CBSE Class 10th Mathematics

Basic Sample Paper - 10

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. Construction of cumulative frequency table is useful to determine

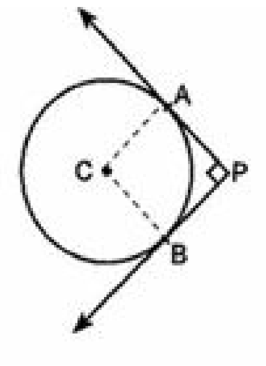
   a. mean

   b. all the three

   c. median

   d. mode

2. In the given figure, the pair of tangents A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm, then the radius of the circle is :



a. 2.5 cm

b. 5 cm

c. 7.5 cm

d. 10 cm

3. What is a lemma?

a. contradictory statement

b. proven statement

c. no statement

d. None of these

4. For what least value of 'n' a natural number, $(24)^n$ is divisible by 8?

a. 1

b. 0

c. 2

d. No value of n is possible

5. The LCM of 24, 60 and 150 is

a. 2400

b. 1800

c. 600

d. 1200

6. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of 'k' is

a. – 10

b. – 5

c. 10

d. 5

7. The number of zeroes of a cubic polynomial is

a. at most 3

b. 3

c. at least 3

d. 2

8. When a die is thrown, the probability of getting an odd number less than 3 is

a. $\frac{1}{3}$

b. $\frac{1}{6}$

c. $\frac{1}{4}$

d. $\frac{1}{2}$

9. If the vertices of a triangle are (1, 1), (– 2, 7) and (3, – 3), then its area is

a. 0 sq. units

b. 2 sq. units

c. 24 sq. units

d. 12 sq. units

10. The distance of the point (– 5, 12) from the y-axis is

    a. 12 units

    b. 5 units

    c. 13 units

    d. - 5 units

11. Fill in the blanks:

    $\triangle$ABC and $\triangle$DEF are similar and $AB = \frac{1}{3}DE$, then ar($\triangle$ABC): ar($\triangle$DEF) is ________.

12. Fill in the blanks:

    The mirror image of (3, 9) on x-axis is ________.

OR

Fill in the blanks:

Coordinates of the mid-point of the line segment joining the points A(x_1, y_1) and B(x_2, y_2) are given by ________.

13. Fill in the blanks:

    Two angles are said to be ________ if their sum is equal to 90^o.

14. Fill in the blanks:

    In right angled triangle, the square of the ________ is equal to the sum of the squares of the other two sides.

15. Fill in the blanks:

    If x tan 45^ocos60^o = sin60^o cot 60^o, then the value of x is ________.

16. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^0. Find the length of the string assuming that there is no slack in the string.

OR

A ladder 15 m long leans against a wall making an angle of 60^0 with the wall. Find the height of the wall from the point the ladder touches the wall.

17. For what value of k, k + 2 ,4k - 6 ,3k - 2 are three consecutive terms of an A.P.

18. A chord of a circle of radius 10 cm subtends a right angle at its centre. What is the length of the chord.

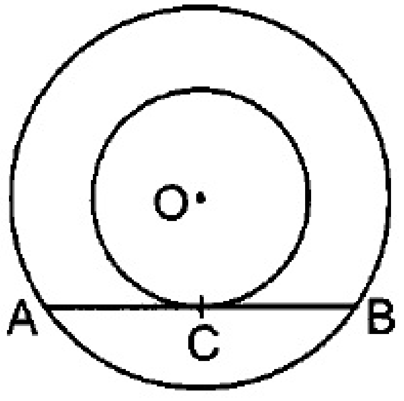
19. If $\triangle$ABC and $\triangle$DEF are triangles such that $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{4}{7}$. Find $\frac{\text{area }\triangle\text{ABC}}{\text{area }\triangle\text{DEF}}$.

20. A die is thrown. Find the probability of getting a number greater than 5.

Section B

21. Find the zeros of q(x) = $\sqrt{3}x^2+10x+7\sqrt{3}$ and verify the relationship between the zeros and their coefficients.

22. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC.



OR

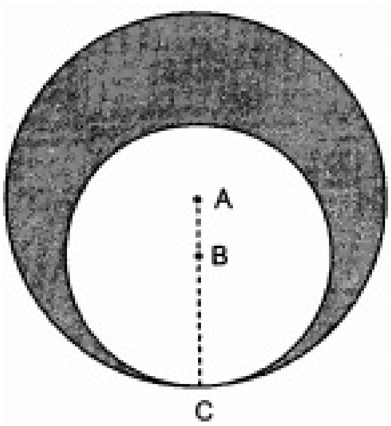
Two concentric circles are of radii 7 cm and r cm respectively where r > 7. A chord of the larger circle of the length 48 cm, touches the smaller circle. Find the value of r.

23. Prove that: $\sin\left(90^0-\theta\right)\cos(90^\circ-\theta)=\frac{\tan\theta}{1+\tan^2\theta}$

OR

Evaluate: $\sec^2 36^o - \cot^2 54^o$

24. In figure, two circles with centres A and B touch each other at the point C. If AC = 8 cm and AB = 3 cm, find the area of the shaded region.



25. Two dice are numbered 1, 2, 3 ,4 ,5 , 6 and 1, 2, 2, 3, 3, 4 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting (i) sum 7 (ii) sum is a perfect square.

26. All kings, jacks and diamonds have been removed from a pack of cards and the remaining cards are well shuffled. A card is drawn at random. Find the probability that it is

i. a red queen
ii. a face card
iii. a diamond
iv. a black card

Section C

27. Find all zeros of $f(x)=2x^4-2x^3-7x^2+3x+6$ if its two zeros are $\sqrt{\frac{3}{2}}\ and\ -\sqrt{\frac{3}{2}}$.

28. Draw a line segment of length 7 cm. Find a point P on it, which divides it in the ratio 3 : 5.

OR

Construct a $\triangle$ABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle$ABC = 60°. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle$ABC.

29. A container open at the top is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs 21 per litre. (Use π = 22/7).

30. Prove the identity:

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$$

OR

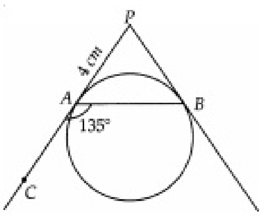
Prove that: (cosecθ - sinθ) (secθ - cosθ) (tanθ + cotθ) = 1.

31. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin?

OR

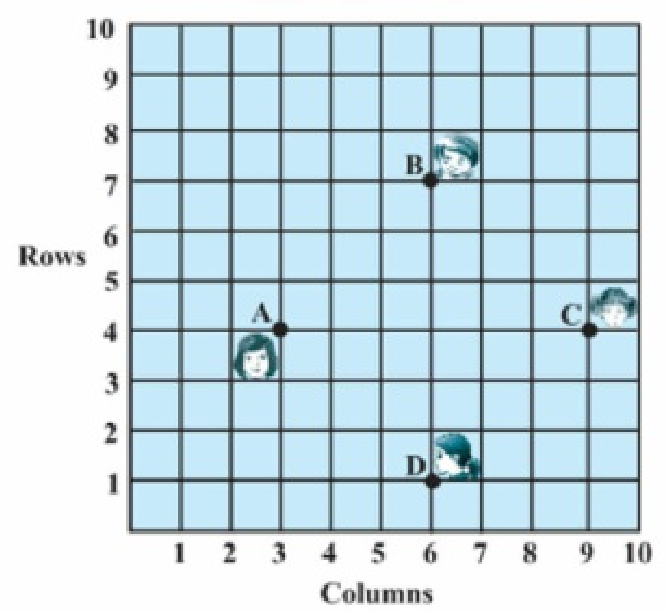
What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

32. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm and $\angle BAC = 135^\circ$. Find the length of chord AB.



33. Read the following passage and answer the question that follows:

In a class room, four student Sita, Gita, Rita and Anita are sitting at A(3, 4), B(6, 7), C(9, 4), D(6, 1) respectively. Then a new student Anjali joins the class.



i. Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.
ii. Calculate the distance between Sita and Anita.
iii. Which two students are equidistant from Gita.

34. A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3}$ = 1.732)

Section D

35. If the roots of the quadratic equation (x - a) (x - b) + (x - b)(x - c) + (x - c)(x - a) = 0 are equal. Then show that a = b = c

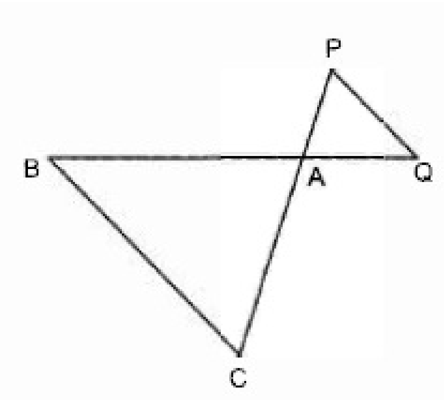
36. If Raghav buys a shop for Rs.1, 20,000. He pays half of the amount in cash and agrees to pay the balance in 12 annual installments of Rs.5000 each. If the rate of interest is 12%, and he pays with the installment the interest due on the unpaid amount. Find the total cost of the shop.

OR

Find the sum of all integers between 100 and 550, which are divisible by 9.

37. A fraction becomes $\frac{9}{11}$ if 2 is added to both 11 numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction by substitution method.

38. In Fig. ΔACB ~ ΔAPQ. If BC= 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ. Also, find the area (ΔACB): area (ΔAPQ).



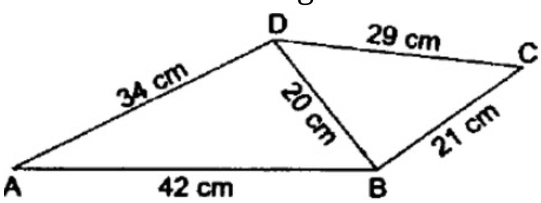
OR

In a ΔABC, D and E are points on AB and AC respectively such that $DE \| BC$. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

39. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km/hr, in how much time will the tank be filled completely?

OR

Find the area of the quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.



40. Compute the median from the following data:

Mid-value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

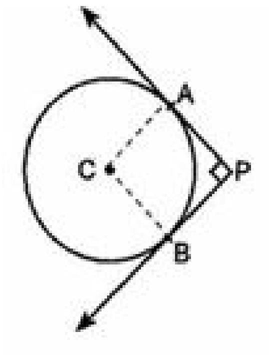
Solution

Section A

1. (c) median
   Explanation:

   A cumulative frequency distribution is the sum of the class and all classes below it in a frequency distribution. Construction of cumulative frequency table is useful to determine Median.

2. (b) 5 cm
   Explanation:

   Construction: Joined OA and OB.
   Here OA$\perp$ AP
   and OB$\perp$BP
   and PA $\perp$PB
   Also AP = PB
   Therefore, APBO is a square.
   $\Rightarrow$ AP = OA = OB = 5 cm

3. (b) proven statement
   Explanation:

   A lemma is a proven statement that is used to prove another statement.

4. (a) 1

Explanation:

$(24)^1 = 24$

$\frac{24}{8} = 3$

Hence, the least value of n is 1.

5. (c) 600

   Explanation:

   $24 = 2^3 \times 3$

   $60 = 2^2 \times 3 \times 5$

   $150 = 2 \times 3 \times 5^2$

   $\therefore LCM(24, 60, 150) = 2^3 \times 3 \times 5^2 = 600$

6. (a) – 10

   Explanation:

   Given Polynomial is $p(x) = x^2 + 3x + k$

   According to question, $p(x) = 0$ (Put x = 2)

   p(2) = 0

   $\Rightarrow (2)^2 + 3 \times 2 + k = 0$

   $\Rightarrow 4 + 6 + k = 0$

   $\Rightarrow k = -10$

7. (a) at most 3

   Explanation:

   The number of zeroes of a cubic polynomial is at most 3 because the highest power of the variable in cubic polynomial is 3, i.e. $ax^3 + bx^2 + cx + d$

8. (b) $\frac{1}{6}$

Explanation:

Odd outcomes are 1 , 3 , 5 but 3 and 5 are not less than 3

Number of possible outcomes whtch are odd and less than 3 = 1

Number of possible outcomes = {1} = 1

Number of Total outcomes = 6

$\therefore$ Required Probability = $\frac{1}{6}$

9. (a) 0 sq. units

   Explanation:

   Given: $(x_1, y_1) = (1, 1), (x_2, y_2) = (-2, 7)$ and $(x_3, y_3) = (3, -3)$, then the Area of triangle

   $\therefore \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
   = $\frac{1}{2}|1(7+3) + (-2)(-3-1) + 3(1-7)|$
   = $\frac{1}{2}|10 + 8 - 18|$

   = $\frac{1}{2}|0|$ = 0 sq. units

   Also therefore the three given points(vertices) are collinear.

10. (b) 5 units

    Explanation:

    The distance of any point from $y-$axis is its abscissa. Therefore, the required distance is 5 units.

11. 1:9

12. (-3, -9)

    OR

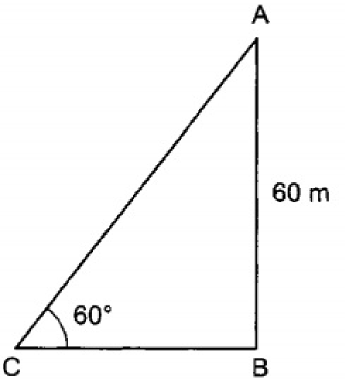
    $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

13. complementary

14. hypotenuse

15. x = 1

16. Let A be the kite and CA be the string attached to the kite such that its one end is tied to a point C on the ground. The inclination of the string CA with the ground is 60^{o}.
In $\triangle ABC$, we are given that $\angle C = 60^{\circ}$ and perpendicular AB = 60 m and we have to find hypotenuse AC. So, we use the trigonometric ratio involving perpendicular and hypotenuse.



In $\triangle ABC$, we have

$\sin C = \frac{AB}{AC}$

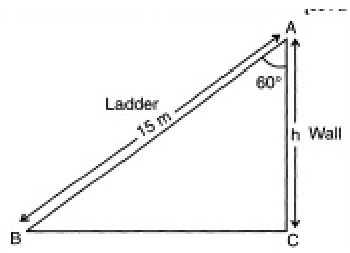
$\Rightarrow \quad \sin 60^{\circ} = \frac{AB}{AC}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$

$\Rightarrow \quad AC = \frac{120}{\sqrt{3}} = 40\sqrt{3}\mathrm{m}.$

Hence, the length of the string is $40\sqrt{3}\mathrm{m}$.

OR



Let ABC be a right angled triangle where AB isis ladder = 15m and angle a = 60°

Let AC be the height of the wall

Therefore by Pythagoras theorem

$\frac{h}{15} = \cos 60^{\circ}$

$\Rightarrow \quad h = 15 \times \cos 60^{\circ}$

$= 15 \times \frac{1}{2}$

= 7.5 m

17. Three consecutive terms k + 2 ,4k - 6 and 3k - 2 are in A.P.

or, $(4k-6)-(k+2)=(3k-2)-(4k-6)$

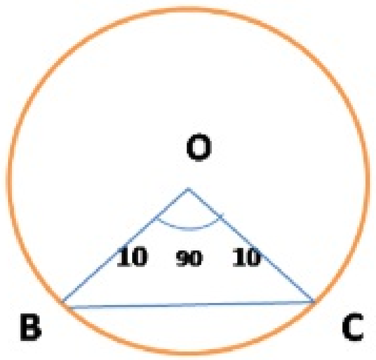
or, $4k-6-k-2=3k-2-4k+6$

or, $3k-8=-k+4$

or, $4k=4+8$

or, $4k=12$

or, k = $\frac{12}{4}$ = 3

18. 

from right angle triangle BOC,

$BC^2 = OB^2 + OC^2$

$\Rightarrow BC^2 = 10^2 + 10^2$

$\Rightarrow BC^2 = 100 + 100$

$\Rightarrow BC^2 = 200$

$\Rightarrow BC = \sqrt{200}$

$\Rightarrow BC = 10\sqrt{2}$

So, the length of the chord is $10\sqrt{2}$ cm.

19. $\because \frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

$\therefore \quad \triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$

Since for similar triangles,the ratio of the areas is the square of their corresponding sides.

$\Rightarrow \quad \frac{area\triangle \mathrm{ABC}}{area\triangle DEF}=\frac{\mathrm{AB}^2}{\mathrm{DE}^2}=\left(\frac{AB}{DE}\right)^2=\left(\frac{4}{7}\right)^2=\frac{16}{49}$

20. When a die is thrown then the possible outcomes are: 1,2,3,4,5,6.

Therefore, the total number of possible outcomes = 6

Clearly, 6 is the only outcomes that is greater than 5, so number of favourable outcomes = 1

Therefore,Required probability = $\frac{no.\ of\ favorable\ outcomes}{total\ possible\ outcomes}=\frac{1}{6}$.

Section B

21. q(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

$= \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}$

$= \sqrt{3}x(x + \sqrt{3})+7(x + \sqrt{3})$

$= (x + \sqrt{3})(\sqrt{3}x+7)$

q(x)=0 if $x+\sqrt{3}=0$ or $\sqrt{3}\ x+7=0$

Zeros of the polynomials are $-\sqrt{3}$ and $\frac{-7}{\sqrt{3}}$

In q(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

Sum of zeros=$-\sqrt{3} - \frac{7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} = \frac{-b}{a}$

Product of zeros= $-\sqrt{3} \times \frac{-7}{\sqrt{3}} = 7 = \frac{7\sqrt{3}}{\sqrt{3}} = \frac{c}{a}$

Hence, the relationship is verified.

22. Construction : Draw OC

Proof : Line AB is tangent to smaller circle at point C.

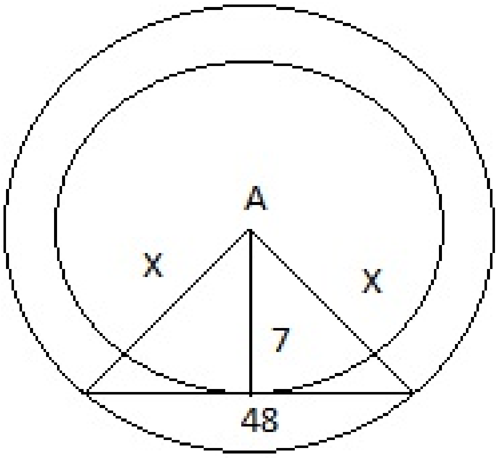
$\therefore$ segment $OC \perp AB$

AB is chord to larger circle and

as perpendicular drawn from centre to chord bisects the chord.

$\therefore AC = CB$

OR



Let us take r = x

Now using Pythagoras theorem

$(x)^2 = 24^2 + 7^2$

$(x)^2 = 576 + 49$

$(x)^2 = 625$

Therefore, x = 25 cm.

r = 25 cm.

23. $\sin\left(90^0-\theta\right)\cos\left(90^0-\theta\right)=\frac{\tan\theta}{1+\tan^2\theta}$

$\text{L. H.S. }=\sin\left(90^0-\theta\right)\cos(90^\circ-\theta)$

$=\cos\theta\sin\theta\left[\because\sin\left(90^0-\theta\right)=\cos\theta\,\text{and}\cos(90^\circ-\theta)=\sin\theta\right]$

$\text{R.H.S. }=\frac{\tan\theta}{1+\tan^2\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{\sec^2\theta}\left[\because 1+\tan^2\theta=\sec^2\text{and}\tan\theta=\frac{\sin\theta}{\cos\theta}\right]$

$=\frac{\sin\theta}{\cos\theta}\cos^2\theta=\sin\theta\cos\theta=\text{ L.H.S. proved}$

OR

$\sec^2 36^o - \cot^2 54^o = [\sec(90^o - 54^o)]^2 - \cot^2 54^o$

$= (\text{cosec } 54^o)^2 - \cot^2 54^o \left[\because\sec\left(90^\circ-\theta\right)=\cos ec\theta\right]$

$= (\text{cosec}^2 54^o - \cot^2 54^o) = 1 \left[\because\cos ec^2\theta-\cot^2\theta=1\right].$

24. AC = 8 cm, AB = 3 cm

Then BC = 8 - 3 = 5 cm

Therefore, radius of big circle=AC=8 cm

and radius of small circle=BC=5 cm

$\therefore$ Area of shaded region = Area of big circle - Area of small circle

$= \pi(AC)^2 - \pi(BC)^2$

$= \frac{22}{7}(8)^2 - \frac{22}{7}(5)^2$

$= \frac{22}{7}[64 - 25]$

$= \frac{22}{7} \times 39$

= 122.57 cm^2

25. Total possible outcomes $= 36$

    i. Sum of number $= 7$

       Favourable outcomes are $(3,4),(4,3),(4,3),(5,2),(5,2),(6,1)$

       Favourable ways $= 6$

       Probability that sum of number is 7 = $\frac{6}{36} = \frac{1}{6}$

    ii. Sum is a perfect square i.e., sum is 4 or 9

       Favourable outcomes are $(1,3),(1,3),(2,2),(2,2),(5,4),(6,3),(6,3) = 7$

       Probability = $\frac{7}{36}$

26. When all kings, jacks and diamonds have been removed, number of cards remaining

    = 52 - (4+4+11) = 52 - 19 = 33

    Total no. of outcomes = 33

    i. Let A be the event of getting a red queen.

       Thus, favorable outcomes = 1

       P(A) = $\frac{1}{33}$

    ii. Let B be the event of getting a face card

       Thus, favorable outcomes = 3

       P(B) = $\frac{3}{33} = \frac{1}{11}$

    iii. Let C be the event of getting a diamond.

       Thus, favorable outcomes = 0 (all diamonds are removed)

       P(C) = 0

    iv. Let D be the event of getting a black card

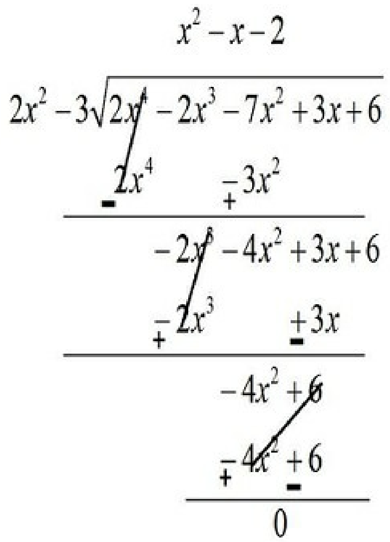
       Black cards left are : 11 clubs+11 spades=22

       Thus, favorable outcomes = 22

       P(D) = $\frac{22}{33} = \frac{2}{3}$

Section C

27. $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$
Two zeros are $\pm\sqrt{\frac{3}{2}}$
$\therefore \left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right) = \frac{1}{2}\left(2x^2-3\right)$
$\therefore \left(2x^2-3\right)$ is the factor of f(x) is



g(x) = x^2 - x - 2

= x^2 - 2x + x - 2

= x(x - 2) + 1(x - 2)

= (x + 1) (x - 2)

$\therefore$ other two zeros are

x + 1 = 0 or x = -1

and x - 2 = 0 or x = 2

$\therefore$ other two zeros are -1 and 2

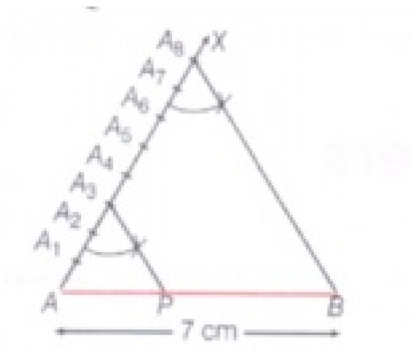
28. We have to draw a line segment of length 7 cm.Then,we have to find a point P on it, which divides it in the ratio 3 : 5.
Steps of construction:

i. Draw a line segment $AB = 7$ cm.
ii. Draw a ray AX, making an acute $\angle BAX$ with AB.
iii. Mark $3 + 5 = 8$ points, i.e, $A_1, A_2, A_3, A_4 \ldots A_8$ on AX, such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 \ldots = A_7A_8$

iv. Join A_8 B

v. From A_3, draw $A_3P||A_8B$ which intersects AB at point P [by making an angle at A_3 equal to $\angle AA_8B$

Then, P is the point on AB which divides it in the ratio 3:5. So, $AP : PB = 3 : 5$



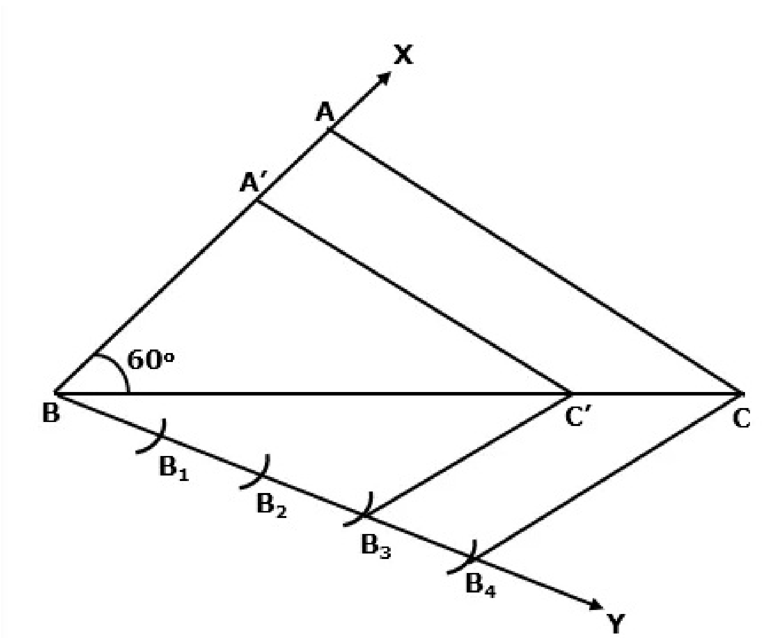
Justification: In $\triangle ABA_8$, we have $A_3P||A_8B$

$\therefore \frac{AP}{PB} = \frac{AA_3}{A_3A_8}$ [by basic proportionality theorem]

By construction, $\frac{AA_3}{A_3A_8} = \frac{3}{5}$

Hence, $\frac{AP}{PB} = \frac{3}{5}$

OR



Steps of construction:

1. Draw a line segment BC = 6.5 cm.
2. At B, construct $\angle$CBX = 60°.

3. With B as centre and radius 4.5 cm, draw an arc intersecting BX at A.
4. Join AC to obtain $\triangle$ABC
5. Below BC, make an acute $\angle$CBY
6. Along BY, mark off 4 points (greater of 3 and 4 in $\frac{3}{4}$)
   B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
7. Join B_4C.
8. From point B_4, draw a line parallel to B_4C intersecting BC at C'.
9. From point C', draw a line parallel to AC intersecting AB at A'.

Thus, $\triangle$A'BC' is the required triangle.

29. According to the question,

Radius of the upper end of container (r_1) = 20 cm

Radius of the lower end of container (r_2) = 8 cm

Height of the container (h) = 24 cm

Therefore,Slant height of frustum $(l) = \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + 24^2}$

$= \sqrt{12^2 + 24^2}$

$= \sqrt{144 + 576}$

= 26.83 cm

Therefore, Capacity of the container = Volume of frustum

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left(20^2 + 8^2 + 20 \times 8\right)$

$= \frac{528}{21} \times (400 + 64 + 160)$

= 15689.14 cm^3

= 15.68914 litres

Cost of 1 litre milk = Rs. 21

Cost of 15.68914 litre milk = 15.68914 $\times$ 21 = Rs.329.47

30. We have,

$\text{LHS} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$\Rightarrow \quad \text{LHS} = \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$ [∵ $\sec^2\theta$ - $\tan^2\theta$ = 1]

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)[1-(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)(1-\sec\theta+\tan\theta)}{(\tan\theta-\sec\theta+1)}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)(\tan\theta-\sec\theta+1)}{(\tan\theta-\sec\theta+1)}$

$\Rightarrow \quad LHS = \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$ = RHS

OR

LHS =$(cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta)$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

= $\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cdot\cos\theta}\right)$

= $= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta\cdot\cos\theta}\right)$ $[\because sin^2\theta + cos^2\theta = 1]$

= $\cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta}$

= 1=RHS

31. The required greatest capacity is the HCF of 120, 180 and 240.

$240 = 180 \times 1 + 60$

$180 = 60 \times 3 + 0$

HCF is 60.

Now HCF of 60, 120

$120 = 60 \times 2 + 0$

∴ HCF of 120, 180 and 240 is 60.

∴ The required capacity is 60 litres.

OR

First of all we subtract the remainders 1,2,and 3 from 626,3127,and 15628 respectively

626 - 1 = 625,

3127 - 2 = 3125 and

15628 - 3 = 15625

Finding the HCF of 625 and 3125 by applying Euclid's division lemma.

3125 = 625 $\times$ 5 + 0

so HCF (625,3125)=625

Finding the HCF of 625 and 15625 by applying Euclid's division lemma.

15625 = 625 $\times$ 25 + 0

So HCF (625,15625)=625

Hence, HCF (625,3125,15625) = 625.

Therefore the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively = 625

32. In the given figure, PA and PB are tangents to a circle from an external point P such that PA =4 cm and $\angle BAC = 135^\circ$. we have to find the length of chord AB.

PA = PB = 4 cm

(Tangents from external point)

$\angle PAB = 180^\circ - 135^\circ = 45^\circ$

(Supplementary angles)

$\angle ABP = \angle PAB = 45^\circ$

(Opposite angles of equal sides)

$\therefore \quad \angle APB = 180^\circ - 45^\circ - 45^\circ$

$= 90^\circ$

So $\triangle ABP$ is a isosceles right angled triangle

$\Rightarrow \quad AB^2 = 2AP^2$

$\Rightarrow \quad AB^2 = 32$

Hence $AB = \sqrt{32} = 4\sqrt{2}\text{cm}$

33. i. Given: A(3, 4), B(6, 7), C(9, 4), D(6, 1)

Using distance formula,

AB = $\sqrt{(6-3)^2+(7-4)^2} = 3\sqrt{2}$ units

BC = $\sqrt{(9-6)^2+(4-7)^2} = 3\sqrt{2}$ units

CD = $\sqrt{(6-9)^2+(1-4)^2} = 3\sqrt{2}$ units

DA = $\sqrt{(3-6)^2+(4-1)^2} = 3\sqrt{2}$ units

AC = $\sqrt{(9-3)^2+(4-4)^2} = 6$ units

BD = $\sqrt{(6-6)^2+(1-7)^2} = 6$ units

As sides AB = BC = CD = DA, and diagonals AC and BD are equal, so ABCD is a square.

Now as diagonals of a square bisect each other, so midpoint of the diagonal gives the position of Anjali to sit in the middle of the four students.

Here diagonal is AC or BD.

So, mid-point of AC = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

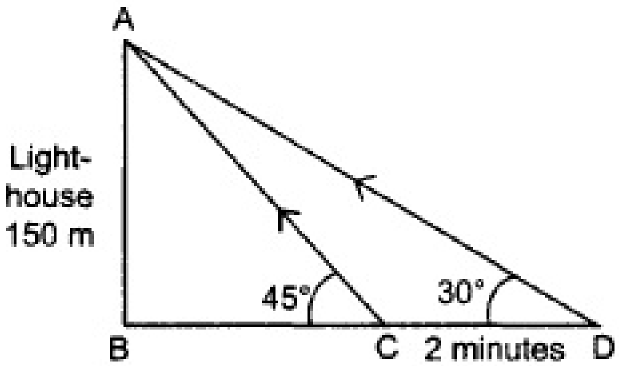
So, position of Anjali is (6, 4).

ii. Position of Sita is at point A i.e. (3, 4) and Position of Anita is at point D i.e. (6, 1).

So, distance between Sita and Anita, AD = $\sqrt{(6-3)^2+(1-4)^2}=3\sqrt{2}$ units

iii. Now, Gita is at position B and as BA and BC are equal and equidistant from point B.

So, we can say Sita and Rita are the two students who are equidistant from Gita.

34. 

AB = 150 m

Initially boat is at C and after 2 minutes it reaches at D.

In right $\triangle$ABC,

$\frac{\text{AB}}{\text{BC}}=\tan 45^{\circ}$

$\Rightarrow \frac{150}{\text{BC}}=1 \Rightarrow \text{BC}=150\text{m}$

In right $\triangle$ABD, $\frac{\text{AB}}{\text{BD}}=\tan 30^{\circ}$

$\Rightarrow \frac{150}{\text{BD}}=\frac{1}{\sqrt{3}} \Rightarrow \text{BD}=150\sqrt{3}$

Distance covered in 2 minutes$=BD-BC$ = $150\sqrt{3}-150=150(\sqrt{3}-1)m$

$\therefore$ speed = $\frac{\text{Distance covered}}{\text{time taken}}=\frac{150(\sqrt{3}-1)}{2}$

= $75\times(1.732-1)$

$=54.9\ m/min$

Section D

35. Given,

(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0

$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$

$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$

For equal roots $B^2 - 4AC = 0$

or, $\{-2(a+b+c)\}^2 = 4 \times 3(ab+bc+ca)$

or, $4(a+b+c)^2 - 12(ab+bc+ca) = 0$

or, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$

or, $\frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ if $a \neq b \neq c$

Since $(a-b)^2 > 0, (b-c)^2 > 0 (c-a)^2 > 0$

Hence, $(a-b)^2 = 0 \Rightarrow a = b$

$(a-c)^2 = 0 \Rightarrow b = c$

$(c-a)^2 = 0 \Rightarrow c = a$

$\therefore a = b = c$ Hence Proved.

36. Raghav pays half of Rs.1,20,000 i.e. Rs.60,000 in cash and the balance Rs.60,000 in 12 annual installments of Rs.5000 each. With each installment, he pays interest on the unpaid amount at the rate of 12% per annum.

$\therefore$ amount of first installment = Rs.5000 + Interest on unpaid amount of Rs.60000
$= Rs.5000 + Rs.\left(\frac{12}{100} \times 60000\right) = Rs.5000 + Rs.7200 = Rs.12200$
$\therefore$ amount of second installment =Rs.5000 + Interest on unpaid amount of Rs.55000
$= Rs.5000 + Rs.\left(\frac{12}{100} \times 55000\right) = Rs.5000 + Rs.6600 = Rs.11600$

$\therefore$ amount of third installment = Rs.5000 + Interest on unpaid amount of Rs.50000

$= Rs.5000 + Rs.\left(\frac{12}{100} \times 50000\right) = Rs.5000 + Rs.6000 = Rs.11000$

Clearly, amount of various installments form an A.P. with first term Rs.12200 and common difference - 600

$\therefore$ total cost of the shop = Rs.[60,000 + Sum of 12 installments]

$= Rs.\left[60,000 + \frac{12}{2}\{2 \times 12200 + (12-1) \times (-600)\}\right]$

= Rs.[60,000 + 6 (24,400 - 6,600)]

= Rs.[60,000 + 1,06,800] = Rs.1,66,800. So total cost of shop is Rs.1,66,800.

OR

According to the question,

All integers between 100 and 550, which are divisible by 9

= 108, 117, 126,.........., 549

First term (a) = 108

Common difference(d) = 117 - 108 = 9

Last term(a_n) = 549

$\Rightarrow$a + (n - 1)d = 549

$\Rightarrow$108 + (n - 1)(9) = 549

$\Rightarrow$108 + 9n - 9 = 549

$\Rightarrow$9n = 549 + 9 - 108

$\Rightarrow$ 9n = 450

$\Rightarrow \quad n = \frac{450}{9} = 50$

Sum of 50 terms = $\frac{n}{2}[a + a_n]$

$= \frac{50}{2}[108 + 549]$

$= 25 \times 657$

= 16425

37. Let the numerator be x and denominator be y

if 2 is added to both numerator and denominator, the fraction becomes $\frac{9}{11}$

$\frac{x+2}{y+2} = \frac{9}{11}$

11(x + 2) = 9(y + 2)

= 11x + 22 = 9y + 18

or, 11x + 22 - 9y - 18 = 0

or, 11 x - 9 y + 4 = 0 ....(i)

If 3 is added to both numerator and denominator the fraction becomes $\frac{5}{6}$

and $\frac{x+3}{y+3}=\frac{5}{6}$

6(x+3) = 5(y+3)

6x +18 = 5y + 15

or, 6x + 18 - 5y - 15 = 0

or, 6x -5y + 3 = 0 ...(ii)

On comparing with ax + by + c = 0

we get $a_1=11, b_1=9, c_1=4$

$a_2=6, b_2=-5$ and $c_2=3$

Now, $\frac{x}{b_2c_1-b_1c_2}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-b_2b_1}$

$\frac{x}{(-9)(3)-(-5)(4)}=\frac{y}{(4)(6)-(11)(3)}$

$=\frac{1}{(11)(-5)-(6)(-9)}$

or, $\frac{x}{-27+20}=\frac{y}{24-33}=\frac{1}{-55+54}$

$\Rightarrow \quad \frac{x}{-7}=\frac{y}{-9}=\frac{1}{-1}$

$\Rightarrow \quad \frac{x}{-7}=-1$

or, x = 7

Hence, x=7, y=9

$\therefore$ Fraction = $\frac{7}{9}$

38. According to the question:

ΔACB ~ ΔAPQ

Therefore, $\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5}$ and $\frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow AC = \frac{10}{5} \times 2.8$ and $AQ = 6.5 \times \frac{5}{10}$

$\Rightarrow$ AC = 5.6 cm and AQ = 3.25 cm

Therefore, By area of similar triangle theorem,we have,

$\frac{Area(\Delta ACB)}{Area(\Delta APQ)} = \frac{BC^2}{PQ^2}$

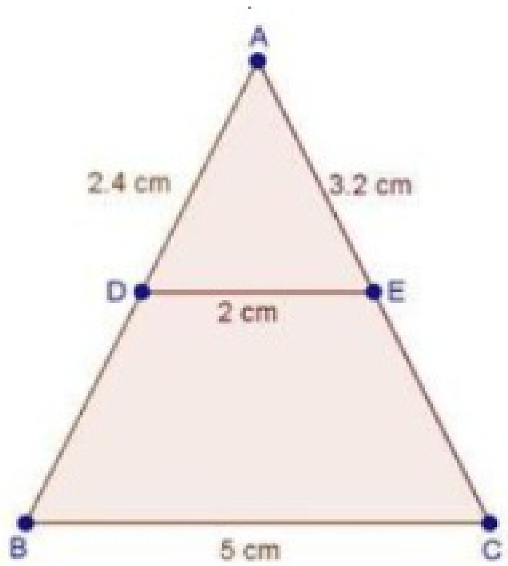
$= \frac{(10)^2}{(5)^2}$

$= \frac{100}{25}$

$= \frac{4}{1}$

OR

We have,



DE | | BC

Now, In $\triangle$ADE and $\triangle$ABC

$\angle A = \angle A$ [common]

$\angle ADE = \angle ABC$ [$\because$ DE | | BC $\Rightarrow$ Corresponding angles are equal]

$\Rightarrow \triangle ADE = \triangle ABC$ [By AA criteria]

$\Rightarrow \frac{AB}{BC} = \frac{AD}{DE}$ [$\because$ Corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{AB}{5} = \frac{2.4}{2}$

$\Rightarrow AB = \frac{2.4 \times 5}{2}$

$\Rightarrow$ AB = 1.2 $\times$ 5

= 6.0 cm

$\Rightarrow$ AB = 6 cm

$\therefore$ BD = AB - AD

= 6 - 2.4

= 3.6 cm

$\Rightarrow$ DB = 3.6 cm

Now,

$\frac{AC}{BC} = \frac{AE}{DE}$ [$\because$ Corresponding sides of similar triangles are equal]

$\Rightarrow \frac{AC}{5} = \frac{3.2}{2}$

$\Rightarrow AC = \frac{3.2 \times 5}{2}$

= 1.6 $\times$ 5

= 8.0 cm

$\Rightarrow$ AC = 8 cm

$\therefore$ CE = AC - AE

= 8 - 3.2

= 4.8 cm

Hence, BD = 3.6 cm and CE = 4.8 cm

39. Let us suppose that x hours be the time taken for the pipe to fill the tank.

$\because$ The water is flowing at the rate of 4 km/hr,

$\therefore$ Length of the water column in x hours is 4x km=4000 =4000 x m.

$\therefore$ The length of the pipe is 4000 x m

The diameter of the pipe = 20 cm

$\Rightarrow$radius = 10 cm

$= \frac{10}{100}\mathrm{m}$ = 0.1m

$\therefore$ Volume of the water flowing through the pipe in x hours = v_1

= $\pi r^2 h$

$= \pi \times (0.1)^2 \times 4000$ .... (i)

According to question it is given that diameter of the cylindrical tank = 10 m

$\Rightarrow$ radius = 5 cm and

Volume of the water that falls into the tank in x hours = v_1

$= \pi r^2 h$

$= \pi \times (5)^2 \times 2$ .... (ii)

$\therefore$ Volume of the water flowing through the pipe in x hours = Volume of the water that falls into the tank in x hours

$\Rightarrow \pi \times (0.1)^2 \times 4000$ x = $\pi \times (5)^2 \times 2$

$\Rightarrow$ 40x = 50

$\Rightarrow x = \frac{50}{40}$ hour

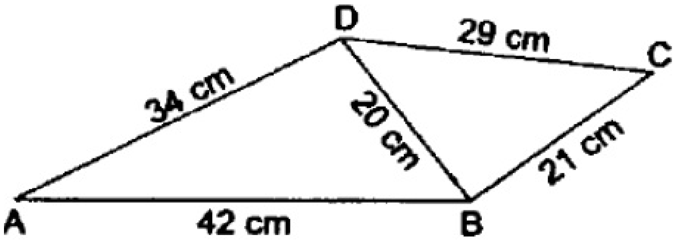
$\Rightarrow x = \frac{50}{40} \times 60$ minutes

$\Rightarrow$ x = 75 minutes = 1 hour 15 mins

Hence, the water in the tank well fill in 1 hour 15 minutes.

OR

Area of quad. ABCD = Area of $\triangle$ABD + Area of $\triangle$DBC



For area of $\triangle$ABD

Let a = 42cm, b = 34 cm, and c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(42+34+20)}{2}$ cm = 48

Then, (s - a) = 6, (s - b) = 14 and (s - c) = 28

Area of $\triangle$ABC = $\sqrt{s \times (s-a)(s-b)(s-c)}$

= $\sqrt{48 \times 6 \times 14 \times 28}$ cm^2

= 336 cm^2

For area of $\triangle$DBC

a = 29 cm, b = 21 cm, c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(29+21+20)}{2}$ cm = 35 cm

Area of $\triangle$DBC = $\sqrt{s \times (s-a)(s-b)(s-c)}$ sq. units

= $\sqrt{35 \times 6 \times 14 \times 15}$ cm^2

= 210 cm^2

Area of quad. ABCD = Area of $\triangle$ABC + Area of $\triangle$DBC

= (336 + 210) cm^2 = 546 cm^2

40. To find the median, we need class intervals but here only the mid-values are given. So, we should first find the upper & lower limits of the various classes. The difference between two consecutive values is h = 125-115 = 10 (difference of two mid values in the given question)

The Lower limit of a class = Mid-value - $\frac{h}{2}$

The Upper limit = Mid-value + $\frac{h}{2}$

Mid-value	Class Groups	Frequency	Cumulative Frequency
115	110-120	6	6
125	120-130	25	6 + 25 = 31
135	130-140	48	31 + 48 = 79
145	140-150	72	79 + 72 = 151
155	150-160	116	151+ 116 = 267
165	160-170	60	267 + 60 = 327
175	170-180	38	327+38=365
185	180-190	22	365+22=387
195	190-200	3	387+3=390
			N = $= \Sigma f_i$ = 390

N = 390

Median= $\frac{N}{2}$ th term $= \frac{390}{2}$ = 195th term

Which lies in the class interval 150 -160, hence the Median class = 150 -160

Here, l = 150, f = 116, h = 10, c.f = 151

Median = $l + \frac{\frac{N}{2} - c.f}{f} \times h$

Median = $150 + \frac{195-151}{116} \times 10$= 153.8

Hence, the median will be 153.8