

Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 3

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The LCM and HCF of two rational numbers are equal, then the numbers must be [1]
 - a) equal
 - b) prime
 - c) co-prime
 - d) composite
2. If $a = 2^3 \times 3, b = 2 \times 3 \times 5, c = 3^n \times 5$ and $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5$, then $n =$ [1]
 - a) 1
 - b) 4
 - c) 3
 - d) 2
3. Two numbers whose sum is 12 and the absolute value of whose difference is 4 are the roots of the equation _____ [1]
 - a) $2x^2 - 24x + 43 = 0$
 - b) $2x^2 - 6x + 7 = 0$
 - c) $x^2 - 12x + 32 = 0$
 - d) $x^2 - 12x + 30 = 0$
4. If $am = bl$ and $bn \neq cm$, then the system of equations [1]
$$ax + by = c$$
$$lx + my = n$$
 - a) Has a unique solution.
 - b) Has infinitely many solutions.
 - c) Has no solution.
 - d) May or may not have a solution.
5. One of the two students, while solving a quadratic equation in x , copied the constant term incorrectly and got the [1] roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The

correct roots are _____.

- a) 6, -1
- b) 3, -2
- c) -3, 2
- d) -6, -1

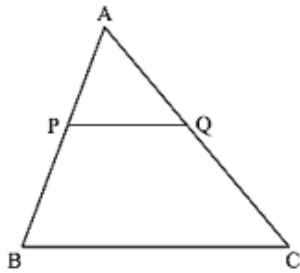
6. The distance of the point (4, 7) from the y-axis is [1]

- a) 11
- b) 4
- c) $\sqrt{65}$
- d) 7

7. $\triangle PQR \sim \triangle XYZ$ and the perimeters of $\triangle PQR$ and $\triangle XYZ$ are 30 cm and 18 cm respectively. If $QR = 9$ cm, then, YZ is equal to [1]

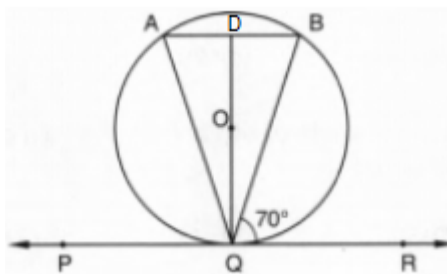
- a) 4.5 cm.
- b) 5.4 cm.
- c) 12.5 cm.
- d) 9.5 cm.

8. In $\triangle ABC$, $PQ \parallel BC$. If $PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm, find the length of AC . [1]



- a) 12 cm
- b) 14 cm
- c) 20 cm
- d) 6 cm

9. In Figure, if PQR is the tangent to a circle at Q whose centre is O , AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to [1]



- a) 40°
- b) 20°
- c) 35°
- d) 45°

10. If $X \sin^3 \theta + Y \cos^3 \theta = \sin \theta \cos \theta$ and $X \sin \theta = Y \cos \theta$, then _____ [1]

- a) $X^4 + Y^4 = 1$
- b) $X^2 + Y^2 = 1$
- c) $X^2 - Y^2 = 1$
- d) $X^3 + Y^3 = 1$

11. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of [1]

The mode of the above data is

- a) 25
- b) 23.5
- c) 24.4
- d) 24

19. **Assertion (A):** Distance between (3, 7) and its image under x-axis is 6 units. [1]

Reason (R): Coordinates of centroid = $\frac{x_1+x_2+x_3}{3}$, $\frac{y_1+y_2+y_3}{3}$

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

20. **Assertion (A):** For any two positive integers a and b, $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$ [1]

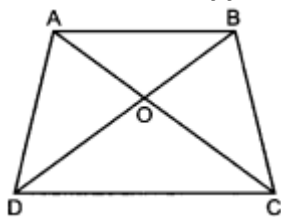
Reason (R): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

Section B

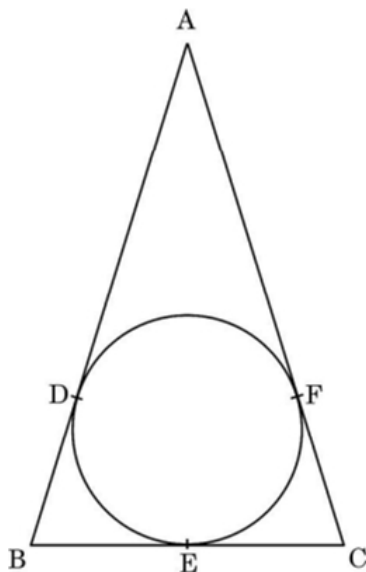
21. Aftab tells his daughter, **Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be.** (Isn't this interesting?) Represent this situation algebraically and graphically by the method of substitution. [2]

22. In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 4 cm. Find the value of DC. [2]



OR

ABC is an isosceles triangle with AB = AC, circumscribed about a circle. Prove that BC is bisected at E.



23. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. [2]
24. Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$ [2]
25. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. [2]

OR

The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.

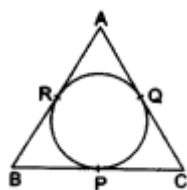
Section C

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point? [3]
27. Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients. [3]
28. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$. [3]

OR

If $x + 1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$.

29. In the given figure, the incircle of $\triangle ABC$ touches the sides BC , CA and AB at P , Q and R respectively. Prove that $(AR + BP + CQ) = (AQ + BR + CP) = \frac{1}{2}(\text{perimeter of } \triangle ABC)$. [3]



30. Find the acute angle θ , when $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$. [3]

OR

Prove: $\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$

31. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of [3]
- i. heart
 - ii. queen
 - iii. clubs.

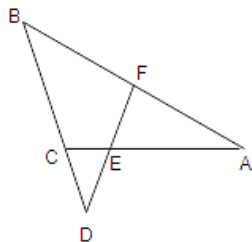
Section D

32. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train. [5]

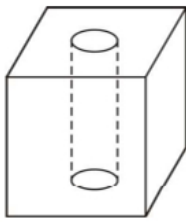
OR

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream, than to return to the same point. Find the speed of the stream and total time of the journey.

33. In the given figure, a $\angle AEF = \angle AFE$ and E is the mid-point of CA . Prove that $\frac{BD}{CD} = \frac{BF}{CE}$ [5]



34. In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid. [5]



OR

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the radius of the ball.

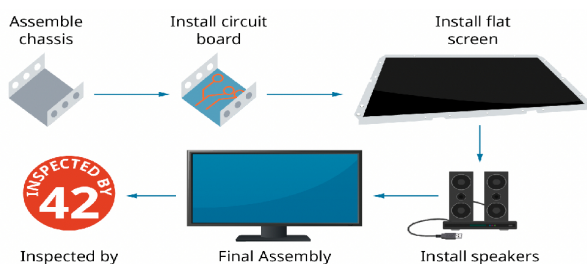
35. The median of the following data is 525. Find the values of x and y , if the total frequency is 100. [5]

Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

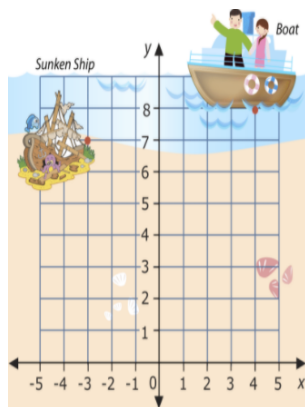
- Find the production in the 1st year. (1)
- Find the production in the 5th year. (1)
- Find the total production in 7 years. (2)

OR

Find in which year 10000 units are produced? (2)

37. Read the following text carefully and answer the questions that follow: [4]

Mary and John are very excited because they are going to go on a dive to see a sunken ship. The dive is quite shallow which is unusual because most sunken ship dives are found at depths that are too deep for two junior divers. However, this one is at 40 feet, so the two divers can go to see it.



They have the following map to chart their course. John wants to figure out exactly how far the boat will be from the sunken ship. Use the information in this lesson to help John figure out the following.

- What are the coordinates of the boat and the sunken ship respectively? (1)
- How much distance will Mary and John swim through the water from the boat to the sunken ship? (1)
- If each square represents 160 cubic feet of water, how many cubic feet of water will Mary and John swim through from the boat to the sunken ship? (2)

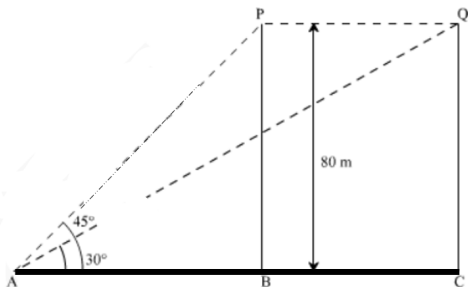
OR

If the distance between the points $(x, -1)$ and $(3, 2)$ is 5, then what is the value of x ? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45° . The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30° . Find the speed of flying of the bird.



- Find the distance between observer and the bottom of the tree? (1)
- Find the speed of the bird? (1)
- Find the distance between second position of bird and observer? (2)

OR

Find the distance between initial position of bird and observer? (2)

Solution

Section A

1. (a) equal

Explanation: If we assume that a and b are equal and consider $a = b = k$

Then,

$$\text{HCF}(a, b) = k$$

$$\text{LCM}(a, b) = k$$

- 2.

(d) 2

Explanation: $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5 \dots (I)$

we have to find the value of n

Also we have

$$a = 2^3 \times 3$$

$$b = 2 \times 3 \times 5$$

$$c = 3^n \times 5$$

We know that while evaluating LCM, we take greater exponent of the prime numbers in the factorisation of the number.

Therefore, by applying this rule and taking $n \geq 1$ we get the LCM as

$$\text{LCM}(a, b, c) = 2^3 \times 3^n \times 5 \dots (II)$$

On comparing (I) and (II) sides, we get:

$$2^3 \times 3^2 \times 5 = 2^3 \times 3^n \times 5$$

$$n = 2$$

- 3.

(c) $x^2 - 12x + 32 = 0$

Explanation: Let the two roots be a and b, then

$$a + b = 12 \dots (i)$$

$$\text{and } a - b = 4 \dots (ii)$$

$$\Rightarrow a = 8 \text{ and } b = 4 \text{ (from (i) and (ii))}$$

$$\therefore \text{Required equation is } x^2 - 12x + 32 = 0$$

- 4.

(c) Has no solution.

Explanation: We have, $ax + by = c$ and $lx + my = n$

$$\text{Now, } \frac{a}{l} = \frac{b}{m} \neq \frac{c}{n} \text{ (given)}$$

$\therefore$ The given system of equations has no solution.

5. (a) 6, -1

Explanation: Let the equation be $x^2 + ax + b = 0$

Its roots are 3 and 2

$$\therefore \text{Sum of roots, } 5 = -a$$

$$\text{and product of roots, } 6 = b$$

$$\therefore \text{Equation is } x^2 - 5x + 6 = 0$$

Now constant term is wrong and it is given that correct constant term is -6.

$$\therefore x^2 - 5x - 6 = 0 \text{ is the correct equation.}$$

Its roots are -1 and 6.

- 6.

(b) 4

Explanation: The distance of the point (4, 7) from y-axis is = 4

7.

(b) 5.4 cm.

Explanation: Given: $\triangle PQR \sim \triangle XYZ$

$$\therefore \frac{\text{Perimeter of } \triangle PQR}{\text{Perimeter of } \triangle XYZ} = \frac{QR}{YZ}$$

$$\Rightarrow \frac{30}{18} = \frac{9}{YZ}$$

$$\Rightarrow YZ = 5.4 \text{ cm}$$

8.

(c) 20 cm

Explanation: In $\triangle ABC$, $PQ \parallel BC$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC} \text{ (By proportionality theorem)}$$

$$\Rightarrow \frac{4}{6} = \frac{8}{QC}$$

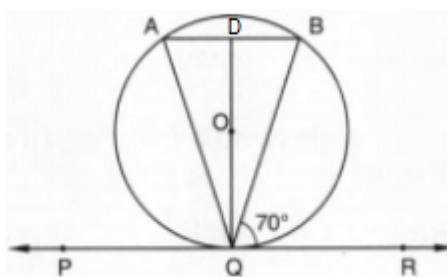
$$\Rightarrow QC = \frac{8 \times 6}{4} = 12 \text{ cm}$$

Now, $AC = AQ + QC$

$$= 8 + 12$$

$$= 20 \text{ cm}$$

9. (a) 40°



Explanation:

Given, $AB \parallel PR$

$$\angle ABQ = \angle BQR = 70^\circ \text{ [alternate angles]}$$

Also QD is perpendicular to AB and QD bisects AB.

In $\triangle ODA$ and $\triangle ODB$

$$\angle QDA = \angle QDB \text{ [each } 90^\circ]$$

$$AD = BD$$

$$QD = QD \text{ [common side]}$$

$$\therefore \triangle ADQ \cong \triangle BDQ \text{ ...[by SAS similarity criterion]}$$

$$\text{Then, } \angle QAD = \angle QBD \text{ ... (i) [c, p, c, t]}$$

$$\text{Also, } \angle ABQ = \angle BQR \text{ [alternate interior angle]}$$

$$\angle ABQ = 70^\circ \text{ ... [} \angle BQR = 70^\circ \text{]}$$

$$\text{Hence, } \angle QAB = 70^\circ \text{ [from Eq. (i)]}$$

Now, in $\triangle ABQ$

$$\angle A + \angle B + \angle Q = 180^\circ$$

$$\Rightarrow \angle Q = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

10.

$$(b) X^2 + Y^2 = 1$$

Explanation: We have, $X \sin^3 \theta + Y \cos^3 \theta = \sin \theta \cos \theta \text{ ... (i)}$

$$X \sin \theta = Y \cos \theta \text{ ... (ii)}$$

Using (ii) in (i), we get

$$\Rightarrow Y \cos \theta \sin^2 \theta + Y \cos^3 \theta = \sin \theta \cos \theta$$

$$\Rightarrow Y \sin^2 \theta + Y \cos^2 \theta = \sin \theta \Rightarrow Y = \sin \theta$$

$$\text{Now, } X \sin \theta = \sin \theta \times \cos \theta \Rightarrow X = \cos \theta$$

$$\therefore X^2 + Y^2 = 1$$

11.

(d) 30°

Explanation: $\tan\theta = \frac{100}{100\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

12. (a) 9

Explanation: Given: $9 \sec^2 A - 9 \tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9 \dots [\because \sec^2 \theta - \tan^2 \theta = 1]$

13.

(b) 3 : 4 : 5

Explanation: Area of sector Having central angle 90 degree : Area of sector Having central angle 120 degree : Area of sector Having central angle 150 degree

$= \frac{90}{360} \times \pi \times 6^2 : \frac{120}{360} \times \pi \times 6^2 : \frac{150}{360} \times \pi \times 6^2$

$= \frac{1}{4} : \frac{1}{3} : \frac{5}{12}$

$= 3 : 4 : 5$

14. (a) 13 cm

Explanation: Radius of wheel = $\frac{91}{2}$ cm

Angle between two adjoining spokes, $\theta = \frac{360^\circ}{22}$

$\therefore$ Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{360^\circ}{360^\circ \times 22} \times 2 \times \frac{22}{7} \times \frac{91}{2} = 13 \text{ cm}$

15.

(d) $\frac{1}{3}$

Explanation: A die is thrown, the possible number of events (n) = 6

Now multiple of 3 are 3, 6 which are 2

$\therefore m = 2$

$\therefore$ Probability = $\frac{m}{n} = \frac{2}{6} = \frac{1}{3}$

16.

(b) 17.5

Explanation: Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval 11.5 - 17.5.

Hence, the upper limit is 17.5.

17. (a) $\frac{9}{4}$ cm

Explanation: Radius of sphere (r_1) = 3 cm

$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 \text{ cm}^3$

$= 36\pi \text{ cm}^3$

$\therefore$ Volume of water in the cylinder = $36\pi \text{ cm}^3$

Radius of cylindrical vessel (r_2) = 4 cm

Let h be its height, then

$$\pi r_2^2 h = 36\pi \Rightarrow \pi(4)^2 h = 36\pi$$

$$\Rightarrow 16\pi h = 36\pi \Rightarrow h = \frac{36\pi}{16\pi} = \frac{9}{4} \text{ cm}$$

18.

(c) 24.4

Explanation: Maximum frequency = 25

Hence, modal class is 22 - 26

$$\text{Now, Mode} = x_k + h \left\{ \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$$

$$= 22 + 4 \left\{ \frac{(25 - 16)}{(2(25) - 16 - 19)} \right\}$$

$$= 22 + 4 \times \frac{9}{15}$$

$$= 22 + 2.4$$

$$= 24.4$$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Distance of point (h, k) from its image under x-axis is 2k units and distance of point (h, k) under y-axis is 2h units.

20.

(c) A is true but R is false.

Explanation: We have,

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

$$\text{LCM} \times 5 = 150$$

$$\text{LCM} = \frac{150}{5} = 30$$

$$\text{LCM} = 30$$

Section B

21. Let the present age of Aftab and his daughter be x and y years respectively. Then, the pair of linear equations that represent the situation is

$$x - 7 = 7(y - 7), \text{ i.e., } x - 7y + 42 = 0 \dots(1)$$

$$\text{and } x + 3 = 3(y + 3), \text{ i.e., } x - 3y = 6 \dots(2)$$

from equation (2), we get $x = 3y + 6$

By putting this value of x in equation (1), we get

$$(3y + 6) - 7y + 42 = 0,$$

$$\text{i.e., } -4y = -48, \text{ which gives } y = 12$$

Again by putting this value of y in equation (2), we get

$$x = 3 \times 12 + 6 = 42$$

So, the present age of Aftab and his daughter are 42 and 12 years respectively.

22. Given: $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 4 cm

To find: DC

Proof: In AOB and COD

$$\frac{AO}{OC} = \frac{BO}{OD} \text{ (given)}$$

and $\angle AOB = \angle COD$ (vertically opposite angles)

$\therefore \triangle AOB \sim \triangle COD$ (SAS similarity)

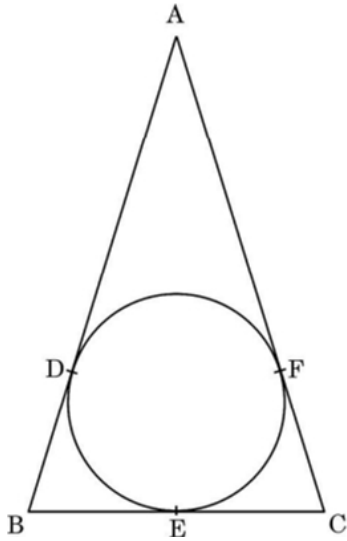
$$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD} \text{ (corresponding sides of similar triangles are proportional)}$$

$$\Rightarrow \frac{1}{2} = \frac{4}{CD}$$

$$\Rightarrow CD = 8 \text{ cm}$$

OR

ABC is an isosceles triangle.



According to the question, $AB = AC$...(i)

$AD = AF$ (Tangents from A) ...(ii)

$$AB - AD = AC - AF$$

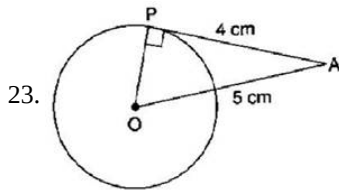
$$\Rightarrow BD = CF \text{ ...(iii)}$$

Now, $BD = BE$ (Tangents from B)

Also, $CF = CE$ (Tangents from C)

$$\Rightarrow BE = CE$$

So, BC is bisected at the point of contact E.



We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ$$

$$\therefore OA^2 = OP^2 + AP^2 \text{ [By Pythagoras theorem]}$$

$$\Rightarrow (5)^2 = (OP)^2 + (4)^2$$

$$\Rightarrow 25 = (OP)^2 + 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = 3 \text{ cm}$$

24. Dividing N^r & D^r by $\sin A$ in LHS

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \operatorname{cosec} A + \cot A$$

25. Here, $r = 14 \text{ cm}$ and $\theta = \frac{90^\circ}{3} = 30^\circ$

$$\therefore \text{Area swept} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{154}{3} \text{ cm}^2$$

OR

We know that:

Angle described by the minute hand in 60 minutes = 360°

Therefore, Angle described by the minute hand in one minute = $\frac{360}{60} = 6^\circ$

Angle described by the minute hand in 35 minutes = $(6 \times 35)^\circ = 210^\circ$

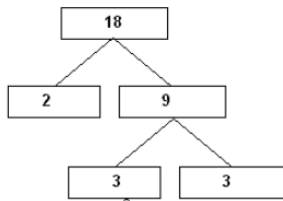
Area swept by the minute hand in 35 minutes = Area of a sector of angle 210° in a circle of radius 10 cm

$$= \left\{ \frac{210}{360} \times \frac{22}{7} \times (10)^2 \right\} \text{ cm}^2 = 183.3 \text{ cm}^2$$

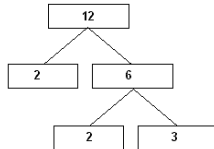
Section C

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$



$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$



$$\text{LCM}(18, 12) = 2^2 \times 3^2 = 36$$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. Here, $p(x) = 3x^2 - 2$.

$$\text{Now } p(x) = 0$$

$$\Rightarrow 3x^2 - 2 = 0$$

$$\Rightarrow 3x^2 = 2$$

$$\Rightarrow x^2 = \frac{2}{3}$$

$$\Rightarrow x = \pm \sqrt{\frac{2}{3}}$$

Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

If $p(x) = 3x^2 - 2$, then $a = 3$, $b = 0$ and $c = -2$

$$\text{Now, sum of zeroes} = \sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0 \dots (i)$$

$$\text{Also, } \frac{-b}{a} = \frac{-0}{3} = 0 \dots (ii)$$

From (i) and (ii)

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$\text{and product of zeroes} = \sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3} \dots (iii)$$

$$\text{Also, } \frac{c}{a} = \frac{-2}{3} \dots (iv)$$

From (iii) and (iv)

$$\text{Product of zeroes} = \frac{c}{a}$$

28. The given pair of linear equations

$$2x + 3y = 11 \dots (1)$$

$$2x - 4y = -24 \dots (2)$$

From equation (1), $3y = 11 - 2x$

$$\Rightarrow y = \frac{11-2x}{3}$$

Substituting this value of y in equation (2), we get

$$2x - 4\left(\frac{11-2x}{3}\right) = -24$$

$$\Rightarrow 6x - 44 + 8x = -72$$

$$\Rightarrow 14x - 44 = -72$$

$$\Rightarrow 14x = 44 - 72$$

$$\Rightarrow 14x = -28$$

$$\Rightarrow x = -\frac{28}{14} = -2$$

Substituting this value of x in equation (3), we get

$$y = \frac{11-2(-2)}{3} = \frac{11+4}{3} = \frac{15}{3} = 5$$

Verification, Substituting $x = -2$ and $y = 5$, we find that both the equations (1) and (2) are satisfied as shown below:

$$2x + 3y = 2(-2) + 3(5) = -4 + 15 = 11$$

$$2x - 4y = 2(-2) - 4(5) = -4 - 20 = -24$$

This verifies the solution,

Now, $y = ax + 3$

$$\Rightarrow 5 = m(-2) + 3$$

$$\Rightarrow -2m = 5 - 3$$

$$\Rightarrow -2m = 2$$

$$\Rightarrow m = \frac{2}{-2} = -1$$

OR

Since $(x + 1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$

$$\Rightarrow x = -1 \text{ is a zero of } 2x^3 + ax^2 + 2bx + 1$$

$$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$$

$$\Rightarrow a - 2b - 1 = 0$$

$$\Rightarrow a - 2b = 1 \dots(i)$$

$$\text{Given that } 2a - 3b = 4 \dots(ii)$$

Multiplying equation (i) by 2, we get

$$2a - 4b = 2 \dots(iii)$$

Subtracting equation (iii) from (ii), we get

$$b = 2$$

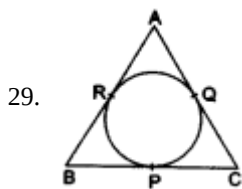
Substituting $b = 2$ in equation (i), we have

$$a - 2(2) = 1$$

$$\Rightarrow a - 4 = 1$$

$$\Rightarrow a = 5$$

Hence, $a = 5$ and $b = 2$.



We know that the lengths of tangents from an exterior point to a circle are equal.

$$\therefore AR = AQ, \dots (i) \text{ [tangents from A]}$$

$$BP = BR, \dots (ii) \text{ [tangents from B]}$$

$$CQ = CP \dots (iii) \text{ [tangents from C]}$$

$$\therefore (AR + BP + CQ) = (AQ + BR + CP) = k \text{ (say).}$$

$$\text{Perimeter of } \triangle ABC = (AB + BC + CA)$$

$$= (AR + BR) + (BP + CP) + (CQ + AQ)$$

$$= (AR + BP + CQ) + (AQ + BR + CP)$$

$$= (k + k) = 2k$$

$$\Rightarrow k = \frac{1}{2} (\text{perimeter of } \triangle ABC).$$

$$\therefore (AR + BP + CQ) = (AQ + BR + CP)$$

$$= \frac{1}{2} (\text{perimeter of } \triangle ABC).$$

30. According to question

$$\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

$$\Rightarrow \frac{(\cos \theta - \sin \theta) + (\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta) - (\cos \theta + \sin \theta)} = \frac{(1 - \sqrt{3}) + (1 + \sqrt{3})}{(1 - \sqrt{3}) - (1 + \sqrt{3})} \text{ [Applying componendo and dividendo]}$$

$$\Rightarrow \frac{2 \cos \theta}{-2 \sin \theta} = \frac{2}{-2\sqrt{3}}$$

$$\Rightarrow \cot \theta = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

OR

$$\text{Given- } \sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$$

Now, taking

$$\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$$

Taking LHS

$$= \sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3$$

$$= (\sin^2 A + \cos^2 A)^3 - 3 \sin^2 A \cos^2 A (\sin^2 A + \cos^2 A) \{ \because a^3 + b^3 = (a + b)^3 - 3ab(a + b) \}$$

$$= (1)^3 - 3\sin^2 A \cos^2 A(1)$$

$$= 1 - 3\sin^2 A \cos^2 A = \text{RHS}$$

31. When king, queen and jack of clubs are removed, number of cards remaining = $52 - 3 = 49$

Total no. of outcomes = 49

i. Let H be the event of getting a heart card.

Thus, favorable outcomes = 13

$$P(H) = \frac{\text{Favorable outcomes}}{\text{Total no. of outcomes}} = \frac{13}{49}$$

ii. Let Q be the event of getting a queen card.

Thus, favorable outcomes = 3 (1 queen of clubs is removed)

$$P(Q) = \frac{\text{Favorable outcomes}}{\text{Total no. of outcomes}} = \frac{3}{49}$$

iii. Let C be the event of getting a clubs card.

Thus out of 49 cards, there are 10 clubs cards, because king, queen and jack of clubs are removed

Hence, favorable outcomes = 10

$$P(C) = \frac{\text{Favorable outcomes}}{\text{Total no. of outcomes}} = \frac{10}{49}$$

Section D

32. Let the usual speed of train be x km/hr

$$\frac{300}{x} - \frac{300}{x+5} = 2$$

$$300(x+5 - x) = 2x(x+5)$$

$$150(5) = x^2 + 5x$$

$$750 = x^2 + 5x$$

$$\text{or, } x^2 + 5x - 750 = 0$$

$$\text{or, } x^2 + 30x - 25x - 750 = 0$$

$$\text{or, } (x + 30)(x - 25) = 0$$

$$\text{or, } x = -30 \text{ or } x = 25$$

Since, speed cannot be negative.

$$\therefore x \neq -30, x = 25 \text{ km/hr}$$

$\therefore$ Speed of train = 25 km/hr

OR

Given:-

Speed of boat = 18 km/hr

Distance = 24 km

Let x be the speed of stream.

Let t_1 and t_2 be the time for upstream and downstream As we know that,

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\Rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

For upstream, Speed = $(18 - x)$ km/hr

Distance = 24 km

Time = t_1

Therefore,

$$t_1 = \frac{24}{18-x}$$

For downstream,

Speed = $(18 + x)$ km/hr

Distance = 24 km

Time = t_2

Therefore,

$$t_2 = \frac{24}{18+x}$$

Now according to the question-

$$t_1 = t_2 + 1$$

$$\frac{24}{18-x} = \frac{24}{18+x} + 1$$

$$\Rightarrow \frac{1}{18-x} - \frac{1}{18+x} = \frac{1}{24}$$

$$\Rightarrow \frac{(18+x)-(18-x)}{(18-x)(18+x)} = \frac{1}{24}$$

$$\Rightarrow 48x = (18-x)(18+x)$$

$$\Rightarrow 48x = 324 + 18x - 18x - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x+54) - 6(x+54) = 0$$

$$\Rightarrow (x+54)(x-6) = 0$$

$$\Rightarrow x = -54 \text{ or } x = 6$$

Since speed cannot be negative.

$$\Rightarrow x \neq -54$$

$$\therefore x = 6$$

Thus the speed of stream is 6 km/hr.

Total time of Journey = $t_1 + t_2$

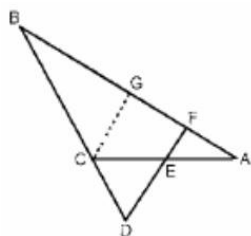
$$= \frac{24}{18-x} + \frac{24}{18+x}$$

$$= \frac{24}{12} + \frac{24}{24} = 2 + 1 = 3 \text{ hrs.}$$

33. Given, $\angle AEF = \angle AFE$ and E is the mid-point of CA.

To prove, $\frac{BD}{CD} = \frac{BF}{CE}$

Construction Draw a line CG parallel to DF ($CG \parallel DF$).



Proof : $\angle AEF = \angle AFE$ and E is the mid-point of CA

$$\therefore CE = AE = \frac{AC}{2} \dots (i)$$

In $\triangle BDF$, $CG \parallel DF$

By Basic proportionality theorem,

$$\frac{BD}{CD} = \frac{BF}{GF} \dots (ii)$$

In $\triangle AFE$,

$$\angle AEF = \angle AFE \text{ [}\therefore \text{given]}$$

$$\Rightarrow AF = AE \text{ [}\therefore \text{Since, sides opposite to equal angles are equal]}$$

$$\Rightarrow AF = AE = CE \text{ [}\therefore \text{From Eq(i)]} \dots (iii)$$

In $\triangle ACG$, E is the midpoint of AC and $EF \parallel CG$,

$$\therefore FG = AF \text{ [}\therefore AE = CE \text{]} \dots (iv)$$

From Eq(ii), Eq(iii) and Eq(iv),

$$\frac{BD}{CD} = \frac{BF}{GF}$$

$$\frac{BD}{CD} = \frac{BF}{CE} \text{ [}\therefore GF = AF = CE \text{]}$$

Hence proved.

34. We have;

A Cube,

$$\text{Cube's } \frac{\text{length}}{\text{Edge}}, a = 7 \text{ cm}$$

A Cylinder:

$$\text{Cylinder's Radius, } r = 2.1 \text{ cm or } r = \frac{21}{10} \text{ cm}$$

$$\text{Cylinder's Height, } h = 7 \text{ cm}$$

$\therefore$ A cylinder is scooped out from a cube,

$\therefore$ TSA of the resulting cuboid:

$$= \text{TSA of whole Cube} - 2 \times (\text{Area of upper circle or Area of lower circle}) + \text{CSA of the scooped out Cylinder}$$

$$= 6a^2 + 2\pi rh - 2 \times (\pi r^2)$$

$$= 6 \times (7)^2 + 2 \times (22 \div 7 \times 2.1 \times 7) - 2 \times [22 \div 7 \times (2.1)^2]$$

$$\begin{aligned}
&= 6 \times 49 + (44 \div 7 \times 14.7) - (44 \div 7 \times 4.41) \\
&= 294 + 92.4 - 27.72 \\
&= 294 + 64.68 \\
&= 358.68 \text{ cm}^2
\end{aligned}$$

Hence, the total surface area of the remaining solid is 358.68 cm^2

OR

According to question it is given that

Radius of cylindrical tub = 12 cm

Depth of cylindrical tub = 20 cm

Let us suppose that (r) be the radius of spherical ball

Again it is given that level of water is raised by 6.75 cm

Now, according to the question,

Volume of spherical ball = Volume of water rise in cylindrical tub

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi (12)^2 \times 6.75$$

$$\Rightarrow \frac{4}{3}r^3 = 12 \times 12 \times 6.75$$

$$\Rightarrow r^3 = \frac{12 \times 12 \times 6.75 \times 3}{4}$$

$$\Rightarrow r^3 = 729$$

$$\Rightarrow r = \sqrt[3]{729} = 9 \text{ cm}$$

Therefore, Radius of the ball = 9 cm

35.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,

$$N = \sum f_i = 100$$

$$\Rightarrow 76 + x + y = 100$$

$$\Rightarrow x + y = 24$$

It is given that the median is 525. Clearly, it lies in the class 500 - 600

$$\therefore l = 500, h = 100, f = 20, F = 36 + x \text{ and } N = 100$$

$$\text{Now, Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$\Rightarrow 525 - 500 = (14 - x)5$$

$$\Rightarrow 25 = 70 - 5x$$

$$\Rightarrow 5x = 45$$

$$\Rightarrow x = 9$$

Putting $x = 9$ in $x + y = 24$, we get $y = 15$

Hence, $x = 9$ and $y = 15$

Section E

36. i. Let production in a 1st year be 'a' unit and increase in production (every year) be 'd' units.

Increase in production is constant, therefore unit produced every year forms an AP.

Now, $a_3 = 6000$

$$a + 2d = 6000 \Rightarrow a = 6000 - 2d \dots(i)$$

$$\text{and } a_7 = 7000 \Rightarrow a + 6d = 7000$$

$$\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000 \text{ [using eq. (i)]}$$

$$\Rightarrow d = 250$$

When $d = 250$, eq. (i) becomes

$$a = 6000 - 2(250) = 5500$$

$\therefore$ Production in 1st year = 5500

- ii. Production in fifth year

$$a_5 = a + 4d = 5500 + 4(250) = 6500$$

- iii. Total production in 7 years = $\frac{7}{2}(5500 + 7000) = 43750$

OR

$$a_n = 1000 \text{ units}$$

$$a_n = 1000$$

$$\Rightarrow 10000 = a + (n - 1)d$$

$$\Rightarrow 1000 = 5500 + 250n - 250$$

$$\Rightarrow 10000 - 5500 + 250 = 250n$$

$$\Rightarrow 4750 = 250n$$

$$\Rightarrow n = \frac{4750}{250} = 19$$

37. i. (4, 8) and (-3, 7)

- ii. 8 units

- iii. 1280 cubic feet

OR

7 or -1

38. i. Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle ABP$

$$\tan 45^\circ = \frac{BP}{AB}$$

$$\Rightarrow 1 = \frac{80}{AB}$$

$$\Rightarrow AB = 80 \text{ m}$$

- ii. The speed of the bird

In $\triangle AQC$

$$\tan 30^\circ = \frac{QC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$$

$$\Rightarrow AC = 80\sqrt{3} \text{ m}$$

$$AC - AB = BC$$

$$\Rightarrow BC = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1) \text{ m}$$

$$\text{Speed of bird} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$$

$$\Rightarrow \text{Speed of the bird} = 29.28 \text{ m/sec}$$

- iii. The distance between second position of bird and observer.

In $\triangle AQC$

$$\sin 30^\circ = \frac{QC}{AQ}$$

$$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$$

$$\Rightarrow AQ = 160 \text{ m}$$

OR

The distance between initial position of bird and observer.

In $\triangle ABP$

$$\begin{aligned}\sin 45^\circ &= \frac{BP}{AP} \\ \Rightarrow \frac{1}{\sqrt{2}} &= \frac{80}{AP} \\ \Rightarrow AP &= 80\sqrt{2}m\end{aligned}$$