CHAPTER 13

Logarithm

Definition

If 'a' is a positive real number, other than 1 and 'b' is a rational number such that $a^b = N$, then we say that logarithm of N to base 'a' is b or 'b' is the logarithm of N to base 'a', written as

$$\log_a N = b$$

So, $$a^b = N \Leftrightarrow \log_a N = b$$

e.g., $$7^0 = 1 \Leftrightarrow \log_7 1 = 0$$

$$81^{1/4} = 3 \Leftrightarrow \log_{81} 3 = \frac{1}{4}$$

$$(1024)^{10} = 2 \Leftrightarrow \log_{1024} 2 = 10$$

Example 1. In the following which have the value of $x = 4$?

(a) $\log_2 x = 3$ (b) $\log_5 x = 3$

(c) $\log_{81} x = \frac{3}{2}$ (d) $\log_{\sqrt{2}} x = 4$

Sol. (*d*) (a) $\log_2 x = 3 \Leftrightarrow x = 2^3 = 8$ (by definition)

(b) $\log_5 x = 2 \Leftrightarrow x = 5^2 = 25$

(c) $\log_{81} x = 3/2 \Leftrightarrow x = (81)^{3/2} = (3^4)^{3/2} \Leftrightarrow x = 3^6 \Leftrightarrow x = 729$

(d) $\log_{\sqrt{2}} x = 4 \Leftrightarrow x = (\sqrt{2})^4$

$$x = (2^{1/2})^4 = 2^2 = 4$$

Example 2. If $\log_5 y = x$ and $\log_2 z = x$, then the value of 20^x in terms of y and z is

(a) zy (b) $z^2 y$ (c) xy^2 (d) z/y

Sol. (*b*) $\log_5 y = x$ and $\log_2 z = x$

$\Rightarrow$ $$y = 5^x \text{ and } z = 2^x \quad \text{...(i)}$$

Now, $$20^x = (2^2 \times 5)^x = 2^{2x} \times 5^x$$

$$= (2^x)^2 \cdot 5^x = z^2 \cdot y \quad \text{[from Eq. (i)]}$$

$\therefore$ $$20^x = z^2 \cdot y$$

Fundamental Laws of Logarithms

1. **First Law** If m, n are positive rational number, then

$$\log_a (mn) = \log_a m + \log_a n$$

Here, if $m_1, m_2, m_3, \ldots, m_n$ are positive rational numbers, then

$$\log_a (m_1 m_2 m_3 \ldots m_n) = \log_a m_1 + \log_a m_2 + \ldots + \log_a m_n$$

2. **Second Law** If m and n are positive rational numbers, then

$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

3. **Third Law** If m and n are positive rational numbers, then

$$\log_a (m^n) = n \log_a m$$

4. **Fourth Law** $\log_a 1 = 0$

5. **Fifth Law** $\log_a a = 1$

6. **Sixth Law** If m is a positive rational number and a, b are positive real numbers such that $a \neq 1, b \neq 1$, then

$$\log_a m = \frac{\log_b m}{\log_b a} \quad \text{...(i)}$$

or $$\log_b a = \frac{\log_b m}{\log_a m}$$

Put $b = m$ in Eq. (i)

$$\log_a m = \frac{\log_m m}{\log_m a}$$

$$\log_a m = \frac{1}{\log_m a}$$

$\Rightarrow$ $$\log_a m \times \log_m a = 1$$

7. **Seventh Law** If 'a' is a positive real number and 'n' is a positive rational number, then

$$\log_{a^y} n^x = \frac{x}{y} \log_a n \quad \text{...(ii)}$$

If $a = n$, then

$$\log_{n^y} n^x = \frac{x}{y} \log_n n \quad \text{[from (ii)]}$$

$$= \frac{x}{y}$$

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8. **Eighth Law** If 'a' is a positive real number and n is a positive rational number, then

$$n\log_a a = n$$

9. **Ninth Law** If 'a' is a positive number and 'n' is a rational number, then

$$\log_a n = \log_{a^2} n^2 = \log_{a^3} n^3 = \ldots = \log_{a^m} n^m$$

Example 3. The value of $\log_{10}(50)$ is

(a) $\log_{10} 2 - \log_{10} 5$ (b) $\log_{10} 5$
(c) $\log_{10} 2 + \log_{10} 5$ (d) $\log_{10} 2 + 2\log_{10} 5$

Sol. (d) $\log_{10} 50 = \log_{10}(2 \times 25)$

$$= \log_{10} 2 + \log_{10} 25 = \log_{10} 2 + \log_{10} 5^2$$
$$= \log_{10} 2 + 2\log_{10} 5$$

Example 4. The value of x in $\frac{\log 256}{\log 16} = \log x$ is

(a) 1 (b) 10 (c) 100 (d) 0

Sol. (c) $\frac{\log 256}{\log 16} = \frac{\log 16^2}{\log 16} = \frac{2\log 16}{\log 16} = 2$

$$\Rightarrow \quad \log x = 2$$
$$\log_{10} x = 2 \qquad (\because \log x = \log_{10} x)$$
$$x = 10^2 = 100$$

Example 5. The value of x in $\log_x 4 + \log_x 16 + \log_x 64 = 12$ is

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (b) $\log_x 4 + \log_x 16 + \log_x 64 = 12$

$$\log_x 2^2 + \log_x 2^4 + \log_x 2^6 = 12$$
$$2\log_x 2 + 4\log_x 2 + 6\log_x 2 = 12$$
$$12\log_x 2 = 12$$
$$\log_x 2 = 1 \Rightarrow x = 2$$

Example 6. The value of x in $\log x - \log(x-1) = \log 3$ is

(a) 2/3 (b) 3/2 (c) 1/2 (d) 1/4

Sol. (b) $\log x - \log(x-1) = \log 3$

$$\log \frac{x}{x-1} = \log 3$$
$$\frac{x}{x-1} = 3$$
$$x = 3(x-1)$$
$$x = 3x - 3$$
$$-2x = -3$$
$$x = 3/2$$

Example 7. The value of $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$ is

(a) 1 (b) 2
(c) −1 (d) None of these

Sol. (b) $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$

$$= \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}$$
$$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2\log 3}{\log 3} = 2$$

Example 8. If $\log_x a, \log_x b, \log_x c$ are in AP, then

(a) a, b, c are in GP (b) a, b, c are in AP
(c) a, b, c are in HP (d) None of these

Sol. (a) As $\log_x a, \log_x b, \log_x c$ are in AP.

$$\Rightarrow \quad 2\log_x b = \log_x a + \log_x c$$
$$2\log_x b = \log_x ac$$
$$\log_x b^2 = \log_x ac$$
$$b^2 = ac$$

$\Rightarrow$ a, b, c are in GP.

Some Useful Results

Result I. If $a > 1$, then

(i) $\log_a x < 0$ if $0 < x < 1$
(ii) $\log_a x = 0$ for $x = 1$
(iii) $\log_a x > 0$ for $x > 1$
(iv) $x > y \Rightarrow \log_a x > \log_a y$

Result II. If $0 < a < 1$, then

(i) $\log_a x > 1$ if $0 < x < a$
(ii) $\log_a x = 1$ if $x = a$
(iii) $\log_a x < 1$ if $x > a$
(iv) $\log_a x < 0$ for all $x > 1$
(v) $\log_a x = 0$ for $x = 1$
(vi) $\log_a x > 0$ for all x satisfying $0 < x < 1$
(vii) $x > y \Rightarrow \log_a x < \log_a y$

Result III. (i) $\log_a x > 1 \Rightarrow \begin{cases} x > a, & \text{where } a > 1 \\ 0 < x < a, & \text{where } 0 < a < 1 \end{cases}$

(ii) $\log_a x < 1 \Rightarrow \begin{cases} 0 < x < a, & \text{if } a > 1 \\ x > a, & \text{if } 0 < a < 1 \end{cases}$

Systems of Logarithms

There are two systems of logarithms which are generally used

1. **Common Logarithms** In this system Base is always taken as 10, also known as Brigg's system.
   e.g., $\log 100 = \log_{10} 100 = 2$
   $\log 1000 = \log_{10} 1000 = 3$

Whenever the base is not mentioned it should be considered as 10.

2. **Natural Logarithms** In this system base of logarithm taken as 'e'. e is irrational number between 2 and 3.

Characteristic and Mantissa of a Logarithm

The logarithm of positive real number 'n' consists of two parts

1. The integral part is known as the characteristic. It is always an integer positive, negative or zero.

2. The decimal part is called as the mantissa. The mantissa is never negative and is always less than one.

To Find the Characteristic

***Case* I.** When the number is greater than 1.

The characteristic is one less than the number of digits in the left of decimal point in the given number.

e.g., 6.125 characteristic is 0.
61.321 characteristic is 1.
725.132 characteristic is 2.

***Case* II.** When the number is less than 1.

The characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

e.g., 0.7684 characteristic is $\bar{1}$.
0.06712 characteristic is $\bar{2}$.
0.00031 characteristic is $\bar{4}$.

> In place of –1 or –2 etc., we use $\bar{1}$ (one bar), $\bar{2}$ (two bar) etc.

Antilogarithm

The positive number 'a' is called the antilogarithm of a number b, if $\log a = b$. If 'a' is antilogarithm of b, we write $a = \text{antilog } b$.

So, $a = \text{antilog } b \Leftrightarrow \log a = b$

Inserting Decimal Point

Two rules are used for inserting a decimal point.

Rule 1 When the characteristic of the logarithm is positive, we insert the decimal point after the $(n+1)$th digit, where n is the characteristic.

Rule 2 When the characteristic of the logarithm is negative, we insert the decimal point such that the first significant figures is at nth place, where $\bar{n}$ is the characteristic.

Example 9. The characteristic of the logarithm of the number 0.00000014 is

(a) 1
(b) 7
(c) –7
(d) None of these

Sol. (*c*) $0.00000014 = 1.4 \times \frac{1}{10000000}$
$= 1.4 \times 10^{-7}$
$\therefore$ The characteristic of $\log 0.00000014 = -7$

Example 10. The characteristic of the logarithm of the number 8.188 is

(a) 2
(b) 1
(c) –1
(d) 0

Sol. (*d*) $8.188 = 8.188 \times 1 = 8.188 \times 10^0$
$\therefore$ The characteristic of $\log 8.188 = 0$

Example 11. The characteristic of the logarithm of the number 566.37 is

(a) 2
(b) –2
(c) 4
(d) None of these

Sol. (*a*) $566.37 = 5.6637 \times 10^2$
$\therefore$ The characteristic of $\log 566.37 = 2$

Example 12. The characteristic of the logarithm of the number 313 is

(a) 3
(b) 2
(c) 1
(d) 0

Sol. (*b*) $313 = 3.13 \times 10^2$
$\therefore$ The characteristic of $\log 313 = 2$

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Exercise

1. If $\log_3 x = -2$, then the value of x is

(a) $\frac{1}{9}$ (b) $-\frac{1}{9}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

2. The value of $\log_{\sqrt{2}} (32)$ is

(a) 15 (b) 10 (c) 5 (d) 16

3. If $\log_a \sqrt{2} = \frac{1}{6}$, then the value of a is

(a) $(\sqrt{2})^6$ (b) $(6)^{1/2}$ (c) 3 (d) –6

4. Find the logarithm of 1728 to the base $2\sqrt{3}$.

(a) 3.124
(b) 3.1732
(c) 6
(d) 5

5. If $\log_x \frac{9}{16} = -\frac{1}{2}$, then x is

(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{81}{256}$ (d) $\frac{256}{81}$

6. The value of $3\log 3 + 2\log 2$ is

(a) log 108
(b) log 106
(c) log 109
(d) None of these

7. What is the value of
$(\log_{1/2} 2)(\log_{1/3} 3)(\log_{1/4} 4)\ldots(\log_{1/1000} 1000)$?
(CDS 2007 II)

(a) 1 (b) –1 (c) 1 or –1 (d) 0

8. One is the value of

(a) $\log_{10} 2$ (b) $\log_{10} 100$ (c) $\log_{10} 10$ (d) $\log_{10} 1000$

9. The value of $\log_2 [\log_2 \log_2 \log_2 (65536)]$ is

(a) 8
(b) 16
(c) 4
(d) 1

10. $\log_y x$ is equal to

(a) $\frac{x}{\log_e y}$ (b) $x \log_e y$ (c) $\frac{\log_e x}{\log_e y}$ (d) $\frac{\log_e y}{\log_e x}$

11. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is
(a) 23 (b) 15 (c) 21 (d) $13 \log_{10} 100$

12. The value of $\left(\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32\right)$ is
(a) $\frac{4}{3}$ (b) 3 (c) 1 (d) 7

13. What is the value of $\log_{100} 0.1$? **(CDS 2008 II)**
(a) 1/2 (b) −1/2 (c) −2 (d) 2

14. If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$, then the value of x is
(a) 4 (b) 8 (c) 16 (d) 1

15. The value of $\log_y x \cdot \log_z y \cdot \log_x z$ is
(a) $\log xyz$ (b) xyz (c) 1 (d) 0

16. The value of $\log_3 (27 \times \sqrt[4]{9} \times \sqrt[3]{9})$ is
(a) 4 (b) $4\frac{1}{3}$ (c) $8\frac{1}{3}$ (d) $4\frac{1}{6}$

17. The value of $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$ is equal to
(a) xyz (b) 2 (c) 0 (d) 1

18. If $\log_4 x + \log_2 x = 6$, then the value of 'x' is
(a) 16 (b) 4 (c) 2 (d) 1

19. What is the value of
$2 \log (5/8) + \log (128/125) + \log (5/2)$? **(CDS 2009 I)**
(a) 0 (b) 1 (c) 2 (d) 5

20. Given $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is
(a) 0.6990 (b) 0.6919 (c) 0.6119 (d) 0.7525

21. If $\log_a x = m$, the value of $\log_{a^2} x$ is
(a) $-\frac{1}{m}$ (b) m
(c) $\frac{m}{2}$ (d) None of these

22. If $\log \frac{x}{y} + \log \frac{y}{x} = \log (x + y)$, then
(a) $x + y = 1$ (b) $x - y = 0$ (c) $x - y = 1$ (d) $x = y$

23. If $\log_r 6 = m$ and $\log_r 3 = n$, then what is $\log_r (r/2)$ equal to? **(CDS 2009 I)**
(a) $m - n + 1$ (b) $m + n - 1$ (c) $1 - m - n$ (d) $1 - m + n$

24. The characteristic in $\log 6.7482 \times 10^{-5}$ is
(a) 6 (b) −4 (c) 5 (d) −5

25. If $10^x = 1.73$ and $\log_{10} 1730 = 3.2380$, then x equals to
(a) 2.380 (b) 0.2380 (c) 2.2380 (d) 1.380

26. $(\log \tan 1° \log \tan 2° \ldots \log \tan 50°)$ is
(a) 1 (b) −1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

27. The value of $\frac{1}{1 + \log_x (yz)} + \frac{1}{1 + \log_y (xz)} + \frac{1}{1 + \log_z (xy)}$ is
(a) 1 (b) $\frac{1}{xy^2}$ (c) $x = yz$ (d) 0

28. What is the value of $[\log_{13} (10)]/[\log_{169} (10)]$? **(CDS 2009 II)**
(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\log_{10} 13$

29. If $2^{2x+3} = 6^{x-1}$, x equals to
(a) $\frac{4 \log 2 + \log 3}{\log 3 - \log 2}$ (b) $\frac{3 \log 2 + 2 \log 3}{\log 3 - 2 \log 2}$
(c) $\frac{\log 48}{\log 7}$ (d) None of these

30. The value of $10^{\log_{10} m + 2 \log_{10} n + 3 \log_{10} p}$
(a) m^2np^3 (b) mn^2p^3
(c) m^3np^2 (d) None of these

31. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 7 = 0.8491$, then $\log_{10} \frac{108}{\sqrt{7}}$ is
(a) 2.6123 (b) 1.6088 (c) 1.6320 (d) 2.4558

32. What is the value of
$\left(\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1\right)$? **(CDS 2010 I)**
(a) 0 (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

33. Which is not correct?
(a) $\log_{10} (1 + 2 + 3) = \log_{10} (1 \cdot 2 \cdot 3)$
(b) $\log_{10} 1 = 0$
(c) $\log_{10} (2 + 3) = \log_{10} 2 \cdot 3$
(d) $\log_{10} 10 = 1$

34. If a, b and c are three consecutive integers, then $\log (ac + 1)$ is equal to
(a) $\log (2b)$ (b) $(\log b)^2$
(c) $2 \log b$ (d) None of these

35. The solution of equation $\log_7 [\log_4 (x^2)] = 0$ is
(a) $x = 1$ (b) $x = 2$ (c) $x = \pm 2$ (d) $x = -2$

36. What is the value of $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$? **(CDS 2010 II)**
(a) 2 (b) 3 (c) 1 (d) 0

37. The value of $7 \log \frac{10}{9} - 2 \log \frac{25}{24} + 3 \log \frac{81}{80}$ is
(a) 2 (b) $\log 2$ (c) 3 (d) $\log 3$

38. If $\log_r p = 2$, $\log_r q = 3$, then the value of $\log_p q$ is equal to
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 6

39. If $\log x^2y^2 = a$ and $\log \frac{x}{y} = b$, then $\frac{\log x}{\log y}$ is equal to
(a) $\frac{a - 3b}{a + 2b}$ (b) $\frac{a + 3b}{a - 2b}$ (c) $\frac{a + 2b}{a - 3b}$ (d) $\frac{a - 2b}{a + 3b}$

40. $2 \log a + 2 \log a^2 + 2 \log a^3 + 2 \log a^4 + 2 \log a^5$ is equal to
(a) 30 (b) 10 (c) $10 \log a$ (d) $30 \log a$

41. If $\log_{10} 5 = 0.70$, $\log_5 10$ is
(a) 1.35 (b) 1.40 (c) 1.43143 (d) 1.56

42. The value of $\log_3\left(1+\frac{1}{3}\right)+\log_3\left(1+\frac{1}{4}\right)+\log_3\left(1+\frac{1}{5}\right)+\ldots+\log_3\left(1+\frac{1}{24}\right)$
(a) $-1+2\log_3 5$ (b) 2
(c) 3 (d) 4

43. What is the value of $[\log_{10}(5\log_{10}100)]^2$? **(CDS 2011 I)**
(a) 4 (b) 3 (c) 2 (d) 1

44. Consider the following statements
I. $(\log_{10} 0.1)^2 + \log_{10} 10 \cdot \log_{10} 100 = 3$
II. $\log_{10}\log_{10} 10 = 1$
III. $\log_{10}\sqrt{10} + \log_{10}\sqrt{10} = 1$
Of the above statements
(a) I and III are correct (b) II and III are correct
(c) I and II are correct (d) All are correct

45. What is $\log_{10}\left(\frac{3}{2}\right)+\log_{10}\left(\frac{4}{3}\right)+\log_{10}\left(\frac{5}{4}\right)+\ldots$ up to 8 terms equal to? **(CDS 2011 II)**
(a) 0 (b) 1
(c) $\log_{10} 5$ (d) None of these

46. If $\log(x+y) = \log x + \log y$ and $x = 1.1568$, then y is equal to
(a) 7.3776 (b) 7.3776 (c) 5.3776 (d) 5.3116

47. If $\log_8 x + \log_4 x + \log_2 x = 11$. The value of x is
(a) 128 (b) 16 (c) 32 (d) 64

48. If $\log a + \log b = \log(a+b)$, then
(a) $ab = 1$ (b) $b = \frac{a}{a-1}$
(c) $b = \frac{a-1}{a}$ (d) $a = b$

49. If $3^x \times 27^x = 9^{x+4}$, then what is x equal to? **(CDS 2011 I)**
(a) 4 (b) 5 (c) 6 (d) 7

50. What is/are the real value(s) of $(256)^{0.16} \times (16)^{0.18}$ **(CDS 2007 II)**
(a) -4 only (b) 4 only (c) $4, -4$ (d) $2, -2$

51. If $(x)^{1/m} = (y)^{1/n} = (z)^{1/p}$ and $xyz = 1$, then what is the value of $m+n+p$? **(CDS 2007 II)**
(a) 0 (b) 1 (c) 2 (d) -1

52. If $a^x = c^q = b$ and $c^y = a^z = d$, then which one of the following is correct? **(CDS 2009 I)**
(a) $\frac{x}{y} = \frac{q}{z}$ (b) $x + y = q + z$
(c) $xy = qz$ (d) $x^y = q^z$

53. If $2^m + 2^{1+m} = 24$, then what is the value of m? **(CDS 2011 II)**
(a) 0 (b) 1/3 (c) 3 (d) 6

54. If $(ab^{-1})^{2x-1} = (ba^{-1})^{x-2}$, then what is the value of x? **(CDS 2008 I)**
(a) 1 (b) 2 (c) 3 (d) 4

55. If $y = (a^x)^{(a^x)\ldots\infty}$, then which one of the following is correct? **(CDS 2008 II)**
(a) $\log y = xy \log a$ (b) $\log y = x + y \log a$
(c) $\log y = y + x \log a$ (d) $\log y = (y+x)\log a$

56. If $p^x = r^y = m$ and $r^w = p^z = n$, then which one of the following is correct? **(CDS 2010 II)**
(a) $xw = yz$ (b) $xz = yw$
(c) $x + y = w + z$ (d) $x - y = w - z$

57. If $a^x = b^y = c^z$ and $abc = 1$, then what is $xy + yz + zx$ equal to **(CDS 2009 II)**
(a) xyz (b) $x + y + z$
(c) 0 (d) 1

58. A ball is dropped from a height 64 m above the ground and every time it hits the ground it rises to a height equal to half of the previous. What is the height attained after it hits the ground for the 16th time? **(CDS 2009 I)**
(a) 2^{-12} m (b) 2^{-11} m (c) 2^{-10} m (d) 2^{-9} m

Answers

1. (a)	2. (b)	3. (a)	4. (c)	5. (d)	6. (a)	7. (b)	8. (c)	9. (d)	10. (c)
11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (d)	17. (b)	18. (a)	19. (a)	20. (a)
21. (c)	22. (a)	23. (d)	24. (d)	25. (b)	26. (c)	27. (a)	28. (b)	29. (a)	30. (b)
31. (b)	32. (c)	33. (c)	34. (c)	35. (c)	36. (c)	37. (b)	38. (c)	39. (c)	40. (d)
41. (c)	42. (a)	43. (d)	44. (a)	45. (c)	46. (a)	47. (d)	48. (b)	49. (a)	50. (b)
51. (a)	52. (c)	53. (c)	54. (a)	55. (a)	56. (a)	57. (c)	58. (c)		

Hints and Solutions

1. $\log_3 x = -2 \Rightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2. Let $\log_{\sqrt{2}} 32 = x \Rightarrow 32 = (\sqrt{2})^x$

$$(2)^5 = (2)^{x/2}$$

On comparing, $5 = \frac{x}{2} \Rightarrow x = 10$

3. $\because \log_a \sqrt{2} = \frac{1}{6}$

$\therefore \quad a^{1/6} = \sqrt{2} \Rightarrow a = (\sqrt{2})^6$

4. Let $\quad \log_{2\sqrt{3}} 1728 = x \Rightarrow (2\sqrt{3})^x = 1728$

$\because \quad 1728 = 2^6(\sqrt{3})^6 = (2\sqrt{3})^6$

$$(2\sqrt{3})^x = (2\sqrt{3})^6$$

On comparing $\Rightarrow x = 6$

5. $\log_x \frac{9}{16} = -\frac{1}{2}$

Hence, $\quad x^{-1/2} = \frac{9}{16}$ (by definition)

$\Rightarrow \quad x^{-1} = \left(\frac{9}{16}\right)^2 = \frac{81}{256} \Rightarrow x = \frac{256}{81}$

6. $3\log 3 + 2\log 2 = \log 3^3 + \log 2^2 = \log 27 + \log 4$

$$= \log(27 \times 4) = \log 108$$

7. $(\log_{1/2} 2)(\log_{1/3} 3)(\log_{1/4} 4)\ldots(\log_{1/1000} 1000)$

$$= \left(\frac{\log 2}{\log 1/2}\right)\left(\frac{\log 3}{\log 1/3}\right)\left(\frac{\log 4}{\log 1/4}\right)\cdots\left(\frac{\log 1000}{\log 1/1000}\right)$$

$$\left(\because \log_b a = \frac{\log a}{\log b}\right)$$

$$= \left(\frac{\log 2}{-\log 2}\right)\left(\frac{\log 3}{-\log 3}\right)\left(\frac{\log 4}{-\log 4}\right)\cdots\left(\frac{\log 1000}{-\log 1000}\right)$$

$$= (-1)\times(-1)\times(-1)\times\ldots\times(-1)$$

($\because$ number of terms is odd)

$$= -1$$

8. $\because \quad \log_x x = 1 \Rightarrow \log_{10} 10 = 1$ (by rule)

9. $\log_2 \log_2 \log_2 \log_2 2^{16} = \log_2 \log_2 \log_2 (16)$

$$= \log_2 \log_2 \log_2 (2^4) = \log_2 \log_2 (4)$$

$$= \log_2 \log_2 (2^2) = \log_2 (2) = 1$$

10. $\log_y x = \frac{\log_e x}{\log_e y}$

11. $\because \log_x x^n = n$, so

$$\log_{10} 10 + \log_{10} 10^2 + \log_{10} 10^3 + \log_{10} 10^4 + \log_{10} 10^5$$

$$= 1 + 2 + 3 + 4 + 5 = 15$$

12. $\because \log_{10} (125)^{1/3} - \log_{10} 4^2 + \log_{10} 2^5$

$$= \log_{10} 5 - \log_{10} 2^4 + 5\log_{10} 2$$

$$= \log_{10} \frac{10}{2} - 4\log_{10} 2 + 5\log_{10} 2$$

$$= \log_{10} 10 - \log_{10} 2 - 4\log_{10} 2 + 5\log_{10} 2 = 1$$

13. $\log_{100} 0.1 = \log_{10^2}\left(\frac{1}{10}\right) = \frac{1}{2}\log_{10}\left(\frac{1}{10}\right)$

$$= \frac{1}{2}\log_{10} (10)^{-1} = -\frac{1}{2}\log_{10} 10 = -\frac{1}{2}$$

14. $\log_4(x^2 + x) - \log_4(x + 1) = 2 \Rightarrow \log_4\left(\frac{x^2 + x}{x + 1}\right) = 2$

$\Rightarrow \quad 4^2 = \frac{x^2 + x}{x + 1} \Rightarrow 16x + 16 = x^2 + x$

$\Rightarrow \quad x^2 - 15x - 16 = 0$

$\Rightarrow \quad x = 16$ or $x = -1$ (not possible)

15. $\frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x} = 1$

16. Let $\log_3 (27 \times \sqrt[4]{9} \times \sqrt[3]{9}) = x$

$\therefore \quad 3^x = 27 \times \sqrt[4]{9} \times \sqrt[3]{9} = 3^3 \times 3^{2/4} \times 3^{2/3}$

$\Rightarrow \quad 3^x = 3^{25/6} \Rightarrow x = \frac{25}{6} = 4\frac{1}{6}$

17. The given expression is

$$= \log_{xyz} (xy) + \log_{xyz} (yz) + \log_{xyz} (zx) = \log_{xyz} (xy\,yz\,zx)$$

$$= \log_{xyz} (xyz)^2 = 2\log_{xyz} xyz = 2$$

18. $\log_4 x + \log_2 x = 6$ or $\frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$

$$\frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 6 \Rightarrow 3\log x = 12\log 2$$

$\Rightarrow \quad \log x = 4\log 2$

$\Rightarrow \quad \log x = \log 16 \Rightarrow x = 16$

19. $2\log\left(\frac{5}{8}\right) + \log\left(\frac{128}{125}\right) + \log\left(\frac{5}{2}\right)$

$$= 2\log 5 - 2\log 8 + \log 128 - \log 125 + \log 5 - \log 2$$

$$- 2\log 5 - 2\log 2^3 + \log 2^7 - \log 5^3 + \log 5 - \log 2$$

$$= (2\log 5 - 6\log 2) + (7\log 2 - 3\log 5) + \log 5 - \log 2$$

$$= 3\log 5 - 3\log 5 - 7\log 2 + 7\log 2 = 0$$

20. $\log_{10} 5 = \log_{10} \frac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - 0.3010 = 0.6990$

21. $\because \log_a x = m \Rightarrow x = a^m \Rightarrow x = (a^2)^{m/2}$

$\therefore \quad \log_{a^2} x = \frac{m}{2}$

22. Here, $\log\frac{x}{y} + \log\frac{y}{x} = \log(x + y)$

$\Rightarrow \quad \log\frac{x}{y}\cdot\frac{y}{x} = \log(x + y) \Rightarrow \log 1 = \log(x + y)$

$\Rightarrow \quad x + y = 1$

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23. Given, $\log_r 6 = m$ and $\log_r 3 = n$

$\because \quad \log_r 6 = \log_r (2\times 3) = \log_r 2 + \log_r 3$

$\therefore \quad \log_r 3 + \log_r 2 = m$

$\Rightarrow \quad n + \log_r 2 = m \Rightarrow \log_r 2 = m - n$

$\therefore \quad \log_r\left(\frac{r}{2}\right) = \log_r r - \log_r 2 = 1 - m + n$

24. The characteristic in $\log 6.7482 \times 10^{-5}$. is -5.

25. $10^x = 1.73$

$x = \log_{10} 1.73 = \log_{10} 1730 - \log_{10} 1000 = 3.2380 - 3 = 0.2380$

26. $(\log\tan 1°)(\log\tan 2°)(\log\tan 3°)$

$\ldots(\log\tan 45°)(\log\tan 46°)\ldots(\log\tan 50°)$

$= [(\log\tan 1°)(\log\tan 2°)\ldots$

$\log(\tan 44°)\log(\tan 46°)..\log\tan 50°]\log\tan 45°$

$= [(\log\tan 1°)\ldots(\log\tan 44°)(\log\tan 46°)$

$\ldots\log\tan 50°] \times 0$

$= 0 \qquad (\because \log\tan 45° = \log 1 = 0)$

27. The given expression is

$$= \frac{1}{\log_x (yz) + \log_x x} + \frac{1}{\log_y (xz) + \log_y y} + \frac{1}{\log_z (xy) + \log_z z}$$

$$= \frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz}$$

$$= \log_{xyz} x + \log_{xyz} y + \log_{xyz} z = \log_{xyz} xyz = 1$$

28. $\dfrac{\log_{13}(10)}{\log_{169}(10)} = \dfrac{\log_{13}(10)}{\log_{13^2}(10)} \qquad \left(\because \log_{a^b} c = \dfrac{1}{b}\log_a c\right)$

$$= \frac{\log_{13} 10}{\frac{1}{2}\log_{13} 10} = \frac{1}{1/2} = 2$$

29. $2^{2x+3} = 6^{x-1}$

$(2x+3)\log 2 = (x-1)\log 6$

$2x \log 2 + 3\log 2 = (x-1)(\log 2 + \log 3)$

$2x\log 2 + 3\log 2 = x(\log 2 + \log 3) - \log 2 - \log 3$

or $\quad x(\log 2 - \log 3) = -4\log 2 - \log 3$

$$x = \frac{4\log 2 + \log 3}{\log 3 - \log 2}$$

30. $10^{\log_{10} m + 2\log_{10} n + 3\log_{10} p} \Rightarrow 10^{\log_{10} m + \log_{10} n^2 + \log_{10} p^3}$

$\Rightarrow \quad 10^{\log_{10} mn^2p^3} = mn^2p^3 \qquad (\because a^{\log_a p} = p)$

31. $\log_{10}\dfrac{108}{\sqrt{7}} = \log_{10} 108 - \log_{10}\sqrt{7}$

$= \log_{10} 2^2 \times 3^3 - \log_{10} 7^{1/2}$

$= 2\log_{10} 2 + 3\log_{10} 3 - \frac{1}{2}\log_{10} 7$

$= 2\times(0.3010) + 3(0.4771) - \frac{1}{2}(0.8491)$

$= 0.6020 + 1.4313 - 0.4245 = 1.6088$

32. $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1$

$= \frac{1}{3}\log_{10}(5)^3 - 2\log_{10}(2)^2 + \log_{10}(2)^5 + 0$

$= \log_{10} 5 - 4\log_{10} 2 + 5\log_{10} 2$

$= \log_{10} 5 + \log_{10} 2 = \log_{10} 10 = 1$

33. (a) $\log_{10}(1+2+3) = \log_{10}(1\cdot 2\cdot 3)$

(b) $\log_{10} 1 = 0$ (c) $\log_{10} 10 = 1$ are true.

(d) $\log_{10}(2+3) = \log_{10} 5 \neq \log_{10} 6 = \log_{10}(2\cdot 3)$ is false.

34. Let $a = a$

Then, $\quad b = a + 1$ and $c = a + 2$

$\therefore \quad ac + 1 = a(a+2) + 1 = a^2 + 2a + 1 = (a+1)^2$

$ac + 1 = b^2$

$\log(ac+1) = \log b^2 = 2\log b$

35. $\log_7[\log_4(x^2)] = 0 = \log_7 1$

$\therefore \quad \log_4(x^2) = 1 = \log_4 4$

$x^2 = 4 \Rightarrow x = \pm 2$

36. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18 = \log_{10} 5 - \log_{10} 9 + \log_{10} 18$

$$= \log_{10}\frac{5\times 18}{9} = \log_{10} 10 = 1$$

37. $7\log\dfrac{10}{9} - 2\log\dfrac{25}{24} + 3\log\dfrac{81}{80}$

$= 7[\log 10 - \log 9] - 2[\log 25 - \log 24] + 3[\log 81 - \log 80]$

$= 7[\log 2 + \log 5 - 2\log 3] - 2[2\log 5 - \log 3 - 3\log 2]$

$+ 3[4\log 3 - 4\log 2 - \log 5]$

$= \log 2$

38. Here, $\log_r p = 2, \log_r q = 3$

By relation, $\log_p q = \dfrac{\log_r q}{\log_r p} = \dfrac{3}{2}$

39. $\log x^2y^2 = a \Rightarrow 2\log x + 2\log y = a$ ...(i)

$\log x - \log y = b$ ...(ii)

Solving $\quad \log x = \dfrac{a+2b}{4}$ and $\log y = \dfrac{a-3b}{4}$

$\therefore \quad \dfrac{\log x}{\log y} = \dfrac{a+2b}{a-3b}$

40. $2\log a + 2\log a^2 + 2\log a^3 + 2\log a^4 + 2\log a^5$

$= 2\log a + 4\log a + 6\log a + 8\log a + 10\log a = 30\log a$

41. $\log_5 10 = \dfrac{\log_{10} 10}{\log_{10} 5} = \dfrac{1}{0.70} = 1.43$

42. $\log_3\left(1+\frac{1}{3}\right) + \log_3\left(1+\frac{1}{4}\right) + \log_3\left(1+\frac{1}{5}\right) + \ldots + \log_3\left(1+\frac{1}{24}\right)$

$= \log_3\frac{4}{3} + \log_3\frac{5}{4} + \log_3\frac{6}{5} + \ldots + \log_3\frac{25}{24}$

$= \log_3 4 - \log_3 3 + \log_3 5 - \log_3 4 + \log_3 6 - \log_3 5$

$+ \ldots + \log_3 25 - \log_3 24$

$= -\log_3 3 + \log_3 25 = -1 + 2\log_3 5$

43. $[\log_{10}(5\log_{10}100)]^2 = [\log_{10}(5\log_{10}10^2)]^2 = [\log_{10}(10\log_{10}10)]^2$

$$= [\log_{10}10]^2 \qquad (\because \log_{10}10 = 1)$$

$$= 1^2 = 1$$

44. II. $\log_{10}\log_{10}10 = \log_{10}(1) = 0 \neq 1$

I. $[\log_{10}(0.1)]^2 + \log_{10}10 \cdot \log_{10}100$

$= [-\log_{10}10]^2 + \log_{10}10 \cdot \log_{10}10^2 = (-1)^2 + (1)\cdot 2\log_{10}10$

$= +1 + (1)\cdot 2 = +1 + 2 = 3$

III. $\log_{10}\sqrt{10} + \log_{10}\sqrt{10} = \frac{1}{2}\log_{10}10 + \frac{1}{2}\log_{10}10 = \frac{1}{2} + \frac{1}{2} = 1$

45. $\log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots$ 8th term

$\because \quad T_n = \log_{10}\left(\frac{n+2}{n+1}\right) \Rightarrow T_8 = \log_{10}\left(\frac{10}{9}\right)$

$= \log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots + \log_{10}\left(\frac{10}{9}\right)$

$= \log_{10}\left(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{10}{9}\right) = \log_{10}\left(\frac{10}{2}\right) = \log_{10}5$

46. $\log(x+y) = \log x + \log y$

$\log(x+y) = \log xy$

$(x+y) = xy$

or $\quad y = \frac{x}{x-1} = \frac{1.1568}{1.1568-1} = \frac{1.1568}{0.1568} = 7.37755 = 7.3776$

47. $\log_8 x + \log_4 x + \log_2 x = 11$

$\Leftrightarrow \quad \frac{\log x}{\log 8} + \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 11$

$\Leftrightarrow \quad \frac{\log x}{3\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 11$

$\Leftrightarrow \quad \frac{11\log x}{6\log 2} = 11 \Leftrightarrow \log x = 6\log 2$

$\Leftrightarrow \quad x = 2^6$

$x = 64$

48. Here, $\quad \log ab = \log(a+b)$

$\Rightarrow \quad ab = a + b \Rightarrow b = \frac{a}{(a-1)}$

49. Given, $3^x \times 27^x = 9^{x+4}$

$\therefore \quad 3^x \times 3^{3x} = 3^{2(x+4)} \Rightarrow 3^{x+3x} = 3^{2(x+4)}$

On comparing, $\quad x + 3x = 2(x+4)$

$\Rightarrow \quad 4x - 2x = 8 \Rightarrow x = \frac{8}{2} = 4$

50. $(256)^{0.16} \times (16)^{0.18} = [(16)^2]^{0.16} \times (16)^{0.18}$

$= (16)^{0.32} \times (16)^{0.18} = 16^{0.5} = [4^2]^{0.5} = 4^1 = 4$

51. Given, $(x)^{1/m} = (y)^{1/n} = (z)^{1/p} = k \quad$ (say)

$\Rightarrow \quad x = k^m, y = k^n, z = k^p$

$\therefore \quad xyz = k^{m+n+p}$

$\Rightarrow \quad 1 = k^{m+n+p} = k^0 \qquad (\because xyz = 1,$ given)

On comparing,

$$m + n + p = 0$$

52. Given, $a^x = c^q = b$ and $c^y = a^z = d \Rightarrow a = b^{1/x}$ and $c = b^{1/q}$

$\because \quad c^y = a^z = d$

$\therefore \quad b^{y/q} = b^{z/x} = d \Rightarrow \frac{y}{q} = \frac{z}{x} \quad$ (on comparing)

$\Rightarrow \quad xy = zq$

53. Given, $\quad 2^m + 2^{1+m} = 24 \therefore 2^m(1+2) = 24$

$\Rightarrow \quad 2^m \times 3 = 24 \Rightarrow 2^m = 8 = 2^3$

$\Rightarrow \quad m = 3$

54. Given, $(ab^{-1})^{2x-1} = (ba^{-1})^{x-2} \Rightarrow \left(\frac{a}{b}\right)^{2x-1} = \left(\frac{b}{a}\right)^{x-2}$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{2x-1}\left(\frac{a}{b}\right)^{x-2} = 1 \Rightarrow \left(\frac{a}{b}\right)^{2x-1+x-2} = \left(\frac{a}{b}\right)^0$

On comparing,

$$3x - 3 = 0 \Rightarrow x = 1$$

55. Given, $y = (a^x)^{(a^x)\cdots\infty}$

$\therefore \quad y = (a^x)^y$

On taking log both sides, we get

$$\log y = y\log a^x \Rightarrow \log y = xy\log a$$

56. Given, $\quad p^x = r^y$

$\Rightarrow \quad r = p^{x/y} \quad$ ...(i)

and $\quad p^z = r^w$

$\Rightarrow \quad r = p^{z/w} \quad$ ...(ii)

From Eqs. (i) and (ii)

$$p^{x/y} = p^{z/w} \Rightarrow \frac{x}{y} = \frac{z}{w} \Rightarrow xw = yz$$

57. Given, $\quad a^x = b^y = c^z = k \quad$ (say)

$\Rightarrow \quad a = k^{1/x}$

$b = k^{1/y}$

and $\quad c = k^{1/z}$

$\therefore \quad abc = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} \Rightarrow 1 = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{k}} = k^0 \quad (\because abc = 1,$ given)

On comparing,

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0 \Rightarrow xy + yz + zx = 0$$

58. After Ist hit ball height will be $= \frac{1}{2}(64)$

After IInd hit ball height will be $= \left(\frac{1}{2}\right)^2(64)$

....

....

After 16th hit ball height will be $= \left(\frac{1}{2}\right)^{16}(64) = \frac{1}{2^{16}}(2$ [illegible] m

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