CBSE Board

Class XII Mathematics

Sample Paper 5

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Part A

Q 1 – Q 20 are multiple choice type questions. Select the correct option.

1. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix?

A. $0°$

B. $30°$

C. $45°$

D. $60°$

2. Integral of $\frac{1}{\sqrt{9-25x^2}}$ is

A. $3\cos^{-1}\left(\frac{3x}{5}\right)+c$

B. $3\sin^{-1}\left(\frac{3x}{5}\right)+c$

C. $\frac{1}{3}\cos^{-1}\left(\frac{5x}{3}\right)+c$

D. $\frac{1}{3}\sin^{-1}\left(\frac{5x}{3}\right)+c$

3. Find the equation of a line through (−2, 1, 3) and parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

A. $\frac{x-2}{3}=\frac{y+1}{5}=\frac{z+3}{6}$

B. $\frac{x+2}{3}=\frac{y-1}{5}=\frac{z-3}{6}$

C. $\frac{x-2}{-3}=\frac{y+1}{-5}=\frac{z+3}{-6}$

D. $\frac{x+2}{-3}=\frac{y-1}{-5}=\frac{z-3}{-6}$

4. A vector normal to the plane x + 2y +3z – 6 = 0 will be

A. $2\hat{i}+3\hat{j}+\hat{k}$

B. $-\hat{i}-2\hat{j}-3\hat{k}$

C. $\hat{i}+2\hat{j}+3\hat{k}$

D. $-2\hat{i}-3\hat{j}-\hat{k}$

5. If the vectors $\vec{a}=3\hat{i}+\hat{j}-2k$ and $\vec{b}=\hat{i}+\lambda\hat{j}-3k$ are perpendicular to each other, then the value of λ is

A. −1

B. 3

C. −9

D. 18

6. The equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane is

A. x = 3

B. y = 3

C. z = 3

D. x = −3

7. Two men A and B appear for an interview in a company. The probabilities of A's and B's selection are $\frac{1}{4}$ and $\frac{1}{6}$ respectively. Then the probability that both of them are selected is

A. $\frac{3}{2}$

B. $\frac{1}{6}$

C. $\frac{1}{4}$

D. $\frac{1}{24}$

8. The value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$ is

A. −1
B. 0
C. 1
D. 2

9. Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{\pi}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $-\frac{\pi}{3}$

10. Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{2}$

11. Let $R = \{(a, b): a = b^2\}$ for all $a, b \in N$. Then R satisfies which of the following?

A. Reflexivity
B. Symmetric
C. Transitivity
D. None of these

12. The value of k, so that the function $f(x)=\begin{cases}\frac{x^2-25}{x-5}, & x\neq 5\\ k & , x=5\end{cases}$ is continuous at x = 5 is

A. 0
B. 10
C. 5
D. 15

13. Range of the real function f defined by $f(x)=\frac{x^2}{(1+x^2)}$, is

A. R
B. $\{y \in R : 0 \leq y < \infty\}$
C. $\{y \in R : 0 \leq y < 1\}$
D. $R - \{1\}$

14. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 40x + 10$. When x = 5, the marginal revenue will be

A. 46
B. –10
C. 60
D. 70

15. Find the area of the curve $y = x^2$ bounded by the line x = y.

A. $\frac{1}{6}$ units
B. $\frac{1}{3}$ units
C. $\frac{1}{2}$ units
D. $\frac{5}{6}$ units

16. Solve the differential equation: $\frac{dy}{dx}+y=1$

A. $\log|1 + y| - x = C$
B. $\log|1 - y| + x = C$
C. $|1 - y| + x + C = 0$
D. $\log|1 + x| - y = C$

17. Find solution of the differential equation: $(x^2+1)\frac{dy}{dx}=xy$

A. $y = C_1\sqrt{x^2+1}$

B. $x = \sqrt{y^2+1}$

C. $y = C_1\sqrt{x^2-1}$

D. $x = C_1\sqrt{y^2-1}$

18. Find $\int \frac{dx}{1+\sin x}$.

A. sec x – tan x + c

B. tan x – sec x + c

C. tan x – cos x + c

D. 2sec x – sin x + c

19. The value of $\frac{d}{dx}\left(e^{x^3}\right)$ is

A. $e^{3x^2}e^{x^3}$

B. e^{x^3}

C. $3x^2e^{x^3}$

D. $3x^2$

20. Find the interval in which the function f given by f(x) = 2x² – 3x is increasing.

A. $\left(-\infty, \frac{3}{4}\right)$

B. $\left(\frac{3}{4}, \infty\right)$

C. $(3, \infty)$

D. $(-\infty, 3)$

Part B

21. Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}dx$

22. Solve the given differential equation $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$ if y (0) = 0.

23. Let A = Q × Q, Q being the set of rational numbers. Let '*' be a binary operation on A, defined by (a, b) * (c, d) = (ac, ad + b). Show that
(i) '*' is not commutative (ii) '*' is associative
(iii) The identity element with respect to '*' is (1, 0)

24. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

25. Find the interval in which the value of the determinant of the matrix A lies, given $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$.

26. If $x = a(\boldsymbol{\theta} - \sin\boldsymbol{\theta})$, $y = a(1 + \cos\boldsymbol{\theta})$, prove that $\frac{d^2y}{dx^2} = -\frac{a}{(2a-y)^2}$

27. Find the distance of the point (2, 4, −1) from the line having the following equation: $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

28. Evaluate $\int \frac{x}{\sqrt{8+x-x^2}}dx$

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

30. Check the continuity and differentiability of the function f(x) = |x – 2| at x = 2.

OR

Show that the function f defined by that $f(x) = \left|1 - x + |x|\right|$, $x \in R$ is continuous.

31. Find the value of λ which makes the vectors $\vec{a}, \vec{b}$, and $\vec{c}$ coplanar, where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$.

OR

Find a, if the points A (10, 3), B (12, –5) and C (a, 11) are collinear.

Part C

32. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

33. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin.

OR

Find the image of the point $2\hat{i} + 3\hat{j} - 4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$.

34. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. How many days shall each work, if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost? Solve the problem graphically.

OR

A catering agency has two kitchen at A and B. From these places, supply is made to three schools situated at P, Q and R for mid-day meals. The monthly requirements of the three schools are respectively 40, 40 and 50 food packets while the production capacity of the kitchens at A and B are 60 and 70 packets respectively. The transportation cost per packet from the kitchens to the schools is given below.

Transportation Cost per packet (in Rupees)		
To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to each school so that the cost of transportation is minimum? Also find the minimum cost.

35. Find the area bounded by the curve $y = 2x - x^2$ and the line $y = -x$.

OR

Find the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

36. Find the intervals on which the function $f(x) = \frac{4x^2 + 1}{x}, (x \neq 0)$ is

(a) increasing (b) decreasing.

37. A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

OR

In a factory which manufactures bulbs, machines X, Y and Z manufacture 1000, 2000, 3000 bulbs respectively. Of their outputs, 1%, 1.5% and 2 % are defective bulbs. A bulb is drawn at random and is found to be defective. What is the probability that the machine X manufactures it?

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Class XII Mathematics

Sample Paper 5 – Solution

Part A

1. Correct option: A

Explanation:-

A is a matrix of order 2 × 2.

For A to be an identity matrix,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\Rightarrow \cos\alpha = 1$ and $\sin\alpha = 0$

$\Rightarrow \cos\alpha = \cos 0°$ and $\sin\alpha = \sin 0°$

$\Rightarrow \alpha = 0°$

Thus, for $\alpha = 0°$, A is an identity matrix

2. Correct option: D

Explanation:-

$$I = \int \frac{1}{\sqrt{9-25x^2}}dx$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}dx$$

$$= \frac{1}{5} \times \frac{5}{3}\sin^{-1}\left(\frac{x}{\frac{3}{5}}\right) + c$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{5x}{3}\right) + c$$

3. Correct option: B

Explanation:-

Equation of a line through (−2, 1, 3) and parallel to $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-1}{5} = \frac{z-3}{6}$$

4. **Correct option:** C
Explanation:-
The plane is x + 2y +3z – 6 = 0

$\Leftrightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)=6$

$\Leftrightarrow \hat{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)=6$

Hence, the vector normal to the plane x + 2y +3z – 6 = 0 is $\hat{i}+2\hat{j}+3\hat{k}$.

5. **Correct option:** C
Explanation:-
Given: $\vec{a}\perp\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b}=0$

$\Rightarrow \left(3\hat{i}+\hat{j}-2k\right)\left(\hat{i}+\lambda\hat{j}-3k\right)=0$

$\Rightarrow 3\times1+1\times\lambda+(-2)\times(-3)=0$

$\Rightarrow \lambda+9=0$

$\Rightarrow \lambda=-9$

6. **Correct option:** B
Explanation:-
The equation of the plane ZOX is y = 0.
A plane parallel to it is of the form, y = a.
As y-intercept of the plane is 3, we have
a = 3.
Thus, the equation of the required plane is y = 3.

7. **Correct option:** D
Explanation:-
Let E_1: A is selected
E_2: B is selected

Now, P (E_1) = $\frac{1}{4}$ and P (E_2) = $\frac{1}{6}$

E_1 and E_2 are independent events.

∴ P (both are selected) = P ($E_1 \cap E_2$) = P (E_1) × P (E_2) = $\frac{1}{24}$.

8. **Correct option:** B
Explanation:-

$$\text{Let } \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta-\sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta-\sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta-\sin\gamma\sin\delta \end{vmatrix}$$

Applying $[C_3 \to C_3 + (\sin\delta)C_1 - (\cos\delta)C_2]$

$$\Rightarrow \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}$$

$\Rightarrow \Delta = 0$ ... [Expanded through C_3]

Hence, $\Delta = 0$.

9. Correct option: B

Explanation:-

Range of principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Let $\sin^{-1}\left(\frac{-1}{2}\right) = \theta$

Then, $\sin\theta = \frac{-1}{2} = \sin\left(-\frac{\pi}{6}\right)$, where $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

10. Correct option: C

Explanation:-

Ranges of principal values of $\sin^{-1}$ and $\cos^{-1}$ are $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively.

Let $\cos^{-1}\left(\frac{1}{2}\right) = \theta_1$

$\Rightarrow \theta_1 = \frac{\pi}{3}$ ... (As $\theta_1 \in [0, \pi]$)

Let $\sin^{-1}\left(\frac{1}{2}\right) = \theta_2$

$\Rightarrow \theta_2 = \frac{\pi}{6}$ .... As $\theta_2 \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{2\pi}{3}$

11. Correct option: D

Explanation:-

Given: $R = \{(a, b): a = b^2\}$

(i) Let $a = 2$,

We know that $2 \neq 2^2 \Rightarrow (2, 2) \notin R$

i.e. 2 is not related to 2.

Therefore, R is not reflexive.

(ii) Now for (4, 2), $4 = 2^2 \Rightarrow 4 R 2$.
But, $2 \neq 4^2 \Rightarrow 2$ is not related to 4.
Therefore, R is not symmetric.
(iii) For (16, 4), $16 = 4^2 \Rightarrow 16 R 4$.
For (4, 2), $4 = 2^2 \Rightarrow 4 R 2$
But $16 \neq 2^2 \Rightarrow 16$ is not related to 2.
Therefore, R is not transitive.

12. Correct option: B
Explanation:-

Given: $f(x)=\begin{cases}\dfrac{x^2-25}{x-5}, & x\neq 5\\ k, & x=5\end{cases}$

f is continuous at x = 5, then

$$\lim_{x\to 5} f(x) = f(5)$$

$$\Rightarrow \lim_{x\to 5} \frac{x^2-25}{x-5} = k$$

$$\Rightarrow \lim_{x\to 5} \frac{(x-5)(x+5)}{x-5} = k$$

$$\Rightarrow \lim_{x\to 5} x+5 = k$$

$$\Rightarrow 10 = k$$

13. Correct option: C
Explanation:-

Let $y = \dfrac{x^2}{(1+x^2)}$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1-y)$$

$$\Rightarrow x = \sqrt{\frac{y}{1-y}}$$

For x to be real, we must have $\dfrac{y}{1-y} \geq 0$ and $1 - y \neq 0$

$\Rightarrow 0 \leq y < 1$
Hence, range (f) = $\{y \in R: 0 \leq y < 1\}$

14. Correct option: D
Explanation:-
Given: $R(x) = 3x^2 + 40x + 10$

$$\Rightarrow MR = \frac{dR}{dx}$$

$$= \frac{d}{dx}\left(3x^2 + 40x + 10\right)$$

$$= 6x + 40$$

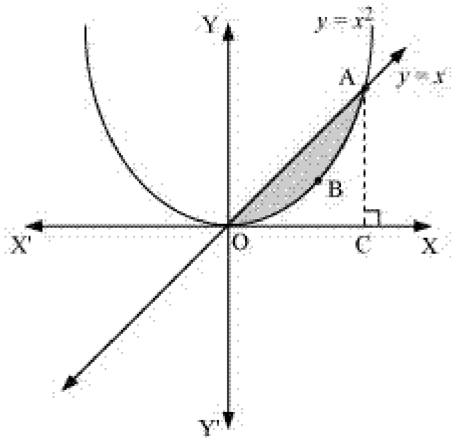
$$\Rightarrow [MR]_{x=5} = 6 \times 5 + 40 = 70.$$

Hence, the marginal revenue is Rs. 70.

15. Correct option: A

Explanation:-

The required area is represented by OBAO below:



Therefore, the area of OBAO = Area (△ OAC) – Area (OCABO).

Required area = $\int_0^1 x\,dx - \int_0^1 x^2\,dx$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6} \text{ units}$$

16. Correct option: B

Explanation:-

Given differential equation is $\frac{dy}{dx} + y = 1$

$$\Rightarrow \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{1-y} = dx$$

$$\Rightarrow \int \frac{dy}{1-y} = \int dx$$

$$\Rightarrow -\log|1-y| = x + C$$

$$\Rightarrow \log|1-y| + x = C$$

17. Correct option: A

Explanation:-

Given differential equation is $\left(x^2+1\right)\frac{dy}{dx} = xy$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{\left(x^2+1\right)}$$

$$\Rightarrow \frac{dy}{y} = \frac{xdx}{\left(x^2+1\right)}$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{x}{\left(x^2+1\right)} dx$$

$$\Rightarrow \int \frac{dy}{y} = \frac{1}{2}\int \frac{2x}{\left(x^2+1\right)} dx$$

$$\Rightarrow \log|y| = \frac{1}{2}\log\left(x^2+1\right) + \log C$$

$$\Rightarrow \log|y| = \log\sqrt{x^2+1} + \log C$$

$$\Rightarrow y = C_1\sqrt{x^2+1}$$

18. Correct option: B

Explanation:-

Let $I = \int \frac{dx}{1+\sin x}$

$$I = \int \frac{dx}{1+\sin x}$$

$$= \int \left(\frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\right) dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx = \int \left(\sec^2 x - \sec x \tan x\right) dx$$

$$= \int \sec^2 x dx - \int \sec x \tan x \cdot dx$$

$$= \tan x - \sec x + c$$

19. Correct option: C

Explanation:-

$$\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3}\left(\frac{d}{dx}x^3\right) = e^{x^3}\left(3x^2\right) = 3x^2 e^{x^3}$$

20. Correct option: B

Explanation:-

Given: f(x) = 2x² – 3x

$\Rightarrow f'(x) = 4x - 3$

Take f'(x) = 0 ⇒ 4x – 3 = 0 i.e. $x = \frac{3}{4}$

Now, the point $x = \frac{3}{4}$ divides the number line in two intervals $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$

f'(x) < 0 in the interval $\left(-\infty, \frac{3}{4}\right)$ and f'(x) > 0 in the interval $\left(\frac{3}{4}, \infty\right)$

Hence, f(x) is increasing in the interval $\left(\frac{3}{4}, \infty\right)$.

Part B

21. $I = \int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\sin^2 x}{\sin 2x}\right)\right)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\sin^2 x}{2\sin x\cos x}\right)\right)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\tan x}{2}\right)\right)dx$ ... (i)

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan\left(\frac{\pi}{2} - x\right)}{2}\right)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cot x}{2}\right)dx$ ... (ii)

Additing (i) & (ii)

$$2I=\int_0^{\frac{\pi}{2}}\left[\log\left(\frac{\tan x}{2}\right)+\log\left(\frac{\cot x}{2}\right)\right]dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log\left[\left(\frac{\tan x}{2}\right)\left(\frac{\cot x}{2}\right)\right]dx$$

$$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\log\left(\frac{1}{4}\right)dx$$

$$\Rightarrow I=\frac{1}{2}\log\left(\frac{1}{4}\right)\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\log\left(\frac{1}{4}\right)^{\frac{1}{2}}\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\log\left(\frac{1}{2}\right)\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$$

OR

$$I=\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \qquad ...(1)$$

Using the proporty $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx \qquad ...(2)$$

Adding (1) and (2),

$$2I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}.\sin x\cos x\right)}{\sin^4 x+\cos^4 x}\,dx$$

$$\Rightarrow I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}}\left[\frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\right]dx$$

$$= \frac{\pi}{4}\int_0^{\frac{\pi}{2}}\left[\frac{\frac{\sin x \cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x} + 1}\right]dx$$

$$= \frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\tan x \sec^2 x}{\tan^4 x + 1}dx$$

Put $\tan^2 x = z$

$\therefore$ $2 \tan x \sec^2 x\, dx = dz$

$\Rightarrow$ $\tan x \sec^2 x\, dx = \frac{dz}{2}$

When $x = 0, z = 0$ and when $x = \frac{\pi}{2}, z = \infty$

$$\therefore I = \frac{\pi}{4}\int_0^{\infty}\frac{\frac{dz}{2}}{z^2 + 1}$$

$$\Rightarrow I = \frac{\pi}{8}\int_0^{\infty}\frac{dz}{1 + z^2}$$

$$= \frac{\pi}{8}\left|\tan^{-1}(z)\right|_0^{\infty}$$

$$= \frac{\pi}{8}\ \tan^{-1}\ \infty - \tan^{-1}\ 0$$

$$= \frac{\pi}{8}\left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi^2}{16}$$

22. Given differential equation is $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$ ... (i)

$$\Rightarrow \frac{dy}{2e^{-y} - 1} = \frac{dx}{(x + 1)}$$

Integrating both sides, we get

$$\int\frac{dy}{2e^{-y} - 1} = \int\frac{dx}{(x + 1)}$$

$$\Rightarrow \int \frac{e^y dy}{2-e^y} = \int \frac{dx}{(x+1)}$$

$$\Rightarrow -\int -\frac{e^y dy}{2-e^y} = \int \frac{dx}{(x+1)}$$

$$\Rightarrow -\log\left|2-e^y\right| = \log|x+1| + c$$

$$\Rightarrow \log\left|(x+1)\left(2-e^y\right)\right| = -c$$

$$\Rightarrow \left|(x+1)\left(2-e^y\right)\right| = e^{-c}$$

$$\Rightarrow (x+1)\left(2-e^y\right) = \pm e^{-c} = A(\text{say})$$

$$\Rightarrow (x+1)\left(2-e^y\right) = A \text{ .... (ii)}$$

$\text{Given}: x = 0, y = 0$

$$(0+1)\left(2-e^0\right) = A$$

$$\Rightarrow 1(2-1) = A$$

$$\Rightarrow A = 1$$

Substituting in (ii), we get

$$(x+1)\left(2-e^y\right) = 1$$

23.

i. (a, b) * (c, d) = (ac, ad + b)
   (c, d) * (a, b) = (ca, cb + d)
   (ac, ad + b) $\neq$ (ca, cb + d)
   So, '*' is not commutative.

ii. Let (a, b) (c, d) , (e, f) $\in$ A, Then
   ((a, b)* (c, d)) * (e, f) = (ac, ad + b) * (e, f) = ((ac)e, (ac)f + (ad + b))
   = (ace, acf + ad + b)
   (a, b)* ((c, d)* (e, f)) = (a, b) * (ce, cf +d) = (a(ce), a(cf + d) +b) = (ace, acf +ad +b)
   ((a, b)* (c, d)) * (e, f) = (a, b) * ((c, d)* (e, f))
   Hence, '*' is associative.

iii. Let (x, y) $\in$ A. Then (x, y) is an identity element, if and only if
   (x, y) * (a, b) = (a, b) = (a, b) * (x, y), for every (a, b) $\in$A
   Consider, (x, y) * (a, b) = (xa, xb + y)
   (a, b) * (x, y) = (ax, ay +b)
   (xa, xb + y) = (a, b) = (ax, ay +b)
   $ax = x\,a = a \Rightarrow x = 1$
   $xb + y = b = ay + b \Rightarrow b + y = b = ay + b \Rightarrow y = 0 = ay \Rightarrow y = 0$
   Therefore, (1, 0) is the identity element.

24. Given: $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

Let $\cos^{-1} x = X, \cos^{-1} y = Y, \cos^{-1} z = Z$

$\Rightarrow x = \cos X, y = \cos Y, z = \cos Z$

Hence, $X + Y + Z = \pi$

Consider $x^2 + y^2 + z^2 + 2xyz$

$= \cos^2 X + \cos^2 Y + \cos^2 Z + 2\cos X \cos Y \cos Z$

$= \frac{1+\cos 2X}{2} + \frac{1+\cos 2Y}{2} + \frac{1+\cos 2Z}{2} + 2\cos X \cos Y \cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + 2\cos X \cos Y \cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [\cos(X+Y) + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [\cos(\pi - Z) + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [-\cos Z + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z + \cos(X-Y)\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z + \cos(X-Y)\cos(\pi - (X+Y))$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z - \cos(X-Y)\cos(X+Y)$

We know that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$\therefore x^2 + y^2 + z^2 + 2xyz$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z - \cos^2 X + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{1+\cos 2Z}{2} - \frac{1+\cos 2X}{2} + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - 1 - \frac{\cos 2Z + \cos 2X}{2} + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{\cos 2Z + \cos 2X}{2} - \cos^2 Y \quad [\because -1 + \sin^2 Y = -\cos^2 Y]$

$\therefore x^2 + y^2 + z^2 + 2xyz$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{\cos 2Z + \cos 2X}{2} - \frac{1+\cos 2Y}{2}$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{1}{2} - \frac{\cos 2Z + \cos 2Y + \cos 2X}{2}$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$

25. Given: $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$

$|A|=\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$

$=1(1+\sin^2\theta)-\sin\theta(-\sin\theta+\sin\theta)+1(1+\sin^2\theta)$

$=2(1+\sin^2\theta)$

The value of $\sin\theta$ lies in the range of -1 and 1.

$\Rightarrow 0\le\sin^2\theta\le 1$

$\Rightarrow 0+1\le(1+\sin^2\theta)\le 1+1$

$\Rightarrow 1\le(1+\sin^2\theta)\le 2$

$\Rightarrow 2(1)\le 2(1+\sin^2\theta)\le 2(2)$

$\Rightarrow 2\le 2(1+\sin^2\theta)\le 4$

$\Rightarrow 2\le|A|\le 4$

$\Rightarrow |A|\in[2,\ 4]$

So value of $|A|$ lies in the interval [2, 4]

26. $x=a(\boldsymbol{\theta}-\sin\boldsymbol{\theta}),\ y=a(1+\cos\boldsymbol{\theta})$

Differentiating both sides w.r.t. $\boldsymbol{\theta}$

$\frac{dx}{d\theta}=a(1-\cos\theta)\ ...(1)$

$\frac{dy}{d\theta}=a(-\sin\theta)\ ...(2)$

Dividing (2) by (1),

$\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=\frac{-a\sin\theta}{a(1-\cos\theta)}$

$=\frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$

$=-\cot\frac{\theta}{2}$

$\Rightarrow\frac{dy}{dx}=-\cot\frac{\theta}{2}$

Differentiating w.r.t. θ,

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(-\cot\frac{\theta}{2}\right)\frac{d\theta}{dx}$

$$= -\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2} \times \frac{1}{a(1-\cos\theta)}$$

$$= -\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2} \times \frac{1}{2a\sin^2\frac{\theta}{2}}$$

$$= -\frac{1}{4a\sin^4\frac{\theta}{2}}$$

As y = a$(1 + \cos\boldsymbol{\theta})$

$$\Rightarrow y = 2a\cos^2\frac{\theta}{2} \Rightarrow \sin^2\frac{\theta}{2} = 1 - \frac{y}{2a} = \frac{2a-y}{2a}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{4a \times \left(\frac{2a-y}{2a}\right)^2} = -\frac{a}{(2a-y)^2}$$

27. The given line is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Given point is (2, 4, −1)

The distance of a point whose position vector is $\overrightarrow{a_2}$

from a line whose vector equation is $\vec{r} = \vec{a_1} + \lambda \vec{v}$ is

$$d = \frac{\left|\vec{v} \times (\vec{a_2} - \vec{a_1})\right|}{\left|\vec{v}\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times\left((2\hat{i}+4\hat{j}-1k)-(-5\hat{i}-3\hat{j}+6k)\right)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times\left((2\hat{i}+4\hat{j}-1k)-(-5\hat{i}-3\hat{j}+6k)\right)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times(7\hat{i}+7\hat{j}-7k)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$\left(\hat{i}+4\hat{j}-9k\right)\times(7\hat{i}+7\hat{j}-7k) = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 4 & -9 \\ 7 & 7 & -7 \end{vmatrix} = 35\hat{i} - 56\hat{j} - 21k$$

$$= \frac{7}{\sqrt{98}}\left|\left(5\hat{i}-8\hat{j}-3k\right)\right| = \frac{7}{\sqrt{98}}\sqrt{98} = 7 \text{ units}$$

28. Take $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

$$\text{Let } x = A\left[\frac{d}{dx}(8+x-x^2)\right]+B$$

$$x = A(1-2x)+B$$

$$\Rightarrow x = -2Ax+(A+B)$$

$$\Rightarrow -2A = 1; A+B = 0$$

$$\Rightarrow A = -\frac{1}{2}; B = \frac{1}{2}$$

$$\therefore I = \int \frac{-\frac{1}{2}(1-2x)+\frac{1}{2}}{\sqrt{8+x-x^2}}dx$$

$$\Rightarrow I = -\frac{1}{2}\int \frac{(1-2x)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{8+x-x^2}}dx = I_1 + I_2(\text{say})$$

$$I_1 = -\frac{1}{2}\int \frac{(1-2x)}{\sqrt{8+x-x^2}}dx;$$

$$\text{Let } t = 8+x-x^2 \therefore dt = (1-2x)dx$$

$$\Rightarrow I_1 = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\left[2\sqrt{t}\right] = -\sqrt{8+x-x^2}$$

$$I_2 = \frac{1}{2}\int \frac{1}{\sqrt{8+x-x^2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\frac{33}{4}-\left(x-\frac{1}{2}\right)^2}} = \frac{1}{2}\sin^{-1}\frac{\left(x-\frac{1}{2}\right)}{\frac{\sqrt{33}}{2}} = \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right)$$

$$\text{So } I = -\sqrt{8+x-x^2}+\frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right)+C$$

29. Let E_1, E_2 and E_3 be the events of a driver being a scooter driver, car driver and truck driver respectively.

Let A be the event that the person meets with an accident

There are 2000 insured scooter drivers, 4000 insured car drivers and 6000 insured truck drivers.

Total number of insured vehicle drivers = 2000 + 4000 + 6000 = 12000

$$\therefore P\ E_1 = \frac{2000}{12000} = \frac{1}{6}, P\ E_2 = \frac{4000}{12000} = \frac{1}{3}, P\ E_3 = \frac{6000}{12000} = \frac{1}{2}$$

Also, we have:

$$P(A|E_1) = 0.01 = \frac{1}{100}$$

$P(A|E_2) = 0.03 = \frac{3}{100}$

$P(A|E_3) = 0.15 = \frac{15}{100}$

Now, the probability that the insured person who meets with an accident is a scooter driver is $P(E_1|A)$.

Using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1) \times P(A|E_1)}{P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2) + P(E_3) \times P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}}$$

$$= \frac{1}{6} \times \frac{6}{52}$$

$$= \frac{1}{52}$$

30. Given: $f(x) = |x - 2|$

Now, $f(2) = |2 - 2| = 0$

$$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2 + h)$$

$$= \lim_{h \to 0} |2 + h - 2|$$

$$= \lim_{h \to 0} |h|$$

$$= \lim_{h \to 0} h$$

$$= 0$$

$$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2 - h)$$

$$= \lim_{h \to 0} |2 - h - 2|$$

$$= \lim_{h \to 0} |-h|$$

$$= \lim_{h \to 0} |h|$$

$$= 0$$

$$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2) = 0.$$

So, $f(x)$ is continuous at $x = 2$.

Now,

$$R.H.D = \lim_{h\to0}\frac{f(2+h)-f(2)}{h}$$
$$= \lim_{h\to0}\frac{|2+h-2|-0}{h}$$
$$= \lim_{h\to0}\frac{|h|}{h} = \lim_{h\to0}\frac{h}{h} = 1$$

And,

$$L.H.D = \lim_{h\to0}\frac{f(2-h)-f(2)}{-h}$$
$$= \lim_{h\to0}\frac{|2-h-2|-0}{-h}$$
$$= \lim_{h\to0}\frac{|-h|}{-h} = \lim_{h\to0}\frac{h}{-h}$$
$$= -1$$

Thus, L.H.D $\neq$ R.H.D.

Hence, f(x) is not differentiable at x = 2.

OR

$f(x) = \left|1-x+|x|\right|,\ x\in R$

Let $g(x) = 1-x+|x|, x\in R$

$h(x) = |x|, x\in R$

$h[g(x)] = h[1-x+|x|]$

$= \left|1-x+|x|\right| = f(x)$

$1-x$, being a polynomial function is continuous

$|x|$, being a modulus function is continuous

If f and g are two continuous functions, then f+g is a continuous function

$\therefore g(x) = 1-x+|x|, x\in R$ is a continuous function

If is continuous function, then $|f|$ is continuous.

In the given problem,

$g(x) = 1-x+|x|$

$\therefore h[g(x)] = |g(x)|$

Since g is a continuous function , $h[g(x)] = |g(x)|$, is a continuous function

$h[g(x)] = f(x) = \left|1-x+|x|\right|$

$\therefore f(x)$ is continuous.

31. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if their scalar product is zero

$\vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

$\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c} = 3\hat{i}-\lambda\hat{j}+5\hat{k}$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -\lambda & 5 \end{vmatrix} = 0$$

$2(10-3\lambda)+1(5+9)+1(-\lambda-6)=0$

$\Rightarrow 20-6\lambda+14-\lambda-6=0$

$\Rightarrow -7\lambda+28=0$

$\Rightarrow -7\lambda=-28$

$\Rightarrow \lambda=4$

OR

Let $\vec{a}=10\hat{i}+3\hat{j}$, $\vec{b}=12\hat{i}-5\hat{j}$ and $\vec{c}=a\hat{i}+11\hat{j}$

are the position vectors of the three points A, B, and C

$\overrightarrow{AB}=\left(12\hat{i}-10\hat{i}\right)+\left(-5\hat{j}-3\hat{j}\right)=2\hat{i}-8\hat{j}$

$\overrightarrow{BC}=\left(a\hat{i}-12\hat{i}\right)+\left(11\hat{j}+5\hat{j}\right)=(a-12)\hat{i}+16\hat{j}$

Since A, B, and C are collinear

So, $\overrightarrow{AB}=m\overrightarrow{BC}$

$\Rightarrow 2\hat{i}-8\hat{j}=m\left[(a-12)\hat{i}+16\hat{j}\right]$

$16m\hat{j}=-8\hat{j} \Rightarrow m=-\frac{1}{2}$

$\Rightarrow -\frac{1}{2}(a-12)=2$

$\Rightarrow a=8$

Part C

32. Let the award money given for honesty, regularity and hard work be Rs. x, Rs. y and Rs. z respectively.

Since total cash award is Rs. 6,000.

$\therefore$ x + y + z = 6,000 ... (1)

Three times the award money for hard work and honesty amounts to Rs. 11,000.

$\therefore$ x + 3z = 11,000

$\Rightarrow$ x + 0×y + 3 z = 11,000 ... (2)

Award money for honesty and hard work is double that given for regularity.

$\therefore$ x + z = 2y

$\Rightarrow$ x – 2y + z = 0 ... (3)

The above system of equations can be written in matrix form AX = B as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$|A| = 1(0+6) - 1(1-3) + 1(-2-0) = 6 \neq 0$$

Thus, A is non-singular. Hence, it is invertible.

$$\text{Adj } A = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 36000-33000+0 \\ 12000+0-0 \\ -12000+33000-0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

Hence, x = 500, y = 2000, and z = 3500.

Thus, award money given for honesty, regularity and hard work is Rs. 500, Rs. 2000 and Rs. 3500 respectively.

The school can include awards for obedience.

33. Equation of the plane passing through the intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 is:

$$(x+y+z-1) + \lambda(2x+3y+4z-5) = 0$$

$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z - (1+5\lambda) = 0$$

This plane has to be perpendicular to the plane x-y+z=0.

Thus,

$$(1+2\lambda)1 + (1+3\lambda)(-1) + (1+4\lambda)1 = 0$$

$$1+2\lambda-1-3\lambda+1+4\lambda = 0$$

$$1+3\lambda = 0$$

$$\lambda = -\frac{1}{3}$$

Thus, the equation of the plane is :

$$\left(1+2\left(-\frac{1}{3}\right)\right)x+\left(1+3\left(-\frac{1}{3}\right)\right)y+\left(1+4\left(-\frac{1}{3}\right)\right)z-\left(1+5\left(-\frac{1}{3}\right)\right)=0$$

$$\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1-\frac{4}{3}\right)z-\left(1-\frac{5}{3}\right)=0$$

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$$x-z=-2$$

Thus, the distance of this plane from the origin is :

$$\left|\frac{-(-2)}{\sqrt{1^2+0^2+1^2}}\right|=\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$$

OR

To find the image of the point $2\hat{i}+3\hat{j}-4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$

So point A (2, 3, −4) and the plane is 2x – y + z = 3.

Let the required image be B (α,β, γ)

Direction ratios of AB are (α – 2, β – 3, γ + 4)

Direction ratios of normal to the plane (2, −1, 1)

AB is parallel to the normal to the plane

⇒ α – 2 = 2λ, β – 3 = −λ, γ + 4 = λ,

⇒ α = 2λ + 2, β = −λ + 3, γ = λ – 4

⇒ (2λ + 2, −λ + 3, λ – 4)

Mid-point of AB is $C\left(\frac{2\lambda+2+2}{2},\frac{-\lambda+3+3}{2},\frac{\lambda-4-4}{2}\right)$

$$C\left(\lambda+2,\frac{-\lambda}{2}+3,\frac{\lambda}{2}-4\right)$$

C lies on plane 2x – y + z = 3

$$2(\lambda+2)-\left(\frac{-\lambda}{2}+3\right)+\left(\frac{\lambda}{2}-4\right)=3$$

$$3\lambda-3=3$$

$$\lambda=2$$

⇒ Image B is given as

B (2 × 2 + 2, −2 + 3, 2 – 4)

B (6, 1, −2)

34. Let the two tailors work for x days and y days respectively,

The problem is to minimise the objective function, $C = 150x + 200y$, subject to the constraints,

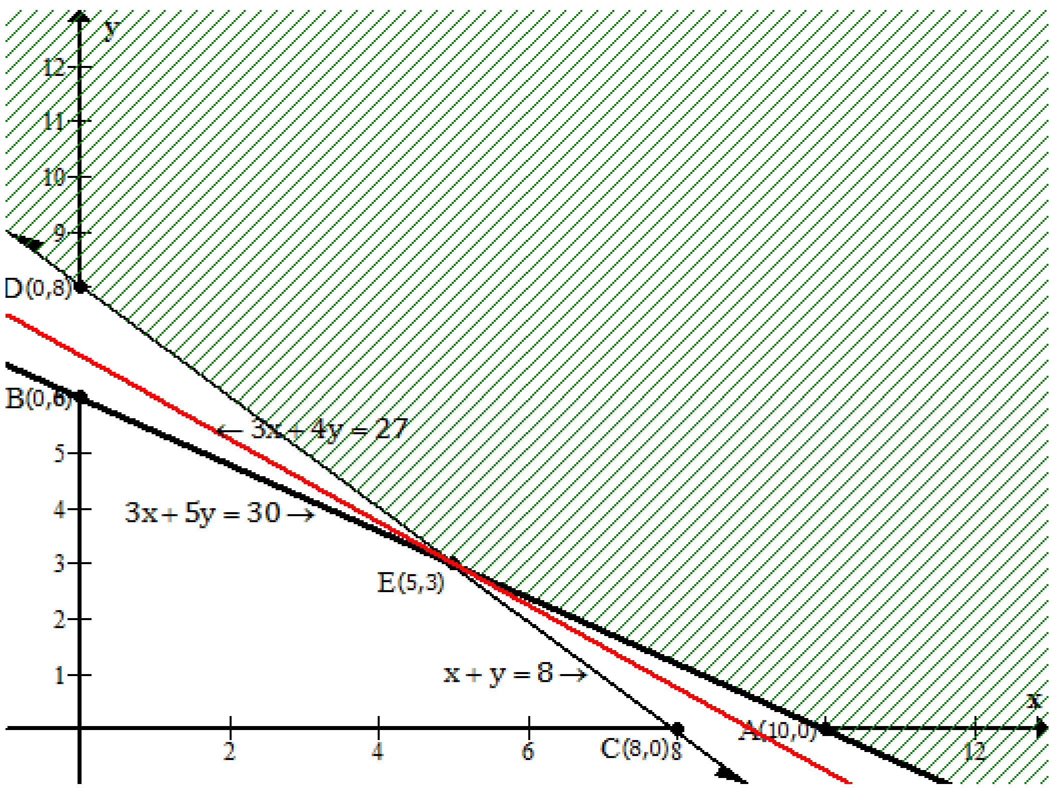
$6x + 10y \geq 60 \Leftrightarrow 3x + 5y \geq 30$

$4x + 4y \geq 32 \Leftrightarrow x + y \geq 8$

And,

$x \geq 0, y \geq 0$

Feasible region is shown shaded.



This region is unbounded.

Corner points	Objective function values $C = 150x + 200y$
A(10, 0)	1500
E(5, 3)	1350
D(0, 8)	1600

The red line in the graph shows the line $150x + 200y = 1350$ or $3x + 4y = 27$

We see that the region $3x + 4y > 27$ has no point in common with the feasible region.

Thus, the function has minimum value at E (5, 3).

Hence, the labour cost is the least when tailor A works for 5 days and Tailor B works for 3 days.

OR

Let x packets be transported to the school at P and y packets to the school at Q from the kitchen at A. Clearly (60 – x – y) packets will be transported to the school at R. Hence, $x \geq 0$, $y \geq 0$, $60 - x - y \geq 0$. The monthly requirements of the school at P is 40 packets and x packets are transported from the kitchen at A, the remaining (40 – x) packets will have to be transported from the kitchen at B. Thus $40 - x \geq 0$.

Similarly (40 – y) and [50 – (60 –x – y)] packets will be transported from the kitchen at B to the schools at Q and R respectively.

So $40 - y \geq 0$

And $50 - (60 - x - y) \geq 0$ i.e $x + y - 10 \geq 0$

Now the objective function or the transportation cost is given by

$C = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$

$= 3x + 4y + 370$

Now our problem is to minimise C subject to the constraints

Minimise $C = 3x + 4y + 370$,

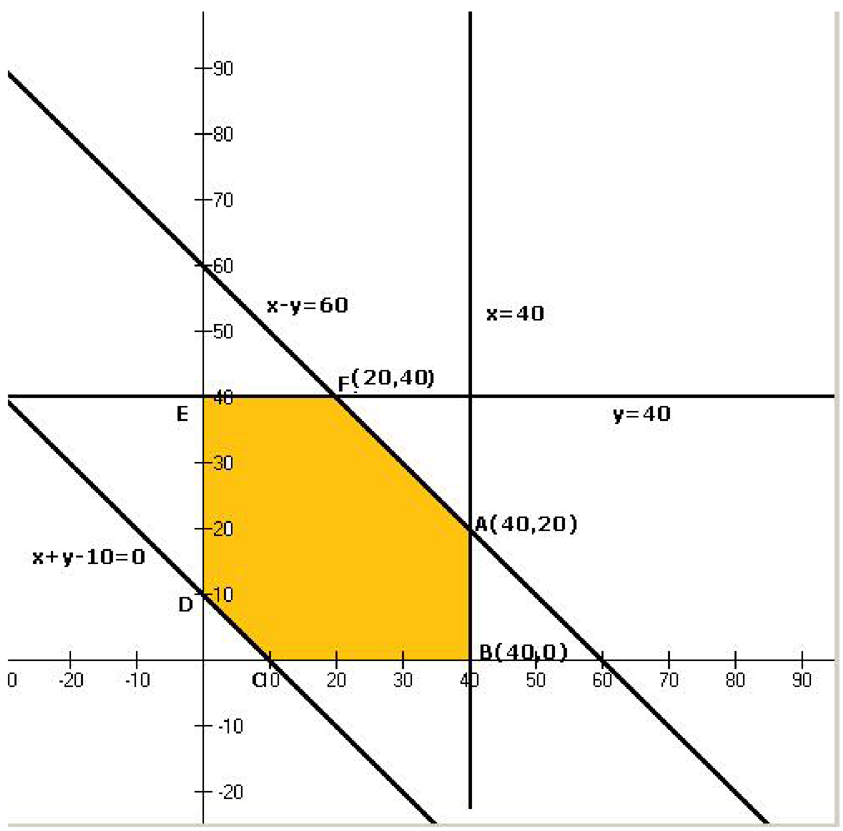
Subject to $60 - x - y \geq 0$

$40 - x \geq 0$

$40 - y \geq 0$

$x + y - 10 \geq 0$

$x \geq 0, y \geq 0$



The shaded region is the feasible region.

The corner points and the corresponding values of transportation cost is in this table.

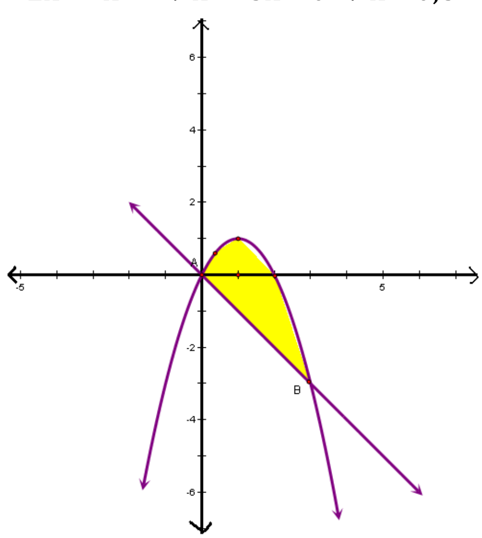
Corner points	Value of transportation cost
(10, 0)	380
(40, 0)	490
(40, 20)	570
(20, 40)	590
(0 , 40)	530
(0, 10)	410

Thus we find that C is minimum at the point (10, 0)
Therefore, for the cost to be minimum 10, 0 and (60 – 10 - 0) packets will be transported from the factory at A and 30, 40 and 0 packets will be transported from the factory at B to the agencies at P, Q and R respectively.
In the tabular form, the number of packets to be transported is

To/ From	A	B
P	10	30
Q	0	40
R	50	0

Minimum cost is given by
$C = 3 \times 10 + 4 \times 0 + 370 = \text{Rs. } 400$

35. The curve is $y = 2x - x^2$
$\Rightarrow (x-1)^2 = -(y-1)$
The points of intersection of the parabola and the line $y = -x$ is:
$-x = 2x - x^2 \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0, 3$



So points of intersection are $(0,0)$ and $(3,-3)$

Required area$=\left|\int_0^3 \left[\left(2x\text{-}x^2\right)-\left(-x\right)dx\right]\right|=\left|\int_0^3 \left[\left(3x-x^2\right)dx\right]\right|$

$=\left|\frac{3x^2}{2}-\frac{x^3}{3}\right|_0^3=\left|\frac{27}{2}-\frac{27}{3}\right|=27\left|\frac{1}{2}-\frac{1}{3}\right|$

$=\frac{9}{2}$sq.units

OR

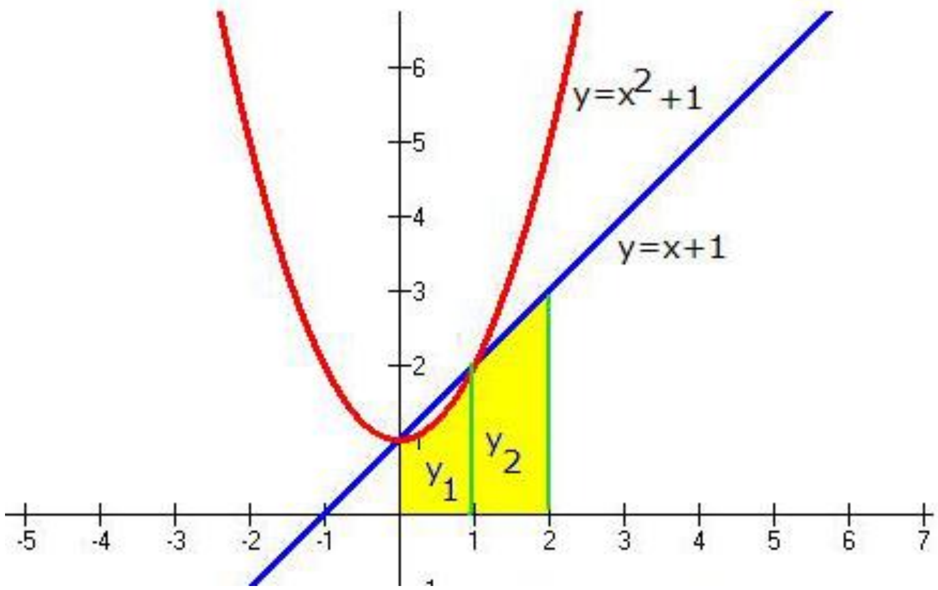
Points of intersection of $y = x^2 + 1$, $y = x + 1$

$x^2 + 1 = x + 1$

$\Rightarrow x\,(x - 1) = 0$

$\Rightarrow x = 0, 1$

So points of intersection are P(0, 1) and Q(1, 2). The graph is represented as



Required area is given by

$A = \int_0^1 y_1\,dx + \int_1^2 y_2\,dx,$

where y_1 and y_2 represent the y co-ordinate of the parabola and straight line respectively.

$\therefore A = \int_0^1 \left(x^2+1\right)dx + \int_1^2 \left(x+1\right)dx$

$=\left(\frac{x^3}{3}+x\right)\Bigg]_0^1 + \left(\frac{x^2}{2}+x\right)_1^2$

$=\left[\left(\frac{1}{3}+1\right)-0\right]+\left[\left(2+2\right)-\left(\frac{1}{2}+1\right)\right]=\frac{23}{6}$ sq. units

36. Given: $f(x)=\dfrac{4x^2+1}{x},(x\neq 0)$

$$\Rightarrow f(x)=\left(4x+\frac{1}{x}\right),(x\neq 0)$$

$$\Rightarrow f'(x)=4-\frac{1}{x^2}$$

$$\Rightarrow f'(x)=\frac{4x^2-1}{x^2} \text{ ... (i)}$$

a) f(x) is increasing

$\Rightarrow f'(x) \geq 0$

$$\Rightarrow \frac{4x^2-1}{x^2}\geq 0 \text{ .... From (i)}$$

$$\Rightarrow 4x^2-1 \geq 0 \text{ ... } [x^2>0]$$

$$\Rightarrow 2x-1 \quad 2x+1 \geq 0$$

$$\Rightarrow 2\left(x-\frac{1}{2}\right)\cdot 2\left(x+\frac{1}{2}\right)\geq 0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)\geq 0$$

$$\Rightarrow \left[\left(x-\frac{1}{2}\right)\geq 0 \text{ and } \left(x+\frac{1}{2}\right)\geq 0\right]$$

$$\text{or } \left[\left(x-\frac{1}{2}\right)\leq 0 \text{ and } \left(x+\frac{1}{2}\right)\leq 0\right]$$

$$\Rightarrow \left[x\geq\frac{1}{2} \text{ and } x\geq-\frac{1}{2}\right]$$

$$\text{or } \left[x\leq\frac{1}{2} \text{ and } x\leq-\frac{1}{2}\right]$$

$$\Rightarrow x\geq\frac{1}{2} \text{ or } x\leq-\frac{1}{2}$$

$$\Rightarrow x\in\left[\frac{1}{2},\infty\right] \text{ or } x\in\left(-\infty,-\frac{1}{2}\right]$$

$$\Rightarrow x\in\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right]$$

$\therefore$ f(x) is increasing on $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right]$.

b) f(x) is decreasing

$\Rightarrow f'(x) \leq 0$

$$\Rightarrow \frac{4x^2-1}{x^2} \le 0 \text{ .... From (i)}$$

$$\Rightarrow 4x^2-1 \le 0 \text{ ... } [x^2 > 0]$$

$$\Rightarrow 2x-1 \quad 2x+1 \le 0$$

$$\Rightarrow 2\left(x-\frac{1}{2}\right)\cdot 2\left(x+\frac{1}{2}\right) \le 0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right) \le 0$$

$$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$$

$$\Rightarrow x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$\therefore$ f(x) is decreasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

37. The events A, E_1, E_2, E_3, and E_4 are given by

A = event when doctor visits patients late

E_1 = doctor comes by train

E_2 = doctor comes by bus

E_3 = doctor comes by scooter

E_4 = doctor comes by other means of transport

So, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$, $P(E_4) = \frac{2}{5}$

P (A/E_1) = Probability that the doctor arrives late, given that he is comes by train.

$= \frac{1}{4}$

Similarly P $(A/E_2) = \frac{1}{3}$, P $(A/E_3) = \frac{1}{12}$, P $(A/E_4) = 0$

Required probability of the doctor arriving late by train by using Baye's theorem,

$$P(E_1/A) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)+P(E_4)P\left(A/E_4\right)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{3}{40}\times\frac{120}{18} = \frac{1}{2}$$

Hence the required probability is $\frac{1}{2}$.

OR

B_1: the bulb is manufactured by machine X

B_2: the bulb is manufactured by machine Y

B_3: the bulb is manufactured by machine Z

P (B_1) = 1000/(1000 + 2000 + 3000) = 1/6

P (B_2) = 2000/(1000 + 2000 + 3000) = 1/3

P (B_3) = 3000/(1000 + 2000 + 3000)= 1/2

P ($E|B_1$) = Probability that the bulb drawn is defective given that it is manufactured by machine X = 1% = 1/ 100

Similarly, P ($E|B_2$) = 1.5% = 15/ 100 = 3/ 200

P ($E|B_3$) = 2% = 2/100

$$P(B_1|E)=\frac{P(B_1)P(E|B_1)}{P(B_1)P(E|B_1)+P(B_2)P(E|B_2)+P(B_3)P(E|B_3)}$$

$$=\frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{200}+\frac{1}{2}\times\frac{2}{100}}$$

$$=\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1}$$

$$=\frac{1}{1+3+6}$$

$$=\frac{1}{10}$$