Mathematics Sample Paper

Max. Marks: 80 **Duration:3 hours**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:

1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:

1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

PART-A

Section-I

1. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then what is the value of a?

OR

Which term of an AP: 21, 42, 63, 84, … is 210?

2. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

A. 0 B. 4

C. 8 D. 0, 8

3. When a die is thrown, what is the probability of getting an odd number less than 3?

4. What is the area of the largest triangle that can be inscribed in a semi – circle of radius r unit?

OR

Find the length of tangent drawn to a circle with radius 8 cm from a point 17 cm away from the center of the circle.

5. Which of the following cannot be the probability of an event?

A. $\frac{1}{3}$ B. 0.1

C. 3 D. $\frac{17}{16}$

6. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

A. 2 B. – 2

C. $\frac{1}{4}$ D. $\frac{1}{2}$

7. The number of polynomials having zeroes as – 2 and 5 is/are:

A. 1 B. 2

C. 3 D. more than 3

8. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

OR

Check whether 6^n can end with the digit 0 for any natural numbers n.

9. If $\cos A = \frac{4}{5}$, then the value of tan A is

A. $\frac{3}{5}$ B. $\frac{3}{4}$

C. $\frac{4}{3}$ D. $\frac{5}{3}$

10. The distance of the point P (2, 3) from the X-axis is

A. 2 B. 3

C. 1 D. 5

11. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha + \beta)$ is equals to ________ .

12. The distance between the points A (0, 6) and B(0, −2) is _________.

13. The value of (tan 1°.tan 2°.tan 3° …… tan 89°) is ____________ .

OR

If A, B, C, are the interior angles of a triangle ABC, prove that

$$\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$$

14. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has ________ roots.

OR

The pair of linear equations 2x + 4y = 3 and 12y + 6x = 6 has/have ________ solutions/s.

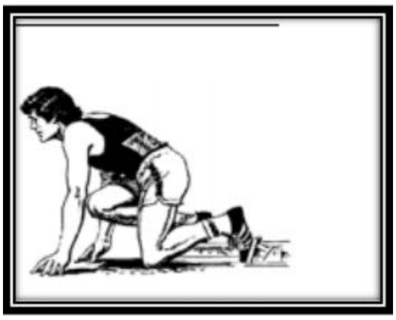
15. If ΔABC ~ ΔPQR with $\frac{BC}{QR} = \frac{1}{3}, then \frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to ______.

16. Given that HCF (306, 657) = 9, find LCM (360, 657).

Section-II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark

17. 100m RACE A stopwatch was used to find the time that it took a group of students to run 100 m.



Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

a. Estimate the mean time taken by a student to finish the race.

(i) 54

(ii) 63

(iii) 43

(iv) 50

b. What will be the upper limit of the modal class?

(i) 20

(ii) 40

(iii) 60

(iv) 80

c. The construction of cumulative frequency table is useful in determining the

(i) Mean

(ii) Median

(iii) Mode

(iv) All of the above

d. The sum of lower limits of median class and modal class is

(i) 60

(ii) 100

(iii) 80

(iv) 140

e. How many students finished the race within 1 minute?

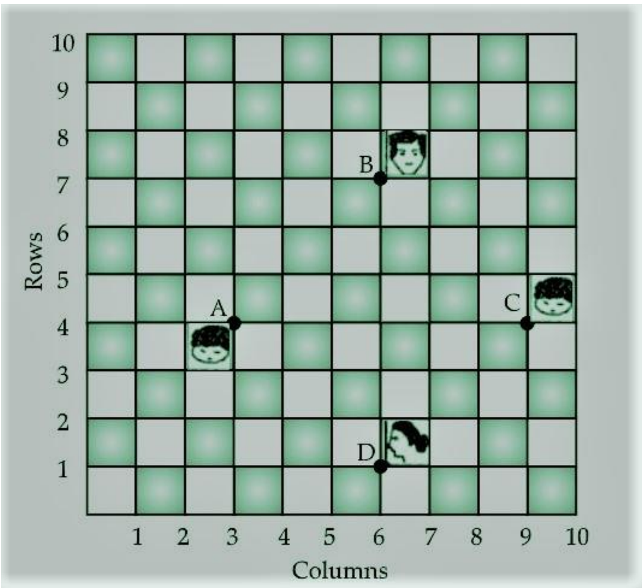
(i) 18

(ii) 37

(iii) 31

(iv) 8

18. In a room, 4 friends are seated at the points A, B, C and D as shown in figure. Reeta and Meeta walk into the room and after observing for a few minutes Reeta asks Meeta.



(a) What is the position of A?

(i) (4, 3)

(ii) (3, 3)

(iii) (3, 4)

(iv) None of these

(b) What is the middle position of B and C?

(i) $\left(\frac{15}{2}, \frac{11}{2}\right)$

(ii) $\left(\frac{2}{15}, \frac{11}{2}\right)$

(iii) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(iv) None of these

(c) What is the position of D?

(i) (6, 0)

(ii) (0, 6)

(iii) (6, 1)

(iv) (1, 6)

(d) What is the distance between A and B?

(i) $3\sqrt{2}$

(ii) $2\sqrt{3}$

(iii) $2\sqrt{2}$

(iv) $3\sqrt{3}$

(e) What is the equation of line CD?

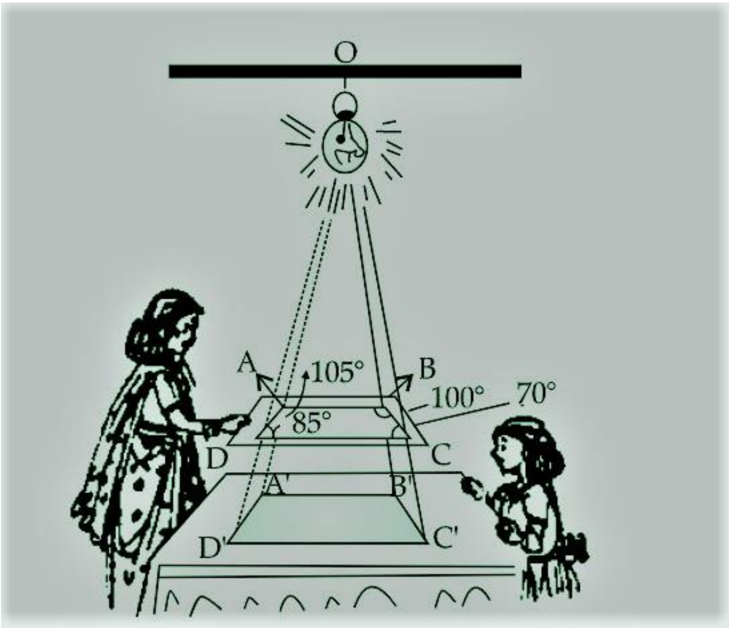
(i) x - y - 5 = 0

(ii) x + y - 5 = 0

(iii) x + y + 5 = 0

(iv) x - y + 5 = 0

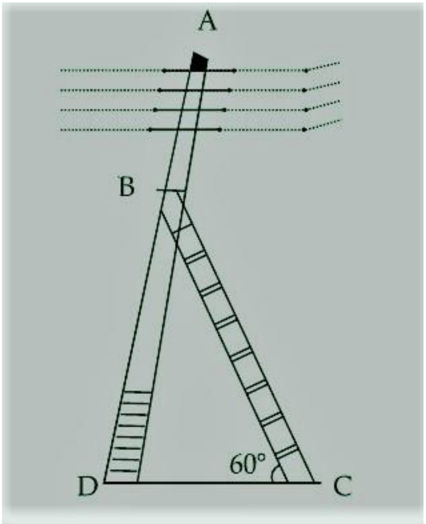
19. Seema placed a lightbulb at point O on the ceiling and directly below it placed a table. Now, she put a cardboard of shape ABCD between table and lighted bulb. Then a shadow of ABCD is casted on the table as A'B'C'D' (see figure). Quadrilateral A'B'C'D' in an enlargement of ABCD with scale factor 1 : 2, Also, AB = 1.5 cm, BC = 25 cm, CD = 2.4 cm and AD = 2.1 cm; $\angle A = 105°$, $\angle B = 100°$, $\angle C = 70°$ and $\angle D = 85°$.



(a) What is the measurement of angle A'?
(i) 105°
(ii) 100°
(iii) 70°
(iv) 80°
(b) What is the length of A'B'?
(i) 1.5 cm
(ii) 3 cm
(iii) 5 cm
(iv) 2.5 cm
(c) What is the sum of angles of quadrilateral A'B'C'D'?
(i) 180°
(ii) 360°
(iii) 270°
(iv) None of these
(d) What is the ratio of sides A'B' and A'D'?
(i) 5: 7
(ii) 7: 5

(iii) 1: 1
(iv) 1 : 2
(e) What is the sum of angles of C’ and D’?
(i) 105°
(ii) 100°
(iii) 155°
(iv) 140°

20. An electrician has to repair an electric fault on the pole of height 5 m. She needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure)



(a) What is the length of BD?

(i) 1.3 m

(ii) 5 m

(iii) 3.7 m

(iv) None of these

(b) What should be the length of Ladder, when inclined at an angle of 60° to the horizontal?

(i) 7.4 m

(ii) $\frac{3.7}{\sqrt{3}}$ m

(iii) 3.7 m

(iv) $\frac{7.4}{\sqrt{3}}$ m

(c) How far from the foot of pole should she place the foot of the ladder? (i) 3.7

(ii) 2.14

(iii) $\frac{1}{\sqrt{3}}$

(iv) None of these

(d) If the horizontal angle is changed to 30°, then what should be the length of the ladder?

(i) 7.4 m

(ii) 3.7 m

(iii) 1.3 m

(iv) 5 m

(e) What is the value of ∠B?

(i) 60°

(ii) 90°

(iii) 30°

(iv) 180°

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

21. What is the radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm?

22. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. What is the probability that the selected ticket has a number which is a multiple of 5?

OR

A coin is tossed two times. Find the probability of getting atmost one head.

23. Calculate the value of the expression

$$\left(\frac{\sin^2 22^o+\sin^2 68^o}{\cos^2 22^o+\cos^2 68^o}+\sin^2 63^o+\cos 63^o \sin 27^o\right)$$

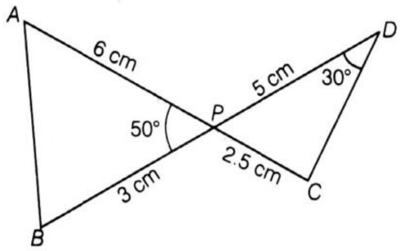
OR

If $\sin\theta - \cos\theta = 0$, then calculate the value of $(\sin^4\theta + \cos^4\theta)$.

24. Calculate the roots of the quadratic equation $x^2 - 3\sqrt{5}x + 10 = 0$

25. For the pair of equations $\lambda x + 3y + 7 = 0$ and $2x + 6y + 14 = 0$. What is the value of λ if the given pair of equations have infinitely many solutions?

26. In figure, two line segments AC and BD intersect each other at the point P such that PA = 6cm, PB = 3cm, PC = 2.5cm, PD = 5cm, ∠APB = 50° and ∠CDP = 30°.then, ∠PBA is equal to



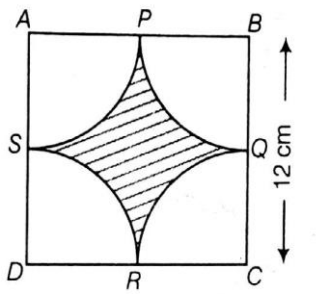
Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

27. Prove that $5\sqrt{2}$ is irrational.

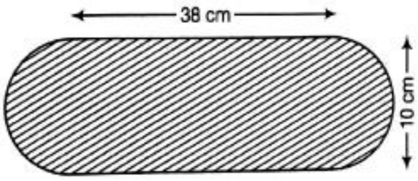
28. Find the area of the shaded region in figure, where arcs drawn with centers A, B, C and D intersect in pairs at mid – point P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD.

(Use π = 3.14)



OR

Find the area of the flower bed (with semi – circular ends) shown in figure. (Use π = 3.14)

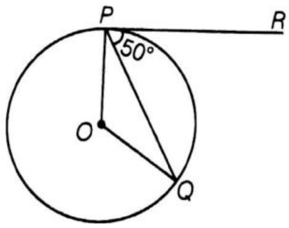


29. Calculate the fourth vertex D of a parallelogram ABCD whose three vertices are A(– 2, 3), B(6, 7) and C(8, 3).
30. Construct a tangent to a circle of radius 4cm from a point which is at a distance of 6cm from its center.

OR

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

31. If angle between two tangents drawn from a point P to a circle of radius a and center O is 90°, then prove that OP = a √2.
32. In the given figure, if O is the center of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then find the measure of ∠POQ.



33. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\text{cosec }\theta$

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

34. The angle of elevation of the tower from certain point is 30^0. If the observer moves 20 m towards the tower, the angle of elevation of the top increase by 15^0. Find the height of the tower.
35. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

OR

How many shots each having diameter of 3 cm can be made from a cuboidal lead solid of dimensions $9\text{ cm} \times 11\text{ cm} \times 12\text{ cm}$?

36. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years):	5-15	15-25	25-35	35-45	45-55	55-65
No. of students:	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

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Hints & Solutions

PART-A

Section-I

1. Solution: As we know, nth term of an AP is

   $a_n = a + (n - 1)d$

   where a = first term

   a_n is nth term

   d is the common difference

   $4 = a + (7 - 1)(- 4)$

   $4 = a - 24$

   $a = 24 + 4 = 28$

   OR

   Solution: Let nth term of the given AP be 210.

   Here, first term, a = 21

   and common difference, $d = 42 - 21 = 21$ and $a_n = 210$

   As we know, nth term of an AP is

   $a_n = a + (n - 1)\ d$

   $210 = 21 + (n - 1)21$

   $189 = (n - 1)21$

   $n - 1 = 9$

   $n = 10$

   So, the 10th term of an AP is 210.

2. Solution: If a quadratic equation has two equal roots, then its discriminant value will be equal to zero i.e.,

   $D = b^2 - 4ac = 0$

   Given, $2x^2 - kx + k = 0$

   For equal roots,

   $D = b^2 - 4ac = 0$

   $\Rightarrow (- k)^2 - 4(2)(k) = 0$

   $\Rightarrow k^2 - 8k = 0$

   $\Rightarrow k\ (k - 8) = 0$

   $\therefore k = 0, 8$

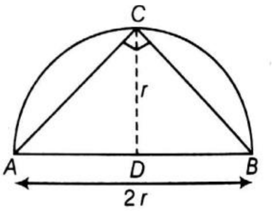
3. Solution: When a die is thrown, then total number of outcomes = 6

Odd number less than 3 is 1 only.

Number of possible outcomes = 1

∴ Required probability = $\frac{1}{6}$

4. Solution: Let ABC be the triangle circumscribed by a triangle of radius r.



Clearly, ∠C = 90° (angle in a semicircle)

So, ΔABC is right angled triangle with base as diameter AB of the circle and height be CD.

Height of the triangle = r

∴ Area of largest ΔABC = $\frac{1}{2}$ × Base × Height

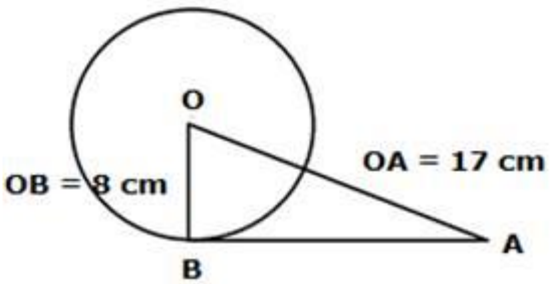
= $\frac{1}{2}$ × AB × CD

= $\frac{1}{2}$ × 2r × r = r^2 sq. units

OR

Solution: Let us consider a circle with center O and radius 8 cm.

The diagram is given as:



Consider a point A 17 cm away from the center such that OA = 17 cm

A tangent is drawn at point A on the circle from point B such that OB = radius = 8 cm

To Find: Length of tangent AB = ?

As seen OB ⊥ AB

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled ΔAOB, By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OA)^2 = (OB)^2 + (AB)^2$

$(17)^2 = (8)^2 + (AB)^2$

$289 = 64 + (AB)^2$

$(AB)^2 = 225$

AB = 15 cm

∴ The length of the tangent is 15 cm.

5. Solution: Since, probability of an event always lies between 0 and 1.
Probability of any event cannot be more than 1 as $\frac{17}{16}$ which is greater than 1.

6. Solution: If $\frac{1}{2}$ is a root of the equation
$x^2 + kx - \frac{5}{4} = 0$ then, substituting the value of $\frac{1}{2}$ in place of x should give us the value of k.
Given, $x^2 + kx - \frac{5}{4} = 0$ where, $x = \frac{1}{2}$
$\Rightarrow \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$
$\Rightarrow \frac{k}{2} = \frac{5}{4} - \frac{1}{4}$
∴ k = 2

7. Solution: Let − 2 and 5 are the zeroes of the polynomials of the form $p(x) = ax^2 + bx + c$.
The equation of a quadratic polynomial is given by x^2 − (sum of the zeroes) x + (product of the zeroes) where,
Sum of the zeroes = − 2 + 5 = 3
product of the zeroes = (− 2)5 = − 10
∴ The equation is $x^2 - 3x - 10$
We know, the zeroes do not change if the polynomial is divided or multiplied by a constant
Therefore, $kx^2 - 3kx - 10k$ will also have − 2 and 5 as their zeroes.
As, k can take any real value, there can be many polynomials having − 2 and 5 as their zeroes.

8. Solution: By definition,

A composite number is a positive integer that has a factor other than 1 and itself. Now considering your numbers,

7×11×13 + 13 may be written as, i.e. 13 * (78). So other than 1 and the number itself, 13 and 78 are also the factors of the number. Further, 78 = 39 x 2. So, 39 and 2 are also its factors. So this number is definitely not prime. Hence its composite number.

Similarly, 7×6×5×4×3×2×1 + 5 can be written as , i.e. 5 * (1009). So, other than the number and 1, it have 5 and 1009 as its factors too. So it is also a composite number.

OR

Solution: If any number ends with the digit 0, it should be divisible by 10 or in other words its prime factorization must include primes 2 and 5 both as 10 = 2 ×5

Prime factorization of $6^n = (2 \times 3)^n$

In the above equation it is observed that 5 is not in the prime factorization of 6^n

By Fundamental Theorem of Arithmetic Prime factorization of a number is unique. So 5 is not a prime factor of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

9. Solution: Given: cos A $= \frac{4}{5}$ ...eq. 1

We know that tan A $= \frac{\sin A}{\cos A}$

We have value of cos A, we need to find value of sin A

Also, we know that, $\sin A = \sqrt{1 - \cos^2 A}$ ...eq. 2

Thus,

Substituting eq. 1 in eq. 2, we get

Sin A = $\sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Therefore, $\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$

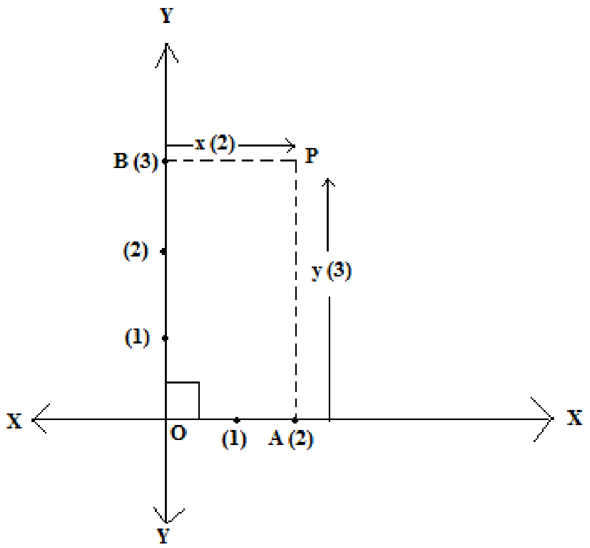
10. Solution: We know that,

(x, y) is any point on the Cartesian plane in first quadrant.

Then,

x = Perpendicular distance from Y-axis and

y = Perpendicular distance from X-axis



So, the distance of the point P (2, 3) from the X-axis = 3

11. Solution: Given: $\cos(\alpha + \beta) = 0$

We can write, $\cos(\alpha + \beta) = \cos 90°$ ($\because \cos 90° = 0$)

By comparing cosine equation on either sides,

We get $(\alpha + \beta) = 90°$

$\Rightarrow \sin(\alpha + \beta) = 1$

12. Solution: By using the distance formula:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Let's calculate the distance between the points (x_1, y_1) and (x_2, y_2)

We have

$x_1 = 0, x_2 = 0$

$y_1 = 6, y_2 = -2$

$d^2 = (0 - 0)^2 + (-2 - 6)^2$

$d = \sqrt{(0)^2 + (-8)^2}$

$d = \sqrt{64}$

d = 8 units

So, the distance between A (0, 6) and B (0, 2) = 8

13. Solution: tan 1°. tan 2°.tan 3° …… tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.tan 45°.tan 46°.tan 47°…tan 87°.tan 88°.tan 89°

= tan1°.tan 2°.tan 3°…tan 43°.tan 44°.1.tan 46°.tan 47°…tan 87°.tan 88°.tan 89° ($\because \tan 45° = 1$)

= tan1°.tan 2°.tan 3°...tan 43°.tan 44°.1.tan(90° – 44°).tan(90° – 43°)...tan(90° – 3°). tan(90° – 2°).tan(90° – 1°)

= tan1°.tan 2°.tan 3°...tan 43°.tan 44°.1.cot 44°.cot 43°...cot 3°.cot 2°.cot 1° (∵ tan(90° – θ)=cot θ)

= tan1°.tan 2°.tan 3°...tan 43°. tan44°.1.$\frac{1}{\tan 44°}.\frac{1}{\tan 43°}\dots\frac{1}{\tan 3°}.\frac{1}{\tan 2°}.\frac{1}{\tan 1°}$

(∵ tan θ = $\frac{1}{\cot\theta}$)

= $(\tan 1° \times \frac{1}{\tan 1°}).(\tan 2° \times \frac{1}{\tan 2°})\dots(\tan 44° \times \frac{1}{\tan 44°})$

= 1

OR

Solution: Since, A, B, C, are the interior angles of a triangle ABC.

Therefore,

$$A + B + C = 180°$$
$$\Rightarrow \quad A + C = 180° - B$$
$$\Rightarrow \quad \frac{A+C}{2} = \frac{180° - B}{2}$$
$$\Rightarrow \quad \tan\left(\frac{A+C}{2}\right) = \tan\left(90° - \frac{B}{2}\right)$$
$$\Rightarrow \quad \tan\left(\frac{A+C}{2}\right) = \cot\left(\frac{B}{2}\right)$$

Hence proved.

14. Solution: The discriminant value of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

D = b^2 – 4ac = 0

Given, $2x^2 - \sqrt{5}x + 1 = 0$

∴ D = b^2 – 4ac

⇒ D = $(-\sqrt{5})^2 - 4(2)(1)$

⇒ D = – 3

Here, D < 0

Hence, the roots of the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ are imaginary.

OR

Solution:

Given pair of equations are,

2x + 4y – 3 = 0 and 6x + 12y – 6 = 0

Here, $a_1 = 2$, $b_1 = 4$, $c_1 = -3$

And $a_2 = 6$, $b_2 = 12$, $c_2 = -6$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution.

15. Solution: Given: In $\Delta ABC \sim \Delta PQR$ and

$$\frac{BC}{QR} = \frac{1}{3}$$

By area property of similar triangles, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

$\Rightarrow \frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \frac{(QR)^2}{(BC)^2}$

$\Rightarrow \frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

16. Solution: We know that LCM × HCF = Product of the numbers

Therefore LCM = $\frac{\textit{Product of the numbers}}{\textit{HCF of the numbers}} = \frac{306\times657}{9}$ = 22338

Section-II

17. a. Answer: C

b. Answer: B

c. Answer: B

d. Answer: C

e. Answer: C

18. (a) Answer: (3,4)

(b) Answer: $\left(\frac{15}{2}, \frac{11}{2}\right)$

(c) Answer: (6, 1)

(d) Answer: $2\sqrt{3}$

(e) Answer: x - y - 5 = 0

19. (a) Answer: 105°

(b) Answer: 3 cm
(c) Answer: 360°
(d) Answer: 5:7
(e) Answer: 155°

20. (a) Answer: 3.7 m
(b) Answer: $\frac{7.4}{\sqrt{3}}$ m
(c) Answer: None of these
(d) Answer: 7.4 m
(e) Answer: 30°

Part –B

21. Solution: Diameter of first circle = d_1 = 36 cm

Diameter of second circle = d_2 = 20 cm

∴ Circumference of first circle = πd_1 = 36π cm

Circumference of second circle = πd_2 = 20π cm

Now, we are given that,

Circumference of circle = Circumference of first circle + Circumference of second circle

$\pi D = \pi d_1 + \pi d_2$

$\Rightarrow \pi D = 36\pi + 20\pi$

$\Rightarrow \pi D = 56\pi$

$\Rightarrow D = 56$

$\Rightarrow$ Radius $= \frac{56}{2} = 28$cm

22. Solution: Number of total outcomes = 40

Multiples of 5 between 1 to 40 = 5, 10, 15, 20, 25, 30, 35, 40

∴ Total number of possible outcomes = 8

∴ Required probability $= \frac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \frac{8}{40} = \frac{1}{5}$

OR

Solution: The possible outcomes, if a coin is tossed 2 times is

S = {(HH), (TT), (HT), (TH)}

Total outcome = 4

Let E = Event of getting at – most one head = {(TT), (HT), (TH)}

Favourable outcome = 3

Hence, required probability $= \frac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$

23. Solution: $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$\Rightarrow \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin(90° - 63°)$

$\Rightarrow \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \cos 63°$

($\because$cos(90° – θ) = sin θ and sin(90° – θ) = cos θ)

$\Rightarrow \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos^2 63°$

$\Rightarrow \frac{1}{1} + 1 = 2$

(Since, $\frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} = 1$ as by identity, $\sin^2 θ + \cos^2 θ = 1$

So, $\sin^2 22° + \cos^2 22° = 1$ and $\sin^2 63° + \cos^2 63° = 1$)

$\therefore \frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27° = 2$

OR

Solution: sin θ – cos θ = 0

$\Rightarrow \sin θ = \cos θ$

$\Rightarrow \frac{\sin θ}{\cos θ} = 1 \Rightarrow \tan θ = 1$

$(\because \tan θ = \frac{\sin θ}{\cos θ})$

And we know, tan 45° = 1

So, tan θ = 1 = tan 45°

By comparing above equation, we get θ = 45°

Thus, $\sin^4 θ + \cos^4 θ = \sin^4 45° + \cos^4 45° = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

24. Solution: Given, $x^2 - 3\sqrt{5}x + 10 = 0$

By using quadratic formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-(-3\sqrt{5}) \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)}$

$= \frac{3\sqrt{5} \pm \sqrt{5}}{2} = 2\sqrt{5}, \sqrt{5}$

25. Solution: The given pair of linear equations

$\lambda x + 3y + 7 = 0$ and 2x + 6y + 14 = 0.

Here, $a_1 = \lambda$, $b_1 = 3$, $c_1 = 7$

And $a_2 = 2$, $b_2 = 6$, $c_2 = +14$

$\frac{a_1}{a_2} = \frac{\lambda}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

For the pair of equations having infinitely many solutions.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Taking $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\frac{\lambda}{2} = \frac{1}{2}$

$\lambda = 1$

26. Solution: In ΔAPB and ΔCPD,

$\angle APB = \angle CPD = 50°$ (vertically opposite angles)

$\frac{AP}{PD} = \frac{6}{5}$ ...(i)

Also, $\frac{BP}{CP} = \frac{3}{2.5}$

Or $\frac{BP}{CP} = \frac{6}{5}$ ...(ii)

From equations (i) and(ii)

$\frac{AP}{PD} = \frac{BP}{CP}$

$\therefore$ ΔAPB ~ ΔDPC [by SAS similarity criterion]

$\therefore \angle A = \angle D = 30°$ [corresponding angles of similar triangles]

In ΔAPB,

$\angle BAP + \angle PBA + \angle APB = 180°$ [Sum of angles of a triangle = 180°]

$\Rightarrow 30° + \angle PBA + 50° = 180°$

$\therefore \angle PBA = 180° - (50° + 30°)$

$\angle PBA = 180 - 80° = 100°$

$\angle PBA = 100°$

Part –B

27. Solution: Let us assume that $5\sqrt{2}$ is a rational number and can be written in the form of $\frac{a}{b}$, where a and b are co – prime.

Therefore, $5\sqrt{2} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{a}{5b}$

Here, $\frac{a}{5b}$ on the right side is a rational number.

This implies that $\sqrt{2}$ is also a rational number but this contradicts the fact that $\sqrt{2}$ is an irrational number.

This contradiction has arisen because of the wrong assumption that we have made in the beginning.

Hence, $5\sqrt{2}$ is an irrational number.

28. Solution: Since P, Q, R and S are the mid points of AB, BC, CD and DA.

$\therefore$ AP = PB = BQ = QC = CR = RD = DS = SA = 6 cm.

Given, side of a square BC = 12 cm

Area of the square = 12 × 12 = 144 cm^2

Area of the shaded region = Area of the square − (Area of the four quadrants)

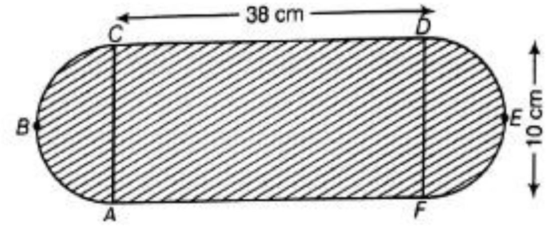
Area of four quadrants = $4 \times \frac{\pi}{4} \times r^2 = \pi r^2 = 3.14 \times (6)^2 = 113.04$ cm^2

Area of the shaded region = $144 - 113.04 = 30.96$ cm^2

OR

Solution: Length and breadth of the rectangular portion AFDC of the flower bed are 38 cm and 10 cm respectively.

Area of the flower bed = Area of the rectangular portion + Area of the two semi – circles.



∴ Area of rectangle AFDC = Length × Breadth

$= 38 \times 10 = 380$ cm^2

Both ends of flower bed are semi – circle in shape.

∴ Diameter of the semi – circle = Breadth of the rectangle AFDC = 10 cm

∴ Radius of the semi-circle = 10/2 = 5 cm

Area of the semi – circle = $\pi r^2/2 = 25\pi/2$ cm^2

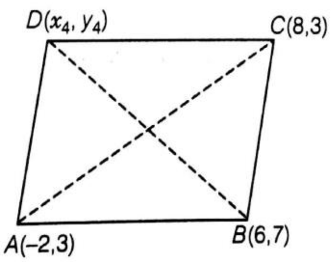
Since there are two semi – circles in the flower bed,

∴ Area of two semi – circles = $2 \times 25\pi/2 = 25 \times 3.14 = 78.5$ cm^2

Total area of flower bed = $380 + 78.5 = 458.5$ cm^2

29. Solution: Given a parallelogram ABCD whose three vertices are

A (− 2, 3), B (6, 7) and C (8, 3)



Let the fourth vertex of parallelogram, D = (x, y) and L, M be the mid points of AC and BD, respectively.

We know that diagonals of a parallelogram bisects each other.

Therefore, mid – point of AC = mid – point of BD

Coordinate of L = Coordinate of M

$$\left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = \left(\frac{6+x}{2}, \frac{7+y}{2}\right)$$

$$(3, 3) = \left(\frac{6+x}{2}, \frac{7+y}{2}\right)$$

Equating the coordinates of both sides.

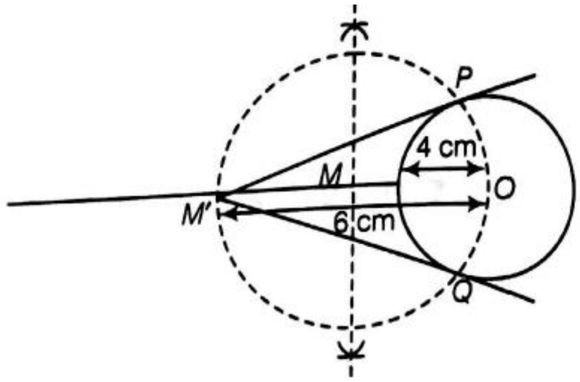
$3 = \frac{6+x}{2}$ and $3 = \frac{7+y}{2}$

⇒ 6 + x = 6 and 7 + y = 6

⇒ x = 0 and y = – 1

Hence, the fourth vertex of parallelogram is D = (0, – 1)

30. Solution:

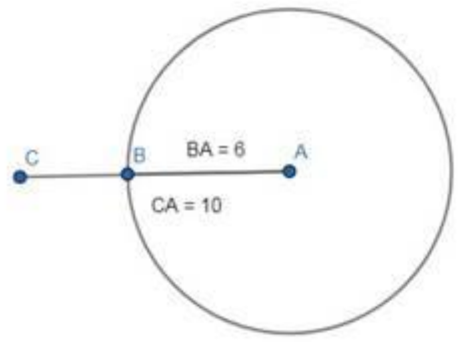


Steps of construction

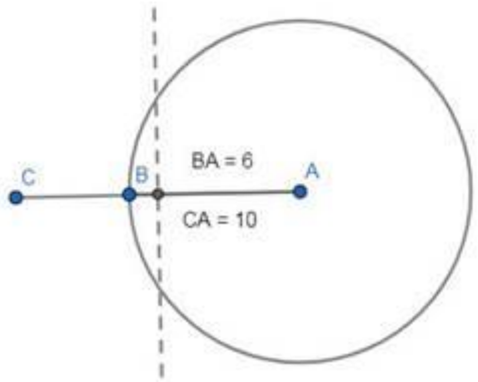
1. Draw a circle of radius 4 cm.
2. Join OM′ and bisect it. Let M be mid – point of OM′.
3. Taking M as center and MO as radius draw a circle to intersect circle (0, 4) at two points P and Q.
4. Join PM′ and QM′. PM′ and QM′ are the required tangents from M′ to circle C (0, 4).

OR

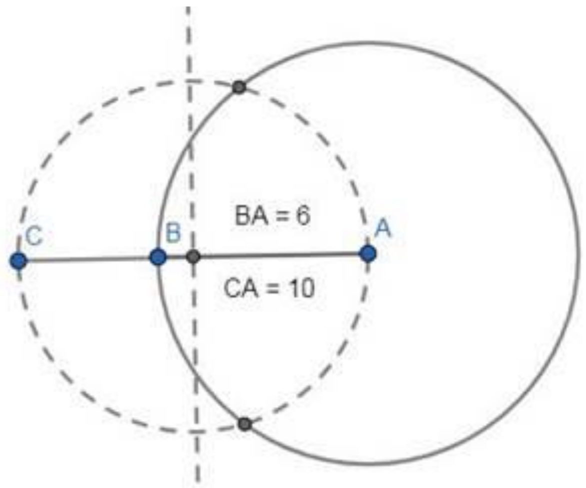
Solution: Step1: Draw circle of radius 6cm with center A, mark point C at 10 cm from the center.



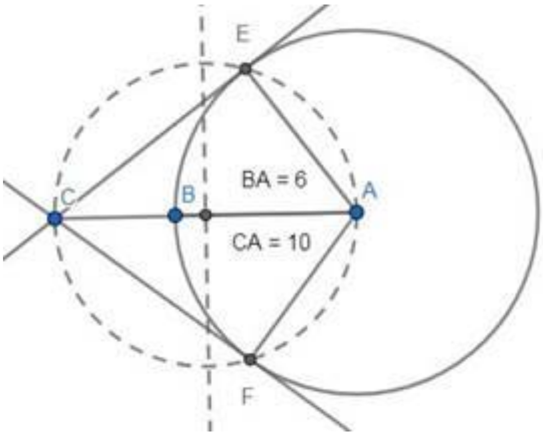
Step 2: find perpendicular bisector of AC



Step3: Take this point as center and draw a circle through A and C



Step4:Mark the point where this circle intersects our circle and draw tangents through C



Length of tangents = 8cm

AE is perpendicular to CE (tangent and radius relation)

In ΔACE

AC becomes hypotenuse

$AC^2 = CE^2 + AE^2$

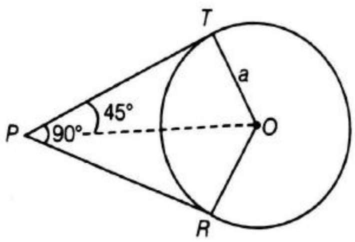
$10^2 = CE^2 + 6^2$

$CE^2 = 100-36$

$CE^2 = 64$

CE = 8cm

31. Solution:



Let us consider a circle with center O and tangents PT and PR and angle between them is 90° and radius of circle is a

To show : $OP = \sqrt{2}a$

Proof :

In △OTP and △ORP

TO = OR [radii of same circle]

OP = OP [common]

TP = PR [tangents through an external point to a circle are equal]

$\triangle OTP \cong \triangle ORP$ [By Side Side Side Criterion]

$\angle TPO = \angle OPR$ [By CPCT] [1]

Now, $\angle TPR = 90°$ [Given]

$\angle TPO + \angle OPR = 90°$

$\angle TPO + \angle TPO = 90°$ [Using 1]

$\angle TP0 = 45°$

Now, OT $\perp$ TP [As tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OTP = 90°$

So $\triangle POT$ is a right – angled triangle

And we know that,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So,

$$\sin\angle TPO = \frac{OT}{OP} = \frac{a}{OP}$$ [As OT is radius and equal to a]

$$\sin 45° = \frac{a}{OP}$$

$$\frac{1}{\sqrt{2}} = \frac{a}{OP}$$

$\Rightarrow OP = a\sqrt{2}$

Hence, Proved.

32. Solution: Given: OP is a radius and PR is a tangent in a circle with center O with $\angle RPQ = 50°$

To find: $\angle POQ$

Solution: Now, OP $\perp$ PR [As tangent to at any point on the circle is perpendicular to the radius through point of contact]

$\angle OPR = 90°$

$\angle OPQ + \angle RPQ = 90°$

$\angle OPQ + 50° = 90°$

$\angle OPQ = 40°$

In $\triangle POQ$

OP = OQ [radii of same circle]

$\angle OQP = \angle OPQ = 40°$ [angles opposite to equal sides are equal] [1]

In $\triangle OPQ$ By angle sum property of a triangle

$\angle OPQ + \angle OPQ + \angle POQ = 180°$ [Using 1]

$40° + 40° + \angle POQ = 180°$

$\angle POQ = 100°$

33. Solution: $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

Taking L.C.M of the denominators,

$\Rightarrow \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta).\sin\theta}$

$\Rightarrow \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta).\sin\theta}$ $[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \frac{1+1+2\cos\theta}{(1+\cos\theta).\sin\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \frac{2+2\cos\theta}{(1+\cos\theta).\sin\theta}$

$\Rightarrow \frac{2(1+\cos\theta)}{(1+\cos\theta).\sin\theta}$

$\Rightarrow \frac{2}{\sin\theta} = 2\ \text{cosec}\ \theta$ = RHS $[\because \frac{1}{\sin\theta} = \text{cosec}\ \theta]$

Hence proved.

Part –B

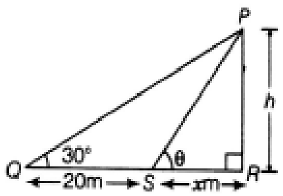
34. Solution: Let PR = h meter, be the height of the tower.

The observer is standing at point Q such that, the distance between the observer and tower is QR = (20 + x) m, where

QR = QS + SR = 20 + x

∠PQR = 30°

∠ PSR = θ



In ΔPQR,

$\tan 30^\circ = \frac{h}{20+x}$ $[\because, \tan\theta = \frac{\text{perpendicular}}{\text{base}}]$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$ $[\because, \tan 30^\circ = \frac{1}{\sqrt{3}}]$

Rearranging the terms,

We get $20 + x = \sqrt{3}\ h$

$\Rightarrow x = \sqrt{3}h - 20$ ...eq. 1

In ΔPSR,

$\tan\theta = \frac{h}{x}$

Since, angle of elevation increases by 15° when the observer moves 20 m towards the tower. We have,

θ = 30° + 15° = 45°

So,

$\tan 45° = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow h = x$

Substituting x=h in eq. 1, we get

$h = \sqrt{3}\,h - 20$

$\Rightarrow \sqrt{3}\,h - h = 20$

$\Rightarrow h(\sqrt{3} - 1) = 20$

$\Rightarrow h = \frac{20}{\sqrt{3}-1}$

Rationalizing the denominator, we have

$\Rightarrow h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{20(\sqrt{3}+1)}{3-1}$

$= \frac{20(\sqrt{3}+1)}{2}$

$= 10(\sqrt{3} + 1)$

Hence, the required height of the tower is $10(\sqrt{3} + 1)$ meter.

35. Solution: We know that,

Volume of cube = a^3,

where a = side of cube

Now,

Side of first cube, $a_1 = 3$ cm

Side of second cube, $a_2 = 4$ cm

Side of third cube, $a_3 = 5$ cm

Now, Let the side of cube recast from melting these cubes is 'a'.

As the volume remains same,

Volume of recast cube = (volume of 1st + 2nd + 3rd cube)

$\Rightarrow a^3 = a_1^3 + a_2^3 + a_3^3$

$\Rightarrow a^3 = (3)^3 + (4)^3 + (5)^3$

$\Rightarrow a^3 = 27 + 64 + 125 = 216$

$\Rightarrow a = 6$ cm

So, side of cube so formed is 6 cm.

OR

Solution: Volume of cuboid = lbh

For cuboidal lead:

Length, l = 9 cm

Breadth, b = 11 cm

Height, h = 12 cm

Volume of lead = 9(11)(12) = 1188 cm^3

Volume of sphere = $\frac{4}{3}\pi r^3$

where r = radius of sphere

For spherical shots,

Diameter = 3 cm

Radius, r = 1.5 cm

Volume of one shot $= \frac{4}{3} \times \frac{22}{7} \times (1.5)^3 = \frac{99}{7}\ \mathrm{cm}^3$

Now,

$$\text{No. of shots can be made} = \frac{\text{Volume of lead}}{\text{Volume of one shot}} = \frac{1188}{\frac{99}{7}} = \frac{1188 \times 7}{99} = 84$$

So, 84 bullets can be made from lead.

36. Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Solution:

We may compute class marks (x_i) as per the relation

$$X_i = \frac{upperclass\ limit + lowerclass\ limit}{2}$$

Now, let assumed mean (A) = 30

Age(in years)	No. of patients(f_i)	Class marks (x_i)	d_i=x_i-30	f_id_i
5-15	6	10	-20	-120
15-25	11	20	-10	-110
25-35	21	30	0	0
35-45	23	40	10	230
45-55	14	50	20	180
55-65	5	60	30	150
Total	80			430

$\Sigma f_i=80$, $\Sigma f_i d_i=430$

Mean=$A + \frac{\Sigma fidi}{\Sigma fi}$

$= 30 + \frac{430}{80} = 30+ 5.375$

=35.38

It represents that on an average the age of patients admitted was 35.38 years. As we can observe that the maximum class frequency 23 belonging to class interval 35-45.

So, modal class= 35-45

Lower limit (l) of modal class =35

Frequency (f_1) of the modal class=23

h=10,

Frequency (f_0)of class preceding the modal class=21

Frequency (f_2) of class succeeding the modal class =14

Now, $Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right) h$

$= 35 + \left(\frac{23-21}{2(23)-21-14}\right) 10$

=35 + 1.81 =36.8years

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