cdot 2^x}{(\log 2)} + \frac{2^x}{(\log 2)^2} + C$

D. none of these

Answer

To find: Value of $\int (x2^x) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int (x2^x) dx$ ... (i)

$\Rightarrow I = x\int 2^x dx - \int \left(\frac{dx}{dx} \int 2^x dx\right) dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \int \left(\frac{2^x}{\log 2}\right) dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\int 2^x dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\frac{2^x}{\log 2}$

$\Rightarrow I = \frac{x.\, 2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$

$\Rightarrow I = \frac{2^x}{(\log 2)^2}(x\log 2 - 1) + c$

Ans) D

21. Question

Mark (√) against the correct answer in each of the following:

$\int x\cot^2 x\, dx = ?$

A. $-x\cot x + \frac{x^2}{2} + \log|\sin x| + C$

B. $-x\cot x - \frac{x^2}{2} + \log|\sin x| + C$

C. $-x\cot x + \frac{x^2}{2} - \log|\sin x| + C$

D. none of these

Answer

To find: Value of $\int x\cot^2 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cot^2 x\, dx$ ... (i)

$\Rightarrow I = x\int \cot^2 x\, dx - \int\left(\frac{dx}{dx}\int \cot^2 x\, dx\right)dx$

$\Rightarrow I = x\int (\text{cosec}^2 x - 1)\, dx - \int\left(1.\int (\text{cosec}^2 x - 1)dx\right)dx$

$\Rightarrow I = x(-\cot x - x) - \int (-\cot x - x)dx$

$\Rightarrow I = -x\cot x - x^2 + \log|\sin x| + \frac{x^2}{2}$

$\Rightarrow I = -x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

Ans) B $-x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

22. Question

Mark (√) against the correct answer in each of the following:

$\int \sin\sqrt{x}\, dx = ?$

A. $-\sqrt{x}\cos\sqrt{x}+C$

B. $-\sqrt{x}\cos\sqrt{x}-2\sin\sqrt{x}-C$

C. $-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}-C$

D. none of these

Answer

To find: Value of $\int \sin\sqrt{x}\,dx$

Formula used: $\int \frac{1}{x}dx=\log|x|+c$

We have, $I = \int \sin\sqrt{x}\,dx$ ... (i)

$\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}}=\frac{dt}{dx}$

$\Rightarrow dx=2\sqrt{x}dt$

$\Rightarrow dx=2tdt$

$I = \int \sin t.\ 2t\,dt$

$I = 2\int t\ .\ \sin t\,dt$

$\Rightarrow I = 2t\int \sin t\,dt - \int\left(\frac{dt}{dt}\int \sin t\,dt\right)dt$

$\Rightarrow I = 2t\,(-\cos t)- \int 1\,(-\cos t)dt$

$\Rightarrow I = 2t\,(-\cos t)+\int \cos t\,dt$

$\Rightarrow I = 2t\,(-\cos t)+\sin t+c$

$\Rightarrow I = -2\sqrt{x}\,\cos\sqrt{x}+\sin\sqrt{x}+c$

Ans) C $2\sqrt{x}\,\cos\sqrt{x}+\sin\sqrt{x}+c$

23. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\sin x}\sin 2x\ dx=?$

A. $(2\sin x)\ e^{\sin x}+C$

B. $(2\cos x)\ e^{\sin x}+C$

C. $2e^{\sin x}(\sin x+1)+C$

D. $2e^{\sin x}(\sin x-1)+C$

Answer

To find: Value of $\int e^{sinx}\sin 2x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^{\sin x}\sin 2x\,dx$ ... (i)

$I = \int e^{\sin x} 2\sin x\cos x\,dx$

Put sinx = t

$\cos x = \frac{dt}{dx}$

$\Rightarrow \cos x\,dx = dt$

$I = 2\int e^{t}.t.dt$

$\Rightarrow I = 2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}\int e^{t}dt\right)dt\right]$

$\Rightarrow I = 2\left[te^{t} - \int 1\,e^{t}dt\right]$

$\Rightarrow I = 2te^{t} - 2e^{t} + c$

$\Rightarrow I = 2e^{t}(t-1) + c$

$\Rightarrow I = 2e^{\sin x}(\sin x - 1) + c$

Ans) D $2e^{\sin x}(\sin x - 1) + c$

24. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}dx = ?$

A. $\frac{\sin^{-1}x}{\sqrt{1-x^2}} - \frac{1}{2}\log\left|1-x^2\right| + C$

B. $x\sin^{-1}x + \frac{1}{2}\log\left|1-x^2\right| + C$

C. $\frac{\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\log\left|1-x^2\right| + C$

D. none of these

Answer

To find: Value of $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$ ... (i)

$$I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$$

$$I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}\,(1-x^2)}dx$$

Putting $\sin^{-1}x = t$, $x = \sin t$

$$\Rightarrow \cos t = \sqrt{1-x^2}$$

$$\Rightarrow \tan t = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$I = \int \frac{t}{\left(1-\sin^2 t\right)}dt$$

$$I = \int \frac{t}{\cos^2 t}dt$$

$$I = \int t.\sec^2 t\ dt$$

$$\Rightarrow I = \left[t\int \sec^2 t\ \ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ \ dt\right)dt\right]$$

$$\Rightarrow I = \left[t\tan t - \int 1\tan t\,dt\right]$$

$$\Rightarrow I = \left[\sin^{-1}x\frac{x}{\sqrt{1-x^2}} - \log\left|\sqrt{1-x^2}\right| + c\right]$$

$$\Rightarrow I = [t\tan t - \log|\cos t| + c]$$

$$\Rightarrow I = 2t\,e^t - 2e^t + c$$

$$\Rightarrow I = 2\,e^t\,(t-1) + c$$

$$\Rightarrow I = 2\,e^{\sin x}\,(\sin x-1) + c$$

Ans) D $2\,e^{\sin x}\,(\sin x-1)+c$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x\tan^{-1}x}{\left(1-x^2\right)^{3/2}}dx = ?$$

A. $\frac{\tan^{-1}x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{1+x^2}} + C$

B. $\frac{-\tan^{-1}x}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+C$

C. $\frac{x\tan^{-1}x}{\sqrt{1+x^2}}+\frac{1}{2}\log\left|\frac{x}{\sqrt{1+x^2}}\right|+C$

D. none of these

Answer

To find: Value of $\int \frac{x\tan^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{x\tan^{-1}x}{(1+x^2)^{\frac{3}{2}}}dx$ ... (i)

$I = \int \frac{x\tan^{-1}x}{\sqrt{1+x^2}\,(1+x^2)}dx$

Putting $\tan^{-1}x = t$, $x = \tan t$

$dx = \sec^2 t\, dt$

When $x = \tan t$

$\Rightarrow 1+x^2 = 1+\tan^2 t$

$\Rightarrow 1+x^2 = \sec^2 t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \cos t$

$\Rightarrow \frac{1}{1+x^2} = \cos^2 t$

$\Rightarrow 1-\frac{1}{1+x^2} = 1-\cos^2 t$

$\Rightarrow \frac{1+x^2-1}{1+x^2} = \sin^2 t$

$\Rightarrow \frac{x}{\sqrt{1+x^2}} = \sin t$

$I = \int \frac{\tan t\, t}{\sec t \sec^2 t}\sec^2 t\, dt$

$I = \int t \sin t dt$

Taking 1^{st} function as t and second function as $\sin t$

$$\Rightarrow I = \left[t\int \sin t\,dt - \int\left(\frac{dt}{dt}\int \sin t\,dt\right)dt\right]$$

$$\Rightarrow I = \left[t(-\cos t) - \int (1\,(-\cos t))dt\right]$$

$$\Rightarrow I = \left[t(-\cos t) + \int \cos t dt\right]$$

$$\Rightarrow I = -t\cos t + \sin t + c$$

$$\Rightarrow I = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

$$\Rightarrow I = \frac{-\tan^{-1} x\,1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

Ans) B $\frac{-\tan^{-1} x\,1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

26. Question

Mark (√) against the correct answer in each of the following:

$\int x \tan^{-1} x\,dx = ?$

A. $\frac{1}{2}\tan^{-1}x + \log\left(1+x^2\right) - \frac{1}{2}x + C$

B. $\frac{1}{2}x^2\tan^{-1}x + \frac{1}{2}x + C$

C. $\frac{1}{2}\left(1+x^2\right)\tan^{-1}x - \frac{1}{2}x + C$

D. none of these

Answer

To find: Value of $\int x\ \tan^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\ \tan^{-1} x\ dx$ ... (i)

Taking 1st function as $\tan^{-1} x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1} x\int x\,dx - \int\left(\frac{d(\tan^{-1} x)}{dx}\int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1} x\frac{x^2}{2} - \int\left(\frac{1}{1+x^2}\ \frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(\frac{x^2+1-1}{1+x^2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1dx - \int \frac{1}{1+x^2}dx\right]\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}[x - \tan^{-1}x]\right] + c$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$$

Ans) C $\frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$

27. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^{-1}\sqrt{x}\,dx = ?$

A. $(x-1)\tan^{-1}\sqrt{x} + \sqrt{x} + C$

B. $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

C. $\frac{1}{2}\sqrt{x}\tan^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x} + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1}\sqrt{x}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1}\sqrt{x}\,dx$ ... (i)

Let $\sqrt{x} = t$,

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$\Rightarrow dx = 2t\,dt$

$$I = \int \tan^{-1}\sqrt{x}\,dx$$

$$\Rightarrow I = \int \tan^{-1}t\;2t\,dt$$

$$\Rightarrow I = 2\int \tan^{-1}t\;t\,dt$$

Taking 1st function as $\tan^{-1}t$ and second function as t

$\Rightarrow I = 2\left[\tan^{-1} t \int t\, dt - \int \left(\frac{d(\tan^{-1} t)}{dt} \int t\, dt\right) dt\right]$

$\Rightarrow I = 2\left[\tan^{-1} t \frac{t^2}{2} - \int \left(\frac{1}{1+t^2}\ \frac{t^2}{2}\right) dt\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int \left(\frac{t^2+1-1}{1+t^2}\right) dt\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left[\int 1dt - \int \frac{1}{1+t^2} dt\ \right]\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}[t - \tan^{-1} t]\right] + c$

$\Rightarrow I = 2\left[\frac{x}{2}\tan^{-1} \sqrt{x} - \frac{1}{2}\sqrt{x} + \frac{1}{2}\tan^{-1} \sqrt{x}\right] + c$

$\Rightarrow I = x\tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c$

$\Rightarrow I = (x+1)\tan^{-1} \sqrt{x} - \sqrt{x} + c$

Ans) B $(x+1)\tan^{-1} \sqrt{x} - \sqrt{x} + c$

28. Question

Mark (√) against the correct answer in each of the following:

$\int \cos^{-1} x\, dx = ?$

A. $x\cos^{-1} x - \sqrt{1-x^2} + C$

B. $x\cos^{-1} x + \sqrt{1-x^2} + C$

C. $x\sin^{-1} x - \sqrt{1-x^2} + C$

D. none of these

Answer

To find: Value of $\int \cos^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int \cos^{-1} x\ dx$ ... (i)

Let $\cos^{-1} x = \theta$, ⇒ x = cosθ

⇒ dx = -sinθ dθ

If x = cosθ ,

Then $\sqrt{1-x^2}$ = sinθ

$I = \int \cos^{-1} x\ dx$

$\Rightarrow I = -\int \theta \sin\theta \, d\theta$

Taking 1st function as θ and second function as $\sin\theta$

$\Rightarrow I = -\left[\theta \int \sin\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = -\left[\theta(-\cos\theta) - \int (-\cos\theta) d\theta\right] + c$

$\Rightarrow I = -[\theta(-\cos\theta) - (-\sin\theta)] + c$

$\Rightarrow I = -[\theta(-\cos\theta) + \sin\theta] + c$

$\Rightarrow I = \theta\cos\theta - \sin\theta + c$

$\Rightarrow I = x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$

Ans) A $x \cdot \cos^{-1} x - \sqrt{1-x^2} + c$

29. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^{-1} x \, dx = ?$

A. $x \tan^{-1} x + \frac{1}{2} \log\left|1 + x^2\right| + C$

B. $x \tan^{-1} x - \frac{1}{2} \log\left|1 + x^2\right| + C$

C. $-x \tan^{-1} x + \frac{1}{2} \log\left|1 + x^2\right| + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1} x \; dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int \tan^{-1} x \, dx$ ... (i)

Let $\tan^{-1} x = \theta$, $\Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta \, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$I = \int \tan^{-1} x \, dx$

$\Rightarrow I = \int \theta \sec^2\theta \, d\theta$

Taking 1st function as θ and second function as $\sec^2\theta$

$\Rightarrow I = \left[\theta \int \sec^2\theta \, d\theta - \int\left(\frac{d\theta}{d\theta} \int \sec^2\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = \left[\theta(\tan\theta) - \int (1\,(\tan\theta))\, d\theta\right] + c$

$\Rightarrow I = [\theta(\tan\theta) - (\log|\sec\theta|)] + c$

$\Rightarrow I = \left[\tan^{-1}x\,(x) - \log\left|\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right|\right] + c$

$\Rightarrow I = \left[x \,.\, \tan^{-1}x - (\log\left|\sqrt{1+x^2}\right|)\right] + c$

$\Rightarrow I = x \,.\, \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$

Ans) B $x \,.\, \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$

30. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^{-1}x \, dx = ?$

A. $x\sec^{-1}x + \log\left|x + \sqrt{x^2-1}\right| + C$

B. $x\sec^{-1}x - \log\left|x + \sqrt{x^2-1}\right| + C$

C. $x\sec^{-1}x + \log\left|x - \sqrt{x^2-1}\right| + C$

D. none of these

Answer

To find: Value of $\int \sec^{-1}x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sec^{-1}x \, dx$ ... (i)

Let $\sec^{-1}x = \theta$, $\Rightarrow x = \sec\theta$

$\Rightarrow dx = \sec\theta \tan\theta \, d\theta$

If $x = \sec\theta$,

Then $\sqrt{x^2-1} = \tan\theta$

$I = \int \sec^{-1}x \, dx$

$\Rightarrow I = \int \theta \sec\theta \tan\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sec\theta \tan\theta$

$$\Rightarrow I = \left[\theta \int \sec\theta \tan\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec\theta \tan\theta \, d\theta\right) d\theta\right]$$

$$\Rightarrow I = \left[\theta(\sec\theta) - \int (1 (\sec\theta)) d\theta\right] + c$$

$$\Rightarrow I = [\theta(\sec\theta) - (\log|\sec\theta + \tan\theta|)] + c$$

$$\Rightarrow I = \left[\sec^{-1}x\,(x) - (\log\left|x + \sqrt{x^2-1}\right|)\right] + c$$

$$\Rightarrow I = x \,.\, \sec^{-1}x - \log\left|x + \sqrt{x^2-1}\right| + c$$

Ans) B $x \,.\, \sec^{-1}x - \log\left|x + \sqrt{x^2-1}\right| + c$

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin^{-1}\left(3x - 4x^3\right) dx = ?$$

A. $3\left[x\sin^{-1}x + \sqrt{1-x^2}\right] + C$

B. $3\left[x\sin^{-1}x - \sqrt{1-x^2}\right] + C$

C. $\frac{3x^2}{2} + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1}(3x - 4x^3)\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int \sin^{-1}(3x-4x^3)\, dx$ ... (i)

Let $x = \sin\theta$, $\Rightarrow \theta = \sin^{-1}x$

$\Rightarrow dx = \cos\theta \, d\theta$

If $x = \sin\theta$,

Then $\sqrt{1-x^2} = \cos\theta$

$$I = \int \sin^{-1}(3x-4x^3)\, dx$$

$$\Rightarrow I = \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta \, d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 3\theta) \cos\theta \, d\theta$$

$\Rightarrow I = \int 3\theta\cos\theta \, d\theta$

$\Rightarrow I = 3\int \theta\cos\theta \, d\theta$

Taking 1st function as θ and second function as $\cos\theta$

$\Rightarrow I = 3\left[\theta\int \cos\theta \, d\theta - \int\left(\frac{d\theta}{d\theta}\int \cos\theta \, d\theta\right)d\theta\right]$

$\Rightarrow I = 3\left[\theta(\sin\theta) - \int(1\,(\sin\theta))\,d\theta\right]$

$\Rightarrow I = 3[\theta(\sin\theta)-(-\cos\theta)]+c$

$\Rightarrow I = 3[\theta(\sin\theta)+\cos\theta]+c$

$\Rightarrow I = 3\sin^{-1}x\,(x)+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3\,x\sin^{-1}x+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

Ans) A $3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

32. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = ?$$

A. $2x\tan^{-1}x + \log|1 + x^2| + C$

B. $2x\tan^{-1}x - \log|1 + x^2| + C$

C. $2x\sin^{-1}x + \log|1 + x^2| + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1}\frac{2x}{1+x^2}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$$

$$\Rightarrow I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\,d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 2\theta)\sec^2\theta\,d\theta$$

$$\Rightarrow I = \int 2\theta\sec^2\theta\,d\theta$$

$$\Rightarrow I = 2\int \theta\sec^2\theta\,d\theta$$

Taking 1st function as θ and second function as $\sec^2\theta$

$$\Rightarrow I = 2\left[\theta\int \sec^2\theta\ d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta\ d\theta\right)d\theta\right]$$

$$\Rightarrow I = 2\left[\theta(\tan\theta) - \int(1\ (\tan\theta))d\theta\right]$$

$$\Rightarrow I = 2[\theta(\tan\theta)-(\log(\sec\theta)]+c$$

$$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log(\sec\left(\sec^{-1}\sqrt{1+x^2}\right))\right]+c$$

$$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log\sqrt{1+x^2})\right]+c$$

$$\Rightarrow I = 2\left[x\,.\,\tan^{-1}x-\frac{1}{2}(\log 1+x^2)\right]+c$$

$$\Rightarrow I = 2x\,.\,\tan^{-1}x-(\log 1+x^2)+c$$

Ans) B $2x\,.\,\tan^{-1}x-(\log 1+x^2)+c$

33. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx = ?$$

A. $\frac{1}{2}x\left(\cos^{-1}x\right)+\frac{1}{2}\sqrt{1-x^2}+C$

B. $\frac{1}{2}x\left(\sin^{-1}x\right)+\frac{1}{2}\sqrt{1-x^2}+C$

C. $\frac{1}{2}x\left(\cos^{-1}x\right)-\frac{1}{2}\sqrt{1-x^2}+C$

D. none of these

Answer

To find: Value of $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\, dx$ ... (i)

Let x = cosθ , ⇒ θ = $\cos^{-1}x$

⇒ dx = -sinθ dθ

If x = cosθ ,

Then $\sqrt{1-x^2}$= sinθ

$$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\, dx$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\ .\ -\sin\theta\, d\theta$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\ .\ -\sin\theta\, d\theta$$

$$\Rightarrow I = \int \tan^{-1}\sqrt{\tan^2\frac{\theta}{2}}\ .\ -\sin\theta\, d\theta$$

$$\Rightarrow I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)\ .\ -\sin\theta\, d\theta$$

$$\Rightarrow I = \int \frac{\theta}{2}\ .\ -\sin\theta\, d\theta$$

$$\Rightarrow I = -\frac{1}{2}\int \theta.\ \sin\theta\, d\theta$$

Taking 1st function as θ and second function as sinθ

$$\Rightarrow I = -\frac{1}{2}\left[\theta\int \sin\theta\, d\theta - \int\left(\frac{d\theta}{d\theta}\int \sin\theta\, d\theta\right)d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) - \int(1(-\cos\theta))d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) + \int(\cos\theta)d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}[\theta(-\cos\theta)+\sin\theta]+c$$

$$\Rightarrow I = \frac{1}{2}\cos^{-1}x\,(x) - \frac{1}{2}\sqrt{1-x^2}+c$$

$$\Rightarrow I = \frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2}+c$$

Ans) C $\frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2}+c$

34. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx = ?$$

A. $3x\ \tan^{-1}x + \frac{3}{2}\log\left(1+x^2\right)+C$

B. $3x\ \tan^{-1}x - \frac{3}{2}\log\left(1+x^2\right)+C$

C. $3x\cos^{-1}x - \frac{3}{2}\sqrt{1-x^2}+C$

D. $3x\ \sin^{-1}x + \frac{3}{2}\sqrt{1-x^2}+C$

Answer

To find: Value of $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x|+c$

We have, $I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$ ... (i)

Let x = tanθ , ⇒ θ = $\tan^{-1}x$

⇒ dx = $\sec^2\theta\ d\theta$

If x = tanθ ,

Then $1 + x^2 = \sec^2\theta$

⇒ θ = $\sec^{-1}\sqrt{1+x^2}$

$$I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$$

$$\Rightarrow I = \int \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)\sec^2\theta\ d\theta$$

$$\Rightarrow I = \int \tan^{-1}(\tan 3\theta)\sec^2\theta\ d\theta$$

$$\Rightarrow I = \int 3\theta\sec^2\theta\ d\theta$$

$$\Rightarrow I = 3\int \theta\sec^2\theta\ d\theta$$

Taking 1st function as θ and second function as $\sec^2\theta$

$\Rightarrow I = 3\left[\theta \int \sec^2\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = 3\left[\theta\tan\theta - \int (\tan\theta) d\theta\right]$

$\Rightarrow I = 3[\theta\tan\theta - (\log\sec\theta)] + c$

$\Rightarrow I = 3\theta\tan\theta - 3\log(\sec\theta) + c$

$\Rightarrow I = 3\tan^{-1}x\tan(\tan^{-1}x) - 3\log\left\{\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right\} + c$

$\Rightarrow I = 3x.\tan^{-1}x - 3\log\left\{\sqrt{1+x^2}\right\} + c$

$\Rightarrow I = 3x.\tan^{-1}x - \frac{3}{2}\log\{1+x^2\} + c$

Ans) B $3x.\tan^{-1}x - \frac{3}{2}\log\{1+x^2\} + c$

35. Question

Mark (√) against the correct answer in each of the following:

$\int x^2 \cos x \, dx = ?$

A. $x^2 \sin x + 2x \cos x - 2 \sin x + C$

B. $2x \cos x - x \sin x + 2 \sin x + C$

C. $x^2 \sin x - 2x \sin x + 2 \sin x + C$

D. none of these

Answer

To find: Value of $\int x^2 cosx \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x^2 cosx \, dx$ ... (i)

Taking 1st function as x^2 and second function as $cosx$

$\Rightarrow I = \left[x^2 \int cosx \, dx - \int \left(\frac{dx^2}{dx} \int cosx \, dx\right) dx\right]$

$\Rightarrow I = \left[x^2 sinx - \int (2x \, sinx) dx\right]$

$\Rightarrow I = \left[x^2 sinx - 2\int (x \, sinx) dx\right]$

Taking 1st function as x and second function as $sinx$

$\Rightarrow I = x^2 sinx - 2\left[x \int sinx \, dx - \int \left(\frac{dx}{dx} \int sinx \, dx\right) dx\right]$

$\Rightarrow I = x^2 sinx - 2\left[x(-cosx) - \int (1 \, (-cosx) dx\right]$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)-(-\sin x)]+c$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)+\sin x]+c$

$\Rightarrow I = x^2\sin x + 2x\cos x - 2\sin x + c$

Ans) A $x^2\sin x + 2x\cos x - 2\sin x + c$

36. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin x \log(\cos x)\, dx = ?$$

A. cos x log (cos x) – cos x + C

B. -cos x log (cos x) + cos x + C

C. cos x log (cos x) + cos x + C

D. none of these

Answer

To find: Value of $\int \sin x \log(\cos x)\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \sin x \log(\cos x)\ dx$ ... (i)

Let cosx = t

-sinx dx = dt

$I = \int \sin x \log(\cos x)\ dx$

$I = -\int \log t\, dt$

$I = -\int \log t \,.\, 1 \,.\, dt$

Taking 1st function as $\log t$ and second function as 1

$\Rightarrow I = -\left[\log t \int 1\, dt - \int \left(\frac{d\log t}{dt} \int 1\, dt \right) dt \right]$

$\Rightarrow I = -\left[\log t \,.\, t - \int \left(\frac{1}{t}\, t \right) dt \right]$

$\Rightarrow I = -\left[\log t \,.\, t - \int 1 dt \right]$

$\Rightarrow I = -[\log t \,.\, t - t]+c$

$\Rightarrow I = -\log t \,.\, t + t + c$

$\Rightarrow I = -\cos x \,.\, \log(\cos x) + \cos x + c$

Ans) B $-\cos x \,.\, \log(\cos x) + \cos x + c$

37. Question

Mark (√) against the correct answer in each of the following:

$\int x \sin x \cos x \, dx = ?$

A. $-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin\ 2x + C$

B. $\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

C. $\frac{1}{4}x\cos 2x - \frac{1}{8}\sin 2x + C$

D. none of these

Answer

To find: Value of $\int x \sin x \cos x \, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int x \sin x \cos x \, dx$ ... (i)

$I = \frac{1}{2}\int x\ 2\sin x \cos x\, dx$

$I = \frac{1}{2}\int x \sin 2x\, dx$

Let 2x = t

2dx = dt

$dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{t}{2} \sin t\ \frac{dt}{2}$

$I = \frac{1}{8}\int t \sin t\, dt$

Taking 1st function as t and second function as sint

$\Rightarrow I = \frac{1}{8}\left[t\int \sin t\ \ dt - \int\left(\frac{dt}{dt}\int \sin t\ \ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{8}\left[t.(-\cos t) - \int(-\cos t)\, dt\right]$

$\Rightarrow I = \frac{1}{8}[-t.\cos t - (-\sin t)] + c$

$\Rightarrow I = \frac{1}{8}[-t.\cos t + \sin t] + c$

$\Rightarrow I = -\frac{1}{8}2x.\cos 2x + \frac{1}{8}\sin 2x + c$

$\Rightarrow I = -\frac{1}{4}x.\cos 2x + \frac{1}{8}\sin 2x + c$

Ans) A $-\frac{1}{4}x.\cos 2x+\frac{1}{8}\sin 2x+c$

38. Question

Mark (√) against the correct answer in each of the following:

$\int x^3 \cos x^2 dx = ?$

A. $x^2 \sin x^2 + \cos x^2 + C$

B. $\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

C. $-\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

D. none of these

Answer

To find: Value of $\int x^3 cosx^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^3 cosx^2 dx$ ... (i)

Let $x^2 = t$

$\Rightarrow xdx = \frac{1}{2}dt$

$I = \int x^3 cosx^2 dx$

$I = \int x.\, x^2 cosx^2 dx$

$I = \int t\cos t\,\frac{1}{2}\,dt$

$I = \frac{1}{2}\int t\cos t\,dt$

Taking 1^{st} function as t and second function as $\cos t$

$\Rightarrow I = \frac{1}{2}\left[t\int \cos t\ dt - \int\left(\frac{dt}{dt}\int \cos t\ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{2}\left[t.\, \sin t - \int \sin t\, dt\right]$

$\Rightarrow I = \frac{1}{2}[t.\, \sin t-(-\cos t)+c]$

$\Rightarrow I = \frac{1}{2}[t.\, \sin t+\cos t+c]$

$\Rightarrow I = \frac{1}{2}x^2.\, \sin x^2+\frac{1}{2}\cos x^2+c$

Ans) B $\frac{1}{2}x^2 . \sin x^2 + \frac{1}{2}\cos x^2 + c$

39. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx = ?$$

A. $2x \tan^{-1} x + \log(1 + x^2) + C$

B. $-2x \tan^{-1} x - 2 \log (1 + x^2) + C$

C. $2x \tan^{-1} x - \log (1 + x^2) + C$

D. none of these

Answer

To find: Value of $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ ... (i)

Let $x = \tan t$, $t = \tan^{-1}x$

$\Rightarrow dx = \sec^2 t\, dt$

If $\tan t = x$,

$\sec t = 1 + x^2$

$$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\sec^2 t\, dt$$

$$I = \int \cos^{-1}(\cos 2t)\sec^2 t\ dt$$

$$I = \int 2t\sec^2 t\ dt$$

$$I = 2\int t\sec^2 t\ dt$$

Taking 1st function as t and second function as $\sec^2 t$

$$\Rightarrow I = 2\left[t\int \sec^2 t\ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ dt\right)dt\right]$$

$$\Rightarrow I = 2\left[t\tan t - \int \tan t\, dt\right]$$

$$\Rightarrow I = 2[t\tan t - \log|\sec t| + c]$$

$\Rightarrow I = 2[\tan^{-1}x\ x - \log|1+x^2| + c]$

$\Rightarrow I = 2x\tan^{-1}x - 2\log|1+x^2| + c$

Ans) D None of these

40. Question

Mark (√) against the correct answer in each of the following:

$\int x\tan^{-1}x\,dx = ?$

A. $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + C$

B. $\frac{1}{2}(x^2-1)\tan^{-1}x - \frac{1}{2}x + C$

C. $\frac{1}{2}(x^2+1)\tan^{-1}x + \frac{1}{2}x + C$

D. none of these

Answer

To find: Value of $\int x\tan^{-1}x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\tan^{-1}x\,dx$ ... (i)

Taking 1st function as $\tan^{-1}x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1}x\int x\,dx - \int\left(\frac{d\tan^{-1}x}{dx}\int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1}x\frac{x^2}{2} - \int\left(\frac{1}{(1+x^2)}\ \frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(\frac{x^2}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1.\,dx + \frac{1}{2}\int\frac{1}{(1+x^2)}.\,dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \left[\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x\right] + c$$

$\Rightarrow I = \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

Ans) A $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

41. Question

Mark (√) against the correct answer in each of the following:

$\int \sin(\log x)dx = ?$

A. $\frac{1}{2}x\sin\log x + \frac{1}{2}x\cos(\log x) + C$

B. $\frac{1}{2}x\sin\log x - \frac{1}{2}x\cos(\log x) + C$

C. $-\frac{1}{2}x\sin\log x + \frac{1}{2}x\cos(\log x) + C$

D. none of these

Answer

To find: Value of $\int \sin(\log x)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin(\log x)\,dx$ ... (i)

$I = \int \sin(\log x)\,.\,1\,.\,dx$

Taking 1st function as sin(logx) and second function as 1

$\Rightarrow I = \left[\sin(\log x)\int 1\,dx - \int\left(\frac{d\sin(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int \frac{\cos(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int \cos(\log x)\,dx\right]$

Taking 1st function as cos(logx) and second function as 1

$\Rightarrow I = \sin(\log x)\,.x - \left[\cos(\log x)\int 1\,dx - \int\left(\frac{d\cos(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x - \int -\frac{\sin(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x + \int \sin(\log x)\,dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - [\cos(\log x).x + I] + c$

$\Rightarrow I = \sin(\log x)\,.\,x - \cos(\log x).x - I + c$

$\Rightarrow 2I = \sin(\log x) . x - \cos(\log x) . x + c$

$\Rightarrow I = \frac{\sin(\log x) . x - \cos(\log x) . x}{2} + c$

$\Rightarrow I = \frac{1}{2}x . \sin(\log x) - x . \frac{1}{2}\cos(\log x) + c$

Ans) B $\frac{1}{2}x . \sin(\log x) - x . \frac{1}{2}\cos(\log x) + c$

42. Question

Mark (√) against the correct answer in each of the following:

$\int (\sin^{-1} x)^2 dx = ?$

A. $\frac{2\sin^{-1} x}{\sqrt{1-x^2}} + C$

B. $\frac{1}{3}(\sin^{-1} x)^3 + \frac{1}{\sqrt{1-x^2}} + C$

C. $x(\sin^{-1} x)^2 + (\sin^{-1} x)\sqrt{1-x^2} + 2x + C$

D. $x(\sin^{-1} x)^2 + 2(\sin^{-1} x)\sqrt{1-x^2} - 2x + C$

Answer

To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int (\sin^{-1} x)^2 dx$ ... (i)

Putting sint = x, ⇒ t = $\sin^{-1} x$

⇒ dx = cost dt

When x = sint then $\sqrt{1-x^2} = \cos t$

$I = \int (\sin^{-1} x)^2 dx$

$\Rightarrow I = \int (\sin^{-1}(\sin t))^2 \cos t \, dt$

$\Rightarrow I = \int t^2 \cos t \, dt$

Taking 1^{st} function as t^2 and second function as cost

$\Rightarrow I = \left[t^2 \int \cos t \, dt - \int \left(\frac{dt^2}{dt} \int \cos t \, dt\right) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - \int (2t \sin t) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - 2\int (t \sin t)dt\right]$

Taking 1^{st} function as t and second function as sint

$\Rightarrow I = t^2 \sin t - 2\left[\int (t \sin t)dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t\int \sin t \ dt - \int \left(\frac{dt}{dt}\int \sin t \ dt\right)dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t(-\cos t) - \int (-\cos t)dt\right]$

$\Rightarrow I = t^2 \sin t - 2[-t\cos t - (-\sin t) + c]$

$\Rightarrow I = t^2 \sin t + 2t\cos t - 2\sin t + c$

$\Rightarrow I = x\left(\sin^{-1} x\right)^2 + 2\sin^{-1} x\sqrt{1-x^2} - 2x + c$

Ans) D $x\left(\sin^{-1} x\right)^2 + 2\sin^{-1} x\sqrt{1-x^2} - 2x + c$

43. Question

Mark (√) against the correct answer in each of the following:

$\int e^x\left\{\frac{1}{x} - \frac{1}{x^2}\right\}dx = ?$

A. $e^x\left\{\log x + \frac{1}{x}\right\} + C$

B. $xe^x - e^x + C$

C. $e^x.\frac{1}{x} + C$

D. none of these

Answer

To find: Value of $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$ ... (i)

Here $f(x) = \frac{1}{x}$

$\Rightarrow f'(x) = -\frac{1}{x^2}$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x} + c$

Ans) C $e^x \frac{1}{x} + c$

44. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx = ?$$

A. $\frac{-e^x}{x^2}+C$

B. $\frac{e^x}{x^2}+C$

C. $e^x\left(\frac{-1}{x}+\frac{1}{x^2}\right)+C$

D. none of these

Answer

To find: Value of $\int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$ ... (i)

Here $f(x) = \frac{1}{x^2}$

$\Rightarrow f'(x) = -\frac{2}{x^3}$

$\Rightarrow I = \int e^x \left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x^2} + c$

Ans) B $e^x \frac{1}{x^2} + c$

45. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x\left\{\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right\}dx = ?$$

A. $e^x . \frac{1}{\sqrt{1-x^2}} + C$

B. $e^x \sin^{-1} x + C$

C. $\frac{-e^x}{\sin^{-1} x} + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$ ... (i)

Here $f(x) = \sin^{-1} x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \sin^{-1} x + c$

Ans) B $e^x \sin^{-1} x + c$

46. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\tan x + \log \sec x) dx = ?$

A. $e^x \log \sec x + C$

B. $e^x \tan x + C$

C. $e^x (\log \cos x) + C$

D. none of these

Answer

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

47. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x (\tan x + \log \sec x) dx = ?$$

A. e^x log sec x + C

B. e^x tan x + C

C. e^x (log cos x) + C

D. none of these

Answer

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

48. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x (\cot x + \log \sin x) dx = ?$$

A. e^x log (sec x + tan x) + C

B. $e^x \sec x + C$

C. $e^x \log \tan x + C$

D. none of these

Answer

To find: Value of $\int e^x(\cot x + \log(\sin x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x+\log(\sin x))dx$ ... (i)

Here $f(x) = \log(\sin x)$

$\Rightarrow f'(x) = \cot x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \log(\sin x) + c$

Ans) D None of these

49. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x\left\{\tan^{-1}x + \frac{1}{(1+x^2)}\right\}dx = ?$$

A. $e^x \cdot \frac{1}{(1+x^2)} + C$

B. $e^x \tan^{-1} x + C$

C. $-e^x \cot^{-1} x + C$

D. none of these

Answer

To find: Value of $\int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$ ... (i)

Here $f(x) = \tan^{-1}x$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x (\tan^{-1} x) + c$

Ans) B $e^x(\tan^{-1} x) + c$

50. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\tan x - \log \cos x) dx = ?$

A. $e^x \tan x + C$

B. $e^x \log \cos x + C$

C. $e^x \log \sec x + C$

D. none of these

Answer

To find: Value of $\int e^x(\tan x - \log(\cos x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int e^x(\tan x - \log(\cos x))dx$ ... (i)

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) C $e^x \log(\sec x) + c$

51. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx = ?$

A. $-e^x \operatorname{cosec}^2 x + C$

B. $e^x \cot x + C$

C. $-e^x \cot x + C$

D. None of these

Answer

To find: Value of $\int e^x(cotx - cosec^2x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(cotx\text{-}cosec^2x)dx$ ... (i)

Here f(x) = cotx

$\Rightarrow f'(x) = -cosec^2x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x cotx+c$

Ans) B $e^x cotx+c$

52. Question

Mark (√) against the correct answer in each of the following:

$\int e^x(\sin x + \cos x)dx = ?$

A. $e^x \sin x + C$

B. $e^x \cos x + C$

C. $e^x \tan x + C$

D. None of these

Answer

To find: Value of $\int e^x(sinx + cosx)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(sinx+cosx)dx$ ... (i)

Here f(x) = sinx

$\Rightarrow f'(x) = cosx$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x sinx+c$

Ans) A $e^x sinx+c$

53. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \sec x(1+\tan x)dx = ?$

A. $e^x (1 + \tan x) + C$

B. $e^x \sec x + C$

C. $e^x \tan x + C$

D. none of these

Answer

To find: Value of $\int e^x secx\,(1 + tanx)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x secx\,(1+tanx)dx$ ... (i)

$I = \int e^x(secx+secxtanx)dx$

Here $f(x) = secx$

$\Rightarrow f'(x) = secxtanx$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x secx+c$

Ans) B $e^x secx+c$

54. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx = ?$$

A. $e^x \cdot \frac{1}{x} + C$

B. $e^x \log x + C$

C. $x e^x \log x + C$

D. None of these

Answer

To find: Value of $\int e^x\left(\frac{1+x\,logx}{x}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1+x\,logx}{x}\right)dx$ ... (i)

$I = \int e^x\left(\frac{1}{x}+logx\right)dx$

Here $f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \log x + c$

Ans) B $e^x \log x + c$

55. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x \cdot \frac{x}{(1+x)^2} dx = ?$$

A. $e^x \cdot \frac{1}{(1+x)} + C$

B. $e^x \cdot \frac{1}{x} + C$

C. $e^x \cdot \frac{x}{(1+x)} + C$

D. None of these

Answer

To find: Value of $\int e^x \frac{x}{(1+x)^2} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int e^x \frac{x}{(1+x)^2} dx$ ... (i)

$I = \int e^x \left(\frac{x+1-1}{(1+x)^2}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right) dx$

Here $f(x) = \frac{1}{(1+x)}$

$\Rightarrow f'(x) = -\frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{(1+x)} + c$

Ans) A $e^x \frac{1}{(1+x)} + c$

56. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = ?$$

A. $e^x \sin\frac{x}{2} + C$

B. $e^x \cos\frac{x}{2} + C$

C. $e^x \tan\frac{x}{2} + C$

D. None of these

Answer

To find: Value of $\int e^x \left(\frac{1+sinx}{1+cosx}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ ... (i)

$$I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$$

Here $f(x) = \tan\frac{x}{2}$

$$\Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$$

$$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$$

$$\Rightarrow I = e^x f(x) + c$$

$$\Rightarrow I = e^x \tan\frac{x}{2} + c$$

Ans) C $e^x \tan\frac{x}{2} + c$