Chapter – 10

Operations Research

Ex 10.1

Question 1.
A company produces two types of pens A and B. Pen A is of superior quality and pen B is of lower quality. Profits on pens A and B are ₹5 and ₹3 per pen respectively. Raw materials required for each pen A is twice as that of pen B. The supply of raw material is sufficient only for 1000 pens per day. Pen A requires a special clip and only 400 such clips are available per day. For pen B, only 700 clips are available per day. Formulate this problem as a linear programming problem.

Solution:
(i) Variables: Let x_1 and x_2 denotes the number of pens in type A and type B.

(ii) Objective function:
Profit on x_1 pens in type A is $= 5x_1$
Profit on x_2 pens in type B is $= 3x_2$
Total profit $= 5x_1 + 3x_2$
Let $Z = 5x_1 + 3x_2$, which is the objective function.
Since the B total profit is to be maximized, we have to maximize $Z = 5x_1 + 3x_2$

(iii) Constraints:
Raw materials required for each pen A is twice as that of pen B.
i.e., for pen A raw material required is $2x_1$ and for B is x_2.
Raw material is sufficient only for 1000 pens per day
$\therefore 2x_1 + x_2 \leq 1000$
Pen A requires 400 clips per day
$\therefore x_1 \leq 400$
Pen B requires 700 clips per day
$\therefore x_2 \leq 700$

(iv) Non-negative restriction:
Since the number of pens is non-negative, we have $x_1 > 0$, $x_2 > 0$.
Thus, the mathematical formulation of the LPP is

Maximize $Z = 5x_1 + 3x_2$
Subject to the constrains
$2x_1 + x_2 \leq 1000$, $x_1 \leq 400$, $x_2 \leq 700$, $x_1, x_2 \geq 0$

Question 2.
A company produces two types of products say type A and B. Profits on the two types of product are ₹ 30/- and ₹ 40/- per kg respectively. The data on resources required and availability of resources are given below.

	Requirements		Capacity available per month
	Product A	Product B	
Raw material (kgs)	60	120	12000
Machining hours / piece	8	5	600
Assembling (man hours)	3	4	500

Formulate this problem as a linear programming problem to maximize the profit.

Solution:
(i) Variables: Let x_1 and x_2 denote the two types products A and B respectively.

(ii) Objective function:
Profit on x_1 units of type A product $= 30x_1$
Profit on x_2 units of type B product $= 40x_2$
Total profit $= 30x_1 + 40x_2$
Let $Z = 30x_1 + 40x_2$, which is the objective function.
Since the profit is to be maximized, we have to maximize $Z = 30x_1 + 40x_2$

(iii) Constraints:
$60x_1 + 120x_2 \leq 12,000$
$8x_1 + 5x_2 \leq 600$
$3x_1 + 4x_2 \leq 500$

(iv) Non-negative constraints:
Since the number of products on type A and type B are non-negative, we have
$x_1, x_2 \geq 0$

Thus, the mathematical formulation of the LPP is
Maximize $Z = 30x_1 + 40x_2$
Subject to the constraints,
$60x_1 + 120x_2 \leq 12{,}000$
$8x_1 + 5x_2 \leq 600$
$3x_1 + 4x_2 \leq 500$
$x_1, x_2 \geq 0$

Question 3.
A company manufactures two models of voltage stabilizers viz., ordinary and autocut. All components of the stabilizers are purchased from outside sources, assembly and testing is carried out at company's own works. The assembly and testing time required for the two models are 0.8 hour each for ordinary and 1.20 hours each for auto-cut. Manufacturing capacity 720 hours at present is available per week. The market for the two models has been surveyed which suggests maximum weekly sale of 600 units of ordinary and 400 units of auto-cut. Profit per unit for ordinary and auto-cut models has been estimated at ₹ 100 and ₹ 150 respectively. Formulate the linear programming problem.

Solution:
(i) Variables : Let x_1 and x_2 denote the number of ordinary and auto-cut voltage stabilized.

(ii) Objective function:
Profit on x_1 units of ordinary stabilizers = $100x_1$
Profit on x_2 units of auto-cut stabilized = $150x_2$
Total profit = $100x_1 + 150x_2$
Let $Z = 100x_1 + 150x_2$, which is the objective function.
Since the profit is to be maximized. We have to
Maximize, $Z = 100x_1 + 15x_2$

(iii) Constraints: The assembling and testing time required for x_1 units of ordinary stabilizers = $0.8x_1$ and for x_2 units of auto-cut stabilizers = $1.2x_2$
Since the manufacturing capacity is 720 hours per week. We get $0.8x_1 + 1.2x_2 \leq 720$
Maximum weekly sale of ordinary stabilizer is 600
i.e., $x_1 \leq 600$

Maximum weekly sales of auto-cut stabilizer is 400
i.e., $x_2 \leq 400$

(iv) Non-negative restrictions:
Since the number of both the types of stabilizer is non-negative, we get $x_1, x_2 \geq 0$.
Thus, the mathematical formulation of the LPP is,
Maximize $Z = 100x_2 + 150x_2$
Subject to the constraints
$0.8x_1 + 1.2x_2 \leq 720$, $x_1 \leq 600$, $x_2 \leq 400$, $x_1, x_2 \geq 0$

Question 4.
Solve the following linear programming problems by graphical method.
(i) Maximize $Z = 6x_1 + 8x_2$ subject to constraints $30x_1 + 20x_2 \leq 300$; $5x_1 + 10x_2 \leq 110$; and $x_1, x_2 \geq 0$.

(ii) Maximize $Z = 22x_1 + 18x_2$ subject to constraints $960x_1 + 640x_2 \leq 15360$; $x_1 + x_2 \leq 20$ and $x_1, x_2 \geq 0$.

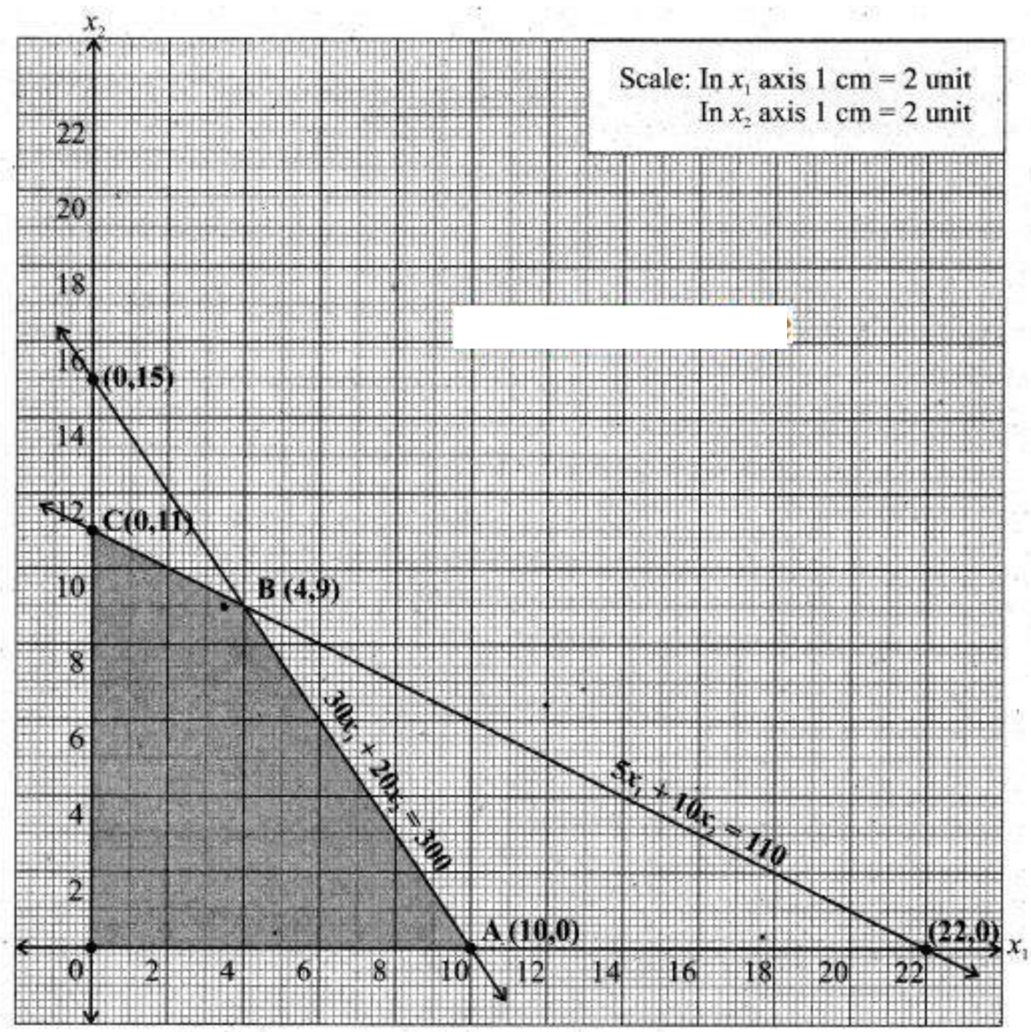
(iii) Minimize $Z = 3x_1 + 2x_2$ subject to the constraints $5x_1 + x_2 \geq 10$; $x_1 + x_2 > 6$; $x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$.

(iv) Maximize $Z = 40x_1 + 50x_2$ subject to constraints $3x_1 + x_2 \leq 9$; $x_1 + 2x_2 \leq 8$ and $x_1, x_2 \geq 0$.

(v) Maximize $Z = 20x_1 + 30x_2$ subject to constraints $3x_1 + 3x_2 \leq 36$; $5x_1 + 2x_2 \leq 50$; $2x_1 + 6x_2 \leq 60$ and $x_1, x_2 \geq 0$.

(vi) Minimize $Z = 20x_1 + 40x_2$ subject to the constraints $36x_1 + 6x_2 \geq 108$; $3x_1 + 12x_2 \geq 36$; $20x_1 + 10x_2 \geq 100$ and $x_1, x_2 \geq 0$.

Solution:
(i) Given that $30x_1 + 20x_2 \leq 300$
Let $30x_1 + 20x_2 = 300$



Therefore

$3x_1 + 2x_2 = 30$

x_1	0	10
x_2	15	0

Also given that $5x_1 + 10x_2 \leq 110$

Let $5x_1 + 10x_2 = 110$

$x_1 + 2x_2 = 22$

x_1	0	22
x_2	11	0

To get point of intersection, (i.e., the to get eo-ordinates of B)

$3x_1 + 2x_2 = 30$ …….(1)

$x_1 + 2x_2 = 22$ ……..(2)

(1) – (2) ⇒ $2x_1 = 8$
$x_1 = 4$
$x_1 = 4$ substitute in (1),
$x_1 + 2x_2 = 22$
$4 + 2x_2 = 22$
$2x_2 = 18$
$x_2 = 9$
i.e., B is (4, 9)
The feasible region satisfying all the given conditions is OABC.
The co-ordinates of the points are O(0, 0), A(10, 0), B(4, 9), C(0, 11).

Corner points	$Z = 6x_1 + 8x_2$
O(0, 0)	0
A(10, 0)	60
B(4, 9)	$6 \times 4 + 8 \times 9 = 96$
C(0, 11)	88

The maximum value of Z occurs at B.
∴ The optimal solution is $x_1 = 4$, $x_2 = 9$ and $Z_{max} = 96$

(ii) Given that $960x_1 + 640x_2 \leq 15360$
Let $960x_1 + 640x_2 = 15360$
$3x_1 + 2x_2 = 48$

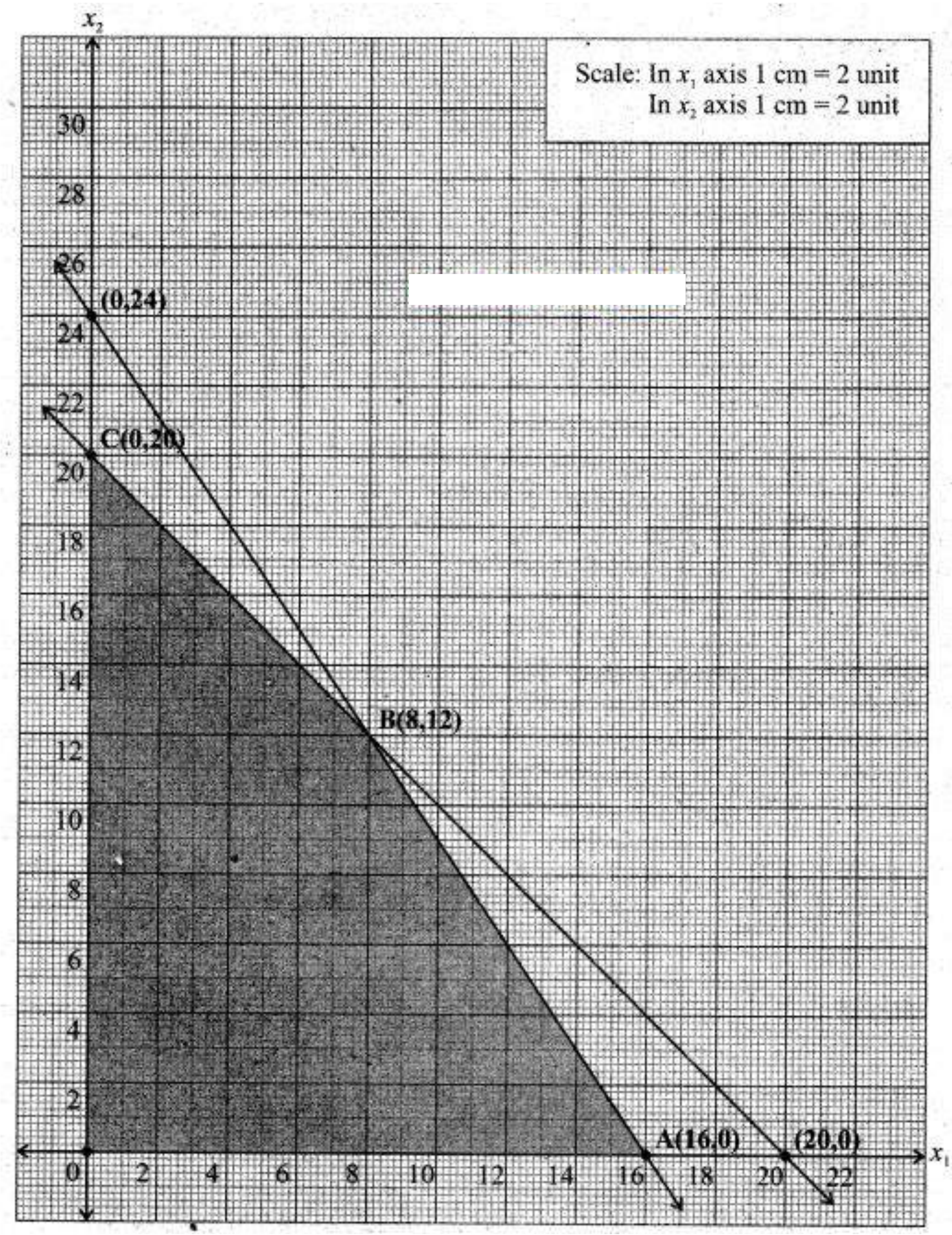
x_1	0	16
x_2	24	0

Also given that $x_1 + x_2 \leq 20$
Let $x_1 + x_2 = 20$

x_1	0	20
x_2	20	0

To get point of intersection
$3x_1 + 2x_2 = 48$ .....(1)
$x_1 + x_2 = 20$ ......(2)
(2) × -2 ⇒ $-2x_1 - 2x_2 = -40$ .....(3)

$(1) + (3) \Rightarrow x_1 = 8$
$x_1 = 8$ substitute in (2),
$8 + x_2 = 20$
$x_2 = 12$



The feasible region satisfying all the given conditions is OABC.
The co-ordinates of the comer points are O(0, 0), A(16, 0), B(8,12) and C(0, 16).

Corner points	$Z = 22x_1 + 18x_2$
O(0, 0)	0
A(16, 0)	352
B(8, 12)	392
C(0, 20)	360

The maximum value of Z occurs at B(8, 12).
∴ The optimal solution is $x_1 = 8$, $x_2 = 12$ and $Z_{max} = 392$

(iii) Given that $5x_1 + x_2 \geq 10$
Let $5x_1 + x_2 = 10$

x_1	0	2
x_2	10	0

Also given that $x_1 + x_2 \geq 6$
Let $x_1 + x_2 = 6$

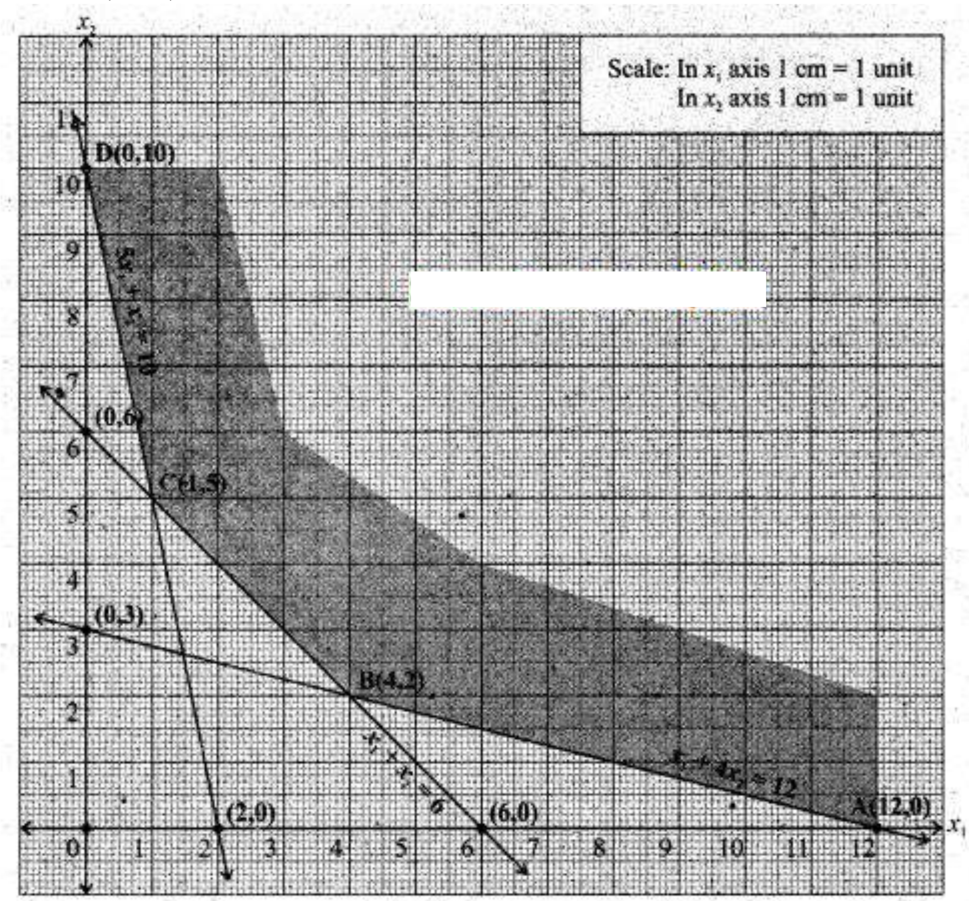
x_1	0	6
x_2	6	0

Also given that $x_1 + 4x_2 \geq 12$
Let $x_1 + 4x_2 = 12$

x_1	0	12
x_2	3	0

To get C
$5x_1 + x_2 = 10$ ……..(1)
$x_1 + x_2 = 6$ ………(2)
(1) – (2) ⇒ $4x_1 = 4$
⇒ $x_1 = 1$
$x = 1$ substitute in (2)
⇒ $x_1 + x_2 = 6$
⇒ $1 + x_2 = 6$
⇒ $x_2 = 5$
∴ C is (1, 5)
To get B

$x_1 + x_2 = 6$
$x_1 + 4x_2 = 12$
$(1) – (2) \Rightarrow -3x_2 = -6$
$x_2 = 2$
$x_2 = 2$ substitute in (1), $x_1 = 4$
$\therefore$ B is (4, 2)



The feasible region satisfying all the conditions is ABCD.
The co-ordinates of the comer points are A(12, 0), B(4, 2), C(1, 5) and D(0, 10).

Corner points	$Z = 3x_1 + 2x_2$
A(12, 0)	36
B(4, 2)	12 + 4 = 16
C(1, 5)	3 + 10 = 13
D(0, 10)	20

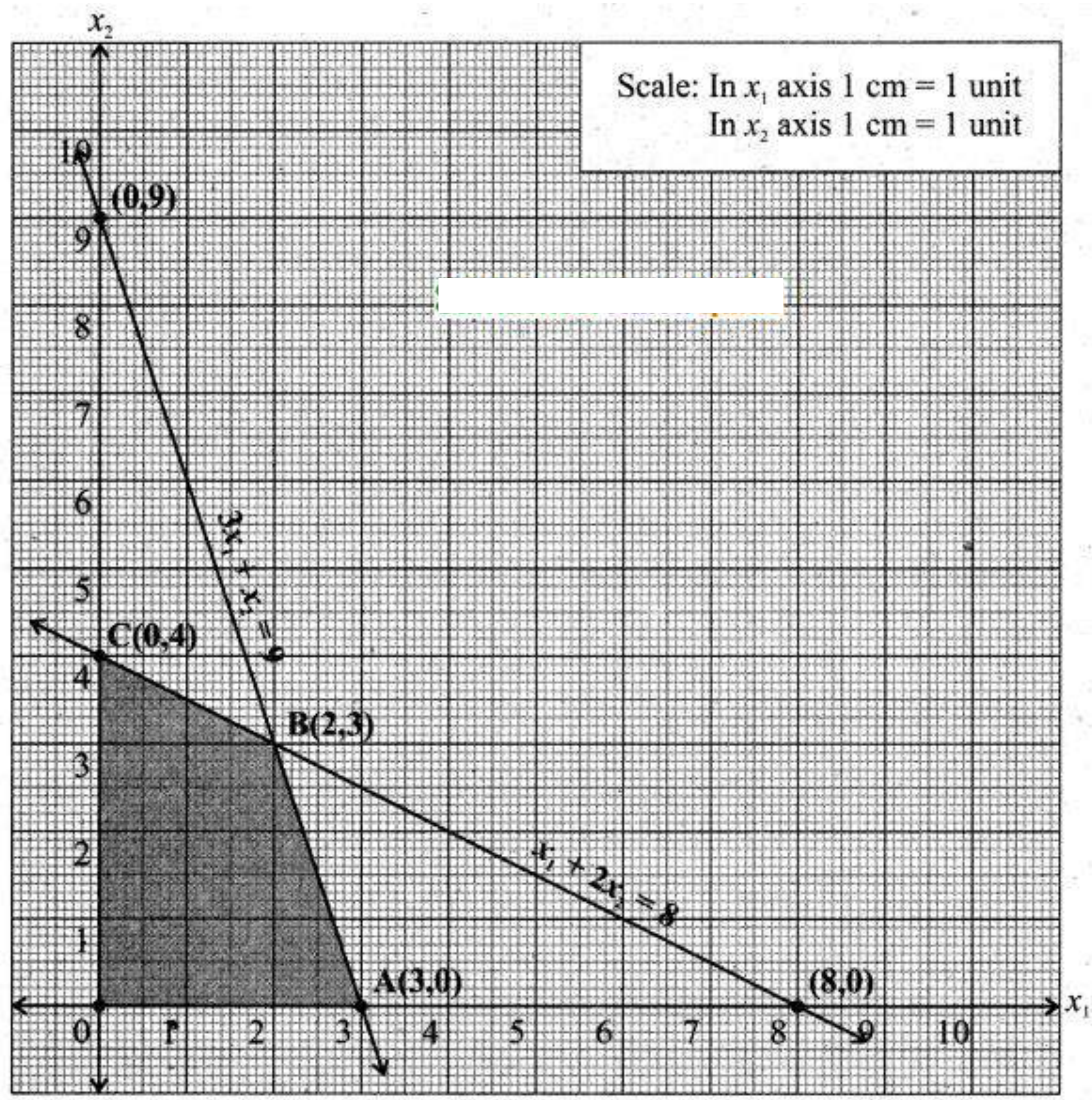
The minimum value of Z occours at C(1, 5).

∴ The optimal solution is $x_1 = 1, x_2 = 5$ and $Z_{min} = 13$

(iv) Given that $3x_1 + x_2 \leq 9$

Let $3x_1 + x_2 = 9$

x_1	0	3
x_2	9	0



Also given that $x_1 + 2x_2 \leq 8$

Let $x_1 + 2x_2 = 8$

x_1	0	8
x_2	4	0

$3x_1 + x_2 = 9$ ………(1)
$x_1 + 2x_2 = 8$ ……..(2)
$(1) \times 2 \Rightarrow 6x_1 + 2x_2 = 18$ ……..(3)
$(2) + (3) \Rightarrow -5x_1 = -10$
$x_1 = 2$
$x_1 = 2$ substitute in (1)
$3(2) + x_2 = 9$
$x_2 = 3$
The feasible region satisfying all the conditions is OABC.
The co-ordinates of the corner points are O(0, 0), A(3, 0), B(2, 3), C(0, 4)

Corner points	$Z = 40x_1 + 50x_2$
O(0, 0)	0
A(3, 0)	120
B(2, 3)	$40\times2+50\times3 = 80 + 150 = 230$
C(0, 4)	200

The maximum value of Z occurs at (2, 3).
∴ The optimal solution is $x_1 = 2$, $x_2 = 3$ and $Z_{max} = 230$

(v) Given that $3x_1 + 3x_2 \leq 36$
Let $3x_1 + 3x_2 = 36$

x_1	0	12
x_2	12	0

Also given that $5x_1 + 2x_2 \leq 50$
Let $5x_1 + 2x_2 = 50$

x_1	0	10	6
x_2	25	0	10

$x_1 + x_2 = 12$ ……(1)
$5x_1 + 2x_2 = 50$ ……….(2)

(1) × 2 ⇒ $2x_1 + 2x_2 = 24$ ………(3)
(2) – (3) ⇒ $3x_1 = 26$

$x_1 = \frac{26}{3} = 8.66$

put $x_1 = \frac{26}{3}$ substitute in (1)

$x_1 + x_2 = 12$

$x_2 = 12 - x_1$

$x_2 = 26 - \frac{26}{3} = \frac{10}{3} = 3.33$

Also given that $2x_1 + 6x_2 \leq 60$
Let $2x_1 + 6x_2 = 60$

$x_1 + 3x_2 = 30$

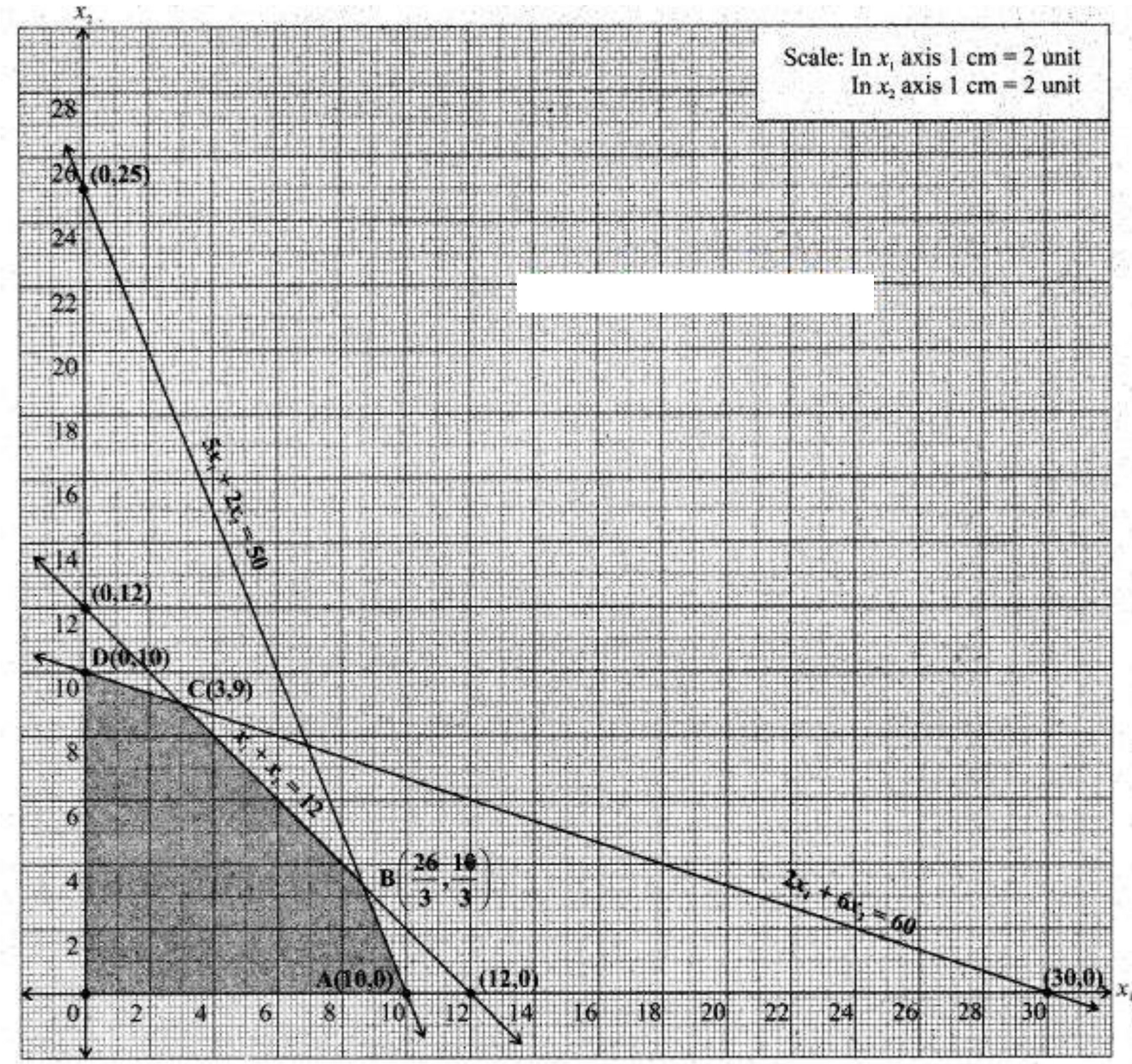
x_1	0	30
x_2	10	0

$x_1 + x_2 = 12$ …….(1)

$x_1 + 3x_2 = 30$ …….(2)

(1) – (2) ⇒ $-2x_2 = -18$
$x_2 = 9$

$x_2 = 9$ substitute in (1) ⇒ $x_1 = 3$



The feasible region satisfying all the given conditions is OABCD.
The co-ordinates of the comer points are O(0, 0), A(10, 0), B(26/3,10/3), and C = (3, 9) and D (0, 10)

Corner points	$Z = 20x_1 + 30x_2$
O(0,0)	0
A(10,0)	200
B$(\frac{26}{3},\frac{10}{3})$	$20 \times \frac{26}{3} + 30 \times \frac{10}{3} = \frac{520+300}{3} = \frac{820}{3} = 273.33$
C(3,9)	$60 + 270 = 330$
D(0,10)	300

The maximum value of Z occurs at C(3, 9)

∴ The optimal solution is $x_1 = 3$, $x_2 = 9$ and $Z_{max} = 330$

(vi) Given that $36x_1 + 6x_2 \geq 108$
Let $36x_1 + 6x_2 = 108$

$6x_1 + x_2 = 18$

x_1	0	3	2
x_2	18	0	6

Also given that $3x_1 + 12x_2 \geq 36$

Let $3x_1 + 12x_2 = 36$

$x_1 + 4x_2 = 12$

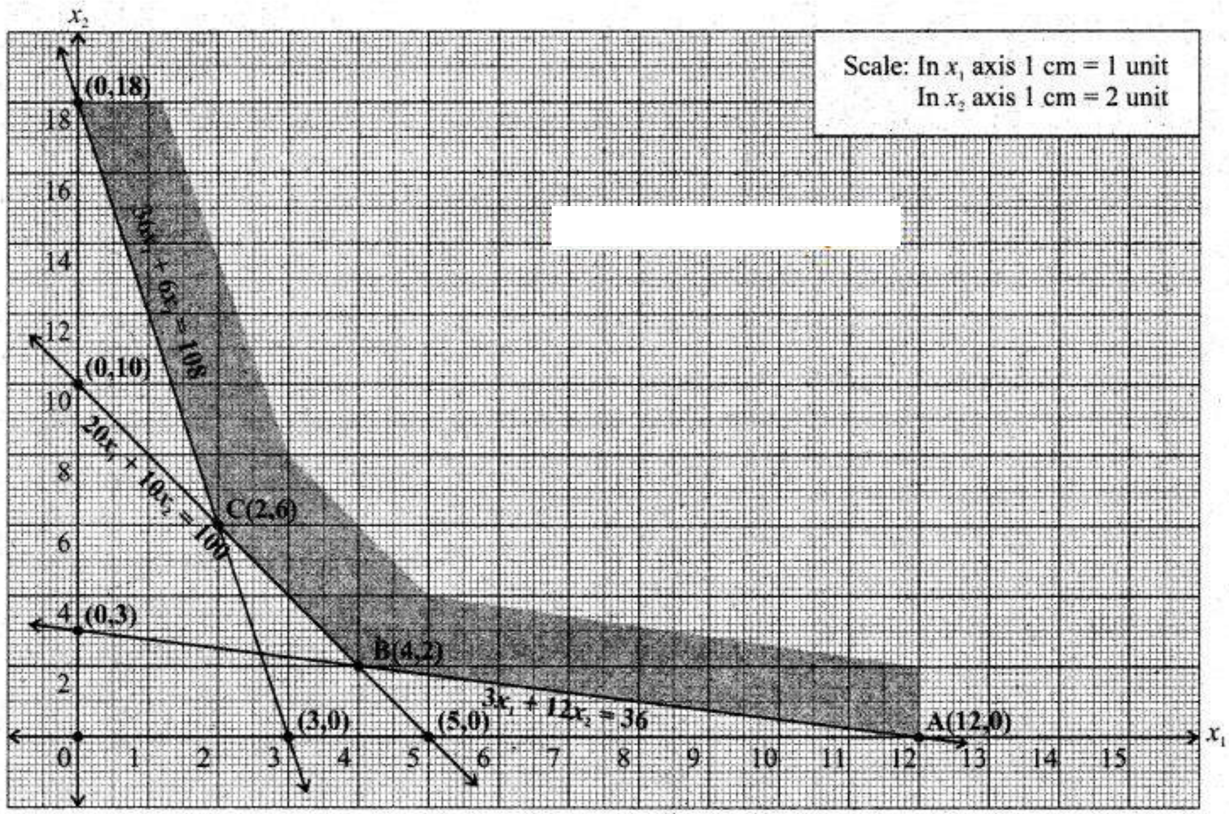
x_1	0	12	4
x_2	3	0	2

Also given that $20x_1 + 10x_2 \geq 100$

Let $20x_1 + 10x_2 = 100$

$2x_1 + x_2 = 10$

x_1	0	5	4
x_2	10	0	2



The feasible region satisfying all the conditions is ABCD.

The co-ordinates of the comer points are A(12, 0), B(4, 2), C(2, 6) and D(0, 18).

Corner points	$Z = 20x_1 + 40x_2$
A(12,0)	240
B(4,2)	80 + 80 = 160
C(2,6)	40 + 240 = 280
D(0,18)	720

The minimum value of Z occurs at B(4, 2)

∴ The optimal solution is $x_1 = 4$, $x_2 = 2$ and $Z_{min} = 160$

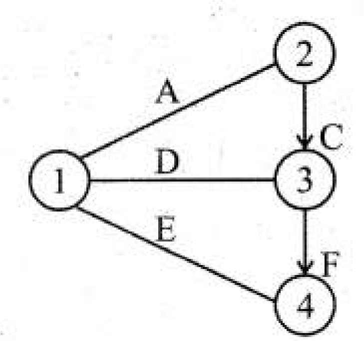
Ex 10.2

Question 1.
Draw the network for the project whose activities with their relationships are given below:
Activities A, D, E can start simultaneously; B, C > A; G, F > D, C; H > E, F.

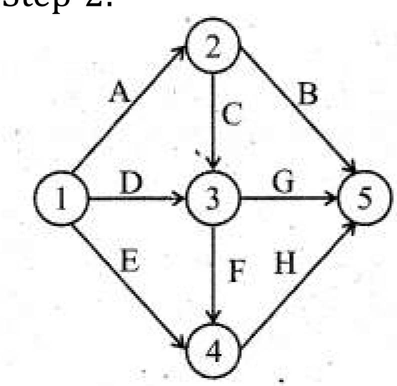
Solution:
Given: (i) A, D, E can start simultaneously.
(iii) A < B, C; C, D < G, F; E, F < H
Working rule:

A < B, C implies activity A is the immediate predecessor of activities B and C. i.e., for activities B and C to occur, activity 'A' has to be completed.

Similarly for activities G, F to occur D and C has to completed for activity H to occur E and F has to be completed.
Step-1:



(∵ A, D and E are independents events)
Step-2:

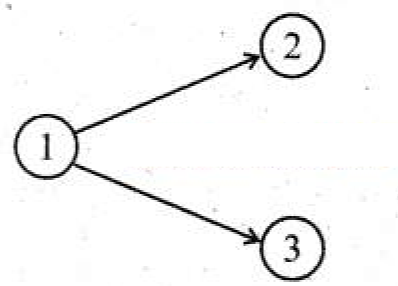


Note: Activities B, G and H are not a part of any activities immediate predecessor. So they have to merge into the lost node 5

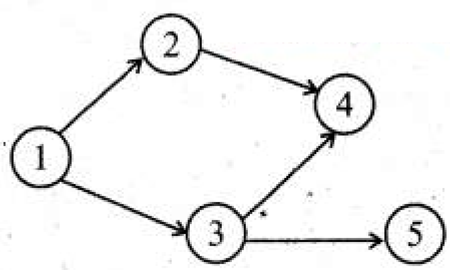
Question 2.
Draw the event-oriented network for the following data:

Events	1	2	3	4	5	6	7
Immediate Predecessors	–	1	1	2,3	3	4,5	5,6

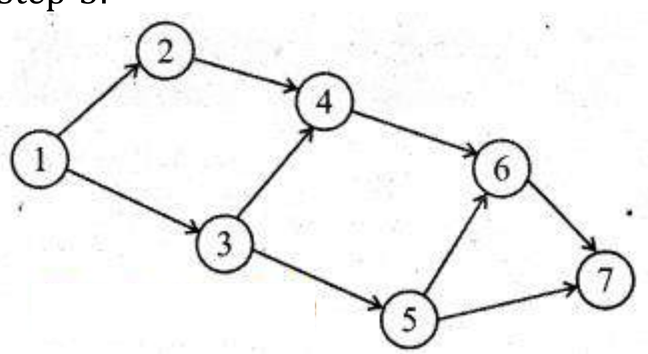
Solution:
Step-1:



Step-2:



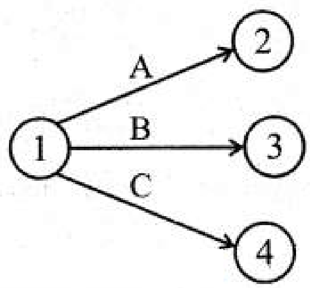
Step-3:



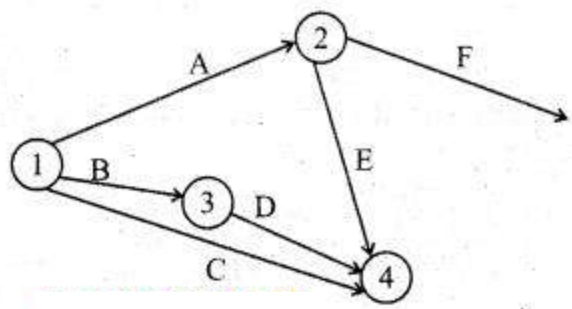
Question 3.
Construct the network for the projects consisting of various activities and their precedence relationships are as given below:
A, B, C can start simultaneously: A < F, E; B < D, C; E, D < G

Solution:

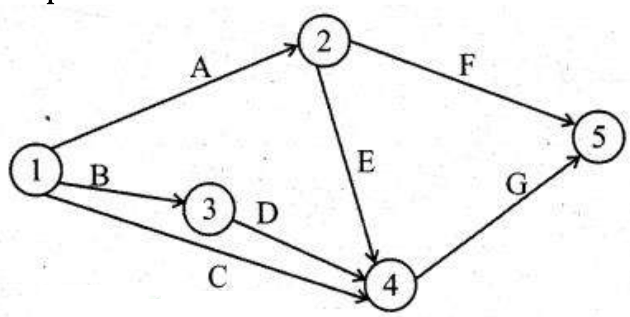
Step-1:



Step-2:



Step-3:



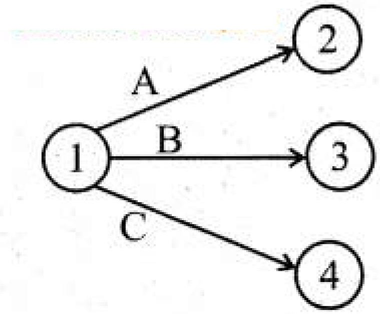
Question 4.

Construct the network for each the projects consisting of various activities and their precedence relationships are as given below:

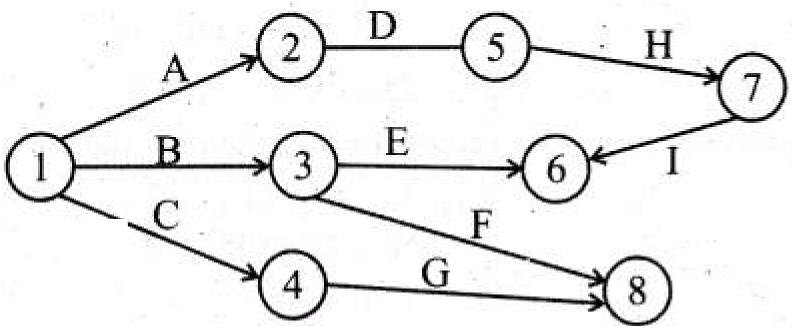
Activity	A	B	C	D	E	F	G	H	I	J	K
Immediate Predecessors	–	–	–	A	B	B	C	D	E	H,I	F,G

Solution:

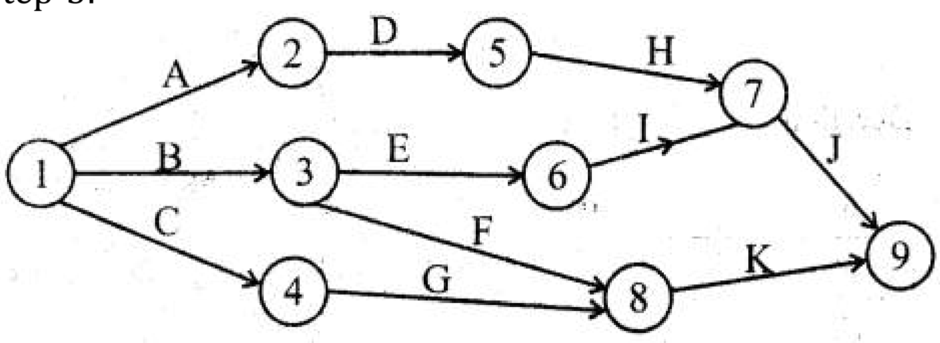
Step-1:



Step-2:



Step-3:



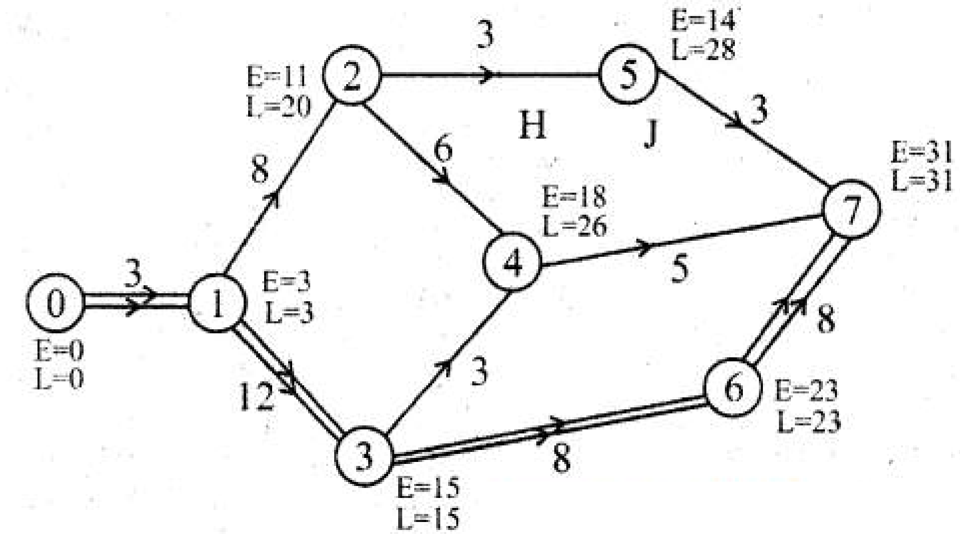
Question 5.

Construct the network for the project whose activities are given below.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (in week)	3	8	12	6	3	3	8	5	3	8

Calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity. Determine the critical path and the project completion time.

Solution:



Forward pass:
$E_1 = 0 + 3 = 3$
$E_2 = E_1 + t_{12} = 8 + 3 = 11$
$E_3 = 3 + 12 = 15$
$E_4 = E_2 + 6$ (or) $E_3 + 3 = 11 + 6$ (or) $15 + 3 = 18$

[We must select a maximum value for forwarding pass]
$E_4 = 15 + 3 = 18$
$E_2 = E_2 + 3 = 11 + 3 = 14$
$E_6 = E_3 + 8 = 15 + 8 = 23$
$E_7 = E_6 + 8 = 23 + 8 = 31$

Backward pass:
$L_7 = 31$
$L_6 = L_7 – 8 = 31 – 8 = 23$
$L_5 = L_7 – 3 = 31 – 3 = 28$
$L_4 = L_7 – 5 = 31 – 5 = 26$
$L_3 = L_6 – 8 = 23 – 8 = 15$
$L_2 = L_5 – 3$ (or) $L_4 – 6 = (28 – 3)$ (or) $(26 – 6) = 25$ (or) 20

[We must select a minimum value for a backward pass and maximum value for forwarding pass]
$L_1 = L_2 – 8$ (or) $L_3 – 12 = 20 – 8$ (or) $15 – 12 = 12$ (or) $3 = 3$
$L_0 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT − t_{ij}	LFT
0-1	3	0	3	3	3
1-2	8	3	11	20 − 8 = 12	20
1-3	12	3	15	15 − 12 = 3	15
2-4	6	11	17	26 − 6 = 20	26
2-5	3	11	14	28 − 3 = 25	28
3-4	3	15	18	26 − 3 = 23	26
3-6	8	15	23	23 − 8 = 15	23
4-7	5	18	23	31 − 5 = 26	31
5-7	3	14	17	31 − 8 = 28	31
6-7	8	23	31	31 − 8 = 23	31

Critical path 0-1-3-6-7 and the duration is 31 works.

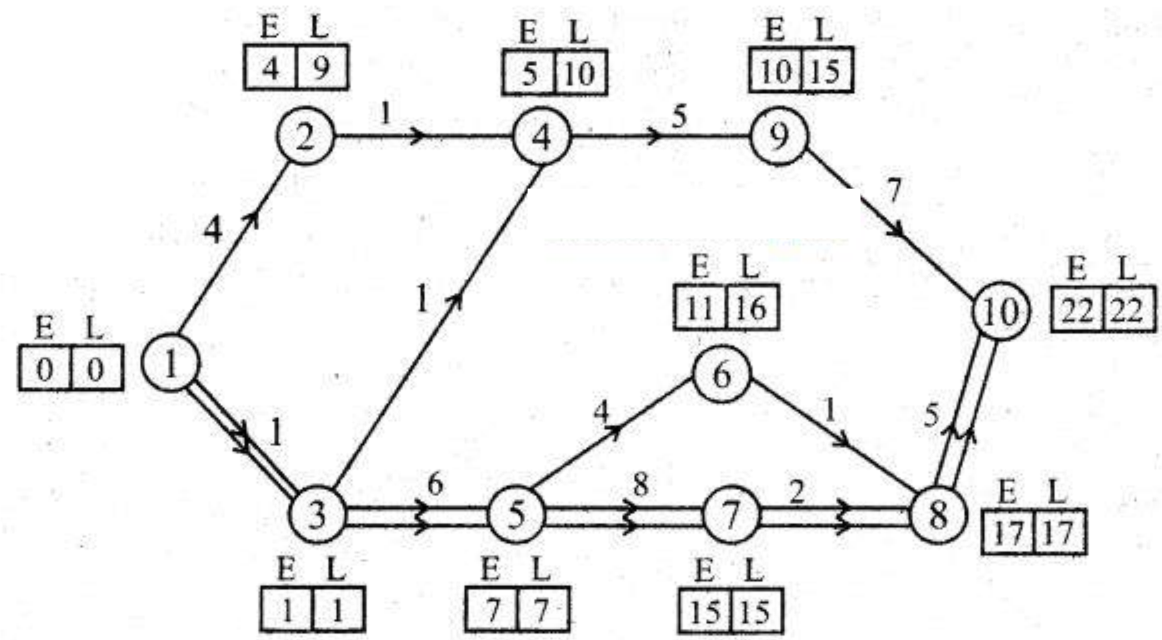
Question 6.

A project schedule has the following characteristics:

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	7

Construct the network and calculate the earliest start time, earliest finish time, latest start time, and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

Solution:



Forward pass:

$E_1 = 0$

$E_2 = 0 + 4 = 4$

$E_3 = 0 + 1 = 1$

$E_4 = (E_2 + 1)$ (or) $(E_3 + 1) = (4 + 1)$ (or) $(1 + 1) = 5$ (or) 2

[whichever is maximum we must select forward pass]

$\therefore E_4 = 5$

$E_5 = 1 + 6 = 7$

$E_6 = 7 + 4 = 11$

$E_7 = 8 + 7 = 15$

$E_8 = (E_7 + 2)$ (or) $(E_6 + 1)$

$= (15 + 2)$ or $(11 + 1)$

$= 17$ or 12 [whichever is maximum]

$\therefore E_8 = 17$

$E_9 = 5 + 5 = 10$

$E_{10} = (E_9 + 7)$ (or) $(E_8 + 5)$

$= (10 + 7)$ (or) $(17 + 5)$

$= 17$ (or) 22 [Which is maximum]

$E_{10} = 22$

Backward pass:

$L_{10} = 22$

$L_9 = 22 - 7 = 15$
$L_8 = 22 - 5 = 17$
$L_7 = 17 - 2 = 15$
$L_6 = 17 - 1 = 16$
$L_5 = (16 - 4)$ (or) $(15 - 8)$ [whichever is minimum]
$L_5 = 7$
$L_4 = 15 - 5 = 10$
$L_3 = (10 - 1)$ (or) $(7 - 6)$ [whichever is minimum]
$L_3 = 1$
$L_2 = 10 - 1 = 9$
$L_1 = 0$

We can also find the critical path by this method, which also helps us to counter check the solution obtained by the table method.

Path	Time
1 - 2 - 4 - 9 - 10	4 + 1 + 5 + 7 = 17
1 - 3 - 4 - 9 - 10	1 + 1 + 5 + 7 = 14
1 - 3 - 5 - 6 - 8 - 10	1 + 6 + 4 + 1 + 5 = 17
1 - 3 - 5 - 7 - 8 - 10	1 + 6 + 8 + 2 + 5 = 22

So critical path is 1-3-5-7-8-10 as it takes 22 units to complete the project.

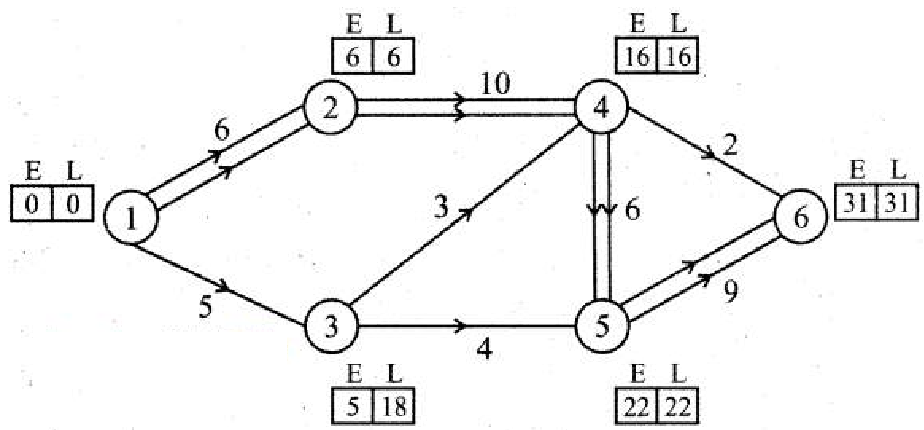
Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT − t_{ij}	LFT
1-2	4	0	4	9 – 4 = 5	9
1-3	1	0	1	1 – 1 = 0	1
2-4	1	4	5	10 – 1 = 9	10
3-4	1	1	2	10 – 1 = 9	10
3-5	6	1	7	7 – 6 = 1	7
4-9	5	5	10	15 – 5 = 10	15
5-6	4	7	11	16 – 4 = 12	16
5-7	8	7	15	15 – 8 = 7	15

6-8	1	11	12	17 – 1 = 16	17
7-8	2	15	17	17 – 2 = 15	17
8-10	5	17	22	22 – 5 = 17	22
9-10	7	16	17	22 – 7 = 15	22

Question 7.
Draw the network and calculate the earliest start time, earliest finish time, latest start time, and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

Jobs	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration	6	5	10	3	4	6	2	9

Solution:



Forward pass:
$E_1 = 0$
$E_2 = 0 + 6 = 6$
$E_3 = 0 + 5 = 5$
$E_4 = 6 + 10 = 16$
$E_5 = (E_4 + 6)$ (or) $(E_3 + 4)$
$E_5 = (16 + 6)$ (or) $(5 + 4)$ [whichever is maximum]
$E_5 = 22$
$E_6 = (E_5 + 9)$(or) $(E_4 + 2)$
$E_6 = (22 + 9)$ (or) $(4 + 2)$ [whichever is maximum]
$E_6 = 31$
Backward pass:
$L_6 = 31$

$L_5 = 31 - 9 = 22$
$L_4 = 22 - 6$ (or) $31 - 2$ [whichever is minimum]
$L_4 = 16$
$L_3 = 22 - 4 = 18$
$L_2 = 16 - 10 = 6$
$L_1 = 6 - 6 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT + t_{ij}	LFT
1-2	6	0	6	$6 - 6 = 0$	6
1-3	5	0	5	$18 - 5 = 13$	18
2-4	10	6	16	$16 - 10 = 6$	16
3-4	3	5	8	$16 - 3 = 13$	16
3-5	4	5	9	$22 - 4 = 18$	22
4-5	6	16	22	$22 - 6 = 16$	22
4-6	2	16	18	$31 - 2 = 29$	31
5-6	9	22	31	$31 - 9 = 22$	31

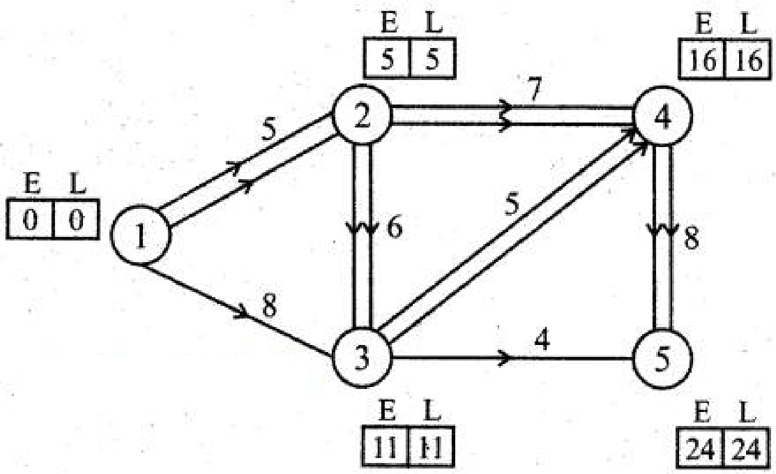
∴ EFT and LFT are the same on 1-2, 2-4, 4-5, 5-6, the vertical path is 1-2-4-5-6 and duration is 31 days to complete.

Question 8.
The following table gives the activities of a project and their duration in days.

Activity	1-2	1-3	2-3	2-4	3-4	3-5	4-5
Duration	5	8	6	7	5	4	8

Construct the network and calculate the earliest start time, earliest finish time, latest start time, and latest finish time of each activity and determine the critical path of the project and duration to complete the project.

Solution:



Forward pass:
$E_1 = 0$
$E_2 = 0 + 5 = 5$
$E_3 = (0 + 8)$ (or) $(5 + 6)$ [whichever is maximum]
$E_3 = 11$
$E_4 = (11 + 5)$ (or) $(5 + 7)$ [whichever is maximum]
$E_4 = 16$
$E_5 = (11 + 4)$ (or) $(16 + 8)$ [whichever is maximum]
$E_5 = 24$

Backward pass:
$L_5 = 24$
$L_4 = 24 - 8 = 16$
$L_3 = (24 - 4)$ (or) $(16 - 5)$ [whichever is minimum]
$L_3 = 11$
$L_2 = (11 - 6)$ (or) $(16 - 7)$ [whichever is minimum]
$L_2 = 5$
$L_1 = (5 - 5)$ (or) $(11 - 8) = 0$ [whichever is minimum]

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT – t_{ij}	LFT
1-2	5	0	5	5 – 5 = 0	5
1-3	8	0	8	11 – 8 = 3	11
2-3	6	5	11	11 – 6 = 5	11

2-4	7	5	12	16 – 7 = 9	16
3-4	5	11	16	16 – 5 = 11	16
3-5	4	11	15	24 – 4 = 20	24
4-5	8	16	24	24 – 8 = 16	24

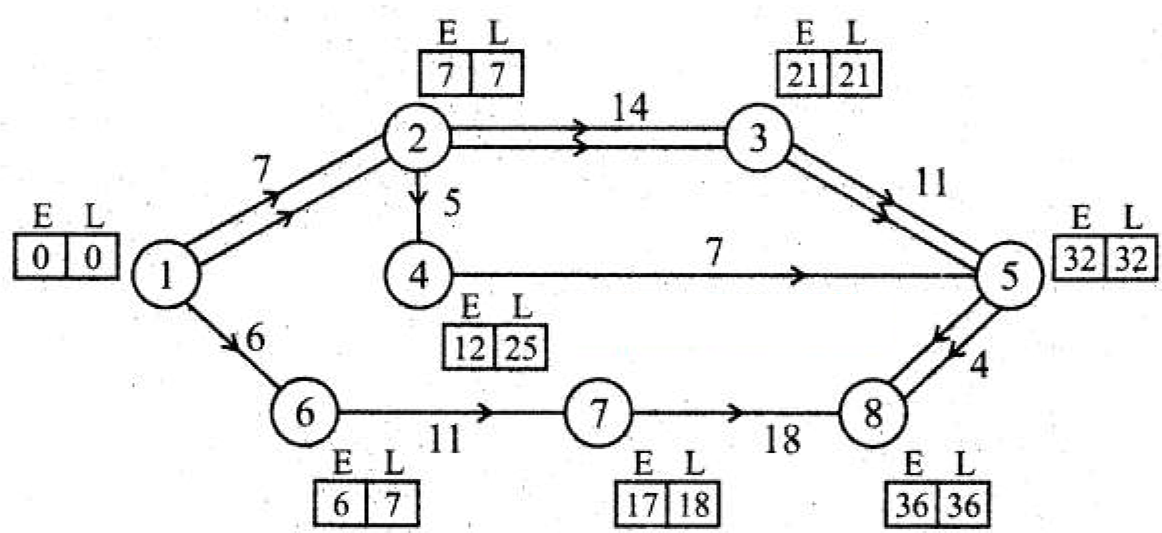
EFT and LFT are the same on 1-2, 2-3, 3-4, and 4-5
So critical path is 1-2-3-4-5 and the time taken is 24 days.

Question 9.
A project has the following time schedule

Activity	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
Duration (in days)	7	6	14	5	11	7	11	4	18

Construct the network and calculate the earliest start time, earliest finish time, latest start time, and latest finish time of each activity and determine the critical path of the project and duration to complete the project.

Solution:



Forward pass:
$E_1 = 0$
$E_2 = 0 + 7 = 7$
$E_3 = 7 + 14 = 21$
$E_4 = 7 + 5 = 12$

$E_5 = (21 + 11)$ (or) $(2 + 7) = 32$ [whichever is maximum]
$E_6 = 0 + 6 = 6$
$E_7 = 6 + 11 = 17$
$E_8 = 32 + 4 = 36$

Backward pass:
$L_8 = 36$
$L_7 = 36 - 18 = 18$
$L_6 = 18 - 11 = 7$
$L_5 = 35 - 4 = 31$
$L_4 = 32 - 7 = 25$
$L_3 = 32 - 11 = 21$
$L_2 = (21 - 14)$ (or) $(25 - 5)$ [whichever is minimum]
$L_2 = 7$
$L_1 = 7 - 7 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT − t_{ij}	LFT
1-2	7	0	7	7 − 7 = 0	7
1-6	6	0	6	7 − 6 = 1	7
2-3	14	7	21	21 − 14 = 7	21
2-4	5	7	12	25 − 5 = 20	25
3-5	11	21	32	32 − 11 = 21	32
4-5	7	12	19	32 − 25 = 7	14
6-7	11	6	17	18 − 11 = 7	18
5-8	4	32	36	36 − 4 = 32	36
7-8	18	17	36	36 − 18 = 18	36

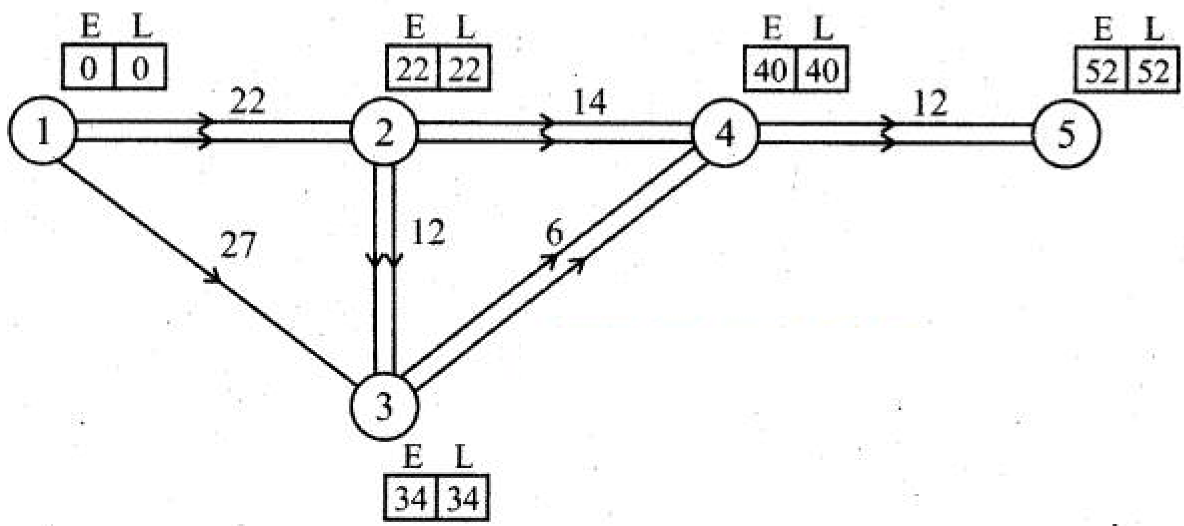
EFT and LFT are same is 1-2, 2-3, 3-5, 5-8.
Hence the critical path is 1-2-3-5-8 and the time to complete is 36 days.

Question 10.
The following table uses the activities in construction projects and relevant information.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration (in days)	22	27	12	14	6	12

Draw the network for the project, calculate the earliest start time, earliest finish time, latest start time, and latest finish time of each activity, and find the critical path. Compute the project duration.

Solution:



Forward pass:
$E_1 = 0$
$E_2 = 0 + 22 = 22$
$E_3 = (0 + 27)$ (or) $(22 + 12) = 34$ [Maximum of both]
$E_4 = (22 + 14)$ (or) $(34 + 16) = 40$
$E_5 = 40 + 12 = 52$

Backward pass:
$L_5 = 32$
$L_4 = 52 – 12 = 40$
$L_3 = 40 – 6 = 34$
$L_2 = (40 – 14)$ (or) $(34 – 12) = 22$ [Minimum of both]
$L_1 = 22 – 22 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT – t_{ij}	LFT
1-2	22	0	22	22 – 22 = 0	22
1-3	27	0	27	34 – 27 = 7	39
2-3	12	22	34	34 – 12 = 22	34
2-4	14	22	36	40 – 14 = 26	40
3-4	6	39	40	40 – 6 = 39	40
4-5	12	40	52	52 – 12 = 40	52

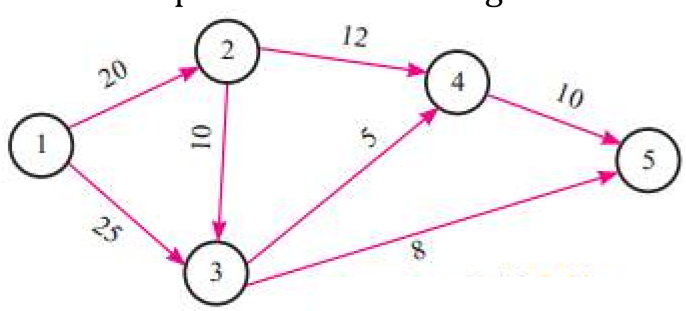
EFT and LFT are the same for 1-2, 2-3, 3-4, and 4-5.

So critical path is 1-2-3-4-5 time to complete is 52 days.

Ex 10.3

Choose the correct answer.

Question 1.
The critical path of the following network is:



(a) 1-2-4-5
(b) 1-3-5
(c) 1-2-3-5
(d) 1-2-3-4-5

Answer:
(d) 1-2-3-4-5
Hint:
1-2-4-5 ⇒ EFT = 20 + 12 + 10 = 42
1-3-5 ⇒ EFT = 25 + 8 = 33

1-2-3-5 ⇒ EFT = 20 + 10 + 8 = 38
1-2-3-4-5 ⇒ EFT = 20 + 10 + 5 + 10 = 45

Question 2.
Maximize: $z = 3x_1 + 4x_2$ subject to $2x_1 + x_2 \leq 40$, $2x_1 + 5x_2 \leq 180$, $x_1, x_2 \geq 0$. In the LPP, which one of the following is feasible comer point?
(a) $x_1 = 18, x_2 = 24$
(b) $x_1 = 15, x_2 = 30$
(c) $x_1 = 2.5, x_2 = 35$
(d) $x_1 = 20.5, x_2 = 19$

Answer:
(c) $x_1 = 2.5, x_2 = 35$
Hint:
$z = 3x_1 + 4x_2$
Let us solve the equations
$2x_1 + x_2 = 40$ ………(1)
$2x_1 + 5x_2 = 180$ ……….(2)
(1) – (2) ⇒ $-4x_2 = -140$
$x_2 = 35$
We have $2x_1 + x_2 = 40$
$2x_1 + 35 = 40$
$2x_1 = 5$
$x_1 = 2.5$

Question 3.
One of the conditions for the activity (i, j) to lie on the critical path is:
(a) $E_j - E_i = L_j - L_i = t_{ij}$
(b) $E_i - E_j = L_j - L_i = t_{ij}$
(c) $E_j - E_i = L_i - L_j = t_{ij}$
(d) $E_j - E_i = L_j - L_i \neq t_{ij}$

Answer:
(a) $E_j - E_i = L_j - L_i = t_{ij}$

Question 4.
In constructing the network which one of the following statement is false?

(a) Each activity is represented by one and only one arrow, (i.e) only one

activity can connect any two nodes.
(b) Two activities can be identified by the same head and tail events.
(c) Nodes are numbered to identify an activity uniquely. Tail node (starting point) should be lower than the head node (end point) of an activity.
(d) Arrows should not cross each other.

Answer:
(b) Two activities can be identified by the same head and tail events.

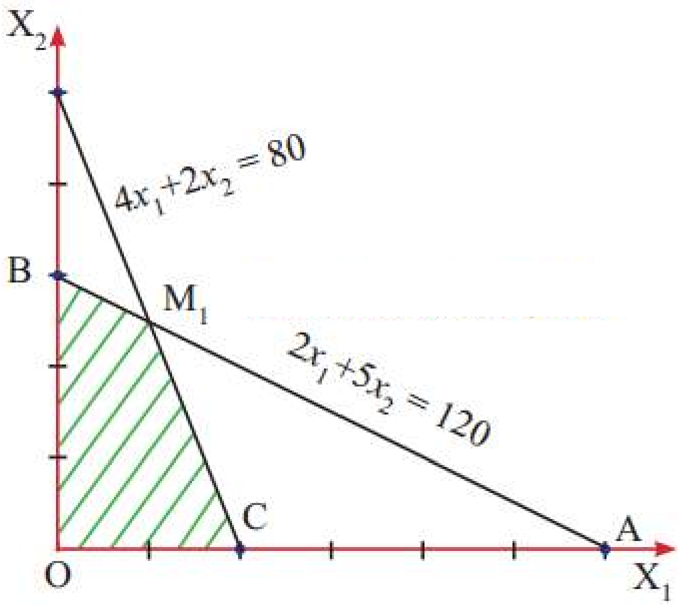
Question 5.
In a network while numbering the events which one of the following statement is false?
(a) Event numbers should be unique.
(b) Event numbering should be carried out on a sequential basis from left to right.
(c) The initial event is numbered 0 or 1.
(d) The head of an arrow should always bear a number lesser than the one assigned at the tail of the arrow.

Answer:
(d) The head of an arrow should always bear a number lesser than the one assigned at the tail of the arrow.

Question 6.
A solution which maximizes or minimizes the given LPP is called:
(a) a solution
(b) a feasible solution
(c) an optimal solution
(d) none of these

Answer:
(a) a solution

Question 7.
In the given graph the coordinates of M_1 are



(a) $x_1 = 5, x_2 = 30$
(b) $x_1 = 20, x_2 = 16$
(c) $x_1 = 10, x_2 = 20$
(d) $x_1 = 20, x_2 = 30$

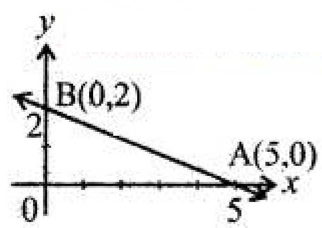
Answer:
(c) $x_1 = 10, x_2 = 20$
Hint:
$4x_1 + 2x_2 = 80$ (or) $2x_1 + x_2 = 40$
$2x_1 + x_2 = 40$ ……(1)
$2x_1 + 5x_2 = 120$ ……(2)
(1) – (2) $\Rightarrow -4x_2 = -80$
$x_2 = 20$
But, $2x_1 + x_2 = 40$
$2x_1 + 20 = 20$
$x_1 = 10$

Question 8.
The maximum value of the objective function $Z = 3x + 5y$ subject to the constraints $x > 0, y > 0$ and $2x + 5y \leq 10$ is:
(a) 6
(b) 15
(c) 25
(d) 31

Answer:
(b) 15
Hint:
Let $2x + 5y = 10$

Corner point	$Z = 3x + 5y$
O(0, 0)	0
A(5,0)	15
B(0,2)	6

x	0	5
y	2	0



∴ Maximum Value = 15

Question 9.
The minimum value of the objective function $Z = x + 3y$ subject to the constraints $2x + y \leq 20$, $x + 2y \leq 20$, $x > 0$ and $y > 0$ is:
(a) 10
(b) 20
(c) 0
(d) 5

Answer:
(c) 0
Hint:
O(0, 0) is a comer point.
So $Z = 0 + 3(0) = 0$
∴ Minimum value is 0

Question 10.
Which of the following is not correct?
(a) Objective that we aim to maximize or minimize
(b) Constraints that we need to specify

(c) Decision variables that we need to determine
(d) Decision variables are to be unrestricted

Answer:
(d) Decision variables are to be unrestricted

Question 11.
In the context of network, which of the following is not correct?
(a) A network is a graphical representation.
(b) A project network cannot have multiple initial and final nodes
(c) An arrow diagram is essentially a closed network
(d) An arrow representing an activity may not have a length and shape

Answer:
(d) An arrow representing an activity may not have a length and shape

Question 12.
The objective of network analysis is to:
(a) Minimize total project cost
(b) Minimize total project duration
(c) Minimize production delays, interruption and conflicts
(d) All the above

Answer:
(b) Minimize total project duration

Question 13.
Network problems have advantage in terms of project:
(a) Scheduling
(b) Planning
(c) Controlling
(d) All the above

Answer:
(d) All the above

Question 14.
In critical path analysis, the word CPM mean:
(a) Critical path method
(b) Crash project management

(c) Critical project management
(d) Critical path management

Answer:
(a) Critical path method

Question 15.
Given an L.P.P maximize $Z = 2x_1 + 3x_2$ subject to the constrains $x_1 + x_2 \leq 1$, $5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$ using graphical method, we observe:
(a) No feasible solution
(b) unique optimum solution
(c) multiple optimum solution
(d) none of these

Answer:
(a) No feasible solution