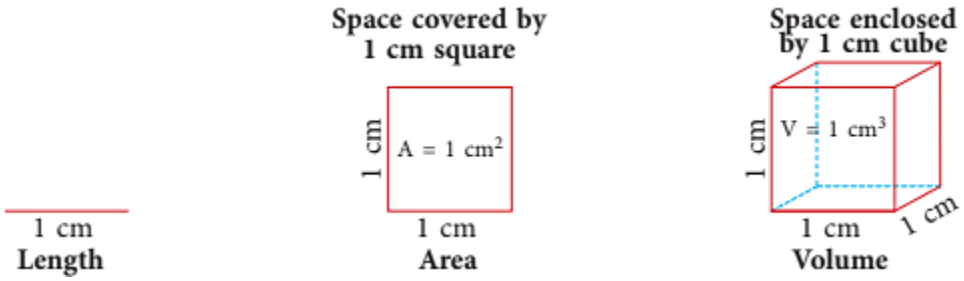
Volume

Volume

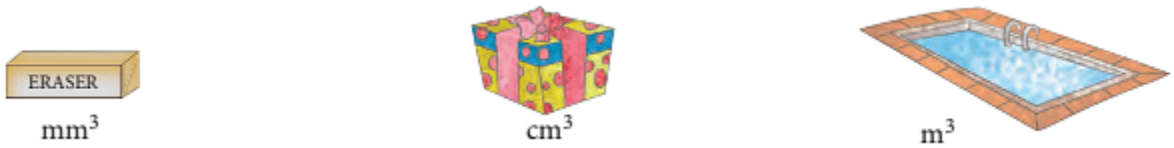
Look at the following solid shapes.

Thus, we observe that solids have three dimensions—length, breadth and height. For example, glass has space to fill water in it, a box has space to fill objects or items in it. The balloon has space to fill the air in it. The space inside the solid is known as the volume of a solid.

Tips: The volume of a solid is the amount of space enclosed by it or the amount of space it takes up.



The other units used for measuring volume are cubic millimetre (mm^3) and cubic metre (m^3). The unit used for measuring volume depends on the size of the solid being measured.



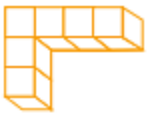
Measurement of Volume

We say that a 1 cm cube is a unit cube and has a volume of 1 cubic centimetre (1 cu cm).
We also write it as 1 cm^3.
To measure the volume of a solid region, we find out the number of unit cubes that make up the solid or fill the solid.
Let us study the following example.

Example 1: Find the volume of these solids by counting the number of cubes in each solid. The volume of each small cube is 1 cm^3.

(a) 

(b) 

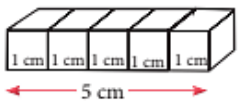
(c) 

(a) There are 5 cubes in the given solid, so, volume = 5 cm^3.

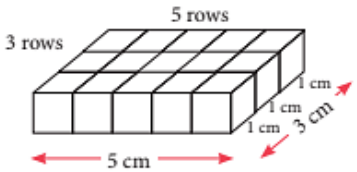
(b) 1st row = 8 cubes; 2nd row = 8 cubes; 3rd row = 8 cubes; 4th row = 8 cubes; 5th row = 8 cubes
∴ Volume = 8 cm^3 + 8 cm^3 + 8 cm^3 + 8 cm^3 + 8 cm^3 = 40 cm^3.
We can also say that there are 5 layers of cubes and each layer consists of 8 cubes. So, Volume = Number of layers × Number of cubes in each layer
V = (5 × 8) cm3 = 40 cm^3.

(c) Cubes in horizontal row = 4 cubes Cubes in vertical row = 2 cubes
Volume = 4 cm^3 + 2 cm^3 = 6 cm^3.

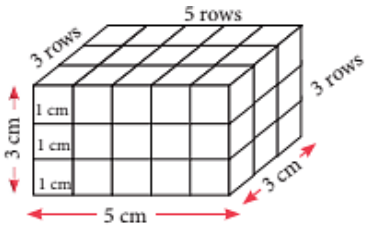
Volume of a Cuboid
A cuboid is a rectangular solid. The following have the shape of a cuboid.
The volume of a cuboid can be found out by finding how many centimetre cubes can be placed in it.
Study the following examples. See how a cuboid is built from 1-centimetre cubes and how we can find its volume.



5 units per row



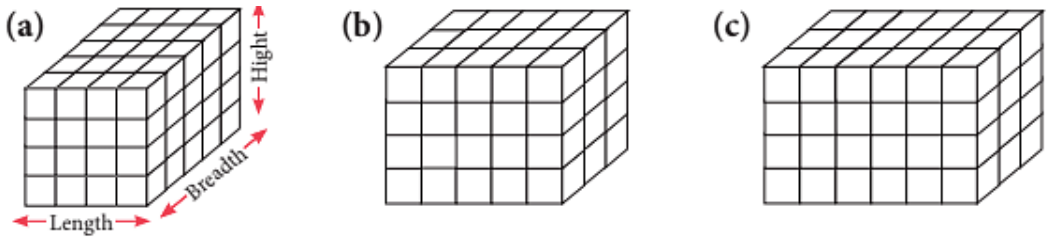
15 units per layer



45 units in all

Length	×	**Breadth**	×	**Height**	=	**Volume**
(Rows)		(Unit cubes in each row)		(Layers)		
5 cm	×	3 cm	×	3 cm	=	45 cm^3

Example 2: The given cuboids are built from 1-centimetre cubes. Find the volume of each solid.



Let us study the table given below using these cuboids as shown.

	Measurements (in centimetres)			Count	Volume
	Length (Rows)	Breadth (Unit cubes in each row)	Height (Layers)	Number of 1 cm cubes	Length × Breadth × Height
(a)	4	5	4	80	4 cm × 5 cm × 4 cm = 80 cm^3
(b)	5	3	4	60	5 cm × 3 cm × 4 cm = 60 cm^3
(c)	6	3	4	72	6 cm × 3 cm × 4 cm = 72 cm^3

From the table, you can conclude that,

• Volume of a cuboid = length × breadth × height

If all the edges of a cuboid are equal, then it is called a cube. So,

• Volume of a cube = side × side × side = $side^3$

Example 3: Find the volume of a cuboid whose length, breadth and height are 15 cm, 11 cm and 6 cm, respectively.

Length of the cuboid = 15 cm

Breadth of the cuboid = 11 cm

Height of the cuboid = 6 cm

Volume of the cuboid = length × breadth × height

= (15 × 11 × 6) cu cm = 990 cu cm.

Example 4: Find the volume of a cube whose one edge measures 2 cm.

Length of each side of the cube = 2 cm.

Volume of the cube = side × side × side

= (2 × 2 × 2) cu cm = 8 cu cm.

Example 5: A box is 20 cm by 18 cm and 40 mm thick. How many cubic centimetres of space will the books keep in it occupy?

Length = 20 cm

Breadth = 18 cm

Height = 40 mm = (40 ÷ 10) cm = 4 cm

∴ Volume of the box = (20 × 18 × 4) cu cm = 1440 cu cm.

Hence, the books will occupy 1440 cu cm of space.

Capacity

The volume is the amount of space a solid takes up or encloses. For example, the amount of space in a swimming pool (in m^3).

The capacity is the amount of liquid that a given solid can hold.
Thus, the capacity of the swimming pool is the amount of water needed to fill the swimming pool.
The capacity of a gas cylinder is the amount of gas it can hold. The capacity of a fuel tank in a vehicle is the amount of petrol or diesel needed to fill it full.
Standard unit of capacity is litre (L). Liquids are often measured in litres, or millilitres.

Tips:

- 1 litre = 1000 mL = 1000 cm^3, 1 mL = 1 cm^3
- 1 cm^3 = 1 mL,
- 1 m^3 = 100 cm × 100 cm × 100 cm
  = 1000000 cm^3
  = 1000000 mL = 1000 L

Example 6: A rectangular tank measures 2.5 m by 3 m by 4 m and is full of water. What is its capacity in litres?
The tank measures 2.5 m by 3 m by 4 m, i.e., 250 cm by 300 cm by 400 cm.
Volume of the tank = (250 × 300 × 400) cm^3
Since 1 litre = 1000 cm^3,
so capacity in litres = 250 × 300 × 400/1000 L = 30000 L.

Example 7: A machine for making ice freezes 5.76 litres of water into ice bricks measuring 3 cm by 2 cm by 1 cm. How many ice bricks will be made?
Volume of 1 ice brick = (3 × 2 × 1) cm^3 = 6 cm^3
Volume of water = 5.76 litres = 5.76 × 1000 cm3 = 5760 cm^3
∴ Number of ice bricks made = 5760 ÷ 6 = 5760/6 = 960.

Example 8: There are 1.75 litres of water in the rectangular container shown. How much more water is needed to fill the container completely?
Capacity of the rectangular container = 15 cm × 10 cm × 20 cm
= 3000 cm^3 = 3 L (Q 1000 cm^3 = 1 L)
Volume of water in the container = 1.75 L
∴ Volume of water needed to fill the container
= 3 L – 1.75 L = 1.25 L
= (1.25 × 1000) mL = 1250 mL.