SAMPLE QUESTION PAPER (STANDARD) - 10

Class 10 - Mathematics

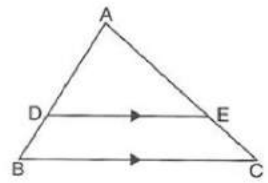
Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

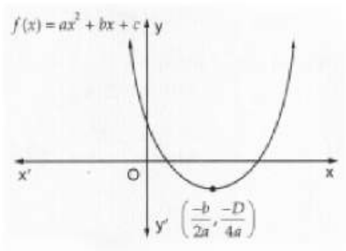
1. In the given figure, $DE||BC$. AB = 15, cm, BD = 6cm, AC = 25 cm, then AE is equal to **[1]**



a) 15 cm. b) 18 cm.

c) 20 cm. d) 10 cm.

2. Figure show the graph of the polynomial f(x) = $ax^2 + bx + c$ for which **[1]**



a) a > 0, b < 0 and c > 0 b) a < 0, b < 0 and c < 0

c) a < 0, b > 0 and c > 0 d) a > 0, b > 0 and c < 0

3. If $2^{x+y} = 2^{x-y} = \sqrt{8}$ then the value of y is **[1]**

a) none of these b) 0

c) $\frac{3}{2}$ d) $\frac{1}{2}$

4. In a given fraction, if 1 is subtracted from the numerator and 2 is added to the denominator, it becomes $\frac{1}{2}$. If 7 is subtracted from the numerator and 2 is subtracted from the denominator, it becomes $\frac{1}{3}$. The fraction is **[1]**

a) $\frac{16}{21}$ b) $\frac{16}{27}$

c) $\frac{15}{26}$ d) $\frac{13}{24}$

5. In △ABC and △PQR, ∠B = ∠Q, ∠R = ∠C and AB = 2QR, then, the triangles are **[1]**

a) Similar but not congruent. b) Neither congruent nor similar.

c) Congruent as well as similar. d) Congruent but not similar.

6. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number, which is a multiple of 7, is **[1]**

a) $\frac{1}{5}$ b) $\frac{1}{8}$

c) $\frac{1}{7}$ d) $\frac{7}{40}$

7. If $2\sin 2\theta = \sqrt{3}$ then θ= ? **[1]**

a) 45° b) 90°

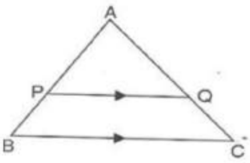
c) 60° d) 30°

8. If the mode of the data: 64, 60, 48, x, 43, 48, 43, 34 is 43, then x + 3 = **[1]**

a) 45 b) 48

c) 44 d) 46

9. In the given figure $PQ||BC$. $\frac{AP}{PB} = 4$, then the value of $\frac{AQ}{AC}$ is **[1]**



a) 5 b) $\frac{4}{5}$

c) 4 d) $\frac{5}{4}$

10. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is **[1]**

a) 1600 b) 8000

c) 1605 d) 320

11. $9x^2 - 6x - 4 = 0$ have **[1]**

a) No Real roots b) Real and Distinct roots

c) Real roots d) Real and Equal roots

12. The coordinates of a point on x-axis which lies on the perpendicular bisector of the line segment joining the points (7, 6) and (-3, 4) are **[1]**

a) (3, 0) b) (0, 2)

c) (0, 3) d) (2, 0)

13. Mode is: **[1]**

a) least frequent value

b) None of these

c) middle most value

d) most frequent value

14. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\theta = q$, then $q(p^2 - 1) =$ **[1]**

a) 2p

b) None of these

c) $\frac{q}{p^2}$

d) 2

15. The angle of elevation of the sun when the shadow of a pole 'h' metres high is $\frac{h}{\sqrt{3}}$ metres long is **[1]**

a) 45°

b) 30°

c) 60°

d) none of these

16. From a point Q, the length of tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm, radius of circle is **[1]**

a) 8 cm

b) 10 cm

c) 6 cm

d) 7 cm

17. If $\triangle ABC \sim \triangle DEF$ then which of the following is true? **[1]**

a) BC.DE = AB.FD

b) AB.EF = AC.DE

c) BC.EF = AC.FD

d) BC.DE = AB.EF

18. The sum of reciprocals of Sharma's age 3 years ago and 5 years from now is $\frac{1}{3}$, then his present age is **[1]**

a) 7 years

b) 10 years

c) 6 years

d) 8 years

19. **Assertion (A):** A quadratic polynomial, sum of whose zeroes is 8 and their product is 12 is $x^2 - 20x + 96$ **[1]**

Reason (R): If α and β be the zeroes of the polynomial f(x), then polynomial is given by $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve equation $6x^2 + (12 - 8a)x - 16a = 0$ using quadratic formula. **[2]**

22. Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram. **[2]**

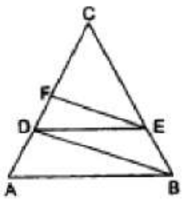
23. Find the largest number which divides 129 and 545, leaving remainders 3 and 5 respectively. **[2]**

24. Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$ **[2]**

OR

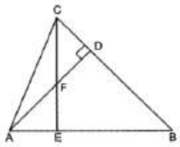
Prove that: (sin α + cos α) (tan α + cot α) = sec α + cosec α

25. In the given figure, $AB \| DE$ and $BD \| EF$ Prove that $\mathrm{DC}^2 = \mathrm{CF} \times \mathrm{AC}$. **[2]**



OR

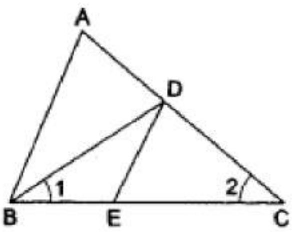
In Fig. AD and CE are two altitudes of $\triangle$ABC. Show that $\triangle ABD \sim \triangle CBE$.



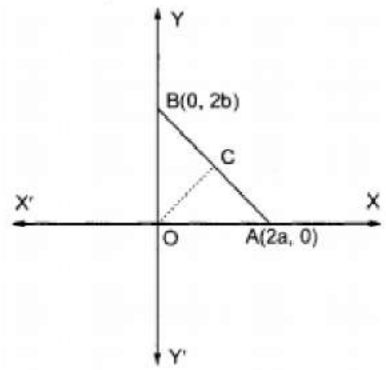
Section C

26. ₹ 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons. **[3]**

27. In the given figure, $\angle 1 = \angle 2$ and $\frac{AC}{BD} = \frac{CB}{CE}$. **[3]**
Prove that $\triangle ACB \sim \triangle DCE$.



28. A right triangle BOA is given. C is the mid-point of the hypotenuse AB. Show that it is equidistant from the vertices O, A and B. **[3]**



OR

A point P divides the line segment joining the points A (3, - 5) and B (- 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k.

29. Find the HCF of the following polynomials: $2\left(x^4 - y^4\right), 3\left(x^3 + 2x^2y - xy^2 - 2y^3\right)$ **[3]**

30. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships are observed from the top of the lighthouse are 60° and 45° respectively. If the height of the lighthouse is 200 m, find the distance between the two ships. (Use $\sqrt{3}$ = 1.73) **[3]**

OR

From the top of hill, the angles of depression of two consecutive kilometer stones due East are found to be 30^o and 45^o. Find the height of the hill.

31. Find the mode of the following distribution: **[3]**

Class Interval	**Frequency**
0 - 10	5
10 - 20	8
20 - 30	7
30 - 40	12
40 - 50	28
50 - 60	20
60 - 70	10
70 - 80	10

Section D

32. Solve graphically the system of linear equation: **[5]**

$4x - 3y + 4=0$

$4x + 3y -20=0$

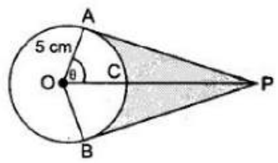
Find the area bounded by these lines and x-axis.

OR

Akhila goes to a fair with Rs. 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically). If each ride costs Rs. 3, and a game of Hoopla costs Rs. 4, how would you find out the number of rides she had and how many times she played Hoopla, provided she spent Rs. 20.

33. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ. **[5]**

34. An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

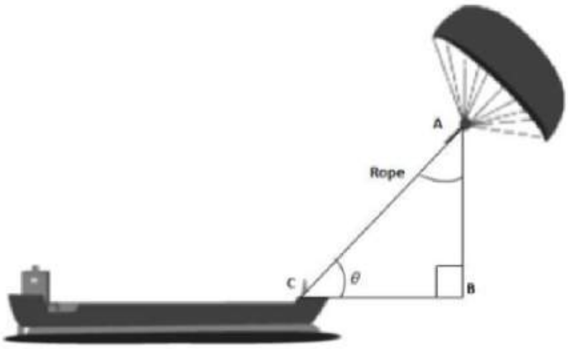
35. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if $\sin\theta = \cos(\theta - 30^\circ)$, where θ and $\theta - 30^\circ$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^\circ)$, where θ and $3\theta - 30^\circ$ are acute angles, then find the value of θ.

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

37. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) How much distance did she cover in pacing 6 flags on either side of center point?

(ii) Represent above information in Arithmetic progression

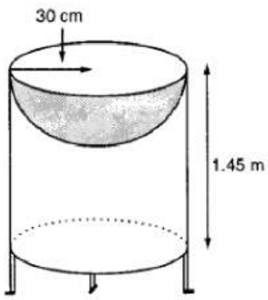
OR

What is the maximum distance she travelled carrying a flag?

(iii) How much distance did she cover in completing this job and returning to collect her books?

38. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

(iii) What is total cost for making the bird bath?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 10

Class 10 - Mathematics

Section A

1. **(a)** 15 cm.

   Explanation: Since $DE||BC$, then using Thales theorem,
   $\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$
   $\Rightarrow \frac{15}{6} = \frac{25}{EC}$
   $\Rightarrow$ EC = 10 cm
   Now, AE = AC – EC = 25 – 10 = 15 cm

2. **(a)** a > 0, b < 0 and c > 0

   Explanation: Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening upwards.
   Therefore, $a > 0$
   The vertex of the parabola is in the fourth quadrant, therefore b < 0
   $y = ax^2 + bx + c$ cuts Y axis at P which lies on OY.
   Putting x = 0 in $y = ax^2 + bx + c$, we get y = c.
   So the coordinates of P is (0, c).
   Clearly, P lies on OY. $\Rightarrow$ c>0
   Hence, a>0, b<0 and c>0

3. **(b)** 0

   Explanation: $2^{x+y} = 2^{x-y} = 2^{3/2} \Rightarrow x + y = \frac{3}{2}$ and $x - y = \frac{3}{2}$. So, by adding above two equations we get and x= y = 0

4. **(c)** $\frac{15}{26}$

   Explanation: Let the fraction be $\frac{x}{y}$
   According to the question,
   $\frac{(x-1)}{(y+2)} = \frac{1}{2}$
   2x - 2 = y + 2
   y = 2x - 4 ...(i)
   And,
   $\frac{(x-7)}{(y-2)} = \frac{1}{2}$
   3x - 21 = y - 2
   3x = y + 19 ...(ii)
   Using (i) in (ii)
   3x = 2x - 4 + 19
   x = 15
   Using value of x in (i), we get
   y = 2 (15) - 4
   y = 30 - 4
   y = 26
   Therefore, required fraction = $\frac{15}{26}$

5. **(a)** Similar but not congruent.

   Explanation: In $\triangle$ABC and $\triangle$PQR $\angle$B = $\angle$Q, $\angle$R = $\angle$C and AB = 2QR
   Then, the triangles are similar, by AA similarity rule, but not congruent because, for congruency, sides should also be equal.

6. **(b)** $\frac{1}{8}$

   Explanation: Total number of tickets = 40.
   Tickets bearing the numbers as multiple of 7 bear the numbers 7,14, 21,28,35.
   Their number is 5.
   $\therefore$ P (getting a multiple of 7) = $\frac{5}{40} = \frac{1}{8}$

7. **(d)** 30°

Explanation: We have, $2 \sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^{\circ}$

$\Rightarrow 2\theta = 60^{\circ}$

$\Rightarrow \theta = 30^{\circ}$

8. **(d)** 46

Explanation: Mode of 64, 60, 48, x, 43, 48, 43, 34 is 43

∵ By definition mode is a number which has maximum frequency which is 43

∴ x = 43

∵ x + 3 = 43 + 3 = 46

9. **(b)** $\frac{4}{5}$

Explanation: Given: $\frac{\text{AP}}{\text{PB}} = \frac{4}{1}$

Let AP = 4x and PB = x, then AB = AP + PB = 4x + x = 5x

Since PQ∥BC, then

$\frac{\text{AP}}{\text{AB}} = \frac{\text{AQ}}{\text{AC}}$ [Using Thales theorem]

$\therefore \frac{\text{AQ}}{\text{AC}} = \frac{\text{AP}}{\text{AB}} = \frac{4x}{5x} = \frac{4}{5}$

10. **(d)** 320

Explanation: Let the two numbers be x and y.

It is given that: x × y = 1600

HCF = 5

We know, HCF × LCM = x × y

⇒ 5 × LCM = 1600

∴ LCM = $\frac{1600}{5}$ = 320

11. **(b)** Real and Distinct roots

Explanation: D = b^2 - 4ac

D = $(-6)^2$ - 4 × 9 × (-4)

D = 36 + 144

D = 180

D > 0. Hence Real and Distinct roots.

12. **(a)** (3, 0)

Explanation: The given point P lies on x-axis

Let the co-ordinates of P be (x, 0)

The point P lies on the perpendicular bisector of the line segment joining the points A(7, 6), B(-3, 4)

$\therefore PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

⇒ -14x + 85 = 6x + 25

⇒ 6x + 14x = 85 - 25 ⇒ 20x = 60

$x = \frac{60}{20} = 3$

∴ co-ordinates of P will be (3, 0)

13. **(d)** most frequent value

Explanation: Mode is the most frequent value of observation or a class.

14. **(a)** 2p

Explanation: Given: $\sin\theta + \cos\theta$ = p

squaring both sides we get

$\sin^2\theta + \cos^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$

$1 + 2\sin\theta\cos\theta = p^2 (\sin^2\theta + \cos^2\theta = 1)$

$2\sin\theta\cos\theta = p^2 - 1$ ... (i)

and also $\sec\theta + \operatorname{cosec}\theta$ = q (given)

$\frac{1}{cos\theta} + \frac{1}{sin\theta} = q$

$\frac{sin\theta + cos\theta}{sin\theta cos\theta} = q$

but $\sin\theta + \cos\theta = p$ ... (given)

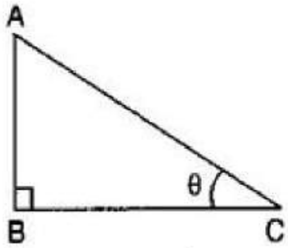
$\frac{p}{sin\theta cos\theta} = q$ ... (ii)

from (i) and (ii) we get

$q(p^2 - 1) = 2p$

15. **(c)** 60°

Explanation:

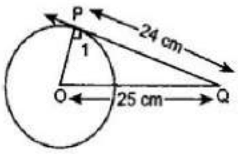


Given: Height of the pole = AB = h meters And the length of the shadow of the pole = BC = $\frac{h}{\sqrt{3}}$ meters $\therefore \tan\theta = \frac{h}{\frac{h}{\sqrt{3}}}$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

16. **(d)** 7 cm

Explanation:



Here $\angle OPQ = 90°$ [Tangent makes right angle with the radius at the point of contact]

in right angled triangle OPQ

$\therefore OQ^2 = OP^2 + PQ^2 \Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576$

$\Rightarrow OP = 7$ cm Therefore, the radius of the circle is 7 cm

17. **(d)** BC.DE = AB.EF

Explanation: If $\triangle ABC \sim \triangle DEF$ then,

$\Rightarrow \frac{BC}{EF} = \frac{AB}{DE}$ (corresponding sides are in proportion)

Here according to the given condition, BC.DE = AB.EF

18. **(a)** 7 years

Explanation: Let Sharma's present age be x years

then, his age 3 years ago is (x - 3) years and 5 years from now is (x + 5) years. According to question,

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2+5x-3x-15} = \frac{1}{3}$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x - 7) + 3(x - 7) = 0$

$\Rightarrow (x + 3)(x - 7) = 0$

$\Rightarrow x + 3 = 0$ and $x - 7 = 0$

$\Rightarrow x = -3$ and $x = 7$

But x = -3 does not satisfy the given condition.

Therefore, Sharma's present age is 7 years.

19. **(d)** A is false but R is true.

Explanation: Reason is correct. If α and β be the zeroes of the required polynomial f(x),

then $(\alpha + \beta)$ = 8 and $\alpha\beta$ = 12

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 8x + 12$

So, Assertion is not correct

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. The given equation is $6x^2 + (12-8a)x - 16a = 0$

Here a = 6, b = 12-8a, c= -16a

$\therefore D = b^2 - 4ac = (12 - 8a)^2 - 4(6)(-16a)$

$= 144 - 192a + 64a^2 + 384a = 64a^2 + 192a + 144$

$= (8a + 12)^2 > 0$

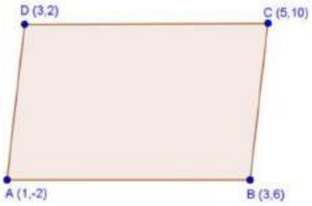
So, the given equation has real roots, given by

$x = \frac{-b\pm\sqrt{D}}{2a} = \frac{-(12-8a)\pm\sqrt{(8a+12)^2}}{2(6)}$

$= \frac{-(12-8a)+(8a+12)}{12}, \frac{-(12-8a)-(8a+12)}{12}$

$= \frac{4a}{3}, -2$

Hence, x=$\frac{4a}{3}$, -2 are the required roots of the given equation.

22. 

Let A(1, -2), B(3, 6), C(5, 10), D(3, 2) be the given points

$AB = \sqrt{(3-1)^2 + (6+2)^2}$

$\Rightarrow \quad AB = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow AB = \sqrt{4 + 64}$

$\Rightarrow \quad AB = \sqrt{68}$

$CD = \sqrt{(5-3)^2 + (10-2)^2}$

$\Rightarrow \quad CD = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow \quad CD = \sqrt{4 + 64}$

$\Rightarrow \quad CD = \sqrt{68}$

$AD = \sqrt{(3-1)^2 + (2+2)^2}$

$\Rightarrow \quad AD = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow \quad AD = \sqrt{4 + 16}$

$\Rightarrow \quad AD = \sqrt{20}$

$BC = \sqrt{(5-3)^2 + (10-6)^2}$

$\Rightarrow \quad BC = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow \quad BC = \sqrt{4 + 16}$

$\Rightarrow \quad BC = \sqrt{20}$

$\therefore$ AB = CD and AD = BC

Since opposite sides of a parallelogram are equal

Hence, ABCD is a parallelogram.

23. Clearly, the required number divides (129 - 3) = 126 and (545 - 5) = 540 exactly.

$\therefore$ required number = HCF(126, 540).

2	126
3	63
3	21
	7

2	540
2	270
3	135
3	45
3	15
	5

Now, $126 = (2 \times 3 \times 3 \times 7) = (2 \times 3^2 \times 7)$

and $540 = (2 \times 2 \times 3 \times 3 \times 3 \times 5)$

$= (2^2 \times 3^3 \times 5)$

$\therefore$ HCF(126, 540) = product of common factors with lowest power

HCF(126, 540)= $\left(2 \times 3^2\right) = (2 \times 9) = 18$.

Hence, the largest number which divides 129 and 545, leaving remainders 3 and 5 respectively is 18.

24. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$

$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)} \left[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)\right)$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \tan\theta + \cot\theta + 1 = 1 + \tan\theta + \cot\theta = RHS$

Hence proved.

OR

LHS = (sin α + cos α) (tan α + cot α)

= (sin α + cos α) $\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$

= (sin α + cos α) $\left(\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha\cos\alpha}\right)$

= (sin α + cos α) $\frac{1}{\sin\alpha\cos\alpha}$ [∵ $\sin^2$ α + $\cos^2$ α = 1]

$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha}$

$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$

= sec α+cosec α

= RHS

Hence, proved.

25. In$\triangle ABC$, $AB \| DE$.

$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB}$ ...(i) [by Thales' theorem]

In $\triangle CDB$, $BD \| EF$

$\therefore \quad \frac{CF}{FD} = \frac{CE}{EB}$ ...(ii) [by Thales' theorem]

From (i) and (ii) we get

$\frac{CD}{DA} = \frac{CF}{FD}$

$\Rightarrow \quad \frac{DA}{DC} = \frac{FD}{CF}$ [taking reciprocals]

$\Rightarrow \quad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$

$\Rightarrow \frac{DA+DC}{DC} = \frac{FD+CF}{CF}$

$\Rightarrow \quad \frac{AC}{DC} = \frac{DC}{CF}$

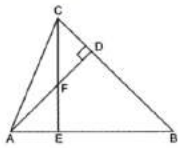
$\Rightarrow \quad DC^2 = CF \times AC$

OR

Given: ABC is a triangle in which AD and CE are two altitudes of △ABC meets each other at F

To Prove $\triangle ABD \sim \triangle CBE$.

Proof: In △'s ABD and CBE, we have



∠ABD = ∠CBE = ∠B [Common angle]

∠ADB = ∠CEB= 90^0 [∵ $AD \perp BC$ and $CE \perp AB$]

Thus, by AA-criterion of similarity, we have

$\triangle ABD \sim \triangle CBE$

Section C

26. Let the original number of persons be x.

Total amount to be divided among all people = Rs. 9000/-

So, Share of each person = Rs. $\frac{9000}{x}$

If the number of persons is increased by 20. Then,

New share of each person = Rs. $\frac{9000}{x+20}$

According to the question ;

$\frac{9000}{x} - \frac{9000}{x+20} = 160$

$\Rightarrow \frac{9000(x+20)-9000x}{x(x+20)} = 160$

$\Rightarrow \frac{9000x+180000-9000x}{x^2+20} = 160$

$\Rightarrow \frac{180000}{x^2+20} = 160$

$\Rightarrow \frac{180000}{160} = x^2 + 20$

$\Rightarrow 1125 = x^2 + 20x$

$\Rightarrow x^2 + 20x - 1125 = 0$

$\Rightarrow x^2 + 45x - 25x - 1125 = 0$

$\Rightarrow x(x + 45) - 25(x + 45) = 0$

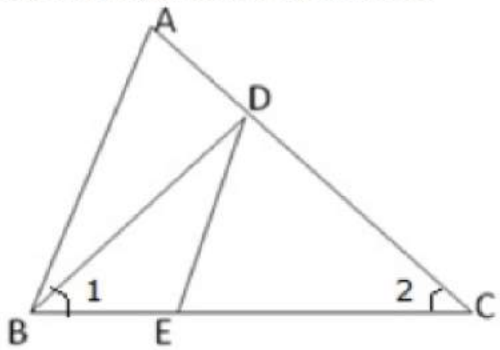
$\Rightarrow (x + 45)(x - 25) = 0$

$\Rightarrow x - 25 = 0$ [$\because$ The number of persons cannot be negative. $\therefore x + 45 \neq 0$]

$\Rightarrow x = 25$

Hence, the original number of persons is 25.

27.



$\angle 1 = \angle 2$ (given)

$\frac{AC}{BD} = \frac{CB}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$ (given)

Also, $\angle 2 = \angle 1$

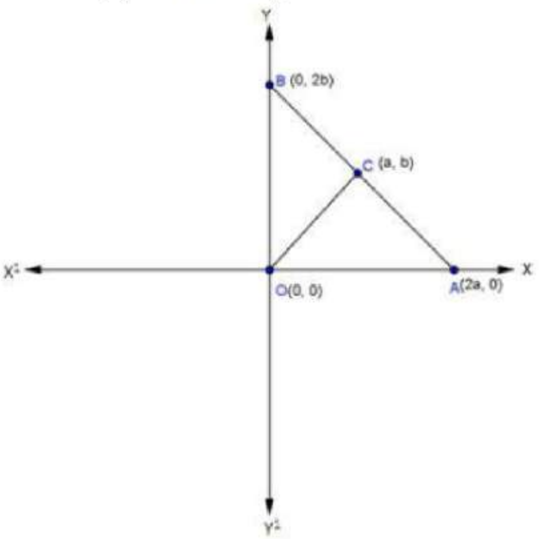
$\therefore$ BD = CD

Thus, $\frac{AC}{CB} = \frac{CD}{CE}$

and $\angle 2 = \angle 1$

Therefore, by SAS similarity criterion $\triangle ACB \sim \triangle DCE$

28.



Given a right triangle BOA with vertices B(0, 2b), O(0, 0) and A(2a, 0)

Since, C is the mid-point of AB

$\therefore$ Coordinates of C are $\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

= (a, b)

Now, $CO = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

$CA = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$CB = \sqrt{(a-0)^2 + (b-2b)^2} = \sqrt{a^2+b^2}$

Since, CO = CA = CB.

$\therefore$ C is equidistant from O, A and B.

OR

Given points are A(3, -5) and B(-4, 8).

P divides AB in the ratio k:1

Using the section formula, we have:

Coordinate of point P are $\left\{\left(\frac{-4k+3}{k+1}\right)\left(\frac{8k-5}{k+1}\right)\right\}$

Now it is given, that P lies on the line x + y =0

Therefore,

$\frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$

$\Rightarrow$ -4k + 3 + 8k - 5 =0

$\Rightarrow$ -4k + 3 + 8k - 5 = 0

$\Rightarrow$ 4k - 2 = 0

$\Rightarrow k = \frac{2}{4}$

$\Rightarrow k = \frac{1}{2}$

Thus, the value of k is 1/2.

29. Let $P(x) = 2\left(x^4 - y^4\right) = 2\left[\left(x^2\right)^2 - \left(y^2\right)^2\right]$

$= 2\left(x^2+y^2\right)\left(x^2-y^2\right)$

$= 2\left(\mathrm{x}^2+\mathrm{y}^2\right)(\mathrm{x}+\mathrm{y})(\mathrm{x}-\mathrm{y})$ Using Identity $\mathrm{a}^2-\mathrm{b}^2 = (\mathrm{a}+\mathrm{b})(\mathrm{a}-\mathrm{b})$

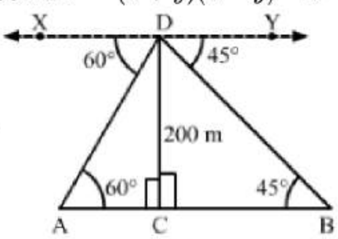
and $Q(x) = 3\left(x^3 + 2x^2y - xy^2 - 2y^3\right)$

$= 3\left[\mathrm{x}^2(\mathrm{x}+2\mathrm{y}) - \mathrm{y}^2(\mathrm{x}+2\mathrm{y})\right]$

$= 3(x+2y)\left(x^2-y^2\right)$

$= 3(x+2y)(x+y)(x-y)$

$\therefore HCF = (x+y)(x-y) = x^2 - y^2$ Using identity $a^2 - b^2 = (a+b)(a-b)$

30. 

Let CD be the lighthouse and A and B be the positions of the two ships.

Height of the lighthouse, CD = 200 m

Now,

$\angle$CAD = $\angle$ADX = 60° (Alternate angles)

$\angle$CBD = $\angle$BDY = 45° (Alternate angles)

In right $\triangle$ACD,

tan 60° = $\frac{\mathrm{CD}}{\mathrm{AC}}$

$\Rightarrow \sqrt{3} = \frac{200}{\mathrm{AC}}$

$\Rightarrow$ AC = $\frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

In right $\triangle$BCD,

tan 45° = $\frac{\mathrm{CD}}{\mathrm{BC}}$

$\Rightarrow$ 1 = $\frac{200}{BC}$

$\Rightarrow$ BC = 200 m

$\therefore$ Distance between the two ships, AB = BC + AC

$= 200 + \frac{200\sqrt{3}}{3}$

$= 200 + \frac{200\times1.73}{3}$

= 200 + 115.33

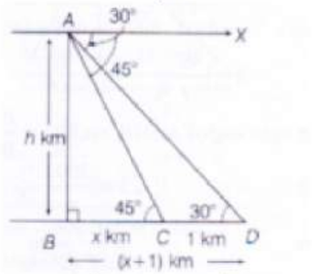
= 315.33 m (approx)

Hence, the distance between the two ships is approximately 315.33 m.

OR

Let AB = h km be the height of the hill, and C,D be two consecutive stones such that CD=1 km.

Let BC be x km, then BD = BC + CD = (x + 1) km



Now, $\angle ADB = \angle XAD = 30^o$ [ailternate angles]

and $\angle ACB = \angle XAC = 45^o$ [alternate angles]

In right angled $\triangle ABC$,

$\tan 45^o = \frac{P}{B} = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = h$

Now, In right angled $\triangle ABD$, $\tan 30^o = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+1}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+1}$ [h = x]

$\Rightarrow \boldsymbol{h + 1 = \sqrt{3}h}$

$\Rightarrow \sqrt{3}h - h = 1$

$\Rightarrow (\sqrt{3} - 1)h = 1$

$\Rightarrow h = \frac{1}{\sqrt{3}-1}$

$\Rightarrow \boldsymbol{h} = \frac{1}{0.732}$

$\Rightarrow \boldsymbol{h} = \frac{1}{0.732}$

Therefore, Height of the hill = 1.366k m

31. Modal Class 40- 50,

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

Mode = $40 + \left(\frac{28-12}{2\times28-12-20}\right) \times 10$

= 40 + $\left(\frac{16}{56-32}\right) \times 10$

= 40 + $\left(\frac{16}{24}\right) \times 10$

= 40 + $\frac{20}{3}$

$= 46.666 = 46.67$

Section D

32. The given system of equation is $4x - 3y + 4 = 0$ and $4x + 3y - 20 = 0$

Now, $4x - 3y + 4 = 0$

$x = \frac{3y-4}{4}$

Solution table for $4x - 3y + 4 = 0$

x	2	-1
y	4	0

We have,

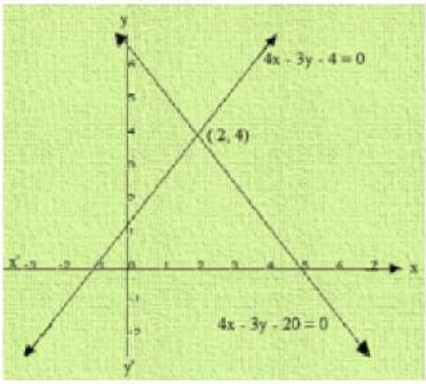
$4x + 3y - 20 = 0$

$x=\frac{20-3y}{4}$

Solution table for $4x+3y-20=0$

x	5	2
y	0	4

Graph of the given system is:



Clearly, the two lines intersect at A(2, 4)

We also observe that the lines meet x - axis B(-1, 0) and C(5, 0)

Thus x = 2 and y = 4 is the solution of the given system of equations.

AD is drawn perpendicular A on x - axis. Clearly we have,

AD = y - coordinate point A(2, 4)

AD = 3 and BC = 5 - (-1) = 4 + 1 = 6

Area of the shaded region = $\frac{1}{2}$ × base × altitude

$=\frac{1}{2}\times 6\times 4$

= 12 sq. units

OR

The pair of equations formed is :

$y=\frac{1}{2}x$

x – 2y = 0 .....(1)

3x + 4y = 20 .....(2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

x	0	2
$y=\frac{1}{2}x$	0	1

x	0	$\frac{20}{3}$	4
$y=\frac{20-3x}{4}$	5	0	2

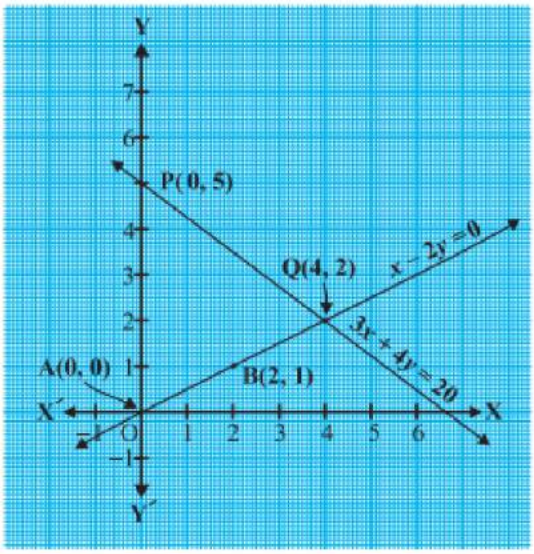
For instance, putting x = 0 in Equation (2), we get 4y = 20, i.e., y = 5.

Similarly, putting y = 0 in Equation (2), we get 3x = 20, i.e., x = $\frac{20}{3}$.

But as $\frac{20}{3}$ is not an integer, it will not be easy to plot exactly on the graph paper.

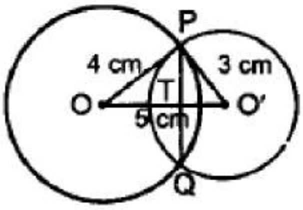
So, we choose y = 2 which gives x = 4, an integral value.

Plot the points A(0, 0), B(2, 1) and P(0, 5), Q(4, 2), corresponding to the solutions in table. Now draw the lines AB and PQ, representing the equations x – 2y = 0 and 3x + 4y = 20



From the above fig. we observe that the two lines representing the two equations are intersecting at the point (4, 2).

33. Given, OP is tangent of the circle having center O'



So, $\angle OPO' = 90^o$

In right angled $\triangle OPO'$

OP = 4 cm

O'P = 3 cm

By pythagoras theorem, we get

$OO'^2 = OP^2 + O'P^2$

$= 4^2 + 3^2$

= 16 + 9 = 25

$OO' = 5cm.$

Let O'T = x , then OT = 5 - x

In right angled $\triangle PTO$

By pythagoras theorem, we get

$OP^2 = OT^2 + PT^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

$PT^2 = 4^2 - (5 - x)^2$ ...(i)

In right angled $\triangle PTO'$

By pythagoras theorem, we get

$O'P^2 = O'T^2 + PT^2$

$\Rightarrow PT^2 = O'P^2 - O'T^2$

$PT^2 = 3^2 - x^2$ ...(ii)

From (i) and (ii) , we get

$3^2 - x^2 = 4^2 - (5 - x)^2$

$9 - x^2 = 16 - 25 - x^2 + 10x$

18 = 10x

$\Rightarrow x = \frac{18}{10} = 1.8$

Substitute x in (ii), we get

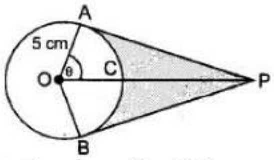
$PT^2 = 3^2 - 1.8^2 = 9 - 3.24 = 5.76$

$PT = \sqrt{5.76} = 2.4$

$\Rightarrow PQ = 2\ PT$

$= 2 \times 2.4$

$\therefore$ PQ= 4.8cm

34. 

$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB = $\frac{2\times3.14\times5\times240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2}\times 5\times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = $\frac{\theta}{360}\times\pi r^2$

= $\frac{25\times3.14\times120}{360}$ = 26.16 cm^2

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77\times42\times24\times11}$

$\Rightarrow \quad \Delta_1 = \sqrt{7\times11\times7\times6\times6\times4\times11} = \sqrt{7^2\times11^2\times6^2\times2^2} = 7\times11\times6\times2 = 924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77\times44\times21\times12} = \sqrt{7\times11\times4\times11\times3\times7\times3\times4} = \sqrt{7^2\times11^2\times4^2\times3^2}$

$\Rightarrow \quad \Delta_2 = 7\times11\times4\times3 = 924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

$\Rightarrow \quad \frac{22}{7}\times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848\times\frac{7}{22} = 3\times4\times7\times7 \Rightarrow r = \sqrt{3\times2^2\times7^2} = 2\times7\times\sqrt{3} = 14\sqrt{3}\text{cm}$

35. The houses are numbered consecutively from 1 to 49.

1, 2, 3....................(x - 1), x, (x+1)..........49

Sum of number of houses preceding x numbered house = Sum of number following x

Sum of number of houses preceding x numbered house=

$S_1 = \frac{x-1}{2}\times(1+x-1) = \frac{x(x-1)}{2} - - - - - - - (1)$

Sum of number following x=

$S_2 = (1+2+3+\dots\dots.49) - \frac{x}{2}\times(x+1)$

$= \frac{49\times50}{2} - \frac{x^2+x}{2} = \frac{2450-x^2-x}{2} - - - - - (2)$

As S_1=S_2

$\frac{2450-x^2-x}{2} = \frac{x(x-1)}{2}$

$2450 - x^2 - x = x^2 - x$

$2x^2 = 2450$

$x^2 = 1225$

$x = 35$

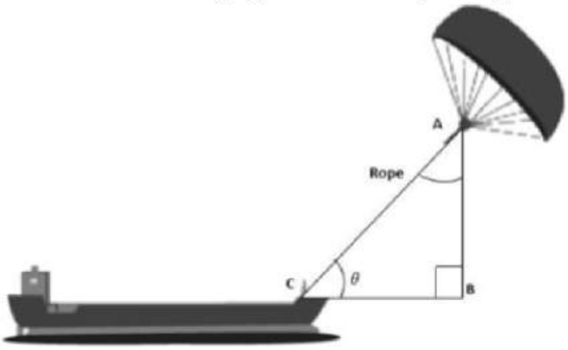
Hence sum of numbers of houses proceeding the house numbered 35 is equal to sum of the numbers of houses following 35

Section E

36. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin\theta = \cos(\theta - 30^o)$

$\cos(90^o - \theta) = \cos(\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^o} = \frac{200}{\frac{\sqrt{3}}{2}} = \frac{200\times 2}{\sqrt{3}}$ = 230.94 m

(iii) $\sin\theta = \cos(3\theta - 30^o)$

$\cos(90^o - \theta) = \cos(3\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o} = \frac{150}{\frac{1}{2}}$ = 150 × 2 = 300 m

37. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

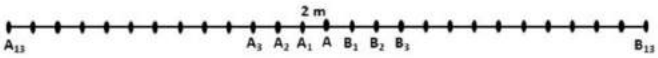
(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii) 

Let A be the position of the middle-most flag.

Now, there are 13 flags (A_1, A_2 ... A_{12}) to the left of A and 13 flags (B_1, B_2, B_3 ... B_{13}) to the right of A.

Distance covered in fixing flag to $A_1 = 2 + 2 = 4$ m

Distance covered in fixing flag to $A_2 = 4 + 4 = 8$ m

Distance covered in fixing flag to $A_3 = 6 + 6 = 12$ m

...

Distance covered in fixing flag to $A_{13} = 26 + 26 = 52$ m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

OR

Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

(iii) $\because$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

= 364

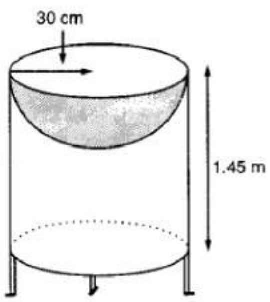
Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

= 364 + 364 = 728 m

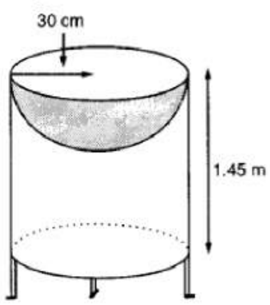
38. **Read the text carefully and answer the questions:**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

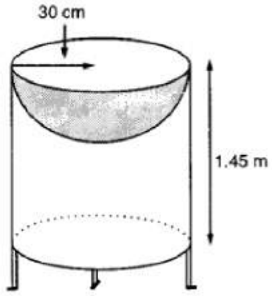


Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



Let S be the total surface area of the birdbath.

S = Curved surface area of the cylinder + Curved surface area of the hemisphere

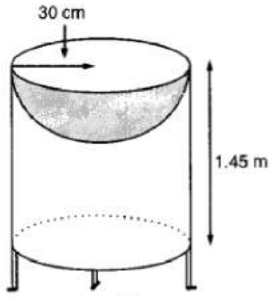
$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30)$ = 33000 cm^2 = 3.3 m^2

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



r = 35 cm = $\frac{35}{100}$ m

We know that S.A = 3.3 m^2

S = $2\pi r(r + h)$

$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$

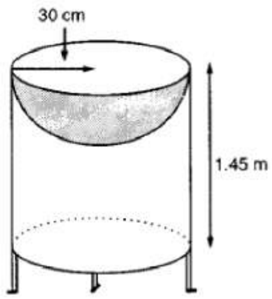
$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$

$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$

$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20}$ = 1.15 m

(iii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



Total Cost of material = Total surface area $\times$ cost per sq m^2

$= 3.3 \times 40 =$ ₹132