AREAS & VOLUMES

1 hectare = 10,000 m^2

1 Acre = 4046.7 m^2 ≅ 4840 sq. yards

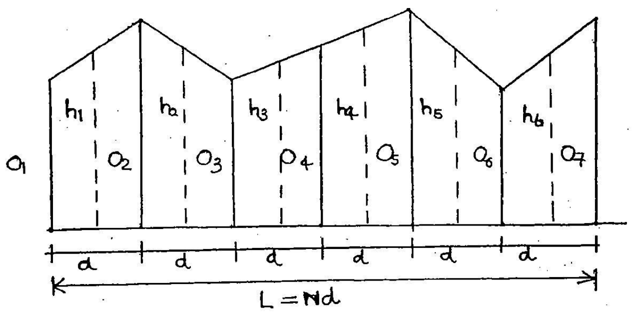
Areas.

Instrumental method.

(i) Cross Staff survey
(ii) Compass survey.
(iii) Plane table survey.
(iv) Theodolite traversing

Graphical method.

(i) Field notes
(ii) End areas. (Boundary areas).
a) Mid Ordinate method.
b) Average Ordinate method.
c) Trapezoidal rule.
d) Simpson's rule.



N → total no. of ~~ordinates + 1~~ common intervals

d → common interval.

Total no. of ordinates = N+1

(i) Mid – Ordinate Method.

$$h_1 = \frac{O_1 + O_2}{2}, \quad h_2 = \frac{O_2 + O_3}{2}$$

$$\boxed{A = d \cdot (h_1 + h_2 + \dots h_n)}$$

(ii) Average – Ordinate method.

$$A = \left(\frac{\text{Sum of Ordinates}}{\text{Total no. of Ordinates}}\right) * \text{Base length}$$

$$\boxed{A = \left(\frac{O_1 + O_2 + O_3 + \dots O_n}{(N+1)}\right) L}$$

(iii) Trapezoidal Rule (Parabolic Rule).

- it is most suitable for curved boundaries.
- there is no restriction to no. of ordinates.

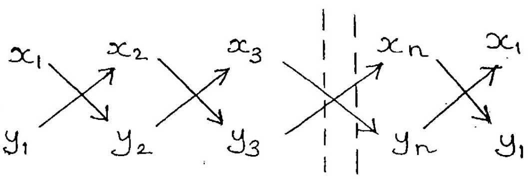
$$\boxed{A = d\left[\left(\frac{O_1 + O_n}{2}\right) + (O_2 + O_3 + \dots O_{n-1})\right]}$$

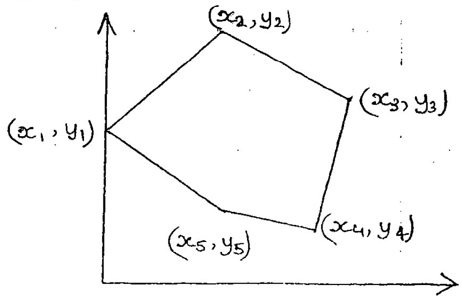
(iv) Simpson's Rule.

- it can be used only for odd number of ordinates
- it can also be used for curved boundaries

$$\boxed{A = \frac{d}{3}\left[(\text{first} + \text{last}) + 4(\text{even}) + 2(\text{odd})\right]}$$

→ Calculation of area from Co-ordinates.



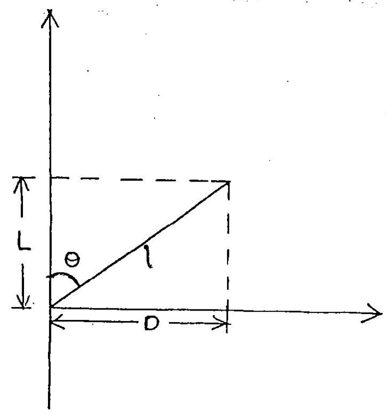


$$A = \frac{1}{2}\left[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - y_2 x_3) + (x_3 y_n - y_3 x_n) + (x_n y_1 - y_1 x_n)\right]$$

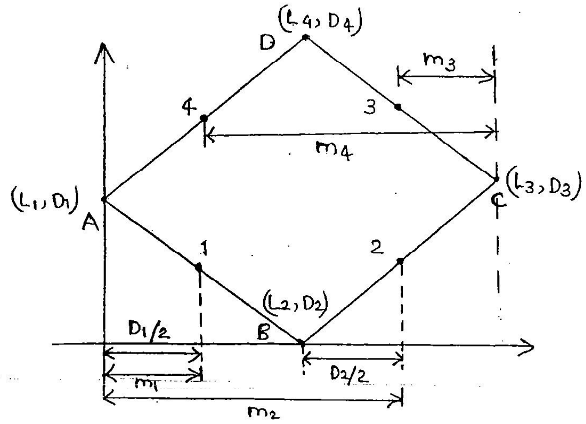
→ Calculation of areas from Latitudes & Departures.

Latitude, $L = l\cos\theta$

Departure, $D = l\sin\theta$



(i) Meridian Distance method.



Meridian distance of a line is defined as the meridian distance of its midpoint.

Meridian distance of a line = meridian distance of preceding line
+ half of departure of preced. lin
+ half of departure of line itself.

$m_1 = \frac{D_1}{2}$; $m_2 = m_1 + \frac{D_1}{2} + \frac{D_2}{2}$

$m_3 = -\frac{D_3}{2}$; $m_4 = -D_3 - \frac{D_4}{2}$

$$\boxed{A = \Sigma mL}$$

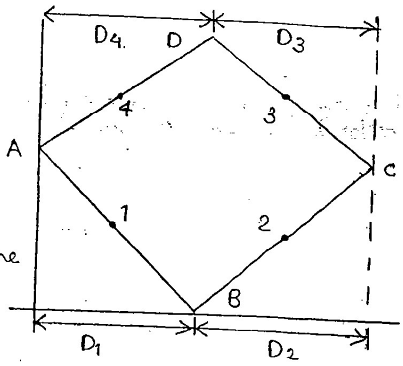
(ii) Double meridian distance Method.

Double meridian distance of a line is equal to sum of the meridian distances of two extremities

$M_1 = D_1$

$M_2 = M_1 + D_1 + D_2$

DMD of a line = DMD of prec. line
+ Departure of prec. line
+ departure of line itself.



$$\boxed{A = \frac{1}{2}\Sigma ML}$$

Line	L	D
AB	108	4
BC	15	249
CD	-123	4
DA	0	-257

Single meridian distance method.

Line	L	D	D/2	m	A = mL
AB	108	4	2	2	216
BC	15	249	124.5	128.5	1927.5
CD	-123	4	2	255	-31365
DA	0	-257	-128.5	128.5	0

Area = ΣmL = $-29221\ m^2$ = $29221\ m^2$

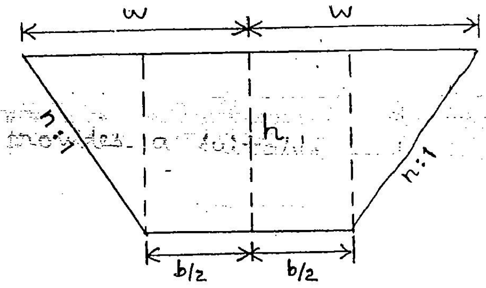
Double meridian distance method.

Line	L	D	M	$A = \frac{ML}{2}$
AB	108	4	4	432/2
BC	15	249	257	3855/2
CD	-123	4	510	-62730/2
DA	0	-257	257	0

Area = $\frac{1}{2}\Sigma ML$ = $29221\ m^2$

→ Volumes

* Calculation of area from Level Sections.

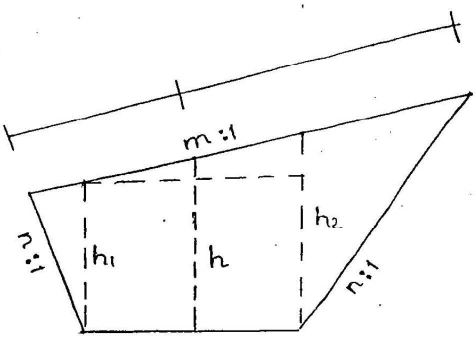


$$A = \frac{1}{2n}\left[\left(\frac{b}{2} + nh\right)(w_1 + w_2) - \frac{b^2}{2}\right]$$

where,

$$w_1 = \left(\frac{b}{2} + nh\right)\left(\frac{m}{m-n}\right)$$

$$w_2 = \left(\frac{b}{2} + nh\right)\left(\frac{m}{m+n}\right)$$



* $w = \frac{b}{2} + nh$

$$A = (b+nh)\,h \quad \{m = 0\}$$

* Calculation of Volumes.

Let $A_1, A_2 \dots A_n$ be the areas of cross sections, h be the contour Interval of cross section. The volume can be calculated by :

1. Mean Area method.

$$\text{Volume, } V = \left(\frac{A_1 + A_2 + A_3 + \dots A_n}{n}\right) * L$$

L → distance b/w end sections.

2. Prismoidal Formula (Simpson's Rule).

$$V = \frac{h}{3}\left[(\text{first} + \text{last areas}) + 4(\text{even areas}) + 2(\text{odd areas})\right]$$

3. Trapezoidal Formula (Average End area method).

$$V = h\left[\left(\frac{\text{first} + \text{last}}{2}\right) + \text{Remaining areas}\right]$$

* Prismoidal correction

- It is the difference b/w prismoidal formula and trapezoidal value
- Prismoidal correction is always negative.
- The volume calculated by trapezoidal formula must be deducted from volume calculated by prismoidal formula.

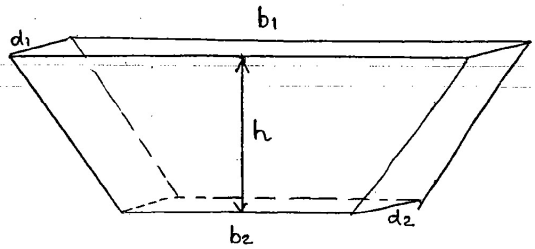
Let $A, w_1, w_2, h_1, h_2 \ldots$ be the c/s at one end and $A_1', w_1', w_2', h_1', h_2' \ldots$ be the c/s at other end.

Prismoidal correction, $C_p = \frac{dn}{6}(h-h')^2 \rightarrow$ single LS

$$C_p = \frac{d}{6n}(w_1-w_1')(w_2-w_2') \rightarrow 2 \text{ LS.}$$

* Volume of water stored.

Volume, $V = \frac{h}{6}\left[A_1 + 4A_m + A_2\right]$



$A_1 = b_1 d_1$

$A_2 = b_2 d_2$

$A_m = \left(\frac{b_1+b_2}{2}\right)\left(\frac{d_1+d_2}{2}\right)$.

Q. A road embankment is 4m wide at formation level with a side slope of 2:1. The average height of embankment is 4m with average gradient as 1 in 30. from 210 m to 330 m contour. Find length of road and quantity of earthwork.

$$\text{Gradient} = \frac{\text{Difference in RL}}{\text{Horizontal distance.}}$$

$$\frac{1}{30} = \frac{330-210}{\text{length of road}} \Rightarrow \text{length of road} = \underline{\underline{3600 \text{ m}}}$$

$$A = (b+nh)h = (12+2\times4)\,4 = 80\ m^2.$$

Volume of earth work, $V = A \times L$

$$= 80 \times 3600 = \underline{288\,000\ m^3}$$

P-75

20 $A_1 = 6\times4 = 24\ m^2$

$A_2 = 4\times2 = 8\ m^2$

$$A_m = \left(\frac{6+4}{2}\right)\left(\frac{4+2}{2}\right) = 15\ m^2$$

$$V = \frac{6}{6}\left(24 + 4\times15 + 8\right) = \underline{92\ m^3}$$

P-76

1 $V = 30\left(\left(\frac{30+108}{2}\right) + 63\right) = \underline{3915\ m^3}$

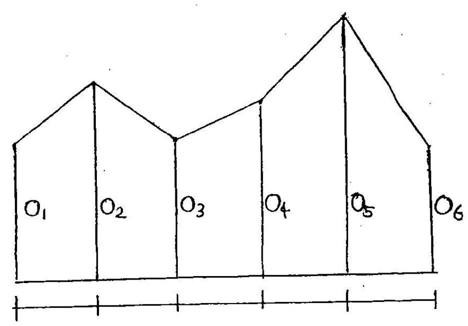
2. $V = \frac{30}{3}\left((30+105) + 4\times63\right) = \underline{3870\ m^3}$

3. Prismoidal correction, $C_p = 3915 - 3870 = \underline{45\ m^3}$

6 $V = \frac{30}{3}\left((20+30) + 4(40+50) + 2\times60\right) = \underline{5300\ m^3}$

7 $h = 5$

$$V = \frac{5}{3}\left((3850+450) + 4(3450+800) + 2(2600)\right) = \underline{44166.66\ m^3}$$



→ Correction for Curvature.

- Prismoidal and trapezoidal formula were derived on the assumption that the end sections are in parallel plane. When the centre line of cutting or the embankment is curved in plan, it is common practise to calculate the volume with the end sections were in parallel planes and then apply correction for curvature.

- No correction for single level sections.

$$C_c = \frac{d}{6R}(w_1^2 - w_2^2)\left(h + \frac{b}{2n}\right).$$

R → radius of curve.