

Chapter : 1. RELATION

Exercise : 1A

Question: 1

Find the domain a

Solution:

$\text{dom}(R) = \{-1, 1, -2, 2\}$ and $\text{range}(R) = \{1, 4\}$

Question: 2

Let $R = \{(a, a$

Solution:

$\text{range}(R) = \{8, 27\}$

Question: 3

Let $R = \{(a, a$

Solution:

(i) $R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

(ii) $\text{dom}(R) = \{2, 3, 5, 7\}$

(iii) $\text{range}(R) = \{8, 27, 125, 343\}$

Question: 4

Let $R = (x, y) :$

Solution:

$\{3, 2, 1\}$

Question: 5

Let $R = \{(a, b) : a$

Solution:

$\text{dom}(R) = \{3, 6, 9\}$ and $\text{range}(R) = \{3, 2, 1\}$

Question: 6

Let $R = \{(a, b) :$

Solution:

$\text{dom}(R) = \{-2, -1, 0, 1, 2\}$ and $\text{range}(R) = \{3, 2, 1, 0\}$

Question: 7

Let $(R) = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}$

Question: 8

Let $R = \{(a, b) :$

Solution:

$\text{dom}(R) = \{1, 2, 3\}$ and $\text{range}(R) = \{6, 7, 8\}$

Question: 9

Let S be the set

Solution:

Let $R = \{(A, B) : A \subset B\}$, i.e., A is a proper subset of B , be a relation defined on S .

Now,

Any set is a subset of itself, but not a proper subset.

$$\Rightarrow (A, A) \notin R \quad \forall A \in S$$

$\Rightarrow R$ is not reflexive.

Let $(A, B) \in R \quad \forall A, B \in S$

$\Rightarrow A$ is a proper subset of B

\Rightarrow all elements of A are in B , but B contains at least one element that is not in A .

$\Rightarrow B$ cannot be a proper subset of A

$$\Rightarrow (B, A) \notin R$$

For e.g., if $B = \{1, 2, 5\}$ then $A = \{1, 5\}$ is a proper subset of B . we observe that B is not a proper subset of A .

$\Rightarrow R$ is not symmetric

Let $(A, B) \in R$ and $(B, C) \in R \quad \forall A, B, C \in S$

$\Rightarrow A$ is a proper subset of B and B is a proper subset of C

$\Rightarrow A$ is a proper subset of C

$$\Rightarrow (A, C) \in R$$

For e.g., if $B = \{1, 2, 5\}$ then $A = \{1, 5\}$ is a proper subset of B .

And if $C = \{1, 2, 5, 7\}$ then $B = \{1, 2, 5\}$ is a proper subset of C .

We observe that $A = \{1, 5\}$ is a proper subset of C also.

$\Rightarrow R$ is transitive.

Thus, R is transitive but not reflexive and not symmetric.

Question: 10

Let A be the set

Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, A be the set of all points in a plane and O be the origin. Then, $R = \{(P, Q) : P, Q \in A \text{ and } OP = OQ\}$

Now,

R is Reflexive if $(P, P) \in R \quad \forall P \in A$

$\forall P \in A$, we have

$$OP = OP$$

$$\Rightarrow (P, P) \in R$$

Thus, R is reflexive.

R is Symmetric if $(P, Q) \in R \Rightarrow (Q, P) \in R \quad \forall P, Q \in A$

Let $P, Q \in A$ such that,

$$(P, Q) \in R$$

$$\Rightarrow OP = OQ$$

$$\Rightarrow OQ = OP$$

$$\Rightarrow (Q,P) \in R$$

Thus, R is symmetric.

R is Transitive if $(P,Q) \in R$ and $(Q,S) \in R \Rightarrow (P,S) \in R \forall P, Q, S \in A$

Let $(P,Q) \in R$ and $(Q,S) \in R \forall P, Q, S \in A$

$$\Rightarrow OP = OQ \text{ and } OQ = OS$$

$$\Rightarrow OP = OS$$

$$\Rightarrow (P,S) \in R$$

Thus, R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation on A.

Question: 11

On the set S of a

Solution:

Let $R = \{(a, b) : a \leq b\}$ be a relation defined on S.

Now,

We observe that any element $x \in S$ is less than or equal to itself.

$$\Rightarrow (x,x) \in R \forall x \in S$$

$\Rightarrow R$ is reflexive.

Let $(x,y) \in R \forall x, y \in S$

$\Rightarrow x$ is less than or equal to y

But y cannot be less than or equal to x if x is less than or equal to y .

$$\Rightarrow (y,x) \notin R$$

For e.g. , we observe that $(2,5) \in R$ i.e. $2 < 5$ but 5 is not less than or equal to 2 $\Rightarrow (5,2) \notin R$

$\Rightarrow R$ is not symmetric

Let $(x,y) \in R$ and $(y,z) \in R \forall x, y, z \in S$

$$\Rightarrow x \leq y \text{ and } y \leq z$$

$$\Rightarrow x \leq z$$

$$\Rightarrow (x,z) \in R$$

For e.g. , we observe that

$$(4,5) \in R \Rightarrow 4 \leq 5 \text{ and } (5,6) \in R \Rightarrow 5 \leq 6$$

And we know that $4 \leq 6 \therefore (4,6) \in R$

$\Rightarrow R$ is transitive.

Thus, R is reflexive and transitive but not symmetric.

Question: 12

Let $A = \{1, 2, 3,$

Solution:

Given that,

$A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(a, b) : a, b \in A \text{ and } b = a + 1\}$.

$$\therefore R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$$

Now,

R is Reflexive if $(a,a) \in R \forall a \in A$

Since, $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \notin R$

Thus, R is not reflexive .

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in A$

We observe that $(1,2) \in R$ but $(2,1) \notin R$.

Thus, R is not symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in A$

We observe that $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$

Thus, R is not transitive.

Exercise : 1B

Question: 1

Define a relation

Solution:

Relation: Let A and B be two sets. Then a relation R from set A to set B is a subset of $A \times B$. Thus, R is a relation to A to B $\Leftrightarrow R \subseteq A \times B$.

If R is a relation from a non-void set B and if $(a,b) \in R$, then we write $a R b$ which is read as 'a is related to b by the relation R'. if $(a,b) \notin R$, then we write $a \not R b$, and we say that a is not related to b by the relation R.

Domain: Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R.

Thus, domain of $R = \{a : (a,b) \in R\}$. The domain of $R \subseteq A$.

Range: let R be a relation from a set A to a set B. then the set of all second component or coordinates of the ordered pairs belonging to R is called the range of R.

Example 1: $R = \{(-1, 1), (1, 1), (-2, 4), (2, 4)\}$.

$\text{dom}(R) = \{-1, 1, -2, 2\}$ and $\text{range}(R) = \{1, 4\}$

Example 2: $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a + 3b = 12\}$

$\text{dom}(R) = \{3, 6, 9\}$ and $\text{range}(R) = \{3, 2, 1\}$

Question: 2

Let A be the set

Solution:

Let $R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$ be a relation defined on A.

Now,

R is Reflexive if $(\Delta, \Delta) \in R \forall \Delta \in A$

We observe that for each $\Delta \in A$ we have,

$\Delta \sim \Delta$ since, every triangle is similar to itself.

$\Rightarrow (\Delta, \Delta) \in R \forall \Delta \in A$

$\Rightarrow R$ is reflexive.

R is Symmetric if $(\Delta_1, \Delta_2) \in R \Rightarrow (\Delta_2, \Delta_1) \in R \forall \Delta_1, \Delta_2 \in A$

Let $(\Delta_1, \Delta_2) \in R \forall \Delta_1, \Delta_2 \in A$

$$\Rightarrow \Delta_1 \sim \Delta_2$$

$$\Rightarrow \Delta_2 \sim \Delta_1$$

$$\Rightarrow (\Delta_2, \Delta_1) \in R$$

$\Rightarrow R$ is symmetric

R is Transitive if $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R \Rightarrow (\Delta_1, \Delta_3) \in R \forall \Delta_1, \Delta_2, \Delta_3 \in A$

Let $(\Delta_1, \Delta_2) \in R$ and $((\Delta_2, \Delta_3) \in R \forall \Delta_1, \Delta_2, \Delta_3 \in A$

$$\Rightarrow \Delta_1 \sim \Delta_2 \text{ and } \Delta_2 \sim \Delta_3$$

$$\Rightarrow \Delta_1 \sim \Delta_3$$

$$\Rightarrow (\Delta_1, \Delta_3) \in R$$

$\Rightarrow R$ is transitive.

Since R is reflexive, symmetric and transitive, it is an equivalence relation on A .

Question: 3

Let $R = \{(a, b) :$

Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, $\forall a, b \in \mathbb{Z}, R = \{(a, b) : (a + b) \text{ is even} \}$.

Now,

R is Reflexive if $(a, a) \in R \forall a \in \mathbb{Z}$

For any $a \in A$, we have

$$a+a = 2a, \text{ which is even.}$$

$$\Rightarrow (a, a) \in R$$

Thus, R is reflexive.

R is Symmetric if $(a, b) \in R \Rightarrow (b, a) \in R \forall a, b \in \mathbb{Z}$

$$(a, b) \in R$$

$$\Rightarrow a+b \text{ is even.}$$

$$\Rightarrow b+a \text{ is even.}$$

$$\Rightarrow (b, a) \in R$$

Thus, R is symmetric .

R is Transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \forall a, b, c \in \mathbb{Z}$

Let $(a, b) \in R$ and $(b, c) \in R \forall a, b, c \in \mathbb{Z}$

$$\Rightarrow a+b = 2P \text{ and } b+c = 2Q$$

Adding both, we get

$$a+c+2b = 2(P+Q)$$

$$\Rightarrow a+c = 2(P+Q)-2b$$

$$\Rightarrow a+c \text{ is an even number}$$

$$\Rightarrow (a, c) \in R$$

Thus, R is transitive on \mathbb{Z} .

Since R is reflexive, symmetric and transitive it is an equivalence relation on \mathbb{Z} .

Question: 4

Let $R = \{(a, b) :$

Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, $\forall a, b \in \mathbb{Z}$, aRb if and only if $a - b$ is divisible by 5.

Now,

R is Reflexive if $(a,a) \in R \forall a \in \mathbb{Z}$

$aRa \Rightarrow (a-a)$ is divisible by 5.

$a-a = 0 = 0 \times 5$ [since 0 is multiple of 5 it is divisible by 5]

$\Rightarrow a-a$ is divisible by 5

$\Rightarrow (a,a) \in R$

Thus, R is reflexive on \mathbb{Z} .

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in \mathbb{Z}$

$(a,b) \in R \Rightarrow (a-b)$ is divisible by 5

$\Rightarrow (a-b) = 5z$ for some $z \in \mathbb{Z}$

$\Rightarrow -(b-a) = 5z$

$\Rightarrow b-a = 5(-z)$ [$\because z \in \mathbb{Z} \Rightarrow -z \in \mathbb{Z}$]

$\Rightarrow (b-a)$ is divisible by 5

$\Rightarrow (b,a) \in R$

Thus, R is symmetric on \mathbb{Z} .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in \mathbb{Z}$

$(a,b) \in R \Rightarrow (a-b)$ is divisible by 5

$\Rightarrow a-b = 5z_1$ for some $z_1 \in \mathbb{Z}$

$(b,c) \in R \Rightarrow (b-c)$ is divisible by 5

$\Rightarrow b-c = 5z_2$ for some $z_2 \in \mathbb{Z}$

Now,

$a-b = 5z_1$ and $b-c = 5z_2$

$\Rightarrow (a-b) + (b-c) = 5z_1 + 5z_2$

$\Rightarrow a-c = 5(z_1 + z_2) = 5z_3$ where $z_1 + z_2 = z_3$

$\Rightarrow a-c = 5z_3$ [$\because z_1, z_2 \in \mathbb{Z} \Rightarrow z_3 \in \mathbb{Z}$]

$\Rightarrow (a-c)$ is divisible by 5.

$\Rightarrow (a, c) \in R$

Thus, R is transitive on \mathbb{Z} .

Since R is reflexive, symmetric and transitive it is an equivalence relation on \mathbb{Z} .

Question: 5

Show that the rel

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and

Transitive.

Given that, $\forall a, b \in A, R = \{(a, b) : |a - b| \text{ is even}\}$.

Now,

R is Reflexive if $(a,a) \in R \forall a \in A$

For any $a \in A$, we have

$|a-a| = 0$, which is even.

$\Rightarrow (a,a) \in R$

Thus, R is reflexive.

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in A$

$(a,b) \in R$

$\Rightarrow |a-b|$ is even.

$\Rightarrow |b-a|$ is even.

$\Rightarrow (b,a) \in R$

Thus, R is symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in A$

Let $(a,b) \in R$ and $(b,c) \in R \forall a, b, c \in A$

$\Rightarrow |a - b|$ is even and $|b - c|$ is even

\Rightarrow (a and b both are even or both odd) and (b and c both are even or both odd)

Now two cases arise:

Case 1 : when b is even

Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow |a - b|$ is even and $|b - c|$ is even

\Rightarrow a is even and c is even [\because b is even]

$\Rightarrow |a - c|$ is even [\because difference of any two even natural numbers is even]

$\Rightarrow (a, c) \in R$

Case 2 : when b is odd

Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow |a - b|$ is even and $|b - c|$ is even

\Rightarrow a is odd and c is odd [\because b is odd]

$\Rightarrow |a - c|$ is even [\because difference of any two odd natural numbers is even]

$\Rightarrow (a, c) \in R$

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

Question: 6

Show that the rel

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, R be the relation in $N \times N$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in

$\mathbb{N} \times \mathbb{N}$.

R is Reflexive if $(a, b) R (a, b)$ for (a, b) in $\mathbb{N} \times \mathbb{N}$

Let $(a, b) R (a, b)$

$$\Rightarrow a+b = b+a$$

which is true since addition is commutative on \mathbb{N} .

$\Rightarrow R$ is reflexive.

R is Symmetric if $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for $(a, b), (c, d)$ in $\mathbb{N} \times \mathbb{N}$

Let $(a, b) R (c, d)$

$$\Rightarrow a+d = b+c$$

$$\Rightarrow b+c = a+d$$

$$\Rightarrow c+b = d+a \text{ [since addition is commutative on } \mathbb{N}]$$

$$\Rightarrow (c, d) R (a, b)$$

$\Rightarrow R$ is symmetric.

R is Transitive if $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$ for $(a, b), (c, d), (e, f)$ in $\mathbb{N} \times \mathbb{N}$

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$\Rightarrow a+d = b+c \text{ and } c+f = d+e$$

$$\Rightarrow (a+d) - (d+e) = (b+c) - (c+f)$$

$$\Rightarrow a-e = b-f$$

$$\Rightarrow a+f = b+e$$

$$\Rightarrow (a, b) R (e, f)$$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

Question: 7

Let S be the set

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, $\forall a, b \in S, R = \{(a, b) : a = \pm b\}$

Now,

R is Reflexive if $(a, a) \in R \forall a \in S$

For any $a \in S$, we have

$$a = \pm a$$

$$\Rightarrow (a, a) \in R$$

Thus, R is reflexive.

R is Symmetric if $(a, b) \in R \Rightarrow (b, a) \in R \forall a, b \in S$

$$(a, b) \in R$$

$$\Rightarrow a = \pm b$$

$$\Rightarrow b = \pm a$$

$$\Rightarrow (b, a) \in R$$

Thus, R is symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in S$

Let $(a,b) \in R$ and $(b,c) \in R \forall a, b, c \in S$

$$\Rightarrow a = \pm b \text{ and } b = \pm c$$

$$\Rightarrow a = \pm c$$

$$\Rightarrow (a, c) \in R$$

Thus, R is transitive.

Hence, R is an equivalence relation.

Question: 8

Let S be the set

Solution:

Given that, $\forall A, B \in S, R = \{(A, B) : d(A, B) < 2 \text{ units}\}$.

Now,

R is Reflexive if $(A,A) \in R \forall A \in S$

For any $A \in S$, we have

$d(A,A) = 0$, which is less than 2 units

$$\Rightarrow (A,A) \in R$$

Thus, R is reflexive.

R is Symmetric if $(A, B) \in R \Rightarrow (B,A) \in R \forall A,B \in S$

$$(A, B) \in R$$

$$\Rightarrow d(A, B) < 2 \text{ units}$$

$$\Rightarrow d(B, A) < 2 \text{ units}$$

$$\Rightarrow (B,A) \in R$$

Thus, R is symmetric .

R is Transitive if $(A, B) \in R$ and $(B,C) \in R \Rightarrow (A,C) \in R \forall A,B,C \in S$

Consider points $A(0,0), B(1.5,0)$ and $C(3.2,0)$.

$$d(A,B)=1.5 \text{ units} < 2 \text{ units and } d(B,C)=1.7 \text{ units} < 2 \text{ units}$$

$$d(A,C)= 3.2 \not< 2$$

$$\Rightarrow (A, B) \in R \text{ and } (B,C) \in R \Rightarrow (A,C) \notin R$$

Thus, R is not transitive.

Thus, R is reflexive, symmetric but not transitive.

Question: 9

Let S be the set

Solution:

Given that, $\forall a, b \in S, R = \{(a, b) : a^2 + b^2 = 1 \}$

Now,

R is Reflexive if $(a,a) \in R \forall a \in S$

For any $a \in S$, we have

$$a^2+a^2 = 2 a^2 \neq 1$$

$$\Rightarrow (a,a) \notin R$$

Thus, R is not reflexive.

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in S$

$$(a,b) \in R$$

$$\Rightarrow a^2 + b^2 = 1$$

$$\Rightarrow b^2 + a^2 = 1$$

$$\Rightarrow (b,a) \in R$$

Thus, R is symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in S$

$$\text{Let } (a,b) \in R \text{ and } (b,c) \in R \forall a, b, c \in S$$

$$\Rightarrow a^2 + b^2 = 1 \text{ and } b^2 + c^2 = 1$$

Adding both, we get

$$a^2 + c^2 + 2b^2 = 2$$

$$\Rightarrow a^2 + c^2 = 2 - 2b^2 \neq 1$$

$$\Rightarrow (a, c) \notin R$$

Thus, R is not transitive.

Thus, R is symmetric but neither reflexive nor transitive.

Question: 10

$$\text{Let } R = \{(a, b) :$$

Solution:

We have, $R = \{(a, b) : a = b^2\}$ relation defined on N.

Now,

We observe that, any element $a \in N$ cannot be equal to its square except 1.

$$\Rightarrow (a,a) \notin R \forall a \in N$$

$$\text{For e.g. } (2,2) \notin R \because 2 \neq 2^2$$

$\Rightarrow R$ is not reflexive.

$$\text{Let } (a,b) \in R \forall a, b \in N$$

$$\Rightarrow a = b^2$$

But b cannot be equal to square of a if a is equal to square of b.

$$\Rightarrow (b,a) \notin R$$

$$\text{For e.g., we observe that } (4,2) \in R \text{ i.e } 4 = 2^2 \text{ but } 2 \neq 4^2 \Rightarrow (2,4) \notin R$$

$\Rightarrow R$ is not symmetric

$$\text{Let } (a,b) \in R \text{ and } (b,c) \in R \forall a, b, c \in N$$

$$\Rightarrow a = b^2 \text{ and } b = c^2$$

$$\Rightarrow a \neq c^2$$

$$\Rightarrow (a,c) \notin R$$

For e.g., we observe that

$$(16,4) \in R \Rightarrow 16 = 4^2 \text{ and } (4,2) \in R \Rightarrow 4 = 2^2$$

But $16 \neq 2^2$

$\Rightarrow (16,2) \notin R$

$\Rightarrow R$ is not transitive.

Thus, R is neither reflexive nor symmetric nor transitive.

Question: 11

Show that the rel

Solution:

We have, $R = \{(a, b) : a > b\}$ relation defined on N .

Now,

We observe that, any element $a \in N$ cannot be greater than itself.

$\Rightarrow (a,a) \notin R \forall a \in N$

$\Rightarrow R$ is not reflexive.

Let $(a,b) \in R \forall a, b \in N$

$\Rightarrow a$ is greater than b

But b cannot be greater than a if a is greater than b .

$\Rightarrow (b,a) \notin R$

For e.g., we observe that $(5,2) \in R$ i.e $5 > 2$ but $2 \ngtr 5 \Rightarrow (2,5) \notin R$

$\Rightarrow R$ is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R \forall a, b, c \in N$

$\Rightarrow a > b$ and $b > c$

$\Rightarrow a > c$

$\Rightarrow (a,c) \in R$

For e.g., we observe that

$(5,4) \in R \Rightarrow 5 > 4$ and $(4,3) \in R \Rightarrow 4 > 3$

And we know that $5 > 3 \therefore (5,3) \in R$

$\Rightarrow R$ is transitive.

Thus, R is transitive but not reflexive not symmetric.

Question: 12

Let $A = \{1, 2, 3\}$

Solution:

Given that, $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$.

Now,

R is reflexive $\because (1,1),(2,2),(3,3) \in R$

R is not symmetric $\because (1,2),(2,3) \in R$ but $(2,1),(3,2) \notin R$

R is not transitive $\because (1,2) \in R$ and $(2,3) \in R \Rightarrow (1,3) \notin R$

Thus, R is reflexive but neither symmetric nor transitive.

Question: 13

Let $A = \{1, 2, 3\}$,

Solution:

Given that, $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\}$.

Now,

R is reflexive $\because (1,1), (2,2), (3,3), (4,4) \in R$

R is not symmetric $\because (1,2), (1,3), (3,2) \in R$ but $(2,1), (3,1), (2,3) \notin R$

R is transitive $\because (1,3) \in R$ and $(3,2) \in R \Rightarrow (1,2) \in R$

Thus, R is reflexive and transitive but not symmetric.

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark the tick aga

Solution:

Given set $A = \{1, 2, 3\}$

And $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since, $(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$

Therefore, R is reflexive (1)

Check for symmetric

Since $(1,3) \in R$ but $(3,1) \notin R$

Therefore, R is not symmetric (2)

Check for transitive

Here, $(1,3) \in R$ and $(3,2) \in R$ and $(1,2) \in R$

Therefore, R is transitive (3)

Now, according to the equations (1), (2), (3)

Correct option will be (B)

Question: 2

Mark the tick aga

Solution:

Given set $A = \{a, b, c\}$

And $R = \{(a, a), (a, b), (b, a)\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since, $(b,b) \notin R$ and $(c,c) \notin R$

Therefore, R is not reflexive (1)

Check for symmetric

Since, $(a,b) \in R$ and $(b,a) \in R$

Therefore, R is symmetric (2)

Check for transitive

Here, $(a,b) \in R$ and $(b,a) \in R$ and $(a,a) \in R$

Therefore, R is transitive (3)

Now, according to the equations (1), (2), (3)

Correct option will be (C)

Question: 3

Mark the tick aga

Solution:

Given set $A = \{1, 2, 3\}$

And $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since, $(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$

Therefore, R is reflexive (1)

Check for symmetric

Since , $(1,2) \in R$ and $(2,1) \in R$

$(2,3) \in R$ and $(3,2) \in R$

Therefore , R is symmetric (2)

Check for transitive

Here , $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$

Therefore , R is not transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 4

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{x, y, z\}$

And $R = \{(x, y), (y, z), (x, z), (y, x), (z, y), (z, x)\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , $(x,x) \notin R$, $(y,y) \notin R$, $(z,z) \notin R$

Therefore , R is not reflexive (1)

Check for symmetric

Since , $(x,y) \in R$ and $(y,x) \in R$

$(z,y) \in R$ and $(y,z) \in R$

$(x,z) \in R$ and $(z,x) \in R$

Therefore , R is symmetric (2)

Check for transitive

Here , $(x,y) \in R$ and $(y,x) \in R$ but $(x,x) \notin R$

Therefore , R is not transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

Question: 5

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{x, y, z\}$

And $R = \{(x, x), (y, y), (z, z)\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , $(x,x) \in R$, $(y,y) \in R$, $(z,z) \in R$

Therefore , R is reflexive (1)

Check for symmetric

Since , $(x,x) \in R$ and $(x,x) \in R$

$(y,y) \in R$ and $(y,y) \in R$

$(z,z) \in R$ and $(z,z) \in R$

Therefore , R is symmetric (2)

Check for transitive

Here , $(x,x) \in R$ and $(y,y) \in R$ and $(z,z) \in R$

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 6

Mark the tick aga

Solution:

According to the question ,

Given set $Z = \{1, 2, 3, 4, \dots\}$

And $R = \{(a, b) : a, b \in Z \text{ and } (a-b) \text{ is divisible by } 3\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Consider, (a, a)

$(a - a) = 0$ which is divisible by 3

$(a, a) \in R$ where $a \in \mathbb{Z}$

Therefore, R is reflexive (1)

Check for symmetric

Consider, $(a, b) \in R$

$\therefore (a - b)$ which is divisible by 3

$-(a - b)$ which is divisible by 3

(since if 6 is divisible by 3 then -6 will also be divisible by 3)

$\therefore (b - a)$ which is divisible by 3 $\Rightarrow (b, a) \in R$

For any $(a, b) \in R$; $(b, a) \in R$

Therefore, R is symmetric (2)

Check for transitive

Consider, $(a, b) \in R$ and $(b, c) \in R$

$\therefore (a - b)$ which is divisible by 3

and $(b - c)$ which is divisible by 3

$[(a - b) + (b - c)]$ is divisible by 3] (if 6 is divisible by 3 and 9 is divisible by 3 then 6+9 will also be divisible by 3)

$\therefore (a - c)$ which is divisible by 3 $\Rightarrow (a, c) \in R$

Therefore $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

Therefore, R is transitive (3)

Now, according to the equations (1), (2), (3)

Correct option will be (D)

Question: 7

Mark the tick aga

Solution:

According to the question,

Given set $N = \{1, 2, 3, 4, \dots\}$

And $R = \{(a, b) : a, b \in N \text{ and } a \text{ is a factor of } b\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Consider, (a, a)

a is a factor of a

$(2, 2), (3, 3), \dots (a, a)$ where $a \in \mathbb{N}$

Therefore, R is reflexive (1)

Check for symmetric

$a R b \Rightarrow a$ is factor of b

$b R a \Rightarrow b$ is factor of a as well

Ex $(2, 6) \in R$

But $(6, 2) \notin R$

Therefore, R is not symmetric (2)

Check for transitive

$a R b \Rightarrow a$ is factor of b

$b R c \Rightarrow b$ is a factor of c

$a R c \Rightarrow a$ is a factor of c also

Ex $(2, 6), (6, 18)$

$\therefore (2, 18) \in R$

Therefore, R is transitive (3)

Now, according to the equations (1), (2), (3)

Correct option will be (B)

Question: 8

Mark the tick aga

Solution:

According to the question,

Given set $Z = \{1, 2, 3, 4, \dots\}$

And $R = \{(a, b) : a, b \in Z \text{ and } a \geq b\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a) (b,b)

$\therefore a \geq a$ and $b \geq b$ which is always true.

Therefore , R is reflexive (1)

Check for symmetric

$a R b \Rightarrow a \geq b$

$b R a \Rightarrow b \geq a$

Both cannot be true.

Ex _ If $a=2$ and $b=1$

$\therefore 2 \geq 1$ is true but $1 \geq 2$ which is false.

Therefore , R is not symmetric (2)

Check for transitive

$a R b \Rightarrow a \geq b$

$b R c \Rightarrow b \geq c$

$\therefore a \geq c$

Ex _ $a=5$, $b=4$ and $c=2$

$\therefore 5 \geq 4$, $4 \geq 2$ and hence $5 \geq 2$

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

Question: 9

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{.....,-2,-1,0,1,2\}$

And $R = \{(a, b) : a, b \in S \text{ and } |a| \leq b\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore |a| \leq a$ and which is not always true.

Ex_ if $a=-2$

$\therefore |-2| \leq -2 \Rightarrow 2 \leq -2$ which is false.

Therefore , R is not reflexive (1)

Check for symmetric

$a R b \Rightarrow |a| \leq b$

$b R a \Rightarrow |b| \leq a$

Both cannot be true.

Ex _ If $a=-2$ and $b=-1$

$\therefore 2 \leq -1$ is false and $1 \leq -2$ which is also false.

Therefore , R is not symmetric (2)

Check for transitive

$a R b \Rightarrow |a| \leq b$

$b R c \Rightarrow |b| \leq c$

$\therefore |a| \leq c$

Ex _ $a=-5$, $b= 7$ and $c=9$

$\therefore 5 \leq 7$, $7 \leq 9$ and hence $5 \leq 9$

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

Question: 10

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{.....,-2,-1,0,1,2\}$

And $R = \{(a, b) : a,b \in S \text{ and } |a - b| \leq 1 \}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a , a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a , b) \in R$, then $(b , a) \in R$

Transitive

Relation is Transitive if $(a , b) \in R$ & $(b , c) \in R$, then $(a , c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore |a - a| \leq 1$ and which is always true.

Ex_ if $a=2$

$\therefore |2-2| \leq 1 \Rightarrow 0 \leq 1$ which is true.

Therefore , R is reflexive (1)

Check for symmetric

$a R b \Rightarrow |a - b| \leq 1$

$b R a \Rightarrow |b - a| \leq 1$

Both can be true.

Ex _ If $a=2$ and $b=1$

$\therefore |2 - 1| \leq 1$ is true and $|1-2| \leq 1$ which is also true.

Therefore , R is symmetric (2)

Check for transitive

$a R b \Rightarrow |a - b| \leq 1$

$b R c \Rightarrow |b - c| \leq 1$

$\therefore |a - c| \leq 1$ will not always be true

Ex _ $a=-5$, $b= -6$ and $c= -7$

$\therefore |6-5| \leq 1$, $|7 - 6| \leq 1$ are true But $|7 - 5| \leq 1$ is false.

Therefore , R is not transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 11

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{.....,-2,-1,0,1,2\}$

And $R = \{(a, b) : a,b \in S \text{ and } (1 + ab) > 0 \}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a , a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a , b) \in R$, then $(b , a) \in R$

Transitive

Relation is Transitive if $(a , b) \in R$ & $(b , c) \in R$, then $(a , c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore (1 + a \times a) > 0$ which is always true because $a \times a$ will always be positive.

Ex_ if $a=2$

$\therefore (1 + 4) > 0 \Rightarrow (5) > 0$ which is true.

Therefore , R is reflexive (1)

Check for symmetric

$$a R b \Rightarrow (1 + ab) > 0$$

$$b R a \Rightarrow (1 + ba) > 0$$

Both the equation are the same and therefore will always be true.

Ex _ If $a=2$ and $b=1$

$\therefore (1 + 2 \times 1) > 0$ is true and $(1 + 1 \times 2) > 0$ which is also true.

Therefore , R is symmetric (2)

Check for transitive

$$a R b \Rightarrow (1 + ab) > 0$$

$$b R c \Rightarrow (1 + bc) > 0$$

$\therefore (1 + ac) > 0$ will not always be true

Ex _ $a=-1$, $b= 0$ and $c= 2$

$\therefore (1 + -1 \times 0) > 0$, $(1 + 0 \times 2) > 0$ are true

But $(1 + -1 \times 2) > 0$ is false.

Therefore , R is not transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 12

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{ \dots \text{All triangles in plane} \dots \}$

And $R = \{ (\Delta_1 , \Delta_2) : \Delta_1 , \Delta_2 \in S \text{ and } \Delta_1 \equiv \Delta_2 \}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a , a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a , b) \in R$, then $(b , a) \in R$

Transitive

Relation is Transitive if $(a , b) \in R$ & $(b , c) \in R$, then $(a , c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (Δ_1, Δ_1)

\therefore We know every triangle is congruent to itself.

$(\Delta_1, \Delta_1) \in R$ all $\Delta_1 \in S$

Therefore , R is reflexive (1)

Check for symmetric

$(\Delta_1, \Delta_2) \in R$ then Δ_1 is congruent to Δ_2

$(\Delta_2, \Delta_1) \in R$ then Δ_2 is congruent to Δ_1

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric (2)

Check for transitive

Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

Then $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1$ is congruent to Δ_2 , and Δ_2 is congruent to Δ_3

$\Rightarrow \Delta_1$ is congruent to Δ_3

$\therefore (\Delta_1, \Delta_3) \in R$

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 13

Mark the tick aga

Solution:

According to the question ,

Given set $S = \{\dots, -2, -1, 0, 1, 2, \dots\}$

And $R = \{(a, b) : a, b \in S \text{ and } a^2 + b^2 = 1\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a, a)

$\therefore a^2 + a^2 = 1$ which is not always true

Ex_ if $a=2$

$\therefore 2^2 + 2^2 = 1 \Rightarrow 4 + 4 = 1$ which is false.

Therefore , R is not reflexive (1)

Check for symmetric

$$a R b \Rightarrow a^2 + b^2 = 1$$

$$b R a \Rightarrow b^2 + a^2 = 1$$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric (2)

Check for transitive

$$a R b \Rightarrow a^2 + b^2 = 1$$

$$b R c \Rightarrow b^2 + c^2 = 1$$

$\therefore a^2 + c^2 = 1$ will not always be true

Ex _a=-1 , b= 0 and c= 1

$\therefore (-1)^2 + 0^2 = 1$, $0^2 + 1^2 = 1$ are true

But $(-1)^2 + 1^2 = 1$ is false.

Therefore , R is not transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 14

Mark the tick aga

Solution:

According to the question ,

$$R = \{(a, b) , (c, d) : a + d = b + c \}$$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a , a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a , b) \in R$, then $(b , a) \in R$

Transitive

Relation is Transitive if $(a , b) \in R$ & $(b , c) \in R$, then $(a , c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , $(a, b) R (a, b)$

$$(a, b) R (a, b) \Leftrightarrow a + b = a + b$$

which is always true .

Therefore , R is reflexive (1)

Check for symmetric

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

$$(c, d) R (a, b) \Leftrightarrow c + b = d + a$$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric (2)

Check for transitive

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

$$(c, d) R (e, f) \Leftrightarrow c + f = d + e$$

On adding these both equations we get , $a + f = b + e$

Also,

$$(a, b) R (e, f) \Leftrightarrow a + f = b + e$$

\therefore It will always be true

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 15

Mark the tick aga

Solution:

According to the question ,

O is the origin

$$R = \{(P, Q) : OP = OQ\}$$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

$$\text{Consider , } (P, P) \in R \Leftrightarrow OP = OP$$

which is always true .

Therefore , R is reflexive (1)

Check for symmetric

$$(P, Q) \in R \Leftrightarrow OP = OQ$$

$$(Q, P) \in R \Leftrightarrow OQ = OP$$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric (2)

Check for transitive

$$(P, Q) \in R \Leftrightarrow OP = OQ$$

$$(Q, R) \in R \Leftrightarrow OQ = OR$$

On adding these both equations, we get , $OP = OR$

Also,

$$(P, R) \in R \Rightarrow OP = OR$$

\therefore It will always be true

Therefore , R is transitive (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 16

Mark the tick aga

Solution:

According to the question ,

Q is set of all rational numbers

$$R = \{(a, b) : a * b = a + 2b\}$$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

$$\text{Consider , } a * b = a + 2b$$

$$\text{And , } b * a = b + 2a$$

Both equations will not always be true .

Therefore , * is not commutative (1)

Check for associative

$$\text{Consider , } (a * b) * c = (a + 2b) * c = a + 2b + 2c$$

$$\text{And , } a * (b * c) = a * (b + 2c) = a + 2(b + 2c) = a + 2b + 4c$$

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 17

Mark the tick aga

Solution:

According to the question ,

$$Q = \{a, b\}$$

$$R = \{(a, b) : a * b = a + ab\}$$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

$$\text{Consider , } a * b = a + ab$$

And , $b * a = b + ba$

Both equations will not always be true .

Therefore , $*$ is not commutative (1)

Check for associative

Consider , $(a * b) * c = (a + ab) * c = a + ab + (a + ab)c = a + ab + ac + abc$

And , $a * (b * c) = a * (b + bc) = a + a(b + bc) = a + ab + abc$

Both the equation are not the same and therefore will not always be true.

Therefore , $*$ is not associative (2)

Now , according to the equations (1) , (2)

Correct option will be (B)

Question: 18

Mark the tick aga

Solution:

According to the question ,

$Q = \{ \text{Positive rationals} \}$

$R = \{(a, b) : a * b = ab/2 \}$

Formula

$*$ is commutative if $a * b = b * a$

$*$ is associative if $(a * b) * c = a * (b * c)$

Check for commutative

Consider , $a * b = ab/2$

And , $b * a = ba/2$

Both equations are the same and will always true .

Therefore , $*$ is commutative (1)

Check for associative

Consider , $(a * b) * c = (ab/2) * c = \frac{ab}{2} \times \frac{c}{2} = abc/4$

And , $a * (b * c) = a * (bc/2) = \frac{a \times bc}{2} = abc/4$

Both the equation are the same and therefore will always be true.

Therefore , $*$ is associative (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

Question: 19

Mark the tick aga

Solution:

According to the question ,

$Q = \{ \text{All integers} \}$

$R = \{(a, b) : a * b = a - b + ab \}$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

Consider , $a * b = a - b + ab$

And , $b * a = b - a + ba$

Both equations are not the same and will not always be true .

Therefore , * is not commutative (1)

Check for associative

Consider , $(a * b) * c = (a - b + ab) * c$

$$= a - b + ab - c + (a - b + ab)c$$

$$= a - b + ab - c + ac - bc + abc$$

And , $a * (b * c) = a * (b - c + bc)$

$$= a - (b - c + bc) + a(b - c + bc)$$

$$= a - b + c - bc + ab - ac + abc$$

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 20

Mark the tick aga

Solution:

According to the question ,

$Q = \{ \text{All integers} \}$

$R = \{(a, b) : a * b = a + b - ab \}$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

Consider , $a * b = a + b - ab$

And , $b * a = b + a - ba$

Both equations are the same and will always be true .

Therefore , * is commutative (1)

Check for associative

Consider , $(a * b) * c = (a + b - ab) * c$

$$= a + b - ab + c - (a + b - ab)c$$

$$= a + b - ab + c - ac - bc + abc$$

And , $a * (b * c) = a * (b + c - bc)$

$$= a + (b + c - bc) - a(b + c - bc)$$

$$= a + b + c - bc - ab - ac + abc$$

Both the equation are the same and therefore will always be true.

Therefore , * is associative (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

Question: 21

Mark the tick aga

Solution:

According to the question ,

$Q = \{ \text{All integers} \}$

$R = \{(a, b) : a * b = a^b\}$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

Consider , $a * b = a^b$

And , $b * a = b^a$

Both equations are not the same and will not always be true .

Therefore , * is not commutative (1)

Check for associative

Consider , $(a * b) * c = (a^b) * c = (a^b)^c$

And , $a * (b * c) = a * (b^c) = a^{(b^c)}$

Ex $a=2$ $b=3$ $c=4$

$(a * b) * c = (2^3) * c = (8)^4$

$a * (b * c) = 2 * (3^4) = 2^{(81)}$

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 22

Mark the tick aga

Solution:

According to the question ,

$R = \{(a, b) : a * b = a + b + ab\}$

Formula

* is commutative if $a * b = b * a$

* is associative if $(a * b) * c = a * (b * c)$

Check for commutative

Consider , $a * b = a + b + ab$

And , $b * a = b + a + ba$

Both equations are same and will always be true .

Therefore , $*$ is commutative (1)

Check for associative

Consider , $(a * b) * c = (a + b + ab) * c$

$$= a + b + ab + c + (a + b + ab)c$$

$$= a + b + c + ab + ac + bc + abc$$

And , $a * (b * c) = a * (b + c + bc)$

$$= a + b + c + bc + a(b + c + bc)$$

$$= a + b + c + ab + bc + ac + abc$$

Both the equation are same and therefore will always be true.

Therefore , $*$ is associative (2)

Now , according to the equations (1) , (2)

Correct option will be (D)