11

Straight Line

11.01 Straight line :

Definiition : A straight line is a locus of a point such that on the line segment joining any two points on it lies on it.

11.02 Equation of straight line :

If every point lying on a line is satisfied by an equation, then that equation is said o be an equation of straight line.

11.03 Definitions :

(A) Intercept : If the straight line AB cuts the ordinate and abscissa at two points namely A and B then -

(i) OA is the intercept made on x-axis by the line AB

(ii) OB is the intercept made on y-axis by the line AB

(iii) OA and OB are the intercepts made on both the axes by the line

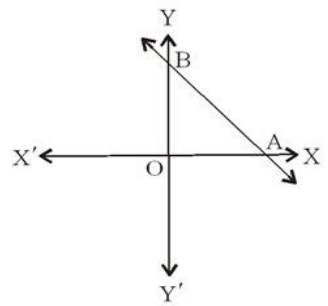

fig. 11.01

Note : If he line AB cuts the OX' , OY' axis, then the intercept will be negative.

(B) Slope of Line : Angle made by any straight line with x-axis in positive direction, then tangent of that angle is called slope or inclination of line.

Slope of line is generally denoted by m. If line AB makes an angle θ with x-axis in positive direction (↶) , then $m = \tan\theta$ shown in fig. 11.02. If line AB makes an angle θ with x-axis in negative direction (↷) then $m = -\tan\theta$ shown in fig. 11.03.

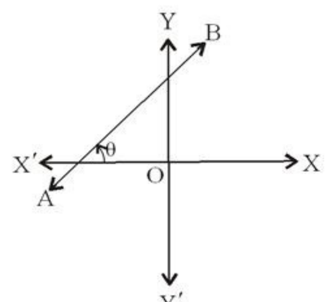

fig. 11.02

Since x-axis or any line parallel to x-axis makes an angle of 0^0 in positive direction with x-axis. So, slope of x-axis or line parallel to x-axis will be $m = \tan 0° = 0$

Similarly, y-axis or line parallel to y-axis makes an angle of 90^0 with x-axis in positive direction. So, slope of y-axis or line parallel to y-axis will be $m = \tan 90° = \infty$

If line makes equal angle with axes, means 45^0 with x-axis in anticlockwise direction, then slope will be m = $\tan 45° = 1$ When it makes an angle with x-axis in clockwise direction, means 135^0 , then slope will be m= $\tan 135° = -1$

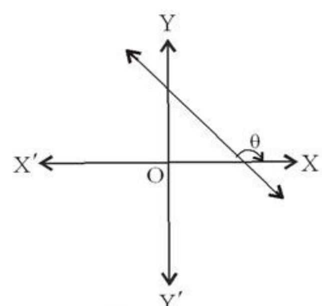

fig. 11.03

Note : Angle made by a line with x-axis in positive direction (measured in anticlockwise direction) is in between 0^0 to $180°$.

11.04 Rectangular axes :

If two lines designating x-axis and y-axis are perpendicular to each other, then they form a rectangular cartesian system. The point of intersection is called origin with coordinates (0, 0). The equation of X-axis is y = 0 and the equation of Y-axis is x = 0 Fig. 11.04

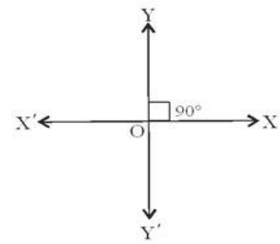

fig. 11.04

11.05 Equation of a line parallel to axes :

(i) Equation of a line parallel to x-axis which is at a distance of 'b' unit :

From figure 11.05, let a line is parallel to x-axis is at a distance 'b' from x-axis which intersect the y-axis at M. So, OM = b

Let any point $P(x, y)$ on line AB. Draw a perpendicular PN on x-axis from P, means for point P, PM = y . But PN = OM.

But PM = OM.

So, OM $= y \therefore y = b$

$\therefore$ Equation of line AB is, $y = $ b

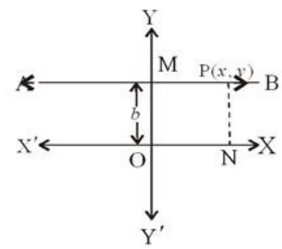

fig. 11.05

Corollary :

(i) From figure 11.06, if line AB is at distance b, below the x-axis then its equation will be y = - b.

(ii) If line AB coincides on x-axis, then b = 0 , then equation of line AB and x-axis will be same as y = 0.

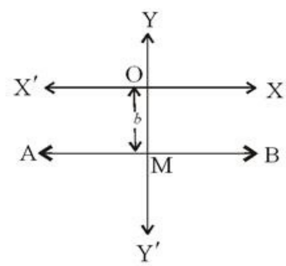

fig. 11.06

(ii) Equation of line parallel to y-axis which is at distance of a unit :

From figure 11.07, let any line AB parallel to y-axis is at distance 'a' which intersect the x-axis at N in positive direction. So, ON $= a$.

Let a variable point $P(x, y)$ is on line AB. Draw a perpenducular PM on y-axis, means for point P, PM = x, but PM = ON.

So, ON $= x$ $\therefore$ $x = a$

Thus, abscissa of all point on line AB is equal to 'a' . So, the equation of line will be $x = a$

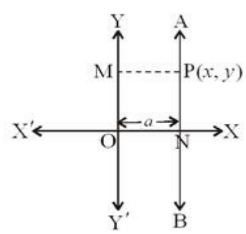

fig. 11.07

Corollary :

(i) From fig. 11.08, if line AB is at distance 'a', left to y-axis, then their equation will be x = -a.

(ii) If line AB coincides with y-axis , then a = 0. So, equation of line AB and y -axis will be same as will be x = 0. Different forms of the equation of straight line.

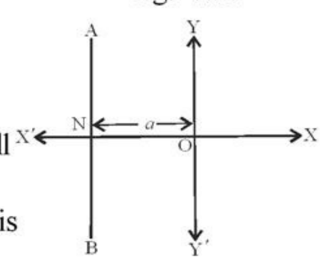

fig. 11.08

11.06 Intercept form :

Let line PQ intersects the x-axis on A and y-axis on B such that OA = 'a' and OB = b. Let any point P (x,

y) is on AB. Join OP and draw perpendicular PM on x-axis from P and PN on y-axis from P.

area of ΔOAB = area of ΔOPA + area of ΔOPB

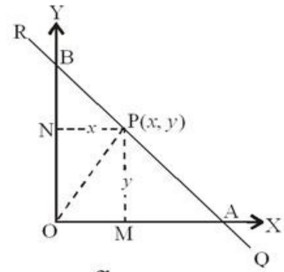

fig. 11.09

$$\frac{1}{2}\times OA\times OB=\frac{1}{2}\times OA\times PM+\frac{1}{2}\times OB\times PN$$

$$\frac{1}{2}\times a\times b=\frac{1}{2}\times a\times y+\frac{1}{2}\times b\times x$$

On dividing by $\frac{1}{2}\times a\times b$ every term, we get

So, $$\frac{x}{a}+\frac{y}{b}=1$$

So, equation of line QR is $\frac{x}{a}+\frac{y}{b}=1$, which is known as intercept form.

टिप्पणी : When a tends to infinity or $a\to\infty$ then equation of line is $\frac{x}{\infty}+\frac{y}{b}=1$, or $\frac{y}{b}=1$ or $y=b$, which is perdict to x-axis. Similarly when b tends to infinity or $b\to\infty$ then equation of line is $\frac{x}{a}+\frac{y}{\infty}=1$, or $\frac{x}{a}=1$, or $x=a$, which is parallel to y-axis.

11.07 Slope form :

The equation of line which makes intercept of length 'c' on y-axis and makes an angle 'θ' with positive x-axis.

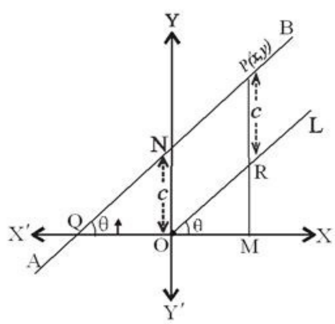

fig. 11.10

Let any point P (x,y) is on line AB. Draw a perpendicular PM on OX from P, which intersects the line OL at R which is parallel to AB. In right angled triangle OMR, we have

$$\tan\theta=\frac{RM}{OM},$$

or $$RM = OM\tan\theta$$

Now $$PM = PR + RM \quad (AB||OL)$$

$$PM = c + OM\tan\theta \quad (ON = RP = c)$$

$\therefore$ $$y = x\tan\theta + c$$

or $$y = m.x + c,$$

Here, $m=\tan\theta$, $m=$ slope of line which is the required equation of line.

Note :

(i) If line passes through origin, then intercept on y-axis is zero. So, equation of line passing through origin is $y = mx$.

(ii) If line intersects y-axis at OY , then value of c is positive and if line intersect at OY' , then c is negative.

(iii) If θ is obtuse angle, then value of slope will be negative and if θ is an acute angle, then slope will be positive. If line makes equal angle with axes then $m = \pm 1$

11.08 Normal form :

The equation of line whose pendicular distance from the origin is 'p' and this perpendicular makes an angle 'α' with x-axis.

Let RS is a line which intersects the x-axis and y-axis at A and B respectively. Draw a perpendicular OM on line from O having length p and it makes an angle α with positive x-axis i.e. $\angle MOA = \alpha$.

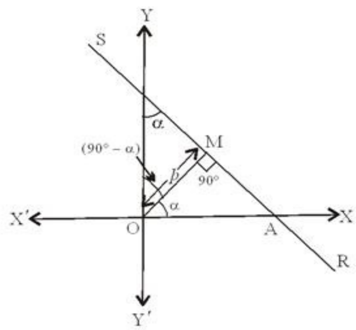

fig. 11.10

In right angled ΔOBM,

$$\begin{aligned}\angle OBM &= 90° - \angle BOM \\ &= 90° - \{90° - \angle MOA\} \\ &= 90° - \{90° - \alpha\} \\ &= \alpha\end{aligned}$$

In right angled ΔOMA,

$$\cos\alpha = \frac{p}{OA} \quad \Rightarrow \quad OA = \frac{p}{\cos\alpha}$$

In right angled Δ OBM ,

$$\sin\alpha = \frac{p}{OB} \quad \Rightarrow \quad OB = \frac{p}{\sin\alpha}$$

So, line cuts intercept OA and OB or $\frac{p}{\cos\alpha}$ and $\frac{p}{\sin\alpha}$ with x-axis and y-axis respectively.

$\therefore$ The equation of line in intercept form is

$$\frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$$

or $\qquad x\cos\alpha + y\sin\alpha = p$

This is the equation of line is normal form.

Note :

(i) Length of perpendicular 'p' is always positive when it is draw from origin to line.

(ii) Angle made by perpendicular on line from origin with positive x-axis is in between 0^0 to 360^0.

(iii) To find an equation of any line, two conditions are equal.

(iv) While writing the equation of line, the abcissa is written in first term and ordinate is written in second term.

Illustrative Examples

Example 1 : Find the equation of the line parallel to y-axis and passing through (4, 3)

Solution : Equation of line parallel to y-axis is

$x = a$

$\therefore$ This passes through (4, 3)

$\therefore$ $x = a$

$\Rightarrow \quad a = 4$

$\therefore \quad$ Required line is $x = 4$

Example 2 : Find the equation of line which is equidistant from the lines $y = 8$ and $y = -14$.

Solution : We know that $y = 8$ and $y = -14$ are always parallel to x-axis. Thus the line equidistant from these lines will also be parallel to x-axis and will be at the distance

$$= \frac{8+(-14)}{2} = -\frac{6}{2} = -3$$

$\therefore \quad$ The required equation, $y = -3$

Example 3 : Find the equation of line which cuts the y-axis below the origin making an intercept of 3 units and inclined at an equal angle with both the axes.

Solution : We know that lines inclined at an equal angle with both the axes and the angle between the axes is 90^0. Thus, the line will make an angle of 45^0 or 135^0. Let the line is

$$y = mx + c \qquad (1)$$

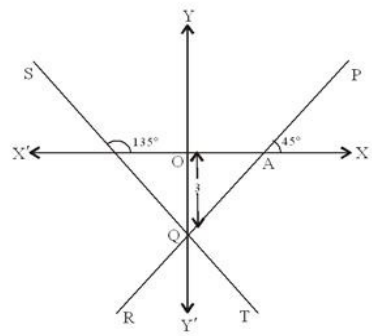

fig. 11.12

Here $c = -3$ and $m = \tan 45°$ or $m = \tan 135°$

$\Rightarrow \quad m = 1$

or $\quad m = \tan(90° + 45°) = -1$

On putting the value of m, c in equation (1),

$y = x - 3 \quad$ or $\quad x + y + 3 = 0$ and

$y = -x - 3 \quad \Rightarrow \quad x + y + 3 = 0$

$\therefore \quad$ Required equation is $x - y - 3 = 0$ or $x + y + 3 = 0$

Example 4 : Find the equation of a line that cuts off equal intercepts with opposite sign on the coordinate axes and passes through the point (2, 3).

Solution : Intercept form of line is

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad (1)$$

According to question, intercepts are equal but with opposite sign, therefore $b = -a$

$\therefore \quad$ Equation of line is

$$\frac{x}{a} + \frac{y}{-a} = 1$$

or $\quad x - y = a \qquad (2)$

the line passes through (2, 3), by putting $x = 2, y = 3$ in equation (2), we get . By substituting the value of a in equation (2) we get

$$x - y = -1$$

or $\quad x - y + 1 = 0$

$\therefore$ required equation is $x - y + 1 = 0$

Example 5 : Point (-4, 1) divides a line segment between the axes in the ratio 1 : 2 . Find equation of the line.

Solution : Let the equation of line cutting intercepts a and b be on x-axis and y-axis respectively is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (1)$$

This line cuts the axes at point A and B respectively and having coordinates (a, 0) and (0, b) . Let the coordinates of point P be (x_1, y_1) which divides the line in the ratio 1 : 2

$$x_1=\frac{2\times a+1\times 0}{2+1},\quad y_1=\frac{2\times 0+1\times b}{2+1}$$

or $\quad x_1=\frac{2a}{3}, \qquad y_1=\frac{b}{3}.$

But the point is $(-4,1)$

$\therefore \quad \frac{2a}{3}=-4$ and $\frac{b}{3}=1$, None $a=-6$ and $b=3$

On putting the values of a, b in (1) we have $\frac{x}{-6}+\frac{y}{3}=1$

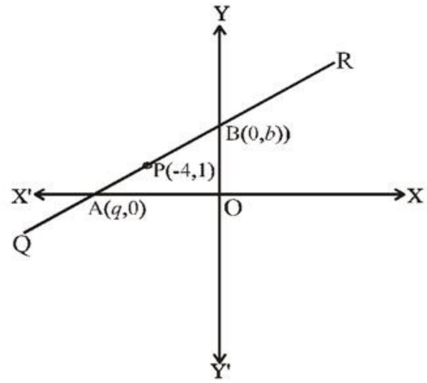

fig. 11.13

Thus, required equation is $x-2y+6=0$

Example 6 : Find the equation of line whose perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 135^0.

Solution : Equation of line in normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (1)$$

here $\alpha=135°$ and $p=5$ units

$\therefore \quad \cos\alpha=\cos 135°=-\frac{1}{\sqrt{2}}$ and $\sin\alpha=\sin 135°=\frac{1}{\sqrt{2}}$

On putting the values , we have $x\left(-\frac{1}{\sqrt{2}}\right)+y\left(\frac{1}{\sqrt{2}}\right)=5$

or, $\quad \frac{-x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=5$ or $x-y+5\sqrt{2}=0$

Thus, required equation is $x-y+5\sqrt{2}=0$

Example 7 : Find the equation of a line which makes a triangle with axes whose area is $\mathbf{54\sqrt{3}}$. Square units and perpendicular drawn from the origin to the line makes an angle of 60^0 with the x-axis.

Solution : Let the length of perpendicular be p units and α be 60^0, then the equation is

$$x\cos 60^0 + y\sin 60^0 = p$$

$\Rightarrow \quad x.\frac{1}{2}+y.\frac{\sqrt{3}}{2}=p \Rightarrow x+\sqrt{3}y=2p \qquad (1)$

Let the line intersets axes at A and B respectively

$\therefore$ Point A is $(2p,0)$ and Point B is $\left(0,\frac{2p}{\sqrt{3}}\right)$

$\therefore$ Area of trianglee $OAB = \frac{1}{2}\times$ Base $\times$ Height

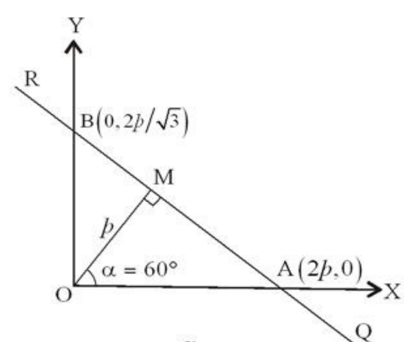

fig. 11.14

$$=\frac{1}{2}\times OA\times OB=\frac{1}{2}.2p.\frac{2p}{\sqrt{3}}=\frac{2p^2}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{2p^2}{\sqrt{3}}=54\sqrt{3} \Rightarrow p=\pm 9$$

But p is positive

$\therefore \quad x+\sqrt{3}y=2\times 9$ or $x+\sqrt{3}y=18$

Thus required equation is $x+\sqrt{3}y=18$.

Exercise 11.1

1. Find the equation of line parallel to x- axis and
   (i) at the distance of 5 units above the origin.
   (ii) at the distance of 3 units below the origin.
2. Find the equation of line parallel to x- axis and at a distance of
   (i) $a+b$ (ii) a^2-b^2 (iii) $b\cos\theta$
3. Find the equation of line parallel to y- axis and whose distance from the origin is
   (i) 5 (ii) –3 (iii) $2/5$
4. Find the equation of line parallel to y- axis and at distance of
   (i) $\sqrt{7}$ (ii) $-\sqrt{3}+2$ (iii) $p+q$
5. Find the equation of lines passing through (-3, 2) and (1) perpendicular to x-axis and (2) parallel to x-axis respectively.
6. Find the equation of lines which are parallel to the axes and passing through $(3, 4)$. Also find the equation of line parallel to these lines and at a distance of 8 units.
7. Find the co-ordinates of the intersecting lines and $y=\pm 3$. Also find the area of the rectangle so formed.
8. Find the equation of line passing through the origin and
   (i) Making an angle of -135^0 with the x-axis.
   (ii) Making an angle of 60^0 with OY in the first quadrant.
   (iii) Cutting an intercept of 5 units in the positive direction of y-axis and is parrallel to the a bisector of angle XOY.
9. Find the equation of line which cuts the following intercepts on x-axis and y- axis respectively .
   (i) 5, 3 (ii) −2, 3
10. Find the equation of line passing through (2, 3) and cuts equal intercepts on the axes.
11. Find the equation of line passing through (1, 2) and whose intercept cut on x-axis is double the intercept cut on y-axis.
12. Find the equation of lines whose perpendicular distance from the origin is 4 units and the angle made by the perpendicular with positive direction of x-axis is 15^0.
13. Find the equation of line passing through the point (4, -3,) and cutting off intercept on the axes whose sum is 6.
14. Prove that the equation of line whose intercepts are reciprocals of a and b is $ax+by=1$
15. A straight line cuts an intercept of 5 and 3 units on the axes. Find the equation of line when the intercepts are

(i) On the positive direction of the axes.

(ii) On the negative direction of the axes.

(iii) First intercept is on the positive direction and another on the negative direction.

16. The perpendicular from the origin to a line makes an angle of 30^0 with the y-axis and whose length is 2 units. Find the equation of the line.

17. Find the length of the intercepts part made by the line $x\sin\alpha + y\cos\alpha = \sin 2\alpha$ between the axes. Also find the co-ordinates of the midpoint of its intercepted part.

18. Find the equation of the line whose perpendicular distance from the origin is $p = 3$ and the angle made by this perpendicular with x-axis is α such that $\cos\alpha = \frac{\sqrt{3}}{2}$

11.09 Staright line and linear equation in x, y :

(a) Each the represents an equation of first order in x and y in given plane :

There is only possibility that any line in plane makes acute angle or obtuse angle or right angle with x-axis.

(i) If line makes acute with x-axis then their equation will be form $y = mx + c$ where $m = \tan\theta$.

(ii) If line makes right angle with x-axis then that line will be parallel to y-axis and their equation will be $x = c$ form.

(iii) If line makes obtusa angle with x-axis then their equation will be also be form of $y = mx + c$

In all three conditions the equation of line is of first order in x and y.

(a) Equation in x and y of first order always represent a straight line :

If, A, B, C are three constant which is independent of x and y then general equation in x and y

$$\mathrm{A}x + \mathrm{B}y + \mathrm{C} = 0 \qquad (1)$$

Here in all terms, the highest order of x and y is 1 and lowest order is zero, means in first and second term the order of x and y in 1 and in constant third term is without x and y in which order of x and y is zero. So, equation (1) is general equation of first order.

In equation (1), A and B both may never be zero because if, A and B both are zero then equation will be form of C = 0 which is not opssible for all values of C due to C is a constant. So, it is meaning less. So, it is necessary in $Ax + By + C = 0$ that $A \neq 0$ or $B \neq 0$.

First Case :

When $\mathrm{A} \neq 0$, $\mathrm{B} = 0$, then the equation (1) we have $\mathrm{A}x + 0 \times y + \mathrm{C} = 0$ or $\mathrm{A}x + \mathrm{C} = 0$ or $x = -\mathrm{C}/\mathrm{A}$ which is the equation of line parallel to y-axis and at a distance of $-\mathrm{C}/\mathrm{A}$ from y-axis. If $\mathrm{C} = 0$ then, $\mathrm{A}x = 0$ or $x = 0$, which is the equation of y-axis.

Second Case :

When $\mathrm{A} = 0, \mathrm{B} \neq 0$, then the equation (1) we have $0 \times x + \mathrm{B}y + \mathrm{C} = 0$ or $\mathrm{B}y + \mathrm{C} = 0$ or $y = -\mathrm{C}/\mathrm{B}$ which is the equation of line parallel to x-axis and at a distance of $-\mathrm{C}/\mathrm{B}$ from x-axis $\mathrm{C} = 0$ then $\mathrm{B}y = 0$ or $y = 0$ which is the equation of x-axis.

If there are three pair of co-ordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ satisfying equation (1) then

$$\mathrm{A}x_1 + \mathrm{B}y_1 + \mathrm{C} = 0 \qquad (2)$$

$$\mathrm{A}x_2 + \mathrm{B}y_2 + \mathrm{C} = 0 \qquad (3)$$

$$\mathrm{A}x_3 + \mathrm{B}y_3 + \mathrm{C} = 0 \qquad (4)$$

Eliminating the constants A,B,C we have

$$x_1(y_2 - y_3) + y_1(x_3 - x_2) + 1(x_2y_3 - y_2x_3) = 0 \tag{5}$$

The LHS of above equation represents the area of triangle whose vertices are given. Since area is zero means, the points are collinear. So, $Ax + By + C = 0$ represents the general form of straight line.

Note :

(i) From equation $Ax + By + C = 0$ it seems that there is 3 constant terms but in real there is only two independent variable because we can write this equation as $\frac{A}{C}x + \frac{B}{C}y + 1 = 0$ in which only two constantans $\frac{A}{C}$ and $\frac{B}{C}$

(ii) If two equation $ax + by + c = 0$ and $a'x + b'y + c' = 0$ represents the same line then $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

11.10 Reduction of general equation of straight line into standard forms :

1. Slope-intercept form : We know that the general form of straight line is $Ax + By + C = 0$

$$By = -Ax - C$$

$$\Rightarrow \quad y = \left(-\frac{A}{B}\right)x - \left(\frac{C}{B}\right) \qquad \text{(while } B \neq 0)$$

$$\Rightarrow \quad y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$$

$$\Rightarrow \quad y = mx + c, \qquad \text{where } m = -\frac{A}{B},\ c = -\frac{C}{B}$$

Note :

(i) Slope of $Ax + By + c = 0$:

$$m = -\frac{A}{B} = -\frac{\text{(coefficient of } x)}{\text{(coefficient of y)}}$$

and intercept on y-axis : $C = -\frac{C}{B} = -\frac{\text{Constant}}{\text{(coefficient of } y)}$

(ii) In equation, $y = mx + c$, the coefficient of y is 1. So,

(a) All terms should be on RHS except y.

(b) If there is any coefficient of y in LHS, then by this coefficient divide both sides.

2. Intercept form $\boldsymbol{\frac{x}{a} + \frac{y}{b} = 1}$: Equation of line

$$Ax + By + C = 0$$

$$\Rightarrow \quad Ax + By = -C$$

$$\Rightarrow \quad \frac{x}{\left(\frac{-C}{A}\right)} + \frac{y}{\left(\frac{-C}{B}\right)} = 1$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 1, \qquad \text{where } a = -\frac{C}{A} \ ; \ b = -\frac{C}{B}$$

length of intercepts made on x-axis and y-axis are $-\frac{C}{A}, -\frac{C}{B}$ respectively.

Note :

(i) Only constant term written in RHS of equation.

(ii) Divide that constant terms in both side that RHS become 1.

(iii) Write the multiple of x and y as inverse in their denominator.

3. Normal Form $x\cos\alpha + y\sin\alpha = p$:

Equation of line

$$Ax + By + C = 0$$

$$\Rightarrow \quad Ax + By = -C \quad (1)$$

Equation of normal form

$$x\cos\alpha + y\sin\alpha = p \text{ in positive} \quad (2)$$

Equation (1) and (2) represent some straight lines, so on comparing the two equations

$$\frac{-C}{p} = \frac{A}{\cos\alpha} = \frac{B}{\sin\alpha} = \pm\frac{\sqrt{A^2+B^2}}{\sqrt{\cos^2\alpha+\sin^2\alpha}},$$

$$\Rightarrow \quad \frac{-C}{p} = \frac{A}{\cos\alpha} = \frac{B}{\sin\alpha} = \pm\frac{\sqrt{A^2+B^2}}{1}$$

$$\Rightarrow \quad p = \frac{-C}{\pm\sqrt{A^2+B^2}}, \quad \cos\alpha = \frac{A}{\pm\sqrt{A^2+B^2}}, \quad \sin\alpha = \frac{B}{\pm\sqrt{A^2+B^2}}$$

substituting in equation (2),

$$x\frac{A}{\pm\sqrt{A^2+B^2}} + y\frac{B}{\pm\sqrt{A^2+B^2}} = \frac{-C}{\pm\sqrt{A^2+B^2}}$$

i.e.

$$\frac{A}{\pm\sqrt{A^2+B^2}}x + \frac{B}{\pm\sqrt{A^2+B^2}}y = \frac{-C}{\pm\sqrt{A^2+B^2}}$$

Thus $x\cos\alpha + y\sin\alpha = p$ is the equation of normal form also for $p = -\frac{C}{\pm\sqrt{A^2+B^2}}$ p should be always positive.

Note :

(i) To convert the equation $Ax + By + C = 0$ into a normal form firstly transfer the constant C onto right hand side.

(ii) Each term is divided by $\sqrt{A^2+B^2}$ by taking the square root of sum of squares of coefficient.

11.11 Straight line passing through one point :

We will now find the equation of line passing through (x_1, y_1) and makes an angle of θ with x-axis thus the slope will be $m = \tan\theta$ now the equation of the line is

$$y = mx + c \quad (1)$$

passing through (x_1, y_1) thus substituting $x = x_1$ and $y = y_1$ in (1)

$$y_1 = mx_1 + c \quad (2)$$

Now substituting (2) from (1) we have

$$y - y_1 = m(x - x_1)$$

This is the required equation of line.

Note : We can find 'm' from any condition given in question which satisfy the line.

11.12 Line passing through two points :

We will now find the equation of line passing through two points (x_1, y_1) and (x_2, y_2). Let the equation of the line is

$$y = mx + c \tag{1}$$

passing through (x_1, y_1) and (x_2, y_2) thus these points satisfy the equation (1)

$$\therefore \quad y_1 = mx_1 + c \tag{2}$$

and
$$y_2 = mx_2 + c \tag{3}$$

now subtracting (2) from (1) we have

$$y - y_1 = m(x - x_1) \tag{4}$$

subtracting (2) from (3)

$$y_2 - y_1 = m(x_2 - x_1) \qquad \text{or} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Putting the value of m in equation (4)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad \text{or} \qquad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

This is the required equation of line

Note : Slope of the line joining the points (x_1, y_1) and (x_2, y_2) None

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Illustrative Examples

Example 8 : Convert the following equation $3x + 4y = 12$ into (i) slope form (ii) intercept form (iii) normal form and find the value of used constant in standard form.

Solution : (i) Given

$$3x + 4y = 12$$

or
$$4y = -3x + 12$$

or
$$y = -\frac{3}{4}x + \frac{12}{4}$$

or
$$y = -(3/4)x + 3$$

It is of the form $y = mx + c$ where $m = -3/4$ and $c = 3$

(ii) Given

$$3x + 4y = 12$$

or

$$\frac{3x}{12} + \frac{4y}{12} = 1$$

or
$$\frac{x}{4} + \frac{y}{3} = 1$$

It is of the form $\frac{x}{a}+\frac{y}{b}=1$ where $a=4, b=3$

(iii) Given

$$3x+4y=12$$

or

$$\sqrt{A^2+B^2}=\sqrt{3^2+4^2}=\sqrt{9+16}=5 \text{ or}$$

or $$\frac{3}{5}x+\frac{4}{5}y=\frac{12}{5}$$

It is fo the form $x\cos\alpha+y\sin\alpha=p$ where

$$\cos\alpha=\frac{3}{5}, \quad \sin\alpha=\frac{4}{5}, \quad p=\frac{12}{5}$$

$\Rightarrow \quad \tan\alpha=\frac{4}{3}$ and $p=\frac{12}{5}$

The length of perpendicular from origin is $12/5$ and the perpendicular makes an angle α with x-axis whole $\tan^{-1}(4/3)$

Example 9 : Reduce the equation $\sqrt{3}\mathbf{x}-\mathbf{y}+\mathbf{2}=\mathbf{0}$ into normal form. Form the perpendicular distance from the origin and angle between perpendicular and the positive x-axis.

Solution : Given equation $\sqrt{3}x-y+2=0$ or $\sqrt{3}x-y=-2$

or

$$-\sqrt{3}x+y=2 \qquad (1)$$

Dividing both the sides by $\sqrt{A^2+B^2}=\sqrt{\left(-\sqrt{3}\right)^2+(1)^2}=\sqrt{3+1}=2$ or

$$-\frac{\sqrt{3}}{2}x+\frac{1}{2}y=1 \qquad (2)$$

comparing with $x\cos\alpha+y\sin\alpha=p$ we have,

$$\cos\alpha=-\frac{\sqrt{3}}{2}, \quad \sin\alpha=\frac{1}{2}, \qquad p=1$$

or $\cos\alpha=-\frac{\sqrt{3}}{2}=-\cos 30°$

$=\cos(180°-30°)$ or $\cos(180°+30°)$

$=\cos 150°$ or $\cos 210°$

therefore $\alpha=150°$ or $210°$ (3)

similarly $\sin\alpha=\frac{1}{2}=\sin 30°=\sin(180°-30°)$

$=\sin 30°$ or $\sin 150°$

therefore $\alpha = 30°$ or $150°$ (4)

Example 10 : Find the equation of line passing through $(3,2)$ and making an angle of 60^0 with x-axis.

Solution : The slope of the line will be $m = \tan 60° = \sqrt{3}$ and the point is (3, 2)

i.e. $x_1 = 3, y_1 = 2$

we know that equation of point slope form is

$$y - y_1 = m(x - x_1)$$

substituting

$$y - 2 = \sqrt{3}.(x-3) \quad \Rightarrow \quad \sqrt{3}x - y + 2 - 3\sqrt{3} = 0$$

Example 11 : Find the equation of line passing through the middle points of the lines joining the points $(4,-7), (-2,3)$ and $(-4,-7), (-2,-3)$

Solution : Co-ordinates of mid points joining $(4,-7)$ and $(-2,3)$

$$= \left(\frac{4+(-2)}{2}, \frac{-7+3}{2}\right) = (1,-2) \qquad (1)$$

Similarly co-ordinates of mid points joining $(-4,-7)$ and $(-2,-3)$

$$= \left(\frac{-4+(-2)}{2}, \frac{-7+(-3)}{2}\right) = (-3,-5) \qquad (2)$$

Thus equation of line passing through $(1,-2)$ and $(-3,-5)$

$$y-(-2) = \frac{(-5)-(-2)}{(-3)-(1)}(x-1)$$

or $$3x - 4y - 11 = 0$$

Exercise 11.2

1. Reduce the following equations into slope intercept and intercept form and find their slopes and the intercepts

   (i) $7x - 13y = 15$ (ii) $5x + 6y + 8 = 0$

2. Find the slope of $x\cos\alpha + y\sin\alpha = p$

3. Find the tangent of angle which line makes with the positive direction of x-axis :

   (i) $\sqrt{3}x - y + 2 = 0$ (ii) $x + \sqrt{3}y - 2\sqrt{3} = 0$

4. Prove that the coordinates of the mid point intercept on the axes by the line $\frac{x}{2x_1} + \frac{y}{2y_1} = 1$ is (x_1, y_1).

5. Find the length and the mid point of the intercept made by the line $3x + 4y = 6$ on the axes.

6. Find a and b if the equation $5x - 4y = 20$ and $ax - by + 1 = 0$ represent the same straight line.

7. Reduce the following equations into $x\cos\alpha + y\sin\alpha = p$ (normal form)

   (i) $x + y + \sqrt{2} = 0$ (ii) $\sqrt{3}x - y + 2 = 0$

8. Reduce the following equations $3x - 4y - 11 = 0$ into normal form. Find the perpendicular distance from the origin and angle between perpendicular and the positive x-axis. Also find the slope.

9. Find a and b, if the equation $\frac{x}{a} + \frac{y}{b} = 1$ and $2x - 3y = 5$ represent the same straight line.

10. If the equations $y = mx + c$ and $x\cos\alpha + y\sin\alpha = p$ represents the same straight line then find the slope from the x-axis and the length of intercept cut on y-axis.

11. Find the equation of the line passing through (2, 3) and , makes an angle 45^0 with the positive direction from x-axis.

12. Find the equation of the line passing through the following pair of points :

(i) $(3,4)$ and $(5,6)$ (ii) $(0,-a)$ and $(b,0)$

(iii) (a,b) and $(a+b,a-b)$ (iv) $(at_1, a/t_1)$ and $(at_2, a/t_2)$

(v) $(a\sec\alpha, b\tan\alpha)$ and $(a\sec\beta, b\tan\beta)$

11.13 Angle between two lines :

Let there be two lines AB and CD whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Suppose these lines make an angle θ_1 and θ_2 with the x-axis. Thus the slope of the lines will be $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$ and the lines intersect each other at point P as shown in fig. so that $\angle BPD = \theta$.

$$\theta + \theta_2 = \theta_1 \qquad \therefore \qquad \theta = \theta_1 - \theta_2 \qquad (1)$$

By (1) $\theta = \theta_1 - \theta_2$

$$\therefore \qquad \tan\theta = \tan(\theta_1 - \theta_2)$$

$$\therefore \qquad \tan\theta = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2} = \frac{m_1 - m_2}{1 + m_1m_2}$$

$$\therefore \qquad \theta = \tan^{-1}\left[\frac{m_1 - m_2}{1 + m_1m_2}\right] \qquad (2)$$

Let the angle between AB and CD be $CPB = \phi$

then $\phi = \pi - \theta$

$$\therefore \qquad \tan\phi = \tan(\pi - \theta)$$

or $\tan\phi = -\tan\theta$

or $$\tan\phi = -\frac{m_1 - m_2}{1 + m_1m_2}$$

$$\therefore \qquad \phi = \tan^{-1}\left[-\frac{m_1 - m_2}{1 + m_1m_2}\right] \qquad (3)$$

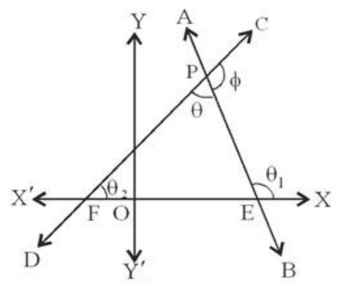

fig. 11.15

Thus the angle between (AB) and (CD) is $= \tan^{-1}\left[\pm\frac{m_1 - m_2}{1 + m_1m_2}\right]$

Note :

(i) If the angle between the lines is acute then $\frac{m_1 - m_2}{1 + m_1m_2}$ is positive. If the angle is obtuse then it is taken negative.

(ii) let a line AB is parallel to y-axis and makes an angle of 90^0 with the x-axis.

$$\theta = \frac{\pi}{2} - \theta_2$$

or $$\tan\theta = \tan\left(\frac{\pi}{2} - \theta_2\right)$$

$\therefore$ $$\tan\theta = \cot\theta_2$$

$\Rightarrow$ $$\tan\theta = \frac{1}{\tan\theta_2} = \frac{1}{m_2}$$

$\therefore$ $$\theta = \tan^{-1}\frac{1}{m_2} \qquad (1)$$

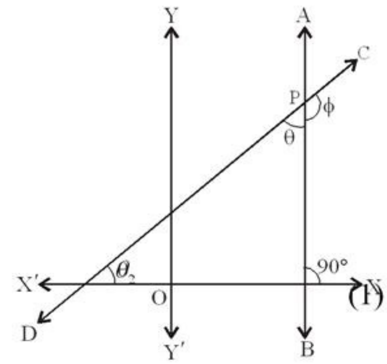

fig. 11.16

when the other angle is $CPB = \phi$

$$\phi = \pi - \theta$$

or $$\phi = \pi - \left(\frac{\pi}{2} - \theta_2\right)$$

or $$\phi = \frac{\pi}{2} + \theta_2$$

or $$\tan\phi = \tan\left(\frac{\pi}{2} + \theta_2\right)$$

or $$\tan\phi = -\cot\theta_2$$

or $$\tan\phi = -\frac{1}{\tan\theta_2}$$

$\Rightarrow$ $$\tan\phi = -\frac{1}{m_2}$$

$\therefore$ $$\phi = \tan^{-1}\left(\frac{-1}{m_2}\right) \qquad (2)$$

Thus from equation (1) and (2) it is clear that if one line is parallel to y-axis then the angle between them is $= \tan^{-1}\left(\pm\frac{1}{m_2}\right)$

11.14 Necessary condition for two lines to be parallel :

If two lines are parallel to each other then there slope will be be equal.

let the equation of the line be $y = m_1x + c_1$ and $y = m_2x + c_2$ and the angle between the lines be θ

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

The lines are parallel therefore $\theta = 0° \Rightarrow \tan\theta = \tan 0 = 0$

$$(m_1 - m_2) = 0 \Rightarrow m_1 = m_2$$

Thus the two lines are parrallel then there slopes will be equal.

11.15 Condition for two lines to be mutually perpendicular :

let the equation of the lines be

$$y = m_1x + c_1 \qquad (1)$$

$$y = m_2x + c_2 \qquad (2)$$

and the angle between the lines be θ then $\tan\theta = \pm\left[\dfrac{m_1 - m_2}{1 + m_1 m_2}\right]$

If the lines are mutually perpendicular then $\theta = 90° \Rightarrow \tan\theta = \tan 90° = \infty$

therefore $\qquad 1 + m_1 m_2 = 0$

or $\qquad m_1 m_2 = -1$

or $\qquad m_2 = -\dfrac{1}{m_1}$

Thus if two lines are perpendicular to each other then the product of their slopes is -1 or the slope of on line is negative reciprocal of another.

11.16 Equation of a line passing through a given point and making a certain angle with the given line :

Let $P(x_1, y_1)$ be any point and let AB be a line which makes an angle θ with the x-axis. Also PQ, PR makes an angle ϕ_1 and ϕ_2 with the x-axis.

$\therefore$ Equation of PQ $\qquad y - y_1 = \tan\phi_1 (x - x_1) \quad (1)$

and equation of PR $\qquad y - y_1 = \tan\phi_2 (x - x_1) \quad (2)$

equation of AB $\qquad y = mx + c \; (\because m = \tan\theta, OT = c)$

makes an angle α with PQ and PR

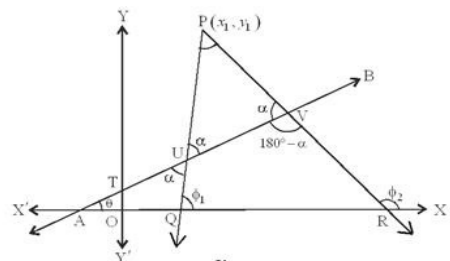

fig. 11.17

ΔAUQ and ΔAVR, $\phi_1 = \theta + \alpha$ and $\phi_2 = \theta + (180 - \alpha)$

$\Rightarrow \qquad \tan\phi_1 = \tan(\theta + \alpha)$

$\Rightarrow \qquad \tan\phi_1 = \dfrac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$

or $\qquad \tan\phi_1 = \dfrac{m + \tan\alpha}{1 - m\tan\alpha} \qquad (3)$

and $\qquad \tan\phi_2 = \tan\left[\theta + (180° - \alpha)\right] = \tan\left[180° + (\theta - \alpha)\right]$

or $\qquad \tan\phi_2 = \tan(\theta - \alpha) = \dfrac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha} = \dfrac{m - \tan\alpha}{1 + m\tan\alpha} \qquad (4)$

Thus

$$y - y_1 = \frac{m + \tan\alpha}{1 - m\tan\alpha}(x - x_1) \qquad (5)$$

or

$$y - y_1 = \frac{m - \tan\alpha}{1 + m\tan\alpha}(x - x_1) \qquad (6)$$

Illustrative Examples

Example 12 : Find the angle between the lines $\mathbf{3x + y - 7 = 0}$ and $\mathbf{x + 2y + 9 = 0}$

Solution : Slope of $3x + y - 7 = 0$

is $\qquad m_1 = -\dfrac{3}{1}$

Similarly slope of $x+2y+9=0$

is $m_2 = -\frac{1}{2}$

Let θ be the angle between them

$$\therefore \qquad \tan\theta = \pm\frac{m_1 - m_2}{1+m_1 m_2} = \pm\frac{-3+\frac{1}{2}}{1+(-3)\left(-\frac{1}{2}\right)} = \pm(-1)$$

$$\therefore \qquad \tan\theta = -1,\ 1$$

$$\therefore \qquad \tan\theta = -\tan 45^\circ \quad \text{and} \quad \tan\theta = \tan 45^\circ$$

$$\therefore \qquad \theta = 135^\circ \quad \text{and} \quad \theta = 45^\circ$$

Example 13 : Prove that the lines $\mathbf{2x - y + 9 = 0}$ and $\mathbf{4x - 2y - 8 = 0}$ are parallel to each other.

Solution : Let m_1 be the slope of $2x - y + 9 = 0$

$$\therefore \qquad m_1 = 2$$

similarly the slope of line $4x - 2y - 8 = 0$, m_2 is

$$\therefore \qquad m_2 = 2$$

$$\therefore \qquad m_1 = m_2$$

As the slopes are equal, hence the lines are parallel.

Example 14 : Find the equation of line passing through (1, 1) and parallel $\mathbf{3x - 4y = 7}$

Solution : Equation of line passing through

$$y - 1 = m(x-1) \qquad (1)$$

Given equation is $3x - 4y = 7$ (2)

Slope $m_1 = \frac{3}{4}$ (3)

Equation (1) and (2) are parallel thus their slopes will be equal

$$\therefore \qquad m = m_1 = 3/4$$

by (2) $y - 1 = (3/4)(x-1)$

or $3x - 4y + 1 = 0$

Example 15 : Find the equation of line passing through (1, 2) and is parallel to the line joining points (4, -3) and (2, 5)

Solution : Equation of line passing through $(1,2)$ is

$$y - 2 = m(x-1) \qquad (1)$$

Slope of line joining the point $(4,-3)$ and $(2,5)$ is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{2-4} = -4$$

Line (1) is parallel to the line joining points (4, -3) and (2, 5)

$$m = m_1 \qquad \therefore \qquad m = -4$$

Putting the value of m in equation (1)

$$y - 2 = -4(x - 1)$$

or $\quad 4x + y - 6 = 0$, which is the required equation.

Example 16 : Find the equation of line passing through $(\mathbf{3,-4})$ and parallel to x-axis

Solution : Equation of line is $y - y_1 = m(x - x_1)$ here $x_1 = 3$, $y_1 = -4$

$\therefore \quad y - (-4) = m(x - 3) \, (m = \tan\theta)$

or $\quad y + 4 = m(x - 3) \qquad (1)$

we know that slope of x-axis is $m = \tan 0 = 0$

(because $x-$ axis makes an angle $0°$ to the $x-$ axis)

Putting the value of m in (1) we have

$$y + 4 = 0(x - 3) \qquad \text{or} \qquad y + 4 = 0 \Rightarrow y = -4$$

Example 17 : Prove that the lines $\mathbf{2x - y + 9 = 0}$ and $\mathbf{x + 2y - 7 = 0}$ are perpendicular to each other.

Solution : Slope of line $2x - y + 9 = 0$

$$m_1 = 2 \qquad (1)$$

Slope of line $x + 2y - 7 = 0$

$$m_2 = -\frac{1}{2} \qquad (2)$$

If the lines are perpendicular then

$m_1 m_2 = -1$, putting the values of m_1 and m_2

$$2 \times \left(-\frac{1}{2}\right) = -1 \quad \Rightarrow \quad = -1 = -1$$

Thus the lines perpendicular to each other.

Example 18 : Find the equation of line passing through (3, 2) and perpendicular to $y = x$

Solution : Equation of line whose slope is m

$y - y_1 = m(x - x_1)$, here $x_1 = 3, y_1 = 2$

$y - 2 = m(x - 3) \qquad (1)$

Slope of line $y = x$ is

$m_1 = 1$

slope of line perpendicular to $x - y = 0$ is $m = -\dfrac{1}{m_1} \Rightarrow m = -\dfrac{1}{1} = -1$

putting $m = -1$ in (1)

$y - 2 = -1(x - 3)$ therefore $\qquad x + y - 5 = 0$

Example 19 : Find the equation of line which is the right bisector of line joining points (2, 1) and (4, 3)

Solution : Slope of line joining points (2, 1) and (4, 3)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 2} = 1$$

Slope of line perpenducular to line joining (2, 1) and (4, 3) is (1)

$$m=-\frac{1}{m_1}\Rightarrow m=-\frac{1}{1}=-1$$

let the mid point of (2, 1) and (4, 3) be $(x_1,\ y_1)$

$$x_1=\frac{2+4}{2},\ y_1=\frac{1+3}{2}\quad\Rightarrow x_1=3,\ y_1=2$$

Thus equation of line passing through (3, 2) with slope -1 is

$y-y_1=m(x-x_1)\quad\Rightarrow y-2=-1(x-3)\ \Rightarrow\qquad x+y-5=0$

This is the required equation

Example 20 : Find the equation of line passing through the point (2, 3) and makes an angle of 45^0 with the line $3x+y=5$

Solution : $\because$ Equation of line passing through (x_1, y_1) and making an angle α with the line $y=mx+c$

$$y-y_1=\frac{m+\tan\alpha}{1-m\tan\alpha}(x-x_1) \tag{1}$$

and

$$y-y_1=\frac{m-\tan\alpha}{1+m\tan\alpha}(x-x_1) \tag{2}$$

Here $(x_1, y_1)=(2,3), \alpha=45^o, \therefore \tan\alpha=\tan 45^o=1$ and m, is the slope of $3x+y=5$

$\therefore\qquad m=-3$

putting the value of (1) and (2) we have

$$x+2y-8=0 \tag{3}$$

and

$$2x-y-1=0 \tag{4}$$

Thus the required equation is $x+2y-8=0$ and $2x-y-1=0$

Example 21 : Prove that the perpendicular dropped from the origin to the line joining the points $(c\cos\alpha, c\sin\alpha)$ and $(c\cos\beta, c\sin\beta)$ bisects the line.

Solution : Slope of $(c\cos\alpha, c\sin\alpha)$ and $(c\cos\beta, c\sin\beta)$

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{c\sin\alpha-c\sin\beta}{c\cos\alpha-c\cos\beta}=\frac{(\sin\alpha-\sin\beta)}{(\cos\alpha-\cos\beta)} \tag{1}$$

Co-ordinates of midpoints are

$$\left(\frac{c\cos\alpha+c\cos\beta}{2},\frac{c\sin\alpha+c\sin\beta}{2}\right)$$

Slope of line joining the points $(0,0)$ and $\left(\frac{c\cos\alpha+c\cos\beta}{2},\frac{c\sin\alpha+c\sin\beta}{2}\right)$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{c\sin\alpha + c\sin\beta}{2} - 0}{\frac{c\cos\alpha + c\cos\beta}{2} - 0} = \frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} \qquad (2)$$

$$\Rightarrow \qquad m_1 m_2 = \frac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta} \times \frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta}$$

$$= \frac{\sin^2\alpha - \sin^2\beta}{\cos^2\alpha - \cos^2\beta} = -\frac{\left(\cos^2\alpha - \cos^2\beta\right)}{\left(\cos^2\alpha - \cos^2\beta\right)} = -1$$

Thus the lines with slopes m_1 and m_2 mutually perpendicular. Therefore the line is bisected.

Exercise 11.3

1. Find the angle between the pair of lines given below :

   (i) $y = \left(2 - \sqrt{3}\right)x + 5$ and $y = \left(2 + \sqrt{3}\right)x - 7$

   (ii) $2y - 3x + 5 = 0$ and $4x + 5y + 8 = 0$

   (iii) $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} - \frac{y}{a} = 1$

2. Prove that the following lines are parallel to each order :

   (i) $2y = mx + c$ and $4y = 2mx$

   (ii) $x\cos\alpha + y\sin\alpha = p$ and $x + y\tan\alpha = 5\tan\alpha$

3. Prove that the lines $4x + 5y + 7 = 0$ and $5x - 4y - 11 = 0$ are perpendicular to each order :

4. Find the equations of lines -

   (i) Passing through the point (4, 5) and parallel to the line $2x - 3y - 5 = 0$

   (ii) Passing through the point (1, 2) and perpendicular to the line $4x + 3y + 8 = 0$

   (iii) Parallel to the line $2x + 5y = 7$ and passing through the midpoint of line joining the points (2, 7) and (-4, 1).

   (iv) Divides the line segment joining the points (-3,7) and (5, -4) in the ratio 4 : 7 and is perpendicular to it.

5. The vertices of a triangle are $(0,0), (4,-6)$ and $(1,-3)$. Find the equation of perpendicular lines dropped on the sides of the triangle from the vertices.

6. Find the co-ordinates of the or the centre of triangle whose vertices. $(2,0), (3,4)$ and $(0,3)$

7. A triangle has two vertices $(3,-1)$ and $(-2,3)$. The orthocentre lies at the origin. Find the third vertex.

8. Find the equations of a right bisector of the line segment joining the points $(2,-3)$ and $(-1, 5)$.

9. Find the equations of a line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ and passes through the point where it meets the x-axis.

10. Determine the equation of a line parallel to the line $2x + 3y + 11 = 0$ and sum of whose intercept cut on the axes is 15.

11. Determine the equation of a line passing through (2, -3) and makes an angle 45^0 with the line $3x-2y=4$

12. Find the equation of a line passing through $(4,5)$ and makes equal angle with the line $3x=4y+7$ and $5y=12x+6$

13. Show the equation of a line passing through the origin and making an angle θ with the line $y=mx+c$ is $\frac{y}{x}=\frac{m+\tan\theta}{1-m\tan\theta}$

14. Prove that the equation of line perpendicular to the line $x\sec\theta+y\,\mathrm{cosec}\,\theta=a$ and passing through the point $\left(a\cos^3\theta, a\sin^3\theta\right)$ is $x\cos\theta-y\sin\theta=a\cos2\theta$

15. One of the vertex of an equilateral triangle is (2, 3) and the equation of its opposite side is $x+y=2$. Find the equations of the remaining two sides.

16. Find the equations of the two lines passing through (3, -2) and makes an angle of 60^0 with the line $x+\sqrt{3}y=1$

Miscellaneous Exercise 11

1. The equation of a line parallel to y-axis and at a distance of 5 units to the left of y-axis.

(A) $y=5$ (B) $x=5$

(C) $x=-5$ (D) $y=-5$

2. The equation of a line passing through (3, -4) and parallel to x-axis.

(A) $x=3$ (B) $y=-4$

(C) $x+3=0$ (D) $y-4=0$

3. Slope of y-axis is :

(A) 1 (B) 0

(C) ∞ (D) $\pi/2$

4. The equation $x\times\frac{1}{2}+y\times\frac{\sqrt{3}}{2}=5$ represents -

(A) Symmetric form (B) Slope-intercept form

(C) Intercept form (D) Perpendicular form

5. Equation of line parallel to $3x-4y=7$ and passing through the origin is :

(A) $3x-4y=1$ (B) $3x-4y=0$

(C) $4x-3y=1$ (D) $3y-4x=0$

6. The length of perpendicular dropped from the origin to the line $x+\sqrt{3}y=1$ is p then the value of p is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$ (D) 1

7. If the lines $y=mx+5$ and $3x+5y=8$ are mutually perpendicular then the value of m :

(A) $\frac{5}{3}$ (B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$ (D) $\frac{3}{5}$

8. Equation of line perpendicular to the line $3x-4y+7=0$ and passing through (1, -2) is :

(A) $4x+3y-2=0$ (B) $4x+3y+2=0$

(C) $4x-3y+2=0$ (D) $4x-3y-2=0$

9. The Obtuse angle between the lines $y=-2$ and $y=x+2$ is :

(A) 145^o (B) 150^o

(C) 135^o (D) 120^o

10. The lengths of intercepts made by the line $3x-4y-4=0$ on x-axis and y-axis are :

(A) $\frac{4}{3}$ and -1 (B) $\frac{4}{3}$ and 1

(C) $\frac{3}{4}$ and -1 (D) $\frac{3}{4}$ and 1

11. Line joining the points $(1,0)$ and $\left(-2,\sqrt{3}\right)$ makes an angle θ with the x-axis then the value of $\tan\theta$ is :

(A) $\sqrt{3}$ (B) $-\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{-\sqrt{3}}$

12. The value of constant when the equation $2x+\sqrt{3}y-4=0$ is converted into a slope intercept form :

(A) $m=2, c=4$ (B) $m=\frac{2}{\sqrt{3}}, c=-\frac{4}{\sqrt{3}}$

(C) $m=\frac{\sqrt{3}}{2}, c=2$ (D) $m=\frac{-2}{\sqrt{3}}, c=\frac{4}{\sqrt{3}}$

13. Find the equation of the line passing through (2, 3) and makes an angle of 45^0 with the x-axis.

14. Find the equation of the line passing through (-3, 2) and makes equal intercepts but with oppisite sign with the axes.

15. If the length of perpendicular dropped from origin to the line $4x+3y+a=0$ is 2 then find the value of a.

16. If the intercept madeby the line on the axes is bisected at the point (5, 2), then find the equation of the line.

17. Find the equation of the line passing through (0, 1) and the intercept made on x-axis is triple the intercept made on y-axis.

18. Straight lines $y=2m+c$ and $2x-y+5=0$ are mutually parallel and perpendicular to each other then find m.

19. The length of perpendicular drawn from origin to the line $4x+3y+a=0$ is 2 then find a.

20. The length of perpendicular drawn from origin on $\frac{x}{2a}+\frac{y}{2b}=4$ is p then prove that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Important Points

1. Linear equation of x and y always represents a straight line.
2. General equation of straight line is $ax+by+c=0$
3. If a passing through the origin, then constant term of equation becomes zero, means general equation will be $ax+by=0$
4. Equation of x–axis by $y=0$
5. Equation of y–axis by $x=0$

6. Equation of line parallel to x-axis at distance b is $y = \pm b$
7. Equation of line parallel to y–axis at distance a in $x = \pm a$
8. If lines makes an angle θ with the positive direction of x-axis, then slope of line is given by $m = \tan\theta$
9. Equation of line in slope form is $y = mx + c$, where m is slope of line and c is intercept on y-axis.
10. Equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are intercept on x–axis and y–axis respectively.
11. Equation of line in normal form is $x\cos\alpha + y\sin\alpha = p$ where p is length of perpendicular from origin and perpendicular make an angle α from x-axis.
12. Equation of line passing through point (x_1, y_1) is $y - y_1 = m(x - x_1)$ where m is slope of line.
13. Equation of line passing through two point (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
14. Slope of line joining two point (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

15. If two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ makes an θ angle then, $\tan\theta = \pm\left[\frac{m_1 - m_2}{1 + m_1m_2}\right]$. It lines are parallel then $m_1 = m_2$ and if they are perpendicular then $m_1m_2 = -1$ and if they co-incide then, $m_1 = m_2$ and $c_1 = c_2$.
16. Usually we find angle between the lines for which we take positive sign of $\tan\theta$.
17. If any of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ parallel to y-axis then above formula will not be use because $\tan\theta$ is non determine at this condition we will use $\tan\theta = \pm\left(\frac{b_1}{a_1}\right)$
18. Equation of line parallel to line $ax + by + c_1 = 0$ is $ax + by + c_2 = 0$ where c_2 is constant term evaluated according to another condition given in question.
19. Equation of line perpendicular to line $ax + by + c_1 = 0$ is $bx - ay + c_2 = 0$ where c_2 is constant term evaluated according to another condition given in question.

Answers

Exercise 11.1

1. (i) $y = 5$ (ii) $y + 3 = 0$
2. (i) $y = a + b$ (ii) $y = a^2 - b^2$ (iii) $y = b\cos\theta$
3. (i) $x = 5$ (ii) $x + 3 = 0$ (iii) $5x - 2 = 0$
4. (i) $x = \sqrt{7}$ (ii) $x = 2 - \sqrt{3}$ (iii) $x = p + q$

5. $x+3=0, \quad y=2$

6. $y=4, x=3, y=12$ and $y+4=0, x=11$ and $x+5=0$

7. $(4,3),(-4,3),(-4,-3),(4,-3)$ Area $=48$ sq.units

8. (i) $x-y=0$ (ii) $x-\sqrt{3}y=0$ (iii) $x-y+5=0$

9. (i) $3x+5y-15=0$ (ii) $3x-2y+6=0$

10. $x+y=5$ 11. $x+2y=5$ 12. $5x+3y+30=0$

13. $3x+2y-6=0, x-2y-10=0$

15. (i) $3x+5y-15=0$ (ii) $3x+5y+15=0$ (iii) $3x-4y-15=0$

16. $x-\sqrt{3}y+4=0$

17. $2,(\cos\alpha,\sin\alpha)$

18. $\sqrt{3}x+y=6, \sqrt{3}x-y=6$

Exercise 11.2

1. (i) $m=\frac{7}{13}, \quad c=-\frac{15}{13}, \quad a=\frac{15}{7}, \quad b=-\frac{15}{13}$

(ii) $m=-\frac{5}{6}, \quad c=-\frac{4}{3}, \quad a=\frac{-8}{5}, \quad b=-\frac{4}{3}$

2. $-\cot\alpha$

3. (i) $\tan 60°$ (ii) $\tan 150°$

5. $\frac{5}{2}; \left(1,\frac{3}{4}\right)$

6. $a=-\frac{1}{4}, b=\frac{1}{5}$

7. (i) $x\cos 225°+y\sin 225°=1$ (ii) $x\cos 150°+y\sin 150°=1$

8. $\frac{11}{5},-\frac{4}{3}$

9. $a=\frac{5}{2} b=-\frac{5}{3}$

10. Slope = 90 + 2, $= p\cot 2$

11. $x-y+1=0$

12. (i) $y-x=1$ (ii) $ax-by=ab$ (iii) $(a-2b)x-by+b^2+2ab-a^2=0$

(iv) $t_1t_2y+x=a(t_1+t_2)$ (v) $bx\cos\frac{\alpha-\beta}{2}-ay\sin\frac{\alpha+\beta}{2}=ab\cos\frac{\alpha+\beta}{2}$

Exercise 11.3

1. (i) 120^o or 60^o (ii) $\tan^{-1}\left(-\frac{23}{2}\right)$ (iii) $90°$

4. (i) $2x-3y+7=0$ (ii) $3x-4y+5=0$ (iii) $2x+5y=18$ (iv) $88x-121y+371=0$

5. $y-x=0,\ x-3y=22,\ 2x-3y=11$

6. $\left(\frac{12}{11},-\frac{30}{11}\right)$

7. $\left(-\frac{36}{7},-\frac{45}{7}\right)$

8. $6x-16y+13=0$

9. $ax+by=a^2$

10. $2x+3y-18=0$

11. $y+5x-7=0$ and $5y-x+17=0$ 12. $9x-7y=1$ and $7x+9y=73$

15. $(2+\sqrt{3})x-y=1+2\sqrt{3}$ and $(2-\sqrt{3})x-y=1-2\sqrt{3}$

16. $x-\sqrt{3}y-3-2\sqrt{3}=0$ and $x-3=0$

Miscellaneous Exercise

1. C 2. B 3. C 4. D 5. B

6. B 7. A 8. B 9. C 10. A

11. D 12. D 13. $x-y+1=0$

14. $x-y+5=0$ 15. 10 16. $2x+5y=20$

17. $x+3y=3$. 18. $1, -\frac{1}{4}$