Chapter-9

Algebraic Expressions and Identities

B3K7K2

Let us refresh ourselves on what we have learnt in our previous class viz.

- ☆ Formation of Algebraic Expression.
- ☆ Terms in expressions and factors of a term.
- ☆ Coefficients, similar terms and unsimilar terms.
- ☆ Monomial, binomials, trinomials and polynomial.
- ☆ Addition and subtraction of algebraic expressions.
- ☆ Determination of the value of an expression for definite value of the variable.

In this chapter, we shall discuss about the multiplication of algebraic expressions. Before that, we collect the rules of addition and subtraction of algebraic expressions.

- ☆ Two similar terms can be added or subtracted.
- ☆ Addition and subtraction of two or more similar terms with numerical co-efficients is done by addition and subtraction of the numerical co-efficients.
  For example , $3x + 2x = (3 + 2)x = 5x$, $7x - 3x = (7 - 3)x = 4x$ etc.
- ☆ Unsimilar terms cannot be added or subtracted.
- ☆ In adding or subtracting two or more algebraic expressions, similar terms are separated and then added or subtracted. Unsimilar terms are kept with their signs as they are.

9.1 Multiplication of algebraic expressions :

You have already learnt the different rules in respect of rational numbers. For example, if a, b, c are any three rational numbers, then

	$a + b = b + a$	[Commutative property of addition]
	$(a + b) + c = a + (b + c)$	[Associative property of addition]
	$a \times b = b \times a$	[Commutative property of multiplication]
	$(a \times b) \times c = a \times (b \times c)$	[Associative property of multiplication]
	$a \times (b + c) = a \times b + a \times c$	[Distributive property of multiplication]
	$1 \times a = a$	[Identity property of multiplication]
Also	$0 \times a = 0$	
	$a + 0 = a = 0 + a$	[Identity property of addition]

Algebraic symbols are used to represent numbers. Therefore, all properties or rules of numbers will be applicable to algebraic expressions also.

Let us recall

Rules, regarding multiplication of positive, negative or both types of expressions.

$(+)\times(+) = +$	$(+)\times(-) = -$	$(-)\times(+) = -$	$(-)\times(-) = +$
$7 \times 3 = 21$	$7 \times (-3) = -21$	$(-7) \times 3 = -21$	$(-7) \times (-3) = 21$

9.1.1 Multiplication of two or more monomial expressions

Look at the following examples

(i) $5 \times 6x = 5 \times (6 \times x) = (5 \times 6) \times x = 30x$ [Associative property of multiplication]

(ii) $x \times 7y = x \times 7 \times y = (x \times 7) \times y = (7 \times x) \times y$ [Commutative property of multiplication]

$= 7 \times x \times y = 7xy$

(iii) $6x \times (-7y) \times 8xy = 6 \times x \times (-7) \times y \times 8 \times x \times y$

$= 6 \times (-7) \times 8 \times x \times y \times x \times y$ [Commutative property of multiplication]

$= (-42) \times 8 \times x \times x \times y \times y$ [Commutative property of multiplication]

$= -336 \times x^2 \times y^2$ [$\because$ $x \times x = x^{1+1} = x^2$ and $y \times y = y^{1+1} = y^2$]

$= -336x^2y^2$

Note :

Numerical co-efficient of the product = Product of their numerical co-efficients.
Algebraic co-efficient of the product = Product of their algebraic co-efficients.

Therefore, for multiplication of two or more monomial expressions, we first write the co-efficients side by side and then the algebraic factors side by side. Then find the product of the co-efficients and product of the algebraic factors by laws of indices. Ultimately, the sign of multiplication (×) is removed from the multiplication of numerical values and variables.

Example 1 : Find the product of $5x$ and $(-8x^3y)$

Solution : $5x \times (-8x^3y)$

$= 5 \times (-8) \times x \times x^3 \times y$

$= -40 \times x^4 \times y$ $[a^m \times a^n = a^{m+n}]$

$= -40x^4y$

Example 2 : Find the product, $3l^2m \times 4lm^2 \times 5mn$

Solution : $3l^2m \times 4lm^2 \times 5mn$

$= 3 \times 4 \times 5 \times l^2 \times l \times m \times m^2 \times m \times n$

$= 60 \times l^3 \times m^4 \times n$

$= 60l^3m^4n$

Activity : Fill up the blanks of the following table of multiplication :

$\times$	$6x$	$-3y$	$7xy$	$-5x^2y$	$8x^2y^3$
$6x$		$6x \times (-3y) = -18xy$			
$-3y$	$(-3y) \times 6x = -18xy$				
$7xy$					
$-5x^2y$				$(-5x^2y) \times (-5x^2y) = 25x^4y^2$	
$8x^2y^3$			$8x^2y^3 \times 7xy = 56x^3y^4$		

9.1.2 Multiplication of monomial expression by binomial or trinomial expression

Look at this example $3 \times 204 = 612$

It can be written in this way also

$3 \times 204 = 3 \times (200 + 4)$

$= (3 \times 200) + (3 \times 4)$ [*Using the law* $a \times (b + c) = a \times b + a \times c$]

$= 600 + 12$

$= 612$

Notice that we have used here the law of distribution of multiplication over addition. In case of mutiplication of monomial, binomial or trinomial expressions, we can apply the same property.

Example 3 : Find the product, $3x \times (9x^2 + 3)$

Solution : $3x \times (9x^2 + 3) = (3x \times 9x^2) + (3x \times 3)$ [Using the law $a\times(b+c) = a\times b + a \times c$]

$= 3 \times 9 \times x \times x^2 + 3 \times 3 \times x$

$= 27x^3 + 9x$

Example 4 : Find the product, $(3x + 5xy) \times 2x^2$

Solution : $(3x + 5xy) \times 2x^2 = (3x \times 2x^2) + (5xy \times 2x^2)$

$= 3 \times 2 \times x \times x^2 + 5 \times 2 \times x \times x^2 \times y$

$= 6x^3 + 10x^3y$

$(a + b) \times c$ $= a \times c + b \times c$

Example 5 : Find the product, $3m^2 \times (5m^2 - 2m + 1)$

Solution : $3m^2 \times (5m^2 - 2m + 1) = (3m^2 \times 5m^2) - (3m^2 \times 2m) + (3m^2 \times 1)$

$= 3 \times 5 \times m^2 \times m^2 - 3 \times 2 \times m^2 \times m + 3m^2$

$= 15m^4 - 6m^3 + 3m^2$

9.1.3 Multiplication of a binomial expression by binomial or trinomial expression

Suppose, we are to multiply the binomial expression $(3x + 2)$ by the binomial expression $(7x + 3y)$. Here, we can use distributive law of multiplication also.

$(3x + 2) \times (7x + 3y) = 3x \times (7x + 3y) + 2 \times (7x + 3y)$

$= (3x \times 7x) + (3x \times 3y) + (2 \times 7x) + (2 \times 3y)$

$= 21x^2 + 9xy + 14x + 6y$

Example 6 : Find the product : $(7x + 2y) \times (11x - 4y)$

Solution : $(7x + 2y) \times (11x - 4y) = 7x \times (11x - 4y) + 2y \times (11x - 4y)$

$= (7x \times 11x) - (7x \times 4y) + (2y \times 11x) - (2y \times 4y)$

$= 77x^2 - 28xy + 22yx - 8y^2$

$= 77x^2 - 28xy + 22xy - 8y^2$ [$\because xy = yx$]

$= 77x^2 - 6xy - 8y^2$

Example 7 : Find the product : $(4xy^2 + 5) \times (3xy - 5xy^2)$

Solution : $(4xy^2 + 5) \times (3xy - 5xy^2)$

$= 4xy^2 \times (3xy - 5xy^2) + 5 \times (3xy - 5xy^2)$

$= (4xy^2 \times 3xy) - (4xy^2 \times 5xy^2) + (5 \times 3xy) - (5 \times 5xy^2)$

$= 12x^2y^3 - 20x^2y^4 + 15xy - 25xy^2$

Here, there is no similar term.

Now, suppose we are to multiply the binomial expression $(a + 2b)$ by the trinomial expression $(2a^2b + 2a + 3b)$. We proceed as below.

$$
\begin{aligned}
(a + 2b)(2a^2b + 2a + 3b) &= a \times (2a^2b + 2a + 3b) + 2b \times (2a^2b + 2a + 3b) \\
&= 2a^3b + 2a^2 + 3ab + 4a^2b^2 + 4ab + 6b^2 \\
&= 2a^3b + 2a^2 + 4a^2b^2 + (3ab + 4ab) + 6b^2 \\
&= 2a^3b + 2a^2 + 4a^2b^2 + 7ab + 6b^2
\end{aligned}
$$

Example 8 : Find the product $(3x^2 + 2x + 5) \times (2x^2 - 3)$

Solution : $(3x^2 + 2x + 5) \times (2x^2 - 3)$

$$
\begin{aligned}
&= 3x^2 \times (2x^2 - 3) + 2x \times (2x^2 - 3) + 5 \times (2x^2 - 3) \\
&= 6x^4 - 9x^2 + 4x^3 - 6x + 10x^2 - 15 \\
&= 6x^4 + 4x^3 - 9x^2 + 10x^2 - 6x - 15 \\
&= 6x^4 + 4x^3 + x^2 - 6x - 15
\end{aligned}
$$

Example 9 : Add $2xy(x - y)$ with $3x(2xy + 5y)$

Solution : First expression $= 2xy(x - y) = 2x^2y - 2xy^2$

Second expression $= 3x(2xy + 5y) = 6x^2y + 15xy$

Adding these two expressions,

$$
\begin{array}{rrrr}
 & 2x^2y & -\,2xy^2 & \\
+ & 6x^2y & & +\,15xy \\
\hline
\text{We get} & 8x^2y & -\,2xy^2 & +\,15xy
\end{array}
$$

Example 10 : Add $(4y + 3)(3y^2 + 5y - 7)$ and $5(y^3 - 4y^2 + 2)$

Solution : First expression

$$
\begin{aligned}
&= (4y + 3)(3y^2 + 5y - 7) \\
&= 4y\,(3y^2+5y-7)+3\,(3y^2+5y-7) && \text{[Using distributive law]} \\
&= 12y^3+20y^2-28y+9y^2+15y-21 && \text{[Using distributive law]} \\
&= 12y^3 + (20y^2 + 9y^2) - (28y - 15y) - 21 \\
&= 12y^3 + 29y^2 - 13y - 21
\end{aligned}
$$

Second expression

$$
\begin{aligned}
&= 5(y^3 - 4y^2 + 2) \\
&= 5y^3 - 20y^2 + 10 && \text{[Using distributive law]}
\end{aligned}
$$

Adding two expressions, we get

$$
\begin{array}{rrrrr}
 & 12y^3 & +\,29y^2 & -13y & -\,21 \\
+ & 5y^3 & -\,20y^2 & & +\,10 \\
\hline
 & 17y^3 & +\,9y^2 & -13y & -\,11
\end{array}
$$

Example 11 : Subtract $m(12l - m)$ from $(l + m)(3l + 2m)$

Solution : First expression $= (l + m)(3l + 2m)$

$= l(3l + 2m) + m(3l + 2m)$

$= 3l^2 + 2ml + 3ml + 2m^2$

$= 3l^2 + 5ml + 2m^2$

Second expression $= m(12l - m)$

$= 12ml - m^2$

Subtracting second expression from the first expression,

$$\begin{array}{rrrrrr} & 3l^2 & + & 5ml & + & 2m^2 \\ (-) & & & 12ml & - & m^2 \\ & & (-) & & (+) & \\ \hline & 3l^2 & - & 7ml & + & 3m^2 \end{array}$$

Example 12 : Simplify

(i) $(2p + 4)(3p + 8)$

(ii) $n(6 + m) - 2(m - n)$

Solution : (i) $(2p + 4)(3p + 8) = 2p(3p + 8) + 4(3p + 8)$

$= 6p^2 + 16p + 12p + 32$

$= 6p^2 + 28p + 32$

(ii) $n(6 + m) - 2(m - n) = 6n + mn - 2m + 2n$

$= 6n + 2n + mn - 2m$

$= 8n + mn - 2m$

Exercise 9.1

1. Find the product :

(i) $3x^2 \times 11xy \times \frac{2}{3}y^2$

(ii) $(-5x) \times 3a^2 \times (-3ax)$

(iii) $(-3pq) \times (-15p^3q^3) \times q^2$

(iv) $3x(5x^2 + 8)$

(v) $\frac{2}{3}y(18y^2 - y)$

(vi) $(-8a^3)(a + 3b + 2c)$

(vii) $(3mn - 2n)(-2m^2n)$

(viii) $(9x^2 + 4x + 3) \times 11x$

(ix) $(20a^2 - 3b^2 + ab) \times (-7b^2)$

(x) $3x^3y^2(xy + xy^3 - 2)$

2. Find the product :

(i) $(x^2 + y)(3x^2y - y^2)$

(ii) $(7x - 2y)(2x + 7y)$

(iii) $\left(\frac{1}{4}a^2 + 3b\right)\left(a^3 + \frac{2}{3}b^2\right)$

(iv) $(1.5x - 2.5y)(2.5x - 1.5y)$

(v) $(3x + 4y)(2x^2 + 3y + xy)$

(vi) $(2xy + 5x^2)(x^5y^4 - x^3y^2 + xy)$

(vii) $(3a^2b^2 - 4c)(a^3b^3 + 2a^4b^3c^3 - 6abc)$

(viii) $(4x^2y - 5xy^2 + 3xy)(3x^3y - 2)$

(ix) $(2x + 3y + z)(5x + 2y + 1)$

(x) $(3x^3 - 2y^2 + z)(3x^3 + 2y^2 - z)$

3. Simplify the following expressions :

(i) $3x(5x + 8) - 10x$

(ii) $(2m + 3m^2)(-2mn)$

(iii) $8(3a + 4b) + 5$

(iv) $2x^2(4x - 1) + 3x(x - 3)$

4. Simplify:

(i) $(p + q^2)(q^2 - p) + 15$

(ii) $(a - b)(a^2 + ab + b^2) + 3b^3$

(iii) $y^2(y^3 + 3x) + y(2xy + y^2)$

(iv) $\left(\frac{2}{3}x^4y^3 + \frac{4}{9}xy^3\right) \times \frac{1}{4} - \frac{1}{6}x^4y^3$

(v) $y^3(4y + 5) - (2y + 1)(y^3 + 2y^2 + 1)$

(vi) $(1.2l - 2.5m)(2.5l + 0.2m + 1.2) + 0.06l + 7m$

9.2 Some Algebraic Identities

A *statement of equality* which is true for any value of the variable is called an '*identity*'.

Now we shall discuss some identities that are generally applied in Algebra. These identities are obtained by multiplying a binomial by another binomial.

9.2.1 Let us first multiply the binomial expression **$(x+a)$** by the binomial expression **$(x+b)$**

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b)\\ &= x^2 + xb + ax + ab\\ &= x^2 + bx + ax + ab\\ &= x^2 + (a + b)x + ab\end{aligned}$$

Distributive law and Commutative law are used here.

$$\boxed{\therefore\ \ (x + a)(x + b) = x^2 + (a + b)x + ab}$$

Left Hand Side (LHS) and Right Hand Side(RHS) of this statement are true for any value of x i.e. the statement is true for any value of x.

So $(x + a)(x + b) = x^2 + (a + b)x + ab$ is an identity.

We can verify this statement geometrically. Let us construct a rectangle ABCD whose length is $(x + a)$ and breadth is $(x + b)$ (Diagram - 1). Find [You can take $x = 5$ cm, $a = 4$ cm, $b = 2$ cm for example]

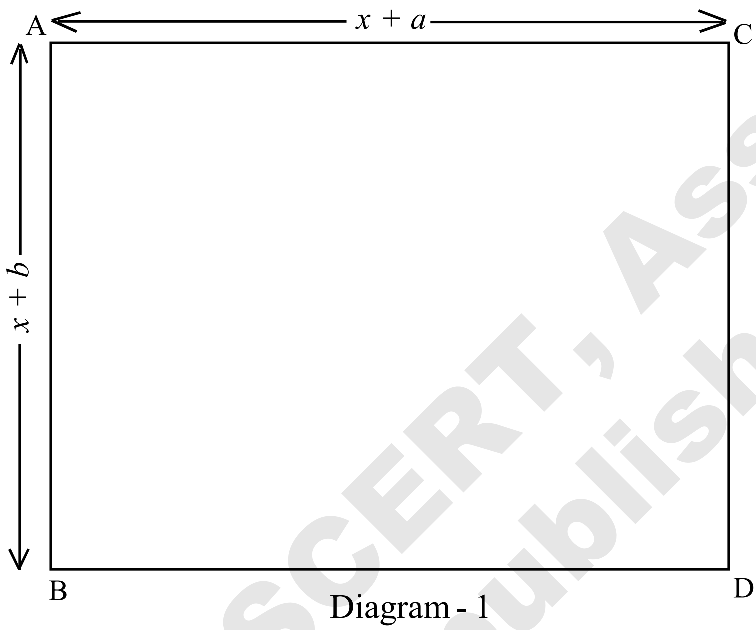


Diagram - 1

Then, divide the rectangle ABCD into four parts as shown in Diagram - 2

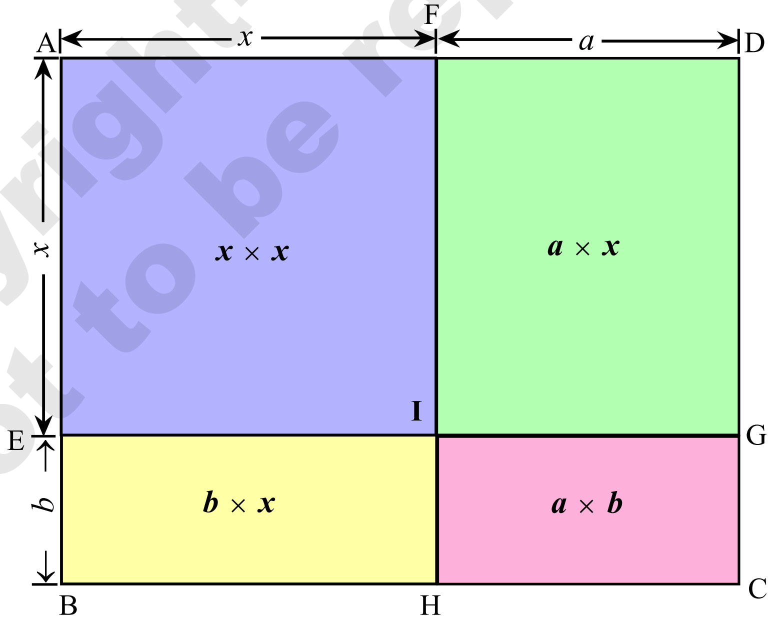


Diagram - 2

Area of the rectangle ABCD = Area of the square AEIF + area of the rectangle FIGD + area of the rectangle EBHI + area of the rectangle IHCG

or, $(x + a) \times (x + b) = x \times x + a \times x + b \times x + a \times b$

or, $(x + a)(x + b) = x^2 + (a + b)x + ab$

Activity : Verify the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ by taking different values of x, a and b.

Example 13 : Find the product by using the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) $(2x + 3)(2x + 7)$

(ii) $(x + 8)(x - 5)$

(iii) $(3x^2 - 5)(3x^2 + 6)$

(iv) $(p^2 - 15)(p^2 - 10)$

(v) 102×97

Solution : (i) $(2x + 3)(2x + 7) = (2x)^2 + (3 + 7)2x + 3 \times 7$

$= 4x^2 + 10 \times 2x + 21$

$= 4x^2 + 20x + 21$

(ii) $(x + 8)(x - 5) = (x + 8)\{x + (-5)\}$

$= x^2 + \{8 + (-5)\}x + 8 \times (-5)$

$= x^2 + (8 - 5)x - 40$

$= x^2 + 3x - 40$

(iii) $(3x^2 - 5)(3x^2 + 6) = \{3x^2 + (-5)\}(3x^2 + 6)$

$= (3x^2)^2 + \{(-5) + 6\} \times 3x^2 + (-5) \times 6$

$= 9x^4 + (6 - 5)3x^2 - 30$

$= 9x^4 + 3x^2 - 30$

(iv) $(p^2 - 15)(p^2 - 10) = (p^2)^2 + \{(-15) + (-10)\} \times p^2 + (-15) \times (-10)$

$= p^4 + (-15 - 10)p^2 + 150$

$= p^4 - 25p^2 + 150$

(v) $102 \times 97 = (100 + 2) \times (100 - 3)$

$= (100)^2 + \{2 + (-3)\} \times 100 + 2 \times (-3)$

$= 10000 + (-1) \times 100 - 6$

$= 10000 - 106$

$= 9894$

9.2.2 Now let us find the value of the square of the binomial expression $(a + b)$

$$
\begin{aligned}
(a+b)^2 &= (a+b)\times(a+b) \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 && [\because \quad a^2 = a \times a\,; \quad b^2 = b \times b] \\
&= a^2 + ab + ab + b^2 && [\text{By Commutative law } ab = ba] \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Therefore $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

or, **(First term + Second term)² = (First term)² + 2 × (First term) × (Second term) + (Second term)²**

Since, the expression in RHS is formed by actual multiplication of the two binomial expressions in LHS, the equality is true for any value of a and b. Hence, it is an identity. To verify this identity geometrically, let us divide a square ABCD of length $(a + b)$ unit into four parts as shown in diagram -2.

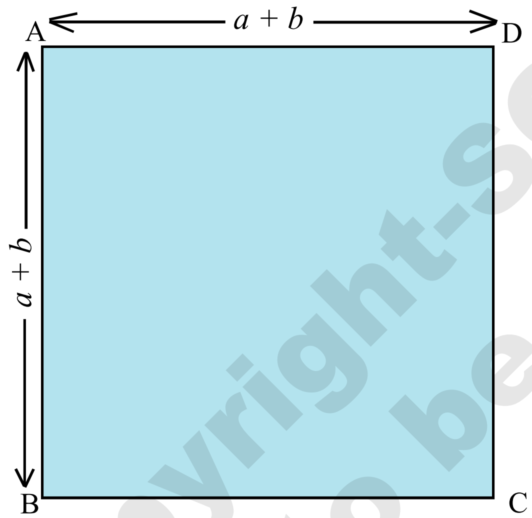


Diagram -1

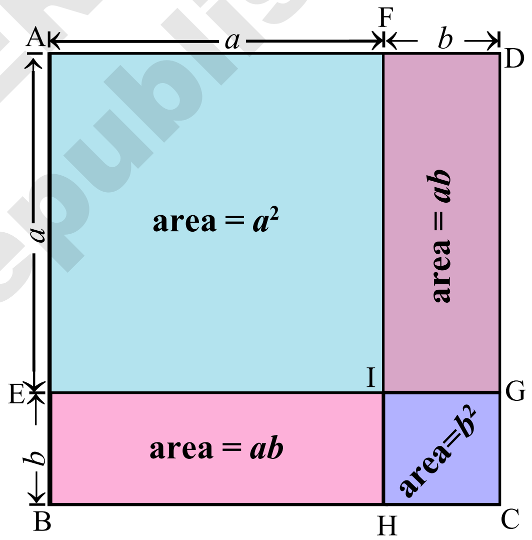


Diagram -2

Area of square ABCD = area of square A EIF + area of rectangle EBHI + area of rectangle FIGD + area of square I HCG

i.e $\quad (a + b) \times (a + b) = a \times a + b \times a + a \times b + b \times b$

$$
\begin{aligned}
\text{or, } (a+b)^2 &= a^2 + ba + ab + b^2 \\
\text{or, } (a+b)^2 &= a^2 + ab + ab + b^2 && [\because\ ab = ba] \\
\text{or, } (a+b)^2 &= a^2 + 2ab + b^2
\end{aligned}
$$

Example 14 : Find the product by using the identity $(a + b)^2 = a^2 + 2ab + b^2$

(i) 203^2 (ii) $(3p + 5)^2$ (iii) $(7x^2 + 2y)^2$

Solution :

(i) $203^2 = (200 + 3)^2$
$= (200)^2 + 2 \times 200 \times 3 + 3^2$
$= 40000 + 1200 + 9$
$= 41209$

(ii) $(3p + 5)^2 = (3p)^2 + 2 \times 3p \times 5 + 5^2$
$= 9p^2 + 30p + 25$

(iii) $(7x^2 + 2y)^2 = (7x^2)^2 + 2 \times 7x^2 \times 2y + (2y)^2$
$= 49x^4 + 28x^2y + 4y^2$

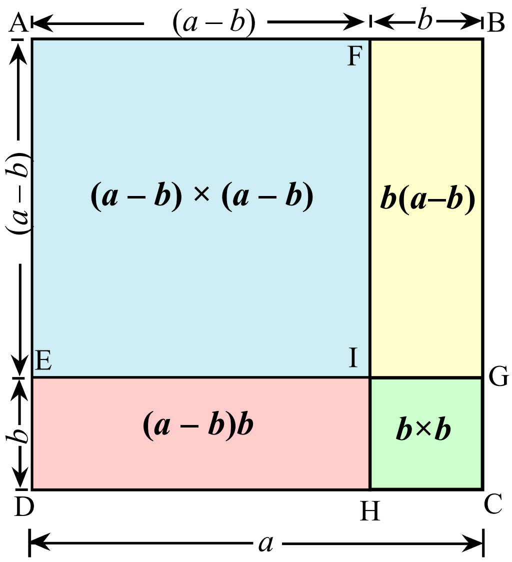
9.2.3 Now, let us find the square of the binomial expression $(a – b)$

$$\begin{aligned}(a - b)^2 &= (a - b) \times (a - b)\\ &= a(a - b) - b(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - ab - ab + b^2 \qquad [\because ab = ba]\\ &= a^2 - 2ab + b^2\end{aligned}$$

Therefore $\boxed{(a - b)^2 = a^2 - 2ab + b^2}$

i.e **(First term – Second term)² = (First term)² – 2 × (First term) × (Second term) + (Second term)²**

For geometrical verification of this identity, let us draw a square ABCD of length a. Then divide this square into two squares of areas $(a - b)^2$ and b^2 and two rectangles, the area of which are $b(a - b)$. From the figure



Area of the square ABCD = area of the square AE I F
+ area of the rectangle FIGB
+ area of the rectangle EDHI
+ area of the square IHCG

or, $a \times a = (a - b)(a - b) + b(a - b) + (a - b)b + b \times b$

or, $a^2 = (a - b)^2 + ba - b^2 + ab - b^2 + b^2$

or, $a^2 = (a - b)^2 + ab + ab - b^2 - b^2 + b^2$

or, $a^2 = (a - b)^2 + 2ab - b^2$

or, $(a - b)^2 + 2ab - b^2 = a^2$

or, $(a - b)^2 = a^2 - 2ab + b^2$

Example 15 : Evaluate the following by applying the identity $(a-b)^2 = a^2 - 2ab + b^2$

(i) 498^2 (ii) $(7a+2b)^2$ (iii) $\left(\frac{2}{3}x^2 - 5\right)^2$

Solution : (i) 498^2

$$= (500-2)^2$$
$$= (500)^2 - 2 \times 500 \times 2 + 2^2$$
$$= 250000 - 2000 + 4$$
$$= 248000 + 4$$
$$= 248004$$

(ii) $(7a-2b)^2$

$$= (7a)^2 - 2 \times 7a \times 2b + (2b)^2$$
$$= 49a^2 - 28ab + 4b^2$$

(iii) $\left(\frac{2}{3}x^2 - 5\right)^2$

$$= \left(\frac{2}{3}x^2\right)^2 - 2 \times \frac{2}{3}x^2 \times 5 + 5^2$$
$$= \frac{4}{9}x^4 - \frac{2 \times 2 \times 5}{3}x^2 + 25$$
$$= \frac{4}{9}x^4 - \frac{20}{3}x^2 + 25$$

9.2.4 **Multiplication of the binomial expression (*a*+*b*) by the binomial expression (*a* – *b*)**

$$(a+b)(a-b) = a(a-b) + b(a-b)$$
$$= a^2 - ab + ba - b^2$$
$$= a^2 - ab + ab - b^2 \quad [\because ab = ba]$$
$$= a^2 - b^2$$

i.e. $\boxed{(a+b)(a-b) = a^2 - b^2}$

Similarly, $\boxed{(a+b)(b-a) = b^2 - a^2}$

i.e **(First term + Second term) × (First term – Second term) = (First term)^2 – (Second term)^2**

This identity can also be verified geometrically.

Draw a square ABCD of side a. Draw another square ALMN so that AN = AL= b ($b < a$) inside ABCD as shown in figure -I.

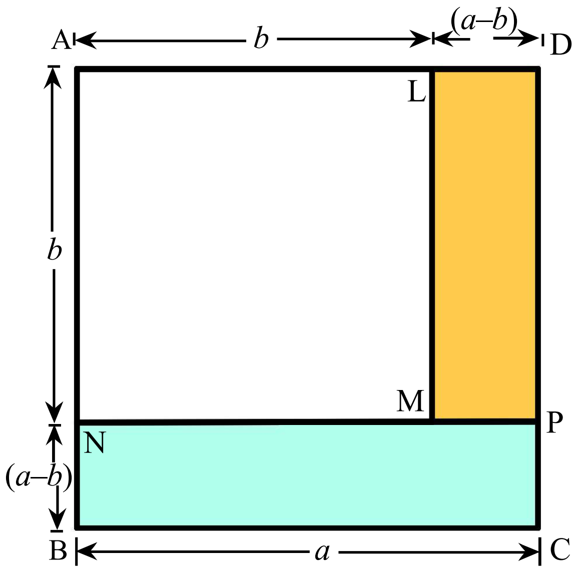


Figure - 1

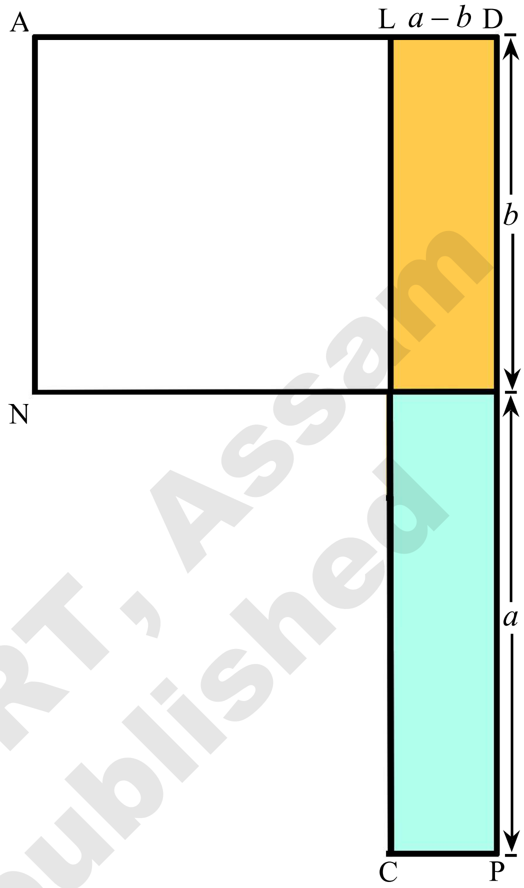


Figure - 3

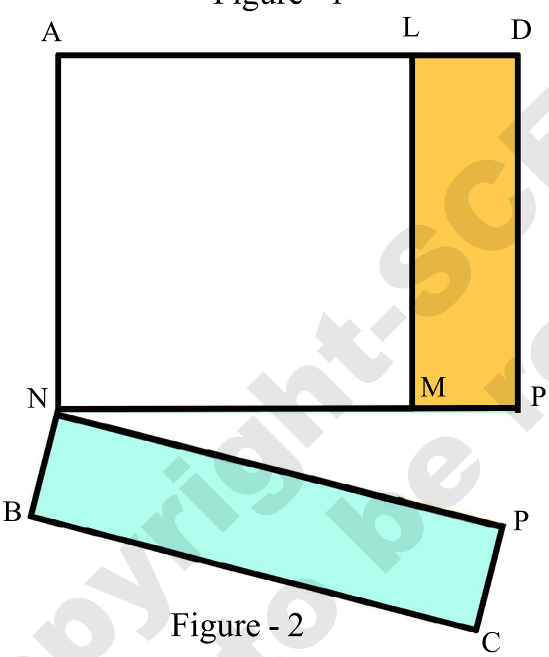


Figure - 2

Now, separate the rectangle NBCP from the square ABCD as shown in figure - 2 and attach the side MP of the rectangle LMPD with the side BN of the rectangle NBCP as shown in Figure - 3

Coloured portion in Figure - 3

= Area of the rectangle LCPD

= LD × DP

$= (a - b)\ (a + b)$

$= (a + b)\ (a - b)$

On the other hand, from the Figure -1, we get ,

Area of coloured portion= Area of the square ABCD – Area of the square ANML

$= a \times a - b \times b$

$= a^2 - b^2$

Since the areas of coloured portions in all figures are equal

$\therefore \quad (a + b)(a - b) = a^2 - b^2$

Example 16 : Find the product of the following using the identity $(a + b)(a - b) = a^2 - b^2$

(i) 102×98 (ii) $(x + 2y)(x - 2y)$ (iii) $(11xy + 3x)(11xy - 3x)$

Solution :

(i) $102 \times 98 = (100 + 2) \times (100 - 2)$
$= (100)^2 - 2^2$
$= 10000 - 4$
$= 9996$

(ii) $(x + 2y)(x - 2y) = x^2 - (2y)^2$
$= x^2 - 4y^2$

(iii) $(11xy + 3x)(11xy - 3x) = (11xy)^2 - (3x)^2$
$= 121x^2y^2 - 9x^2$

Identities used frequently in Algebra are –

1. $(x + a)(x + b) = x^2 + (a + b)x + ab$
2. $(a + b)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2 = a^2 - 2ab + b^2$
4. $(a + b)(a - b) = a^2 - b^2$

Exercise 9.2

1. Find the product using the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) $(x + 7)(x + 5)$ (ii) $(7x + 2y)(7x + 6y)$

(iii) $(4x^3 + 8)(4x^3 + 10)$ (iv) $(4k^2 - 3k)(4k^2 - 7k)$

(v) $\left(\frac{a}{2}+\frac{1}{2}\right)\left(\frac{a}{2}-\frac{1}{4}\right)$ (vi) $\left(\frac{n^2}{5}-0.6\right)\left(\frac{n^2}{5}+1.6\right)$

(vii) 98×97 (viii) 501×503

2. Evaluate the following using the identity $(a + b)^2 = a^2 + 2ab + b^2$

(i) $(x + 5)^2$ (ii) $(5x + 4y)^2$ (iii) $(3a^3 + 4a^2)^2$

(iv) $\left(3x+\frac{1}{3x}\right)^2$ (v) $\left(\frac{p}{q}+\frac{q}{p}\right)^2$ (vi) 502^2

(vii) $(9.5)^2$ (viii) $\left(4\frac{1}{8}\right)^2$

3. Evaluate the following using the identity $(a - b)^2 = a^2 - 2ab + b^2$

(i) $(x - 7)^2$ (ii) $(6x - 5)^2$ (iii) $(10x^2 - 3y)^2$

(iv) $(p^2 - q^2)^2$ (v) $(a^2x - ax^2)^2$ (vi) $\left(x^2-\frac{1}{x^2}\right)^2$

(vii) 296^2 (viii) 1999^2

4. Find the product of the following expressions using the identity $(a + b)(a - b) = a^2 - b^2$

(i) $(y + 11)(y - 11)$ (ii) $(2x + 3)(2x - 3)$
(iii) $(6 + m^2)(m^2 - 6)$ (iv) $(ax^2 - by)(ax^2 + by)$
(v) $(1 - x^m)(1 + x^m)$ (vi) 61×59
(vii) 106×94 (viii) 9.5×8.5

5. Find the product of the following expressions using proper identities.

(i) $(3x - 5m)(3x - 5m)$ (ii) $(4m + 3)(4m + 2)$

(iii) $(9 + 4n)^2$ (iv) $\left(6x+\frac{1}{3}\right)(6x + 3)$

(v) $(4ab - c)(4ab + c)$ (vi) $\left(x-\frac{x}{2}\right)^2$

(vii) $\left(\frac{a^2}{2}+\frac{b^2}{4}\right)\left(\frac{a^2}{2}+\frac{b^2}{4}\right)$ (viii) $(0.5x^2 - 0.2y^2)^2$

(ix) $(-9x^2 + y^3)(9x^2 + y^3)$ (x) $\left(\frac{y^2}{2}-4\right)\left(\frac{y^2}{2}+6\right)$

(xi) $\left(7x^2+\frac{1}{3}\right)^2$ (xii) $(x + y + z)(x + y - z)$

(xiii) 1002×999 (xiv) $(10.2)^2$

(xv) 79^2 (xvi) 6.2×5.8

6. Simplify.

(i) $\left(x+\frac{1}{x}\right)\left(2x+\frac{1}{x}\right)$

(ii) $(2l + m)^2 - (2l - m)^2$

(iii) $(a^2b + ab^2)^2 - 6a^3b^3$

(iv) $(x + y)(x - y) + (y + z)(y - z) + (z + x)(z - x)$

(v) $(5a - 6b)^2 + 20ab - (6b + 5a)^2$

(vi) $(4p^2 + 5q^2)(4p^2 - 5q^2) + (2p^2 - 5q^2)^2$

(vii) $(2x - 5)(2x + 3) - (x - 2)^2 + 29$

(viii) $\left(\frac{x}{3}-\frac{3y}{4}\right)\left(\frac{x}{3}+\frac{3y}{4}\right)+\left(\frac{3y}{4}+3x\right)\left(\frac{3y}{4}+x\right)$

(ix) $\left(\frac{x}{5}+\frac{y}{5}\right)^2+2\left(\frac{x}{5}+\frac{y}{5}\right)\left(\frac{x}{5}-\frac{y}{5}\right)+\left(\frac{x}{5}-\frac{y}{5}\right)^2$

(x) $2.89 \times 2.89 + 0.22 \times 2.89 + 0.0121$

(xi) $\frac{0.25-2\times 0.5\times 3.5+12.25}{3}$

(xii) $\frac{4.68\times 4.68-3.32\times 3.32}{1.36}$

7. Show that

(i) $(a - b + c - d)^2 - (a + b - c + d)^2 + 4a(b + d) = 4ca$

(ii) $(1.5x^2 + 1.2y)^2 - (1.5x^2 - 1.2y)^2 = 7.2x^2y$

(iii) $\left(\frac{2}{3}x^2+5\right)^2-25=\frac{4}{9}x^4+\frac{20}{3}x^2$

(iv) $(a^2 + b^2)(a^2 - b^2) + (b^2 + c^2)(b^2 - c^2) + 2c^2(c^2 - a^2) = (c^2 - a^2)^2$

8. Solve the following problems using suitable identities

(i) Find the area of a rectangular playground of length $(x + 8)$ metres and breadth 3 metres less than the length.

(ii) Find the area of a square garden whose length is $\left(2x+\frac{1}{4}\right)$ metres.

(iii) In a field, a cultivator cultivates two varieties of Paddy namely *Joha* and *Bora* in two square shaped land. The length of the field where *Joha* is cultivated is 5 m more than that where *Bora* is cultivated. Find the difference between the areas of two fields.

(iv) The cost of painting a wall is Rs. 9.00 per square metre. Determine the cost of painting a wall of length 107 m and breadth 93 m.

(v) Find the area of a square field whose length is 197 m.

(vi) If $x+\frac{1}{x}=3$, find the values of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$.

(vii) If $2x-\frac{1}{2x}=2$, then find the values of $4x^2+\frac{1}{4x^2}$ and $16x^4-\frac{1}{16x^4}$.

(viii) If $a-b=10$, $ab=11$, find the value of $a+b$.

1. Numerical co-efficient in multiplication of two or more algebraic expressions = Product of their numerical co-efficients.

2. Some important identities.

   (i) $(a+b)^2 = a^2+2ab+b^2$

   (ii) $(a-b)^2 = a^2-2ab+b^2$

   (iii) $(a+b)(a-b) = a^2-b^2$

   (iv) $(x+a)(x+b) = x^2+(a+b)x+ab$

   These identities can be represented geometrically.

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