CBSE Class 11 Mathematics
Sample Papers 04 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Write the interval [-20, 3) in set-builder form.

OR

Write the subsets of R as intervals: $\{x : x \in R, 3 \leq x \leq 4\}$ Also, find the length of interval.

2. In which octant does the given point (3, - 2, -5) lie.
3. Prove the identities: $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 = \tan^2 x + \cot^2 x + 7$.

OR

Prove that $\frac{\sin(180^\circ+\theta)\cos(90^\circ+\theta)\tan(270^\circ-\theta)\cot(360^\circ-\theta)}{\sin(360^\circ-\theta)\cos(360^\circ+\theta)\operatorname{cosec}(-\theta)\sin(270^\circ+\theta)} = 1$

4. Simplify: (1 - i) - (-3 + 6i)
5. Find the number of ways in which a committee of 2 teachers and 3 students can be formed out of 10 teachers and 20 students. In how many of these committees a particular teacher is included?

OR

If ${}^{10}C_x = {}^{10}C_{x+4}$, find the value of x.

6. Find the 12^{th} term from the end of the arithmetic progressions: 1, 4, 7, 10, ..., 88.
7. Show that the line joining the points (2, -5) and (-2, 5) is perpendicular to the line joining the points (6, 3) and (1, 1).

OR

Reduce the equation into slope-intercept form and find the slope and the y-intercept.
y = 0

8. For the following parabolas find the coordinates of the foci, the equations of the directrices and the lengths of the latus-rectum: $y^2 = -12x$.
9. If A = {x : x ∈ N}, B = {x : x ∈ N and x is even}, C = {x : x ∈ N and x is odd} and D = { x : x ∈ N and x is prime} then find: C ∩ D

OR

Show that ϕ, {0} and 0 are all different.

10. An experiment consists of recording boy-girt composition of families with 2 children. What is the sample space if we are interested in knowing whether it is a boy or girl in the

order of their births?

11. Name the octant in (2, -5, -7) point lies.
12. In how many ways can 6 persons stand in a queue?
13. Convert the product into the sum or difference of sines and cosines: 2 sin 5x cos x
14. Evaluate: cos (-870°)
15. Solve: $2(3 - x) \geq \frac{x}{5} + 4$
16. State whether the given set is finite or infinite: A = set of all triangles in a plane.

Section - II

17. **Read the Case study given below and attempt any 4 subparts:**

On the roof of Monesh's house, a water tank of capacity 9000 litres is installed. A water pump fills the tank, the pump uses water from the municipality water supply, In the beginning, the water flow of the pump remains 100 litres/hour for the first hour.

The water flow from the pump is 1.25th after each 1 hour.

Once Monesh's mother was not at home and told him to switch off the pump when the tank is almost full.

He calculated that after how many hours should he stop the pump so water does not get overflow in the next one hour.

Now answer the following questions:

i. After how many hours Monesh should stop the pump so that in the next hour the tank does not get overflow?

a. 15 hours

b. 14 hours
c. 16 hours
d. 13 hours

ii. After 10 hours how much water was filled in the tank?
a. 3000 Liters
b. 3300 Liters
c. 3200 Liters
d. 3325.29 Liters

iii. In 7th hour how much water was filled in the tank?
a. 400 Liters
b. 381.47 Liters
c. 375.25 Liters
d. 450 Liters

iv. What was the water flow in 5th hour?
a. 244.14 Liters/hr
b. 250 Liters/hr
c. 300 Liters/hr
d. 400 Liters/hr

v. After 15 hours how much water would have filled in the tank?
a. 11235.32 Liters
b. 10025.47 Liters
c. 10968.68 Liters
d. 11968.68 Liters

18. **Read the Case study given below and attempt any 4 sub parts:**

A restaurant offers 5 choices of appetizer, 10 choices of the main meal, and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all the three courses. Assuming all choices are available.

Using the above information answer the following questions:

i. If the customer eats a 1-course meal, the number of ways of doing this is:

   a. 200
   b. 110
   c. 19
   d. 120

ii. If the customer has a 2-course meal, the number of ways of doing this is:

   a. 38
   b. 110
   c. 200
   d. 329

iii. If the customer has a 3-course meal, the number of combinations is:

   a. 200
   b. 57

c. 110

d. 300

iv. How many different possible meals do the restaurant offer i.e.The number of possible meals is:

a. 329

b. 310

c. 200

d. 300

v. A person who eats an appetizer and the main meal has:

a. 50 choices.

b. 60 choices

c. 20 choices

d. 40 choices

Part - B Section - III

19. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee. Find how many drink coffee? How many drink coffee but not tea?

20. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(a, b): a, b $\in$A, b is exactly divisible by a}.

i. Write R in roster form

ii. Find the domain of R

iii. Find the range of R.

OR

Let A = {x $\in$ W : x < 2}, B $\{x \in N : 1 < x \leq 4\}$ and C = {3, 5}, Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hint: A = {0, 1}, B = {2, 3, 4} and C = {3, 5}

21. Express $\left[\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right]-\left(-\frac{4}{3}+i\right)$ in the form of a + ib.

22. For any two complex numbers z_1 and z_2, prove that: $|z_1+z_2| \geq |z_1|-|z_2|$.

23. Solve $\sqrt{2}\,x^2 + x + \sqrt{2} = 0$.

OR

If z = -5 + 3i, find the value of $(z^4 + 9z^3 + 26z^2 - 14z + 8)$

24. If $\frac{5}{14}$ is the probability of occurrence of an event, find
    i. the odds in favour of its occurrence.
    ii. the odds against its occurrence.
25. Evaluate: $\lim\limits_{x\to\infty} \frac{5x-6}{\sqrt{4x^2+9}}$.
26. What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29th February?
27. Calculate mean deviation about mean from the following data:

x_i	3	9	17	23	27
f_i	8	10	12	9	5

28. Prove that: $\cos x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 7x \sin 8x$.

OR

Solve the following equation $\tan\theta + \tan 2\theta + \sqrt{3} \tan\theta \times \tan 2\theta = \sqrt{3}$

Section - IV

29. If $\overline{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, x_3, \ldots, x_n$, then prove that the mean and variance of the observations $ax_1, ax_2, \ldots, ax_n$ are $a\overline{x}$ and $a^2\sigma^2$, respectively (where, $a \neq 0$).
30. Let f = {(1, 1), (2, 3), (0, - 1), - 1, - 3)} be a function from Z to Z defined by f (x) = ax + b for some integers a, b. Determine a,b.
31. Let a, b, c be respectively the p^{th}, q^{th}, r^{th} terms of an AP. Prove that a(q-r) + b (r-p) + c (p-q) = 0

OR

Sum the following series to n terms: 4 + 6 + 9 + 13 + 18 + ...

32. Find the equation of the ellipse whose foci are at $(\pm 1, 0)$ and e = $\frac{1}{2}$.
33. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$. find r.
34. Find the distance between the points L(a $\cos\alpha$, a $\sin\alpha$) and M(a $\cos\beta$, a $\sin\beta$).

OR

Find the distance of the point (2, 3) from the line 2x - 3y + 9 = 0 measured along a line making an angle of 45° with the x-axis.

35. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 4 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only?

Section - V

36. i. Find the derivative of $\frac{\sin x+\cos x}{\sin x-\cos x}$.

ii. Let $f(x)=\begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\leq x<3 \end{cases}$, find quadratic equation whose roots are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$.

OR

Differentiate: $\frac{e^x \sin x}{\sec x}$

37. For any four sets A, B, C, D, prove that $(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$.

OR

Draw the graph of the modulus function, defined by f : R ⇒ R : f(x) = Ixl =

$\begin{cases} x, \text{ when } x\geq 0 \\ -x, \text{ when } x<0 \end{cases}$

38. Solve the following system of linear inequalities

$\frac{4x}{3}-\frac{9}{4}$ < x + $\frac{3}{4}$ and $\frac{7x-1}{3}$ - $\frac{7x+2}{6}$ > x.

OR

Represent to the solution set of the inequations graphically in two-dimensional plane: 0 $\geq$ 2x - 5y +10.

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Solution

Part - A Section - I

1. Therefore, [-20, 3) = $\{x : x \in R \text{ and } -20 \leq x < 3\}$

OR

Therefore, $\{x : x \in R, 3 \leq x \leq 4\}$ = [3, 4]. Length = 4 - 3 = 1

2. Point(3, -2, -5) lies in octant VIII

3. LHS = $(\sin x + \text{cosec } x)^2 + (\cos x + \sec x)^2$

$= \sin^2 x + \text{cosec}^2 x + 2 \sin x \text{ cosec } x + \cos^2 x +^2 \sec x + 2 \cos x \sec x$

$= (\sin^2 x + \cos^2 x) + (\text{cosec}^2 x + \sec^2 x) + 2 + 2$

$= 1 + (1 + \cot^2 x) + (1 + \tan^2 x) + 4 = \tan^2 x + \cot^2 x + 7$ = RHS

OR

Take L.H.S we have

L.H.S = $\frac{\sin(180^\circ+\theta)\cos(90^\circ+\theta)\cdot\tan(270^\circ-\theta)\cot(360^\circ-\theta)}{\sin(360^\circ-\theta)\cos(360^\circ-\theta)\cdot\text{cosec}(-\theta)\sin(270^\circ+\theta)}$

$= \frac{\sin\theta\cdot-\sin\theta\cdot\cot\theta\cdot-\cot\theta}{-\sin\theta\cdot-\sin\theta-\text{cosec}\,\theta\cdot-\cos\theta}$

$= \cot\theta\cdot\tan\theta.\cot\theta.\tan\theta = 1$

Hence proved.

4. let, z = (1 - i) - (-3 + 6i)

= 1 - i + 3 - 6i

z = 4 - 7i.

5. Since a committee is to be formed of 2 teachers and 3 students

When a particular teacher is included

No. of ways in which committee can be formed = ${}^9C_1 \times {}^{20}C_3$

= 10260 ways

OR

We have given that,

${}^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3.$

6. 1, 4, 7, 10, ..., 88

   Consider the given progression with 88 as the first term and -3 as the common difference.

   According to the given conditions we can write ,

   12 th term from the end = 88 + (12 - 1)(-3) = 55.

7. Suppose A(2, -5), B (-2, 5), C(6, 3) and D(1, 1) be the given points

   Suppose m_1 and m_2 be the slopes of AB and CD respectively. Then,

   m_1 = slope of AB$= \frac{5-(-5)}{-2-2} = \frac{10}{-4} = -\frac{5}{2}$

   m_2 = slope of CD $= \frac{1-3}{1-6} = \frac{-2}{-5} = \frac{2}{5}$

   $\therefore \quad m_1 m_2 = \left(\frac{-5}{2}\right) \times \frac{2}{5} = -1$

   Therefore, AB $\perp$ CD.

OR

Here y = 0

$\Rightarrow y = 0 . x + 0$

Which is required slope intercept form,

Comparing it with y = mx + c, we have

m = 0 and c = 0

8. The given parabola $y^2 = -12x$

   comparing above equation with $y^2 = -4ax$, we get,

   $4a = 12 \Rightarrow a = 3.$

   the coordinates of the focus are (-a, 0) = (-3, 0) and

   the equation of the directrix is x = a i.e. x = 3.

   Length of the latus-rectum = 4a = 12

9. Given: A = {x : x $\in$ N}, B = {x : x $\in$ N and x is even}, C = {x : x $\in$ N and x is odd} and D = { x

: x $\in$ N and x is prime}

Therefore, C $\cap$ D = {x : x $\in$ N and x is prime and x $\neq$ 2}

OR

We know that ϕ is a set containing no element at all.

And {0} is a set containing one element, namely 0.

Also, we know that 0 is a number, not a set.

Therefore, ϕ {0} and 0 are all different.

10. There are two children in a family which are to be boys (B) or girls (G).

    Hence sample space S = {BB, BG, GB, GG}

11. x coordinate is +ve

    y coordinate is -ve

    z coordinate is -ve

    Therefore, this point lies in XOY'Z' octant.

12. Required number of ways = number of arrangements of 6, different things taking all at a time.... that is

    $^6P_6$ = 6! = $(6\times5\times4\times3\times2\times1)$ = 720.

13. Using 2 sin A cos B = sin (A + B) + sin (A - B), we obtain

    2 sin 5x cos x = sin (5x + x) + sin (5x - x) = sin 6x + sin 4x

14. $cos(-870^o) = cos870^o$ [$\because cos(-\theta) = cos\ \theta$]

    cos $\left(\frac{\pi}{2} \times 10 - 30\right)$ = $\pm$ cos 30^0 [$\because$ n is even]

    Now, $\alpha = 870^o = \frac{\pi}{2} \times 10 - 30^o$

    It lies in II quadrant in which cos θ is negative.

    So, cos (- 870^0) = - cos 30^0 = - $\frac{\sqrt{3}}{2}$

15. $\Rightarrow 6 - 2x \geq \frac{x}{5} + 4$

    $\Rightarrow$ -2x - $\frac{x}{5}$ $\geq$ 4 - 6

    $\Rightarrow \frac{-11x}{5} \geq -2$

    $\Rightarrow \frac{11x}{5} \leq 2$

    $\Rightarrow$ x $\leq \frac{10}{11}$

    Therefore, $\left(-\infty, \frac{10}{11}\right]$ is the solution set.

16. Now we have,the set of all triangles in a plane is an infinite set because in a plane there is an infinite number of triangles.

Section - II

17. i. (b) 14 hours
    ii. (d) 3325.29 liters
    iii. (b)381.47 liters
    iv. (a) 244.14 liters/hr
    v. (c) 10968.68 liters
18. i. (c) 19
    ii. (b) 110
    iii. (a) 200
    iv. (a) 329
    v. (a) 50 choices

Part - B Section - III

19. n (T) = 100

    n (T - C) = 65

    n (T ∪ C) = 210

    n (T - C) = n (T) - n (T ∩ C)

    65 = 100 - n (T ∩ C)

    n (T ∩ C) = 35

    n (T ∪ C) = n (T) + n (C) - n (T ∩ C)

    210 = 100 + n (C) - 35

    n (C) = 145.

    Now,

    n(C - T) = n(C) - n(C ∩ T)

    n(C - T) = 145 - 35

    n(C - T) = 110
20. Here A = {1, 2, 3, 4, 6}

    We have to form a set of ordered pairs (a, b) where b is exactly divisible by a.

    i. R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
    ii. Domain of R = {1, 2, 3, 4, 6}
    iii. Range of R = {1, 2, 3, 4, 6}

OR

Here we have, A = {x ∈ W : x < 2}, B $\{x \in N : 1 < x \leq 4\}$ and C = {3, 5}

Now, A = {x ∈ W : x < 2} where W = set of whole numbers (non-negative integers)

∴ A = {0, 1} [∵ x < 2 and the whole numbers which are less than 2 are 0, 1]

$B = \{x \in N : 1 < x \leq 4\}$ where, N = set of natural numbers.

∴ B = {2, 3, 4} [∵ the value of x is greater than 1 and less than or equal to 4]

and C = {3, 5}

L.H.S $= A \times (B \cap C)$

According to the definition of the intersection of two sets,

$(B \cap C) = \{3\}$

Now $A \times (B \cap C) = \{0,1\} \times \{3\}$ = {(0, 3), (1, 3)}

R.H.S $= (A \times B) \cap (A \times C)$

Now, $A \times B = \{0,1\} \times \{2,3,4\}$ = {(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)}

and $A \times C = \{0,1\} \times \{3,5\}$ = {(0, 3), (0, 5), (1, 3), (1, 5)}

So, according to the definition of the intersection of two sets,

$(A \times B) \cap (A \times C)$ = {(0, 3), (1, 3) = L.H.S

∴ L.H.S = R.H.S is verified.

21. We have,

$\left[\left(\frac{1}{3} + \frac{7}{3}i\right) + \left(4 + \frac{1}{3}i\right)\right] - \left(-\frac{4}{3} + i\right)$

$= \frac{1}{3} + \frac{7}{3}$ i + 4 + $\frac{1}{3}$ i + $\frac{4}{3}$ - i

$= \left(\frac{1}{3} + 4 + \frac{4}{3}\right) + i\left(\frac{7}{3} + \frac{1}{3} - 1\right)$

$= \left(\frac{1+12+4}{3}\right) + i\left(\frac{7+1-3}{3}\right)$

$= \frac{17}{3} + \frac{5}{3}i$

= a + ib

where, a = $\frac{17}{3}$ and b = $\frac{5}{3}$

22. We have,

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$\because -1 \leq \cos(\theta_1 - \theta_2) \leq 1$

$\Rightarrow \cos(\theta_1 - \theta_2) \geq -1$

$\Rightarrow 2|z_1||z_2|\cos(\theta_1 - \theta_2) \geq -2|z_1||z_2|$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) \geq |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow |z_1 + z_2|^2 \geq (|z_1| - |z_2|)^2$

$\Rightarrow |z_1 + z_2| > |z_1| - |z_2|$

Hence proved.

23. We have, $\sqrt{2}x^2 + x + \sqrt{2} = 0$...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

$a = \sqrt{2}$, b = 1 and $c = \sqrt{2}$

Then, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-1\pm\sqrt{(1)^2-4\times\sqrt{2}\times\sqrt{2}}}{2\times\sqrt{2}}$

$= \frac{-1\pm\sqrt{1-8}}{2\sqrt{2}}$

$= \frac{-1\pm\sqrt{-7}}{2\sqrt{2}}$

$= \frac{-1\pm\sqrt{7}i}{2\sqrt{2}}$

$x = \frac{-1+\sqrt{7}i}{2\sqrt{2}}$ or $x = \frac{-1-\sqrt{7}i}{2\sqrt{2}}$

Hence, the roots of the given equation are $\frac{-1+\sqrt{7}i}{2\sqrt{2}}$ and $\frac{-1-\sqrt{7}i}{2\sqrt{2}}$.

OR

We have,

$z = -5 + 3i \Rightarrow (z + 5) = 3i$

$\Rightarrow (z + 5)^2 = 9i^2$

$\Rightarrow z^2 + 10z + 25 = -9$

$\Rightarrow z^2 + 10z + 34 = 0$ ...(i)

Now, $z^4 + 9z^3 + 26z^2 - 14z + 8$

$= z^2(z^2 + 10z + 34) - z(z^2 + 10z + 34) + 2(z^2 + 10z + 34) - 60$

$= (z^2 \times 0) - (z \times 0) + (2 \times 0) - 60 = -60$ [using (i)].

Hence, the value of $z^4 + 9z^3 + 26z^2 - 14z + 8$ is -60

24. i. We know that,

If odds in favor of the occurrence an event are a:b, then the probability of an event to occur is $\frac{a}{a+b}$

Given, probability = $\frac{5}{14}$

We know, probability of an event to occur = $\frac{a}{a+b}$

Here, a = 5 and a + b = 14 i.e. b = 9

So, $\frac{a}{a+b}$ = $\frac{5}{14}$

odds in favor of its occurrence = a : b = 5 : 9

Conclusion: Odds in favor of its occurrence is 5: 9

ii. As we solved in part (i), a = 5 and b = 9

Also, we know, odds against its occurrence is b: a = 9: 5

Conclusion: Odds against its occurrence is 9 : 5

25. We have to find the value,

$\lim_{x \to \infty} \frac{5x-6}{\sqrt{4x^2+9}} = \lim_{x \to \infty} \frac{5-\frac{6}{x}}{\sqrt{4+\frac{9}{x^2}}}$ [Dividing each term in N' and D' by x]

$= \frac{5-0}{\sqrt{4+0}} = \frac{5}{2}$

26. We know that,

Probability of occurring = 1 - the probability of not occurring

So we have to find the probability of not occurring, i.e. probability such that both of them don't have a birthday on the same day.

Let us suppose the first person has a birthday on a particular day then the other person can have a birthday in the remaining 364 days

The Probability of not having the same birthday = $\frac{364}{365}$

The Probability of having same birthday = 1 - the probability of not having the same Birthday

= 1 - $\frac{364}{365}$

= $\frac{1}{365}$

Conclusion: Probability of two persons having the same birthday is $\frac{1}{365}$

27. To calculate the mean deviation about the mean, we need to calculate the mean first.

Consider the following table.

x_i	f_i	$f_i x_i$	$\|x_i - 15\|$	$f_i \|x_i - 15\|$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60

	$N = \Sigma f_i = 44$	$\Sigma f_i x_i = 660$		$\Sigma f_i \lvert x_i - 15 \rvert = 312$

Now,

$Mean = \overline{X} = \frac{1}{N}(\Sigma f_i x_i) = \frac{660}{44} = 15$

The mean deviation for the given data,

$M.D. = \frac{1}{N} \Sigma f_i \lvert x_i - 15 \rvert = \frac{312}{44} = 7.09$

28. We have to prove: cos x cos $\frac{x}{2}$ - cos 3x cos $\frac{9x}{2}$ = sin 4x sin 7x

LHS = cos x cos $\frac{x}{2}$ - cos 3x cos $\frac{9x}{2}$

= $\frac{1}{2}\left[2\cos x \cos\frac{x}{2}\right.$ - $\left.2\cos 3x \cos\frac{9x}{2}\right]$

= $\frac{1}{2}\left[\cos\left(x+\frac{x}{2}\right)\right.$ + $\cos\left(x-\frac{x}{2}\right)$ - $\cos\left(3x+\frac{9x}{2}\right)$ - $\cos\left(3x-\frac{9x}{2}\right)$

= $\frac{1}{2}\left[\cos\frac{3x}{2} + \cos\frac{x}{2}\right.$ - $\left.\cos\frac{15x}{2} - \cos\frac{3x}{2}\right]$

= $\frac{1}{2}\left[\cos\frac{x}{2} - \cos\frac{15x}{2}\right]$

= $\frac{1}{2}$ [-2 sin $\left(\frac{x+15x}{4}\right)$ sin $\left(\frac{x-15x}{4}\right)$]

= $\frac{1}{2}$ [-2 sin (4x) sin $\left(-\frac{7x}{2}\right)$]

= sin 4x sin 7x = RHS

Hence proved.

OR

Given: $tan\theta + tan2\theta$ + $\sqrt{3}\, tan\theta \times tan2\theta$ = $\sqrt{3}$

$\Rightarrow tan\theta + tan2\theta = \sqrt{3}$ (1 - tanθ × tan 2θ)

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \cdot \tan 2\theta}$ = $\sqrt{3}$

[$\because tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$]

$\Rightarrow tan(\theta$ + 2θ) = $\sqrt{3}$

$\Rightarrow tan3\theta$ = $\sqrt{3}$

$\Rightarrow tan3\theta$ = tan $\frac{\pi}{3}$

[$\because$ tan $\frac{\pi}{3}$ = $\sqrt{3}$]

$\Rightarrow$ 3θ = $n\pi + \frac{\pi}{3}$

$\because \theta$ = $\frac{n\pi}{3} + \frac{\pi}{9}, n \in Z$

Section - IV

29. Given, Mean $\overline{x}$ = $\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$ ...(i)

Now, Mean of $ax_1, ax_2, \dots ax_n$

= $\frac{ax_1+ax_2+ax_3+\dots+ax_n}{n} = \frac{a(x_1+x_2+x_3+\dots+x_n)}{n}$

= a$\overline{x}$ [Using Eq.(i)]

Hence proved.

Now, Variance, $\sigma^2 = \frac{\sum(x_i-\overline{x})^2}{n}$ ...(ii)

$\therefore$ Variance of $ax_1, ax_2, ax_3, \dots ax_n = \frac{\sum(ax_i-a\overline{x})^2}{n}$

= $\frac{(ax_1-a\overline{x})^2+(ax_2-a\overline{x})^2+\dots+(ax_n-a\overline{x})^2}{n}$

= $\frac{a^2(x_1-\overline{x})^2+a^2(x_2-\overline{x})^2+\dots+a^2(x_n-\overline{x})^2}{n}$

= $\frac{a^2\sum(x_i-\overline{x})^2}{n} = a^2\sigma^2$ [using Eq.(ii)]

Hence proved.

30. Here f(x) = ax + b

f = {(1, 1), (2, 3), (0, -1), (-1, -3)}

$\Rightarrow$ f (1) = 1, f (2) = 3, f(0) = -1, f(-1) = -3

Now f(1) = 1 $\Rightarrow a \times 1 + b = 1 \Rightarrow$ a+ b = 1 . . . (i)

$f(2) = 3 \Rightarrow a \times 2 + b = 3 \Rightarrow 2a + b = 3$ . . . (ii)

Subtracting (i) from (ii) we get

2a+b-(a+b)=3-1 $\Rightarrow$ a = 2

Putting a = 2 in (i)

2 + b = 1 $\Rightarrow$ b = -1

31. Let A be the first term and d be a common difference.

Since, T_p = a $\Rightarrow$ A + (p - 1)d = a ....(i)

T_q = b $\Rightarrow$ A + (q - 1)d = b ...(ii)

and T_r = c $\Rightarrow$ A + (r - 1)d = c ...(iii)

On multiplying Eq. (i) by (q - r), Eq. (ii) by (r - p) and Eq. (iii) by (p - q), we get

(q - r) A + (p - 1) (q - r)d = a (q - r) ...(iv)

(r - p) A + (q - 1) (r - p)d = b (r - p) ...(v)

and (p - q) A + (r - 1) (p - q)d = c (p - q) ...(vi)

On adding Eqs. (iv), (v) and Eq. (vi), we get

(q - r) A + (p - 1) (q - r)d + (r - p) A + (q - 1) (r - p)d + (p - q) A + (r - 1) (p - q)d = a (q - r) + b(r - p) + c (p - q)

$\Rightarrow$ A [(q - r) + (r - p) + (p - q)] + [(p - 1) (q - r) + (q - 1) (r - p) + (r - 1) (p - q)]d = a (q - r) + b (r -

p) + c (p - q)

$\Rightarrow$ A (0) + (0)d = a (q - r) + b (r - p) + c (p - q)

$\Rightarrow$ a (q - r) + b (r - p) + c (p - q) = 0

Hence proved.

OR

According to the question , we can state,

Let T_n be the nth term and S_n be the sum of n terms of the given series.

Thus, we have:

$S_n = 4 + 6 + 9 + 13 + 18 + ... + T_{n-1} + T_n$ ...... (i)

Equation (i) can be rewritten as:

$S_n = 4 + 6 + 9 + 13 + 18 + ... + T_{n-1} + T_n$ ...... (ii)

On subtracting (ii) from (i), we get:

$S_n = 4 + 6 + 9 + 13 + 18 + ... + T_{n-1} + T_n$

$S_n = 4 + 6 + 9 + 13 + 18 + ... + T_{n-1} + T_n$

$0 = 4 + [2 + 3 + 4 + 5 + 6 + ... + (T_n - T_{n-1})] - T_n$

The sequence of difference between successive terms is 2, 3, 4, 5, ...

We observe that it is an AP with common difference 1 and first term 2.

Now,

$4 + \left[\frac{(n-1)}{2}\{4 + (n-2)1\}\right] - T_n = 0$

$\Rightarrow 4 + \left[\frac{(n-1)}{2}(n+2)\right] - T_n = 0$

$\Rightarrow 4 + \left[\frac{n^2+n}{2} - 1\right] - T_n = 0$

$\Rightarrow \left[\frac{n^2}{2} + \frac{n}{2} + 3\right] = T_n$

$\because S_n = \sum_{k=1}^{n} T_k$

$\therefore S_n = \sum_{k=1}^{n} \left(\frac{k^2}{2} + \frac{k}{2} + 3\right)$

$= \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$

$= \frac{n(n+1)(2n+1)}{2\times 6} + \frac{n(n+1)}{2\times 2} + 3n$

$= n\left(\frac{2n^2+3n+1+3n+3+36}{12}\right)$

$= \frac{n}{12}(2n^2 + 6n + 40)$

$= \frac{n}{6}(n^2 + 3n + 20)$

32. Given that:

Coordinates of foci $= (\pm 1, 0)$ ... (i)

Eccentricity $= \frac{1}{2}$ ... (ii)

Let the equation of the required ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

we know that, Coordinates of foci $= (\pm c, 0)$ ... (iii)

∴ From eq. (i) and (iii), we get

==> c = 1

Eccentricity $= \frac{c}{a} \Rightarrow \frac{1}{2} = \frac{2}{a} \Rightarrow a = 2 \Rightarrow a^2 = 4$ [∵ c = 1]

Now,

$c^2 = a^2 - b^2 \Rightarrow (1)^2 = (2)^2 - b^2 \Rightarrow 1 = 4 - b^2$

$\Rightarrow b^2 = 4 - 1 \Rightarrow b^2 = 3$

Putting the value of a^2 and b^2 in the equation of an ellipse, we get

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

33. Here ${}^{22}P_{r+1}$: ${}^{20}P_{r+2} = 11 : 52$

$\Rightarrow \frac{22!}{(21-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$

$\Rightarrow = \frac{22 \times 21 \times 20!}{(21-r)(20-r)(19-r)(18-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$

$\Rightarrow \frac{22 \times 21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$

$\Rightarrow (21-r)(20-r)(19-r) = 2 \times 21 \times 52$

$\Rightarrow (21-r)(20-r)(19-r) = 14 \times 13 \times 12$

$\Rightarrow (21-r)(20-r)(19-r) = (21-7)(20-7)(19-7)$

$\Rightarrow r = 7$

34. The distance between L and M is

LM = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a\cos\beta - a\cos\alpha)^2 + (a\sin\beta - a\sin\alpha)^2}$

$= \sqrt{a^2(\cos^2\beta + \cos^2\alpha - 2\cos\alpha\cos\beta) + a^2(\sin^2\beta - \sin^2\alpha - 2\sin\beta\sin\alpha)}$

$= a\sqrt{(\cos^2\beta + \sin^2\beta) + (\cos^2\alpha + \sin^2\alpha) - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$

$= a\sqrt{1 + 1 - 2\cos(\alpha - \beta)} = a\sqrt{2[1 - \cos(\alpha - \beta)]}$ $[\because \cos^2\theta + \sin^2\theta = 1$ and cos A cos B + sin A sin B = cos (A - B)]

= $a\sqrt{2 \times 2\sin^2\left(\frac{\alpha-\beta}{2}\right)}$ $[\because \cos\theta = 1 - 2\sin^2\theta/2] = 2a\sin\left(\frac{\alpha-\beta}{2}\right)$

OR

We have, (x_1, y_1) = A(2, 3), θ = 45°

So, the equation of the line passing through (2, 3) and making an angle of 45° with the x-axis is

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$

$\Rightarrow \frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ}$

$\Rightarrow \frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-2}{\frac{1}{\sqrt{2}}}$

⇒ x - y + 1 = 0

Let, x - y + 1 = 0 intersect the line 2x - 3y = 9 = 0 at point P.

Let, AP = r

Then, the coordintes of P are given by $\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ}$ = r

⇒ x = $2 + \frac{r}{\sqrt{2}}$ and y = $3 + \frac{r}{\sqrt{2}}$

Hence, the coordinates of P are $\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$

Clearly, Point P lies on the line 2x - 3y + 9 = 0

$\therefore 2\left(2 + \frac{r}{\sqrt{2}}\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9$ = 0

$\Rightarrow 4 + \frac{2r}{\sqrt{2}} - 9 - \frac{3r}{\sqrt{2}} + 9$ = 0

$\Rightarrow \frac{r}{\sqrt{2}}$ = 4

⇒ r = $4\sqrt{2}$

Therefore, the distance of the point from the given line is $4\sqrt{2}$

35. Here

n (A) = a + b + c + d = 21 .... (i)

n(B) = b +c + f + g = 26 .... (ii)

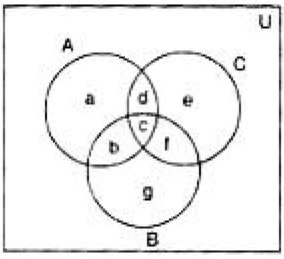
n(C) = c +d + e + f = 29 .... (iii)

$n(A \cap B) = b + c = 14$....... (iv)

$n(C \cap A) = c + d = 12$........(v)

$n(B \cap C) = c + f = 14$....... (vi)

$n(A \cap B \cap C) = c = 8$....... (vii)



Putting value of c in (iv), (v) and (vi)

b + 8 = 14 ⇒ b = 6

8 + d = 12 ⇒ d = 4

8 + f = 14 ⇒ f = 6

Putting value of c, d, f in (iii),

8 + 4 + e + 6 = 29 ⇒ e = 29 - 18 = 11

Number of people who like product C only = 11

Section - V

36. i. Let $y = \frac{\sin x + \cos x}{\sin x - \cos x}$

On differentiating both sides of y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\left[(\sin x + \cos x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)\right]}{(\sin x - \cos x)^2}$$

[by quotient rule of derivative]

$$= \frac{[\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(cosx + sin]}{(\sin x - \cos x)^2}$$

$$= \frac{-(\cos x - \sin x)(\cos x - \sin x) - (\cos x + \sin x)^2}{(\sin x - \cos x)^2}$$

$$= \frac{-(\cos x - \sin x)^2 - (\cos x + \sin x)^2}{(\sin x - \cos x)^2}$$

$$= \frac{[-(\cos^2 x + \sin^2 x - 2\cos x \sin x) + (\cos^2 x + \sin^2 x + 2\cos x \sin x)]}{(\sin x - \cos x)^2}$$

$$= \frac{-[1+1]}{(\sin x - \cos x)^2} = \frac{-2}{(\sin x - \cos x)^2}$$

ii. Given, $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$

At x = 2,

RHL = $\lim_{x \to 2^+} f(x)$

$= \lim_{h \to 0} f(2 + h)$

$= \lim_{h \to 0}$ 2(2 + h) + 3

= 2(2 + 0) + 3

= 4 + 3 = 7 = α [say]

[∵ f(x) = 2x + 3]

LHL = $\lim_{x\to 2^-} f(x) = \lim_{h\to 0}$ f(2 - h)

$= \lim_{h\to 0}$ (2 - h)^2 - 1 = (2 - 0)^2 - 1

= 4 - 1 = 3 = β [say] [∵. f(x) = x^2 - 1]

If a quadratic euation has root α and β, then the equation is

x^2 - (Sum of roots) x + Product of roots = 0

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., x^2 - (7 + 3)x + 7 × 3 = 0

⇒ x^2 - 10x + 21 = 0

OR

Let us take u = (e^x sin x) and v = (sec x)

$u' = \frac{du}{dx} = \frac{d(e^x \sin x)}{dx}$

Applying Product rule

(gh)' = g'h + gh'

Taking g = e^x and h = sin x

u'= e^x sin x + e^x cos

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$\left[\frac{e^x \sin x}{\sec x}\right] = \frac{(e^x \sin x + e^x \cos x)\times(\sec x) - (e^x \sin x)\times(\sec x \tan x)}{(\sec x)^2}$

$= \frac{(e^x \sin x + e^x \cos x) - (e^x \sin x)\times(\tan x)}{(\sec x)}$

= cos x[(e^x sin x + e^x cos x) - (e^x sin x) × (tan x)]

= [(e^x sin x cos x + e^x $\cos^2$ x) - (e^x sin x cos x) × (tan x)]

= [(e^x sin x cos x + e^x $\cos^2$ x) - (e^x $\sin^2$ x)]

= (e^x sin x cos x + e^x $\cos^2$ x - e^x $\sin^2$ x)

= (e^x sin x cos x + e^x $\cos^2$ x - e^x $\sin^2$ x)

= e^x sin x cos x + e^x cos 2x

= e^x(sin x cos x + cos 2x)

37. Suppose (a, b) be an arbitrary element of $(A \times B) \cap (C \times D)$.

$\Rightarrow (a, b) \in (A \times B) \cap (C \times D)$

$\Rightarrow \quad (a, b) \in A \times B$ and $(a, b) \in C \times D$

$\Rightarrow$ (a $\in A$ and $b \in B$) and (a $\in$ C and b $\in$ D)

$\Rightarrow$ ($a \in A$ and $a \in C$) and ($b \in B$ and $b \in D$)

$\Rightarrow \quad a \in (A \cap C)$ and $b \in (B \cap D)$

$\Rightarrow \quad (a, b) \in (A \cap C) \times (B \cap D)$

$\therefore \quad (A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$..... (i)

Similarly, $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$..... (ii)

From Equations (i) and (ii),

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Hence proved.

OR

Here it is given that: f : R $\Rightarrow$ R : f(x) = Ixl = $\begin{cases} x, \text{ when } x \geq 0 \\ -x, \text{ when } x < 0 \end{cases}$

We have

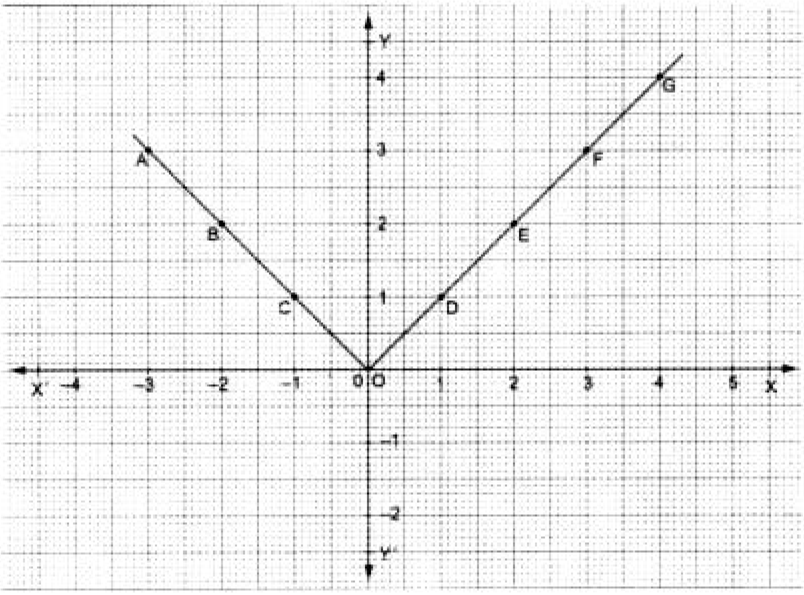
x	-3	-2	-1	0	1	2	3	4
f(x) = |x|	3	2	1	0	1	2	3	4

On a graph paper draw the horizontal line X' OX as the x-axis and the vertical line YOY' as the y-axis.

Take the scale: 10 small divisions = 1 unit.

On this graph paper, plot the points A(-3, 3), B(-2, 2), C(-1, 1), 0(0, 0), D(1, 1), E(2, 2), F(3, 3) and G(4, 4).

Join these points successively to obtain the graph lines ABCO and ODEFG, as shown below.

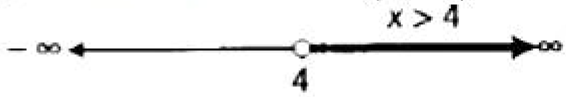


Graph of f(x) = |x|

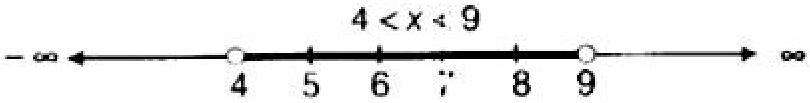
38. We have, $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4}$ ... (i)
and $\frac{7x-1}{3}-\frac{7x+2}{6}>x$ ... (ii)
From inequality (i), we get
$\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4} \Rightarrow \frac{16x-27}{12}<\frac{4x+3}{4}$
$\Rightarrow 16x - 27 < 12x + 9$ [multiplying both sides by 12]
$\Rightarrow 16x - 27 + 27 < 12x + 9 + 27$ [adding 27 on both sides]
$\Rightarrow 16x < 12x + 36$
$\Rightarrow 16x - 12x < 12x + 36 - 12x$ [subtracting 12x from bot sides]
$\Rightarrow 4x < 36 \Rightarrow x < 9$ [dividing both sides by 4]
Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$

From inequality (ii) we get,
$\frac{7x-1}{3}-\frac{7x+2}{6}>x \Rightarrow \frac{14x-2-7x-2}{6}>x$
$\Rightarrow 7x - 4 > 6x$ [multiplying by 6 on both sides]
$\Rightarrow 7x - 4 + 4 > 6x + 4$ [adding 4 on both sides]
$\Rightarrow 7x > 6x + 4$
$\Rightarrow 7x - 6x > 6x + 4 - 6x$ [subtracting 6x from both sides]
$\therefore x > 4$
Thus, any value of x greater than 4 satisfies the inequality.

So, the solution set is $x \in (4, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.

Hence, the solution of the given system is, 4 < x < 9 i.e., $x \in (4, 9)$

OR

Given inequation is 2x - 5y + 10 $\geq$ 0

Consider given inequation as strict equation 2x - 5y + 10 = 0

Points passing through line are shown below,

x	0	5	-5
y	2	4	0

plot a line using the above points.

so, now line divide xy-plan in two parts

let point (0,0) which are not on a line. put this point in given inequation

0 + 10 $\geq$ 0 => 10 $\geq$ 0

we observe that it satisfy the given equation. so the shaded region will be towards the origin.

this shaded region shows the solution of the inequality.