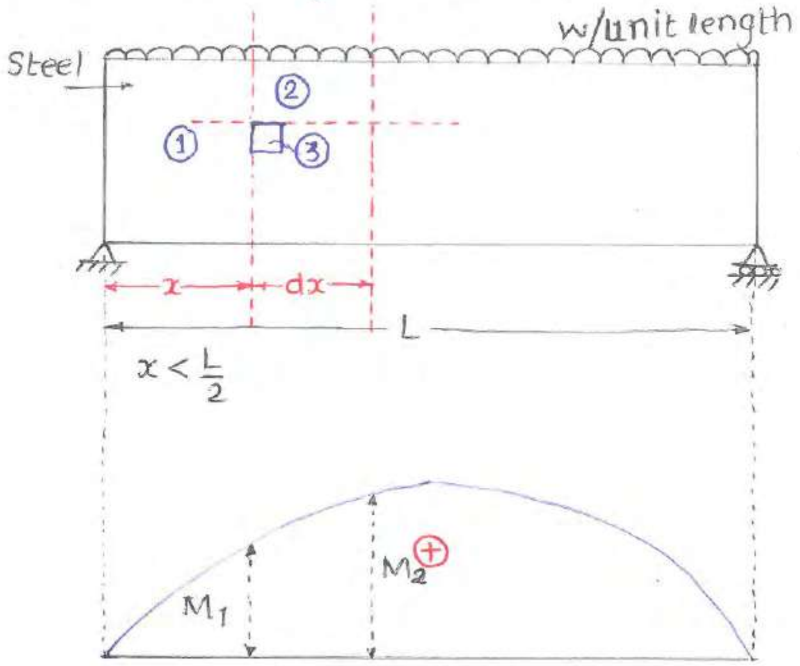
CHAPTER 7

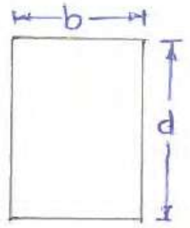
Shear

CONTENTS

7. Shear

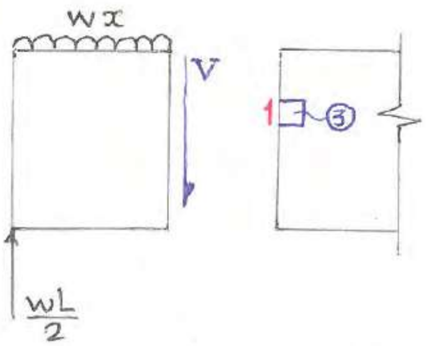
7.1 Shear stress variation over Rectangular Section of Linearly Elastic Homogeneous material :





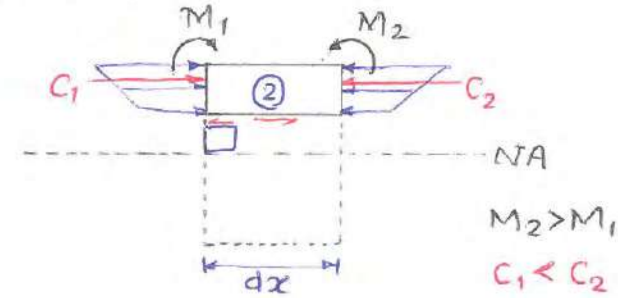
BMD

FBD of ①:



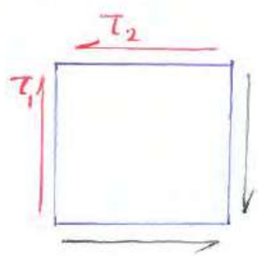
Only vertical equilibrium is represented

FBD of ②



Only horizontal equilibrium is represented.

FBD of ③:



For rotational equilibrium,

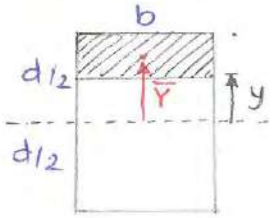
$$\tau_1 = \tau_2$$

We are interested in calculation of τ_1 since, $\tau_1 = \tau_2$ τ_2 is being calculated as follows -

for horizontal equilibrium of (2)

$$C_1 - C_2 + \tau_2 (b \cdot dx) = 0$$

$$\tau_2 = \frac{VA\bar{Y}}{Ib}$$



$$\tau_{2y} = \frac{VA\bar{Y}}{Ib}$$

V = SF on section

A = Area of hatched portion

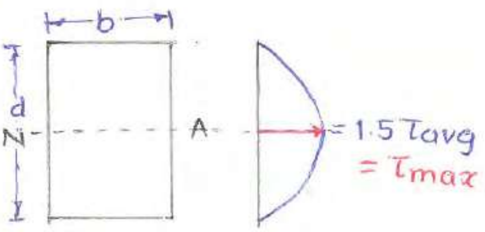
$\bar{Y}$ = distance of C.G. of hatch portion from NA

I = MI of section

b = width of section at level 'y'.

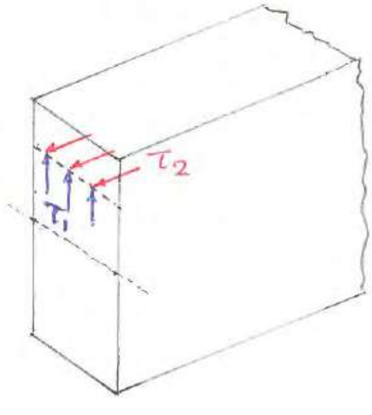
$$\tau_{2y} = \frac{V \times \left[b \cdot \left(\frac{d}{2} - y\right)\right] \times \left[\frac{d}{2} - \left(\frac{d/2 - y}{2}\right)\right]}{\frac{bd^3}{12} \times b}$$

$$\tau_{2y} = \frac{6V}{bd^3}\left[\left(\frac{d}{2}\right)^2 - y^2\right]$$

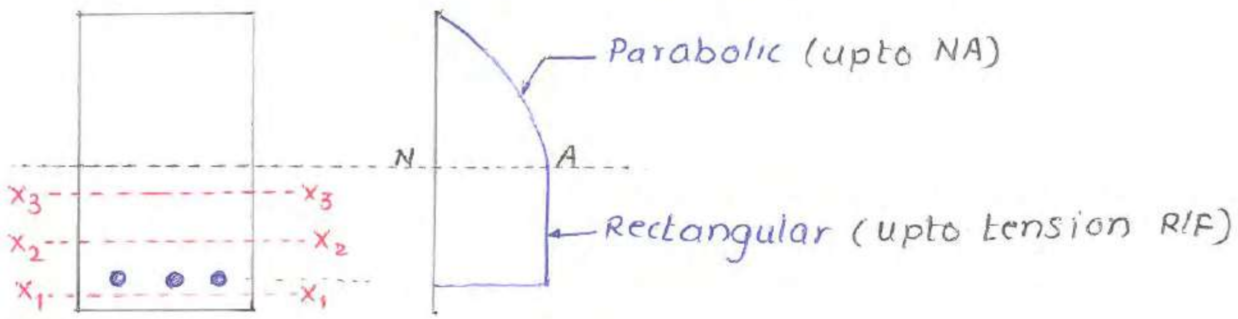


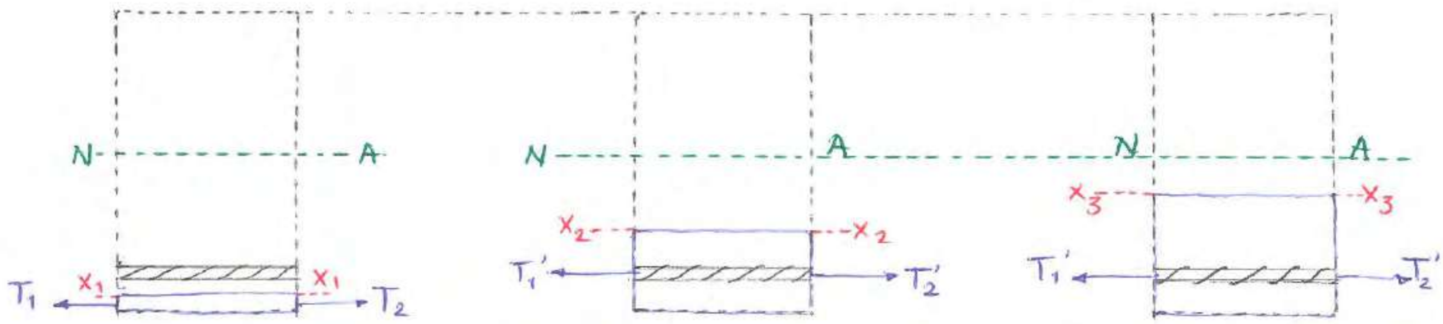
At $y = \pm \frac{d}{2}$, $\tau_{2y} = 0$

at $y = 0$, $\tau_{2y} = \frac{3}{2} \frac{V}{bd} = 1.5\, \tau_{avg}$



7.2 Shear-stress Variation over Rectangular Section of RCC :





Concrete below NA is cracked so, $T_1 = T_2 = 0$ It means shear stress is zero

Concrete below NA is cracked so, entire force will be taken by steel only. It means difference of T_2 & T_1 below (NA) $x_2 - x_2$ and $x_3 - x_3$ remains constant. So shear stress also remains constant.

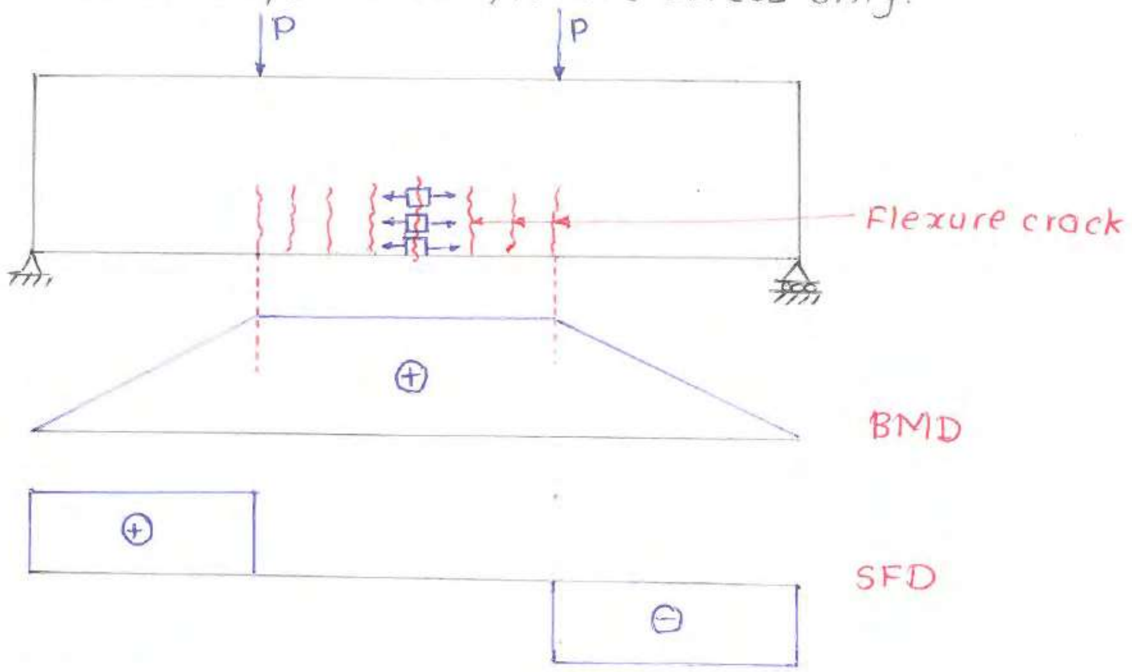
*** Conclusion:**

Shear stress variation over rectangular section of RCC beam subjected to sagging BM is parabolic from top fibre to NA and remains constant upto tension reinforcement

7.3 Types of Crack :

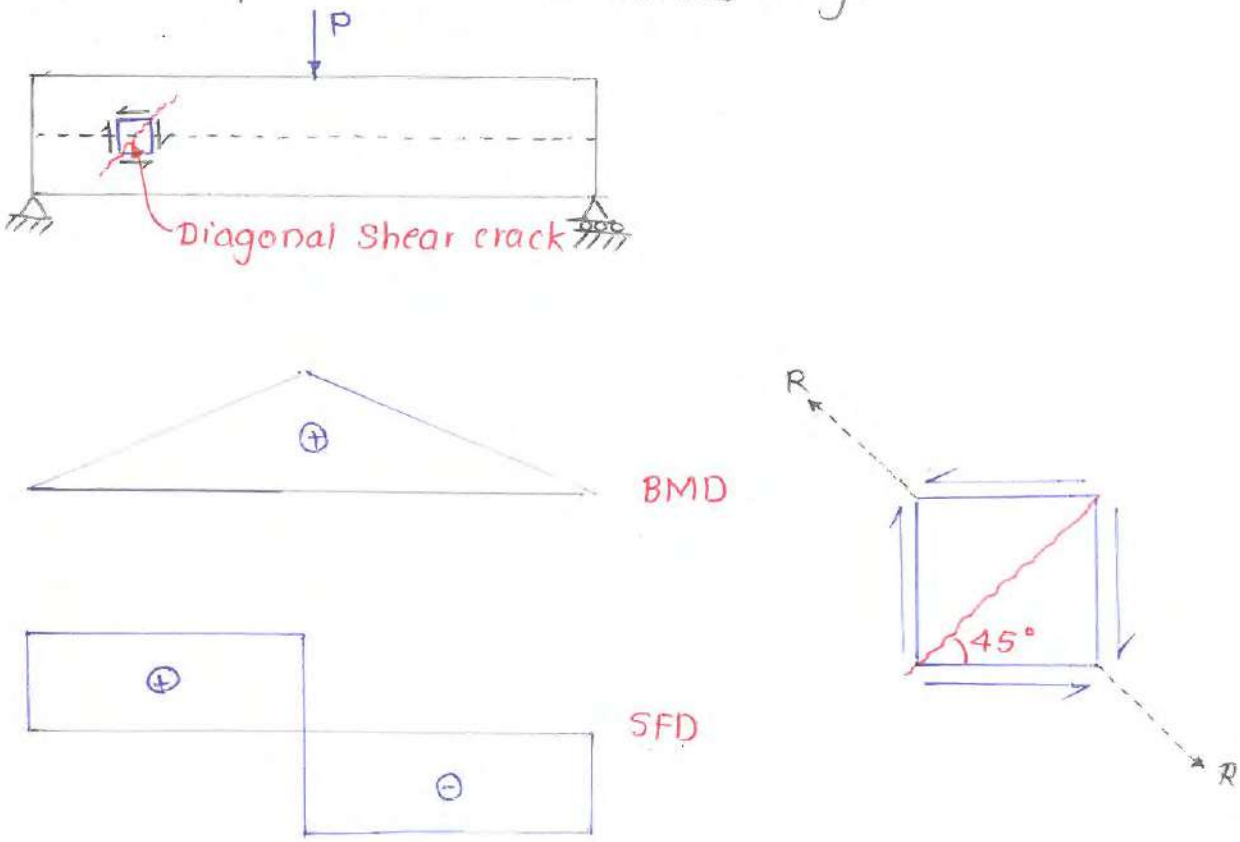
7.3.1 Flexure Crack :

It develops due to flexure stress only.



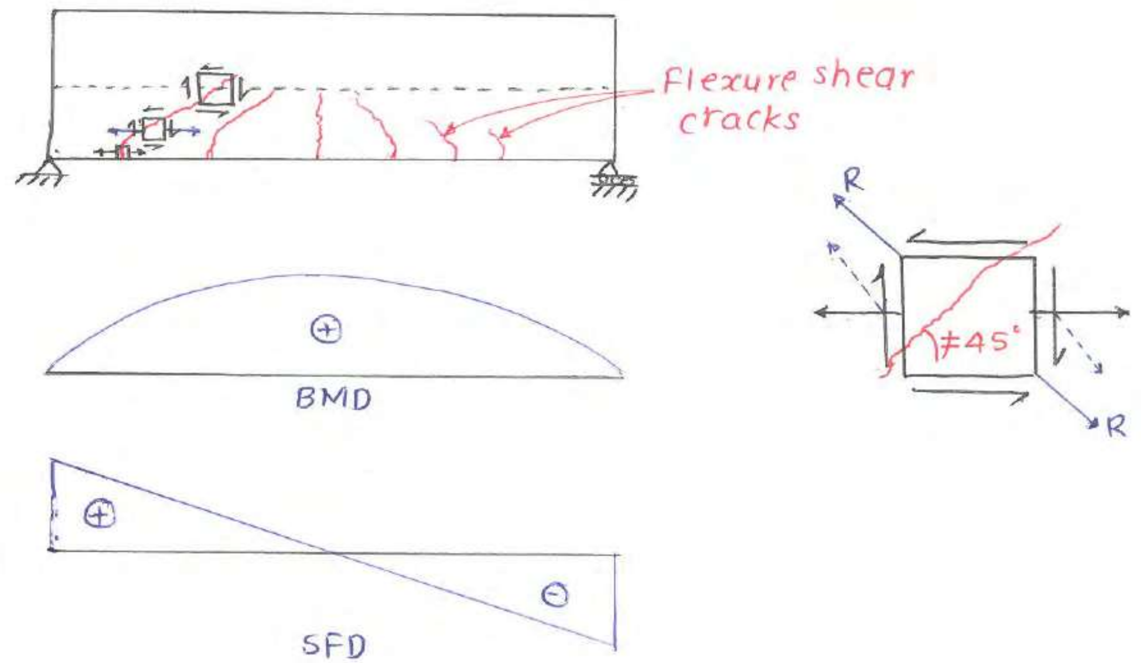
7.3.2 Shear Crack :

It develops due to shear stress only.

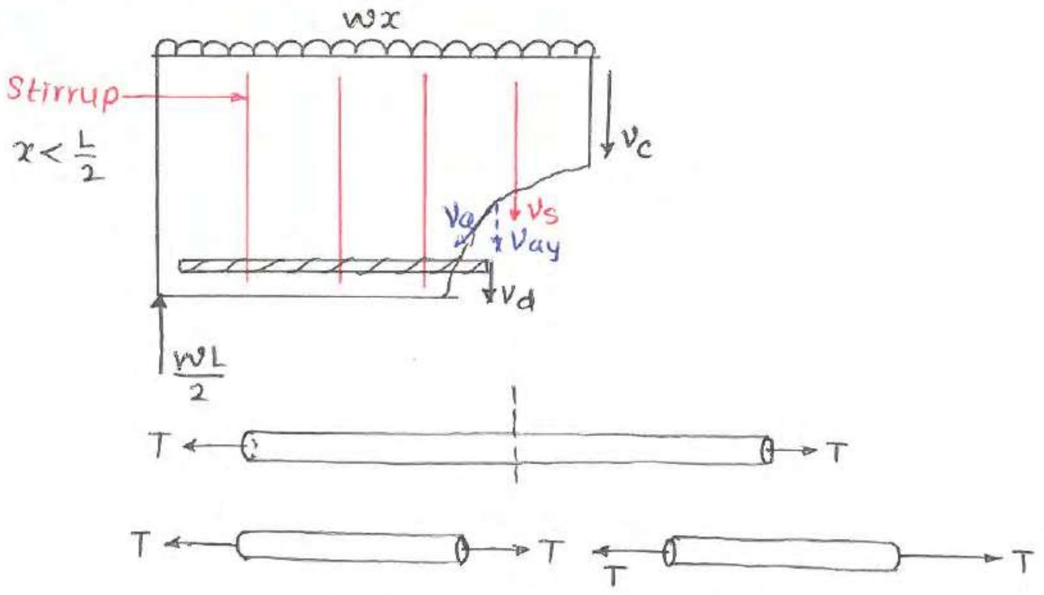


7.3.3 Flexure Shear Crack:

It develops due to combined effect of flexure and shear stress.



7.4 Shear Transfer Mechanism:



All forces are away from section.

Total Shear Resistance of section $= (V_c + V_{ay} + V_d) + V_s$.

V_c = Shear resistance offered by uncracked concrete.

V_{ay} = Vertical component of resistance offered by aggregate interlocking.

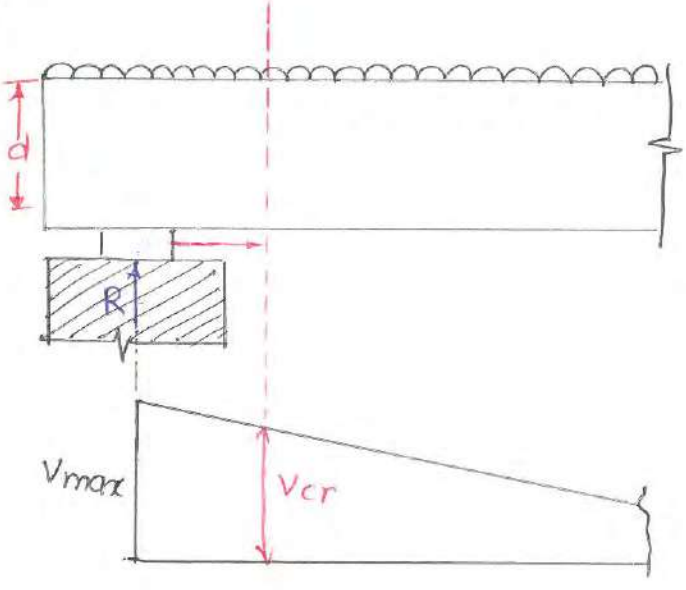
V_d = Shear Resistance offered by longitudinal tension R/F by dowel action

V_s = Shear resistance offered by shear reinforcement.

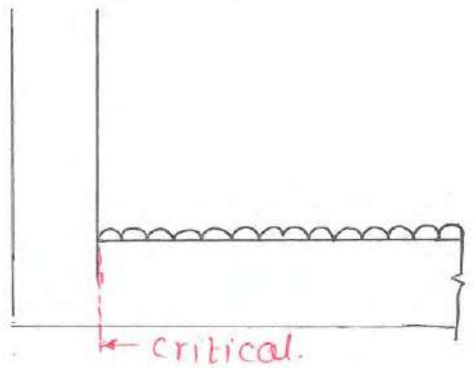
Combined effect of first 3-components $(V_c + V_{ay} + V_d)$ is shear resistance of section without shear reinforcement. Its value is $\tau_c bd$ where, τ_c is design shear strength of concrete and its value is given in Table 19 of IS 456 (Pg 73) corresponding to grade of concrete and % of longitudinal tension reinforcement.

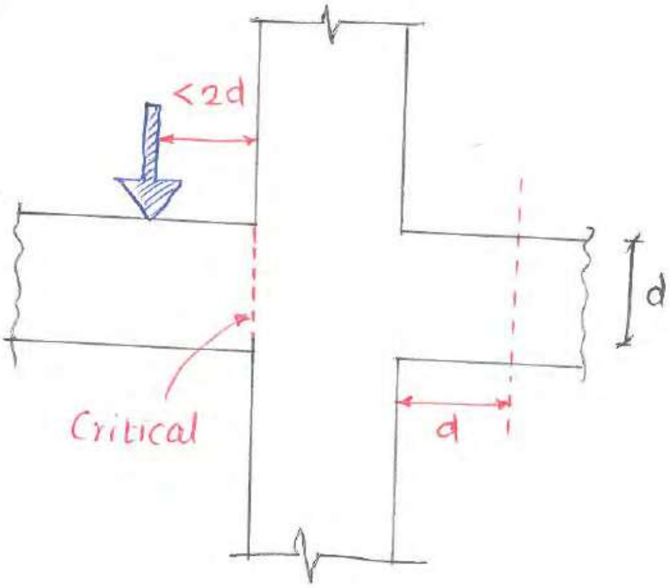
7.5 Critical Section for Shear.

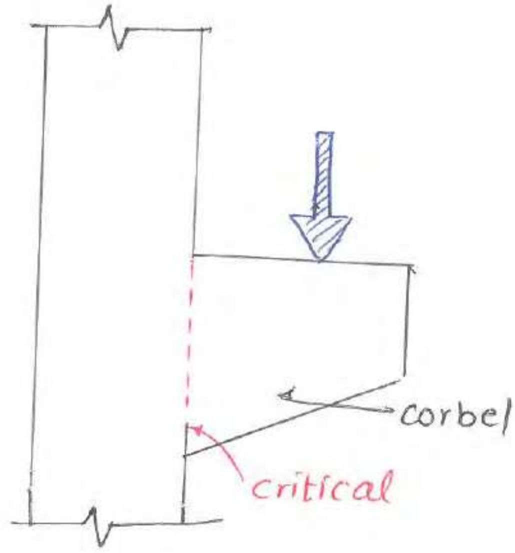
It is that section where chance of failure due to shear is maximum.



If support reaction provides compression to the end of member then critical section for shear is at a distance 'd' from face of of support.







7.6 Nominal Shear Stress:

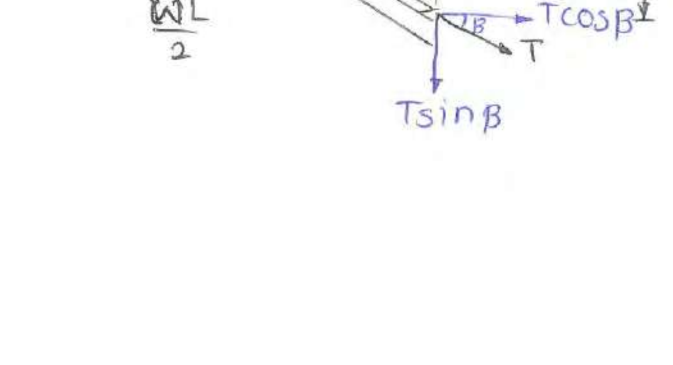
IS 456 provides uniform stress & Shear Stress distribution instead of as disscussed in 7.2

7.6.1 Beam of Uniform Depth:

$$\text{Nominal Shear Stress} = \tau_v = \frac{V_u}{bd}$$

7.6.2 Beam of Varying Depth:

Case I: Bending Moment and depth are increasing in same direction



Applied BM = Resistance of section

$$M_u = T\cos\beta \times LA$$

$$M_u \simeq T\cos\beta \times d \quad (\because LA \simeq d)$$

$$T = \frac{M_u}{d\cos\beta}$$

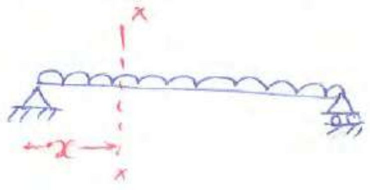
Now,

$$\Sigma F_y = 0$$

$$\frac{wL}{2} - wx - V - T\sin\beta = 0$$

$$\Rightarrow V = \underbrace{\frac{\omega L}{2} - \omega x}_{} - T\sin\beta$$

$$V = V_u - \frac{M_u}{d}\tan\beta$$

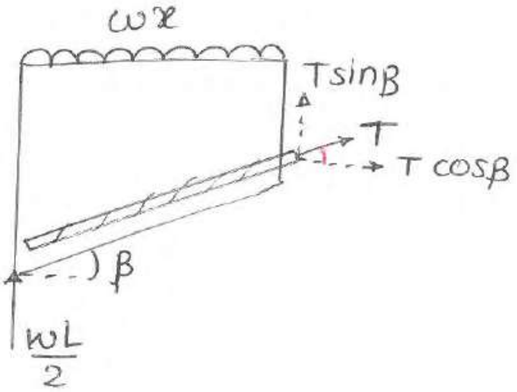


$$V_u = \frac{\omega L}{2} - \omega x$$

then,

$$\text{Nominal Shear Stress} = \tau_v = \frac{V}{bd} = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

Case II : BM and depth are increasing in opposite direction:



$$\text{Nominal Shear Stress} = \tau_v = \frac{V}{bd} = \frac{V_u + \frac{M_u}{d}\tan\beta}{bd}$$

where β is angle between top and bottom fibres

* Sign convention:

$\longrightarrow$ BM (↑)
$\longrightarrow$ d (↑) (−ve)

$\longrightarrow$ BM (↑)
d (↑) $\longleftarrow$ (+ve)

7.7 Design of Section for Shear:

Step 1: Calculate design/factored/ultimate shear force at critical section.

Step 2: Calculate Nominal Shear Stress.

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd} \leq \tau_{c,max}$$

Where $\tau_{c,max}$ is maximum shear st capacity of section with shear reinforcement its value is given in Table 20 of IS 456 (Page 73) corresponding to grade of concrete.

$$\tau_{c,max} \approx 0.63\sqrt{f_{ck}} \ \ N/mm^2$$

If $\tau_v > \tau_{c,max}$ then section size is increased.

Step 3: Take value of of τ_c from Table 19 of IS 456, corresponding to grade of concrete and % of tension reinforcement.

τ_c is modified as $\delta\tau_c$ for member subjected to axial compression.

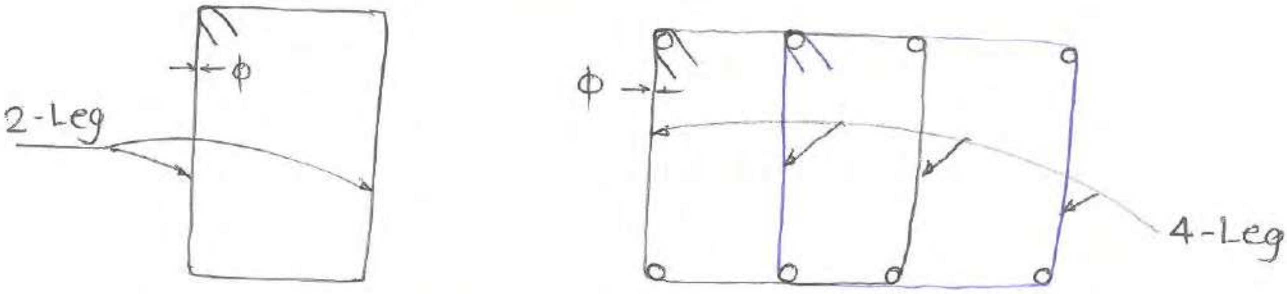
$$\delta = \text{Minimum} \begin{cases} \text{i)} \ 1 + \dfrac{3P_u}{f_{ck}A_g} \\ \text{ii)} \ 1.5 \end{cases}$$

τ_c is modified as $k\tau_c$ for slab thickness

$$\begin{aligned} k &= 1.3 && D \leq 150mm \\ &= 1.6 - 0.002D && 150 < D < 300 \\ &= 1.0 && 300mm \leq D \end{aligned}$$

Step 4: If $\tau_v < \frac{\tau_c}{2}$ then nominal shear stirrup is provided in primary members and no shear stirrup is provided in member of minor importance. (lintel)

$$\boxed{\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}}$$



$A_{sv} = 2 \times \left(\frac{\pi}{4} \times \phi^2\right)$ $A_{sv} = 4 \times \left(\frac{\pi}{4} \times \phi^2\right)$

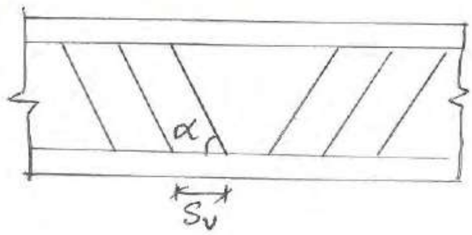
If $\frac{\tau_c}{2} \leq \tau_v < \tau_c$ then nominal shear stirrup is provided in all types of member.

$$\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}$$

If $\tau_c \leq \tau_v \leq \tau_{c,max}$ then shear reinforcement is designed for shear force $(\tau_v - \tau_c) bd$

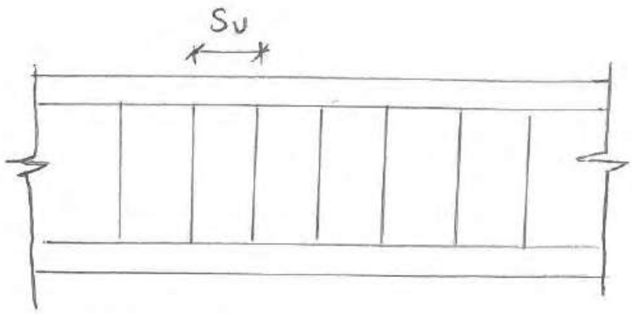
Step 5: Shear reinforcement is provided by following three ways.

(1) Inclined Stirrup.



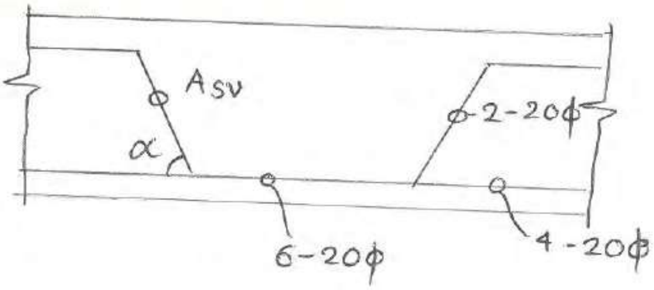
$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}(\sin\alpha + \cos\alpha)$$

② Vertical Stirrup:



$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

③ Bent-up Bars:



$$V_{us} = 0.87 f_y \cdot A_{sv} \cdot \sin\alpha$$

*Note:-

- Not more than 50% of shear force taken by shear reinforcement $[(\tau_v - \tau_c) bd]$ is assumed to be taken by bent ~~to~~up bars

Bent up bars are always provided with shear stirrups so shear stirrups are designed for maximum of following shear forces.

- $\frac{(\tau_v - \tau_c) bd}{2}$

- $(\tau_v - \tau_c) bd$ – Capacity of bent-up bars.

	Case I	Case II
V_u	200kN	200kN
V_c	30kN	30kN
Shear force taken by Shear R/F	200-30 = 170kN	170kN
Capacity of bent-up bars	250kN	40kN
Shear force taken by Stirrup	$\frac{170}{2} = 85$kN	170-40 = 130kN

- ~~α sh~~ $\boxed{\alpha \nless 45^\circ}$ (preferably $45^\circ - 60^\circ$)

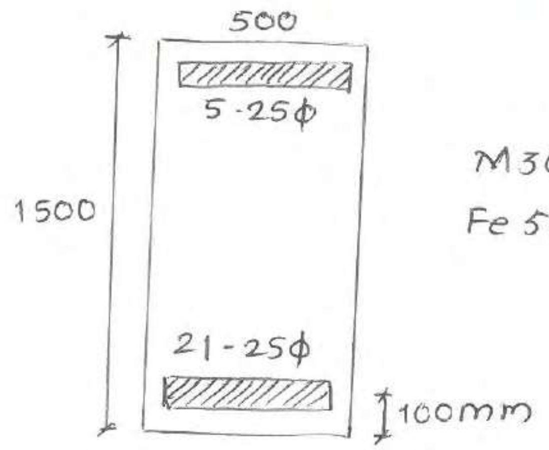
- f_y is not assumed to be more than 415 N/mm². This limitation is irrespective of grade of steel. This limitation is imposed to control crack width.

  - Fe 250 ⟶ 250 N/mm²
  - Fe 415 ⟶ 415 N/mm²
  - Fe 500 ⟶ 415 N/mm².

Step 6: Maximum permissible spacing of stirrups.

$S_v \leq$ Minimum
- as per step (5)
- $\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}$
- 0.75d and d for vertical & inclined stirrup respectively
- 300mm

* Ex. Design 4-legged vertical stirrups for section given below which is subjected to ultimate shear force 1000 kN.



M30

Fe 500.

⇒

Step 1: $V_u = 1000\,kN$

Step 2 $\tau_v = \frac{V_u}{bd} = \frac{1000 \times 10^3}{500 \times 1400}$

$\tau_v = 1.43\ N/mm^2\ (< \tau_{max} = 3.5\ N/mm^2)$

Step 3: $P_t = \frac{A_{st}}{bd} \times 100$

$= \frac{21 \times \frac{\pi}{4} \times 25^2 \times 100}{500 \times 1400}$

$P_t = 1.47\%$ $\Rightarrow$ $\tau_c = 0.75\ N/mm^2$ [Table 19 of IS456 $P_t = 1.47\%$ & M30]

Step 4: Since $\tau_v > \tau_c$ so shear R/F is designed for

$SF = (\tau_v - \tau_c)\, b\, d$

$= (1.43 - 0.75) \times 500 \times 1400$

$SF = 476\ kN.$

Step 5: $\phi\ (\uparrow) \longrightarrow S_v\ (\uparrow)$

No. of legs $(\uparrow) \longrightarrow S_v\ (\uparrow)$

Assuming 4-legged - 8 φ

$V_{us} = \frac{0.87\, f_y A_{sv} \cdot d}{S_v}$

$$476 \times 10^3 = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 8^2 \times 1400}{S_v}$$

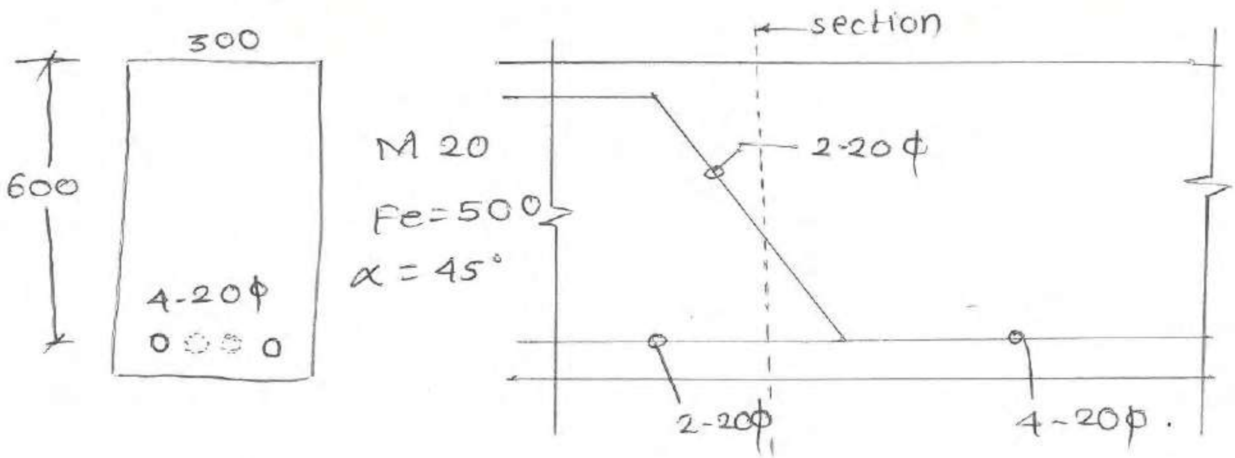
$$S_v = 213.5 \text{ mm}$$

Step 6: Spacing:

$$S_v \leq \text{Minimum} \begin{cases} \bullet \ 213.5 \text{ mm} \\ \bullet \ \frac{A_{sv}}{b s_v} \geqslant \frac{0.4}{0.87 f_y} \Rightarrow S_v \leq 362.96 \text{ mm} \\ \bullet \ 0.75 d = 1050 \text{ mm} \\ \bullet \ 300 \text{ mm} \end{cases}$$

Providing 4-legged 8 mm ϕ @ 200 c/c

*Ex. Design shear reinforcement for section given below which is subjected to working load shear force 200 kN. Assume 2-bars used for bent up bars.



⇒

Step 1: $V_u = 1.5 \times 200 = 300$ kN

$$\tau_v = \frac{V_u}{bd} = \frac{300 \times 10^3}{300 \times 600}$$

$$\tau_v = 1.67 \text{ N/mm}^2 < \tau_{c,max} \ (2.8 \text{ N/mm}^2)$$

Step3 $P_t = \frac{A_{st}}{bd} \times 100$

$= \frac{2 \times \frac{\pi}{4} \times 20^2}{300 \times 600} \times 100$

$P_t = 0.35\%$

87 $\tau_c = 0.39 \ N/mm^2$ (from Table 19 $P_t = 0.35\%$ & M20)

Step4: Since $\tau_v > \tau$ so section is designed for

$SF = (\tau_v - \tau_c) \cdot bd$

$= (1.67 - 0.39) \times 300 \times 600$

$SF = 230.4 \ kN$

Step5: Capacity of bent-up bars

$= 0.87 f_y \cdot A_{sv} \cdot \sin\alpha$

$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 \times \sin 45°$

$= 160.41 \ kN$

Bent-up bars are always provided with stirrups so, stirrups should be designed for maximum of following.

- $\frac{(\tau_v - \tau_c) bd}{2} = \frac{230.4}{2} = 115.2 kN$

- $(\tau_v - \tau_c) bd$ − Capacity of bent up bars $= 230.4 - 160.4$ $= 69.99 kN$

Assuming 2-legged 8 ϕ

$V_{us} = \frac{0.87 f_y \cdot A_{sv} \cdot d}{S_v}$

$\Rightarrow S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 \times 600}{115.2 \times 10^3}$

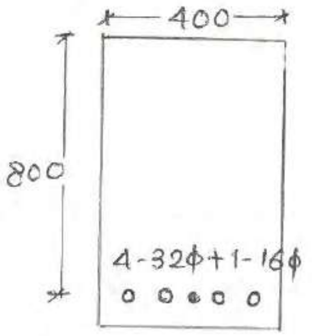
$S_v = 189.04 \ mm$

Step 6 :

$$S_v \leq \text{Minimum} \begin{cases} \bullet\ 189.04\,mm \\ \bullet\ \dfrac{A_{sv}}{b S_v} \geq \dfrac{0.4}{0.87 f_y} \Rightarrow S_v \leq 302.4\,mm \\ \bullet\ 0.75d = 0.75 \times 600 = 450\,mm \\ \bullet\ 300\,mm \end{cases}$$

$\Rightarrow$ Providing 2-legged 8ϕ @ 175 mm c/c.

* Ex. Design shear reinforcement for the ~~g~~ beam given in ex. of chapter 6. Assume $2\text{-}32\phi$ bars are curtailed before critical section for shear.



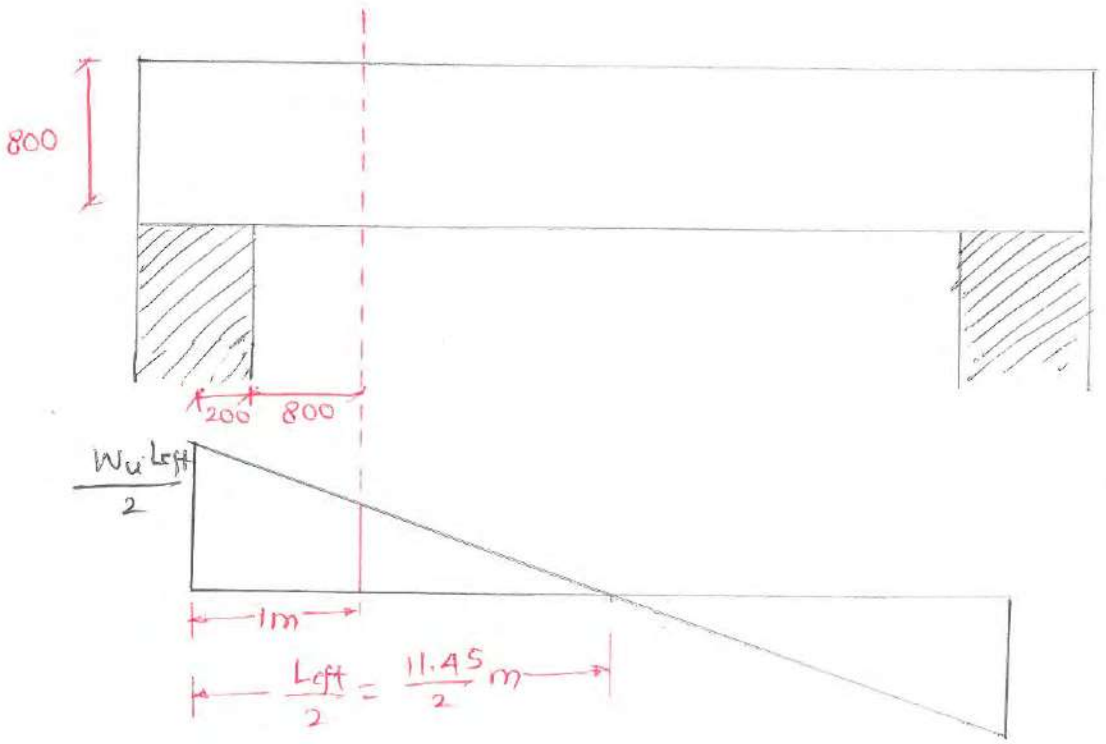
M30

Fe500

$W_u = 58.125\ kN/m$

$L_{eff} = 11.45\ m$

Support width = 400 & 500mm.



$$V_{cr\,(x=1m)} = \frac{W_u L}{2} - wx$$

$$= \frac{58.125 \times 11.45}{2} - 58.125$$

$$V_{cr} = 274.64\,kN$$

Step 2: $\tau_v = \frac{V_u}{bd} = \frac{274.64 \times 10^3}{400 \times 800} = 0.85\,N/mm^2 < \tau_{c,max}$ $(3.5\,N/mm^2)$

Step 3: $P_t = \frac{A_{st}}{bd} \times 100$

$$= \frac{2 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}{400 \times 800} \times 100$$

$P_t = 0.56\%$ $\left[\begin{array}{c} \text{from Table 19 of IS456} \\ P_t = 0.56\% \text{ \& M30} \end{array} \right]$

Step 4: Since $\tau_v > \tau_c$ so shear reinforcement is designed for $SF = (\tau_v - \tau_c)bd$

$$= (0.85 - 0.52) \times 400 \times 800$$

$$SF = 105.6\,kN$$

Step 5: Assuming 2-legged 10ϕ

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

$$105.6 \times 10^3 = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 800}{S_v}$$

$$\Rightarrow S_v = 429.6\,mm$$

* Note:

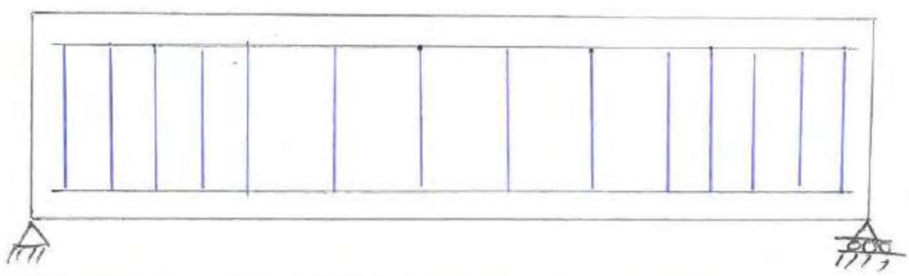
8mm dia should have taken for more appropriate result.

Step6; Spacing:

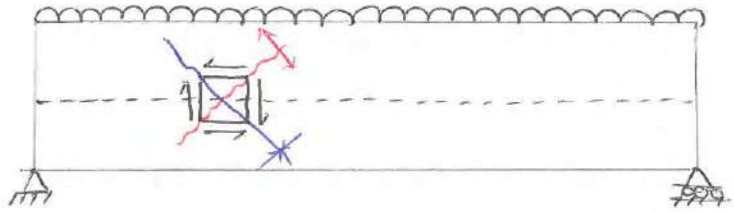
$$S_v = \text{Minimum}\begin{cases} \bullet\ 429.6\ mm \\ \bullet\ \dfrac{A_{sv}}{bS_v} \geq \dfrac{0.4}{0.87 f_y} \Rightarrow S_v \leq 354.46 mm \\ \bullet\ 0.75d = 0.75 \times 800 = 600 mm \\ \bullet\ 300 mm \end{cases}$$

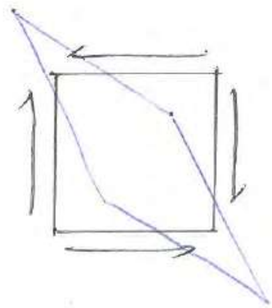
Providing 2-legged 10ϕ @ 300 mm c/c

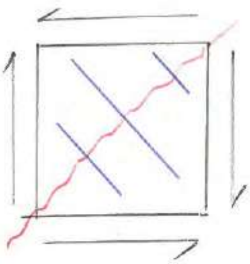
* Note:
Spacing of stirrups is increased in middle portion because shear force is less as compared to supports.



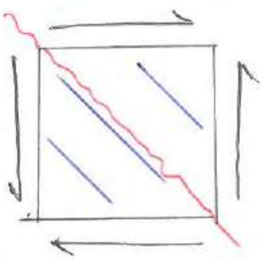
7.8 Reason of $\tau_{c,max}$:



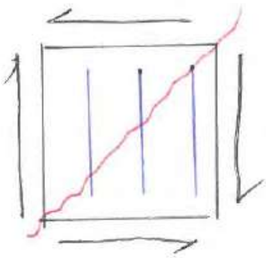




(a) Inclined stirrups



(b) Reversal of stress

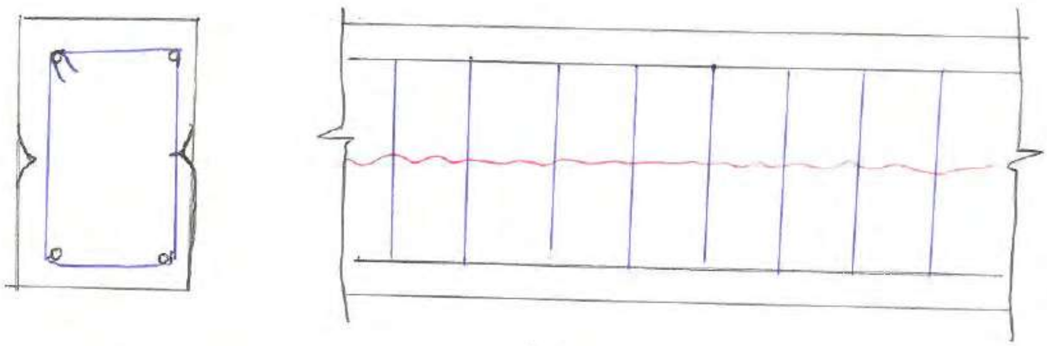


c) Vertical stirrups

- As load increases on beam, τ_v also increases. This τ_v produces tension on one diagonal plane and compression on anothe diagonal plane.
- Diagonal tension is taken by shear reinforcement and diagonal compression is taken by concrete
- For $\tau_v > \tau_{c,max}$, concrete under diagonal compression get crushed so τ_v should not be more than $\tau_{c,max}$
- Vertical shear stirrups are preferable than ~~inde~~ inclined stirrups because it takes care of reversal of stress.

7.9 Advantages of Minimum Shear Stirrup:

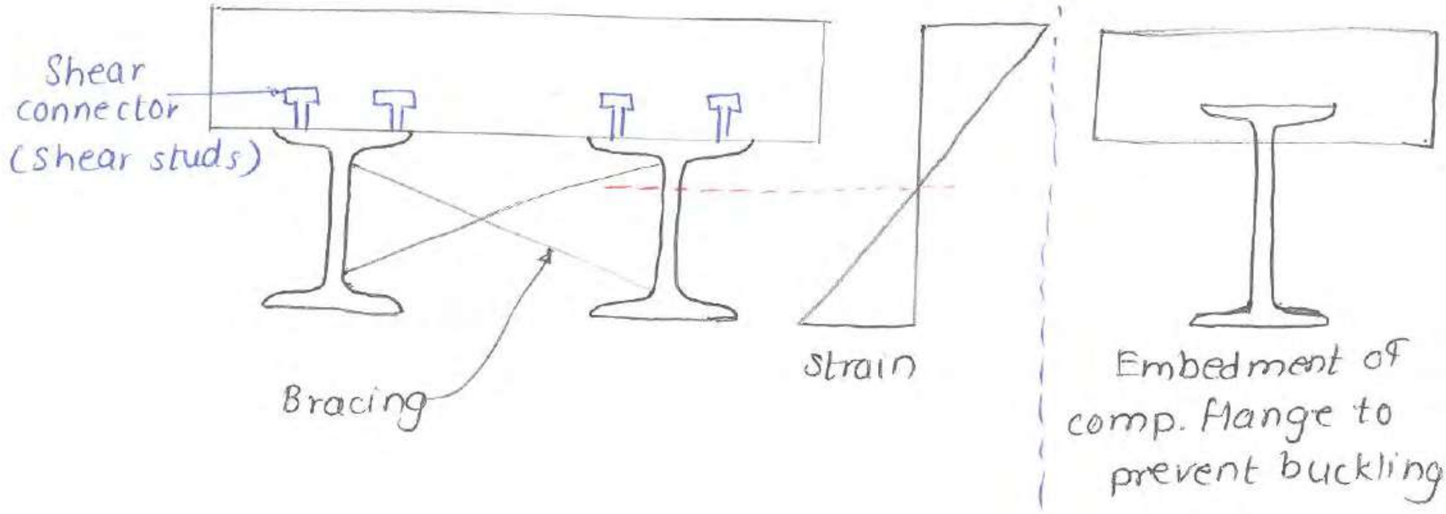
1) It safegaurds against any sudden failure of member. In other words, shear stirrup provides ductility to the member.
2) It prevents horizontal cracks due to shrinkage.



3) confinement of concrete.
4) Enhances performance of member subjected to torsion.
5) Enhances dowel action of tension reinforcement

7.10 Shear Connector:

It is provided for monolithic action between concrete slab and steel beam which makes this section as composite. It also prevents lateral buckling of beam.



7.11 Deep Beams:

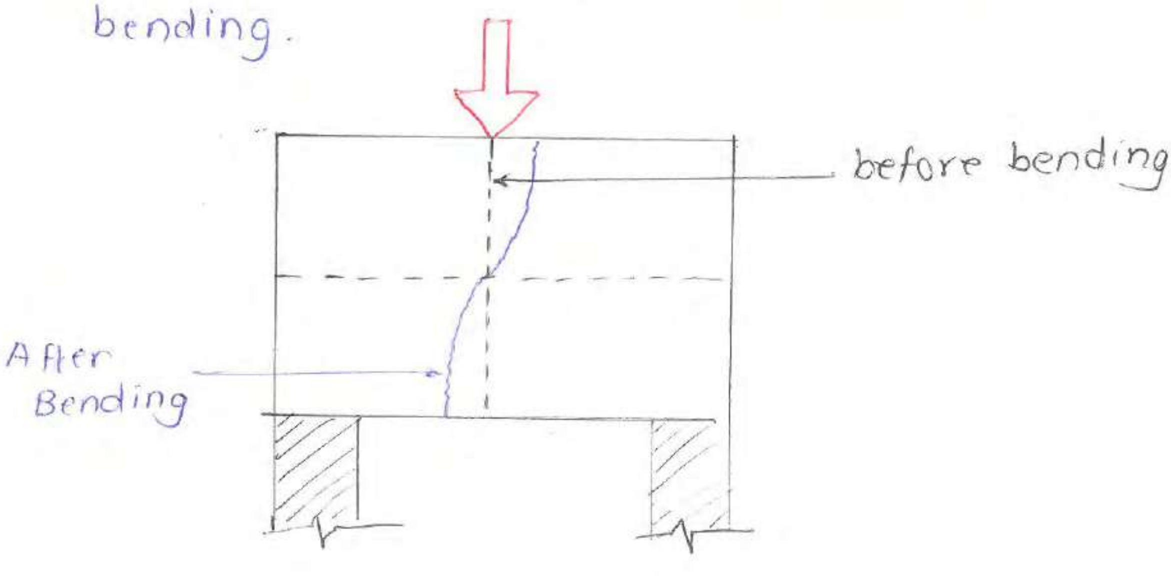
A beam is classified as deep beam based on $\frac{L_{eff}}{D}$ ratio. It is provided to carry a very high value of point load.

1) For S.S. beam, $\frac{L_{eff}}{D} < 2$

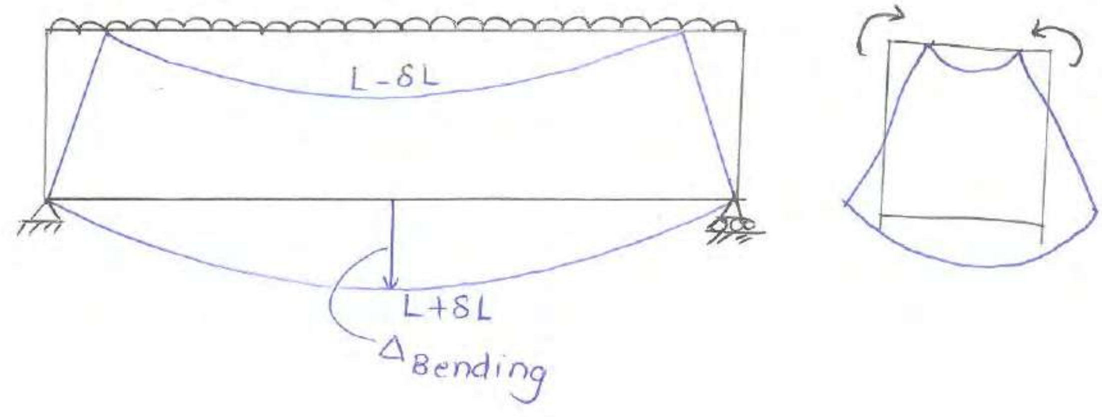
continuous, $\frac{L_{eff}}{D} < 2.5$

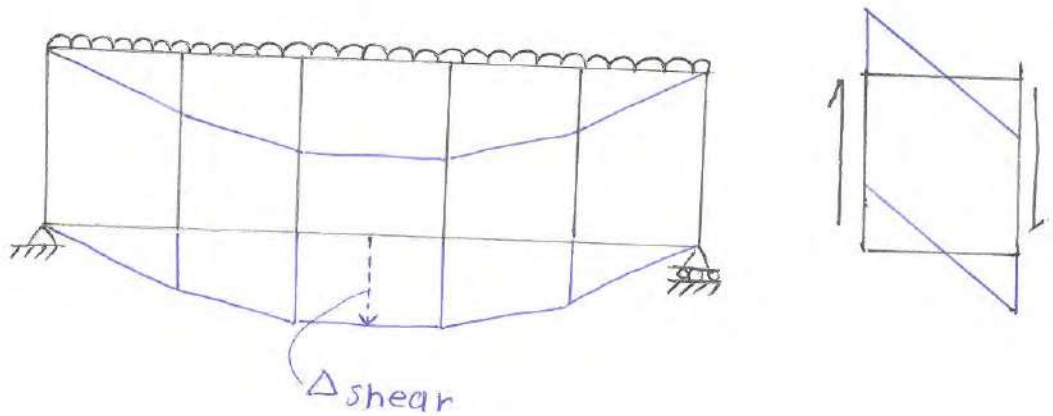
2) L_{eff} = Minimum $\begin{cases} \bullet\ 1.15\ L_{clear} \\ \bullet\ c/c \text{ distance b/w supports.} \end{cases}$

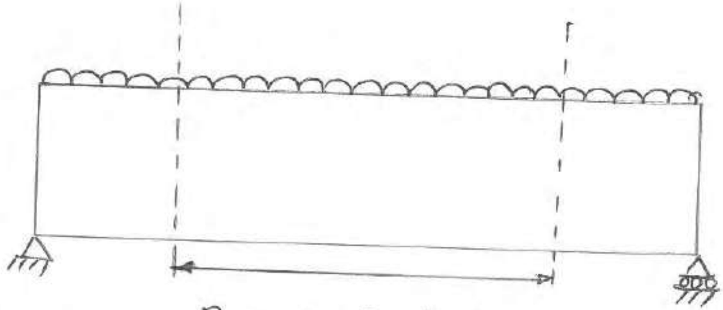
3) Plane section no longer remains plane after bending.



4) Deflection in deep Beam:



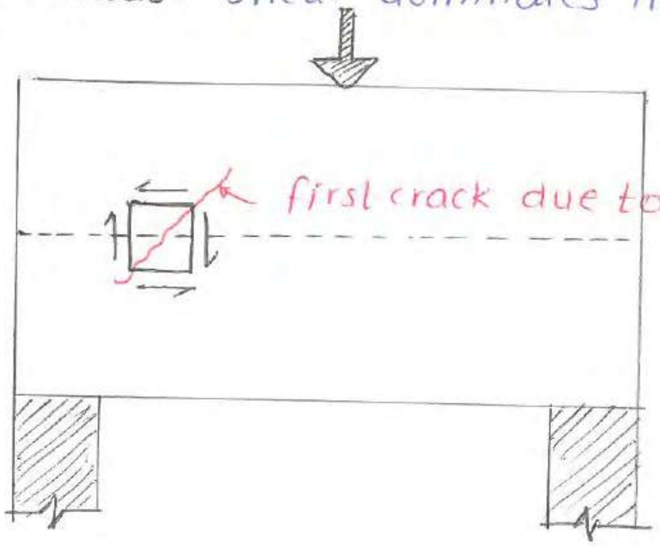




Region of plane section remains plane practically.

- In shallow beams, $\Delta_{bending} >> \Delta_{shear}$ so Δ_{shear} is neglected. but in the case of deep beams, Δ_{shear} dominates.

5> First crack in deep beams develop due to diagonal tension. because shear dominates in deep beams.



... Chapter 7 Ends Here...