Electrostatics

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Source of Charges

1. **Point Charge**

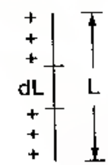
   If charge is put at a point only $\bullet^{+Q}$

2. **Line charge density**

   It is charge per unit length.

$$\boxed{\frac{dQ}{dL} = \rho_L} \text{ C/m}$$

$$\boxed{Q = \int_L \rho_L dL} \text{ C}$$

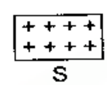


3. **Surface charge density**

   It is the charge per unit area

$$\boxed{\frac{dQ}{dS} = \rho_S} \text{ C/m}^2$$

$$\boxed{Q = \iint_S \rho_S dS} \text{ C}$$

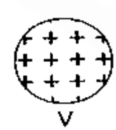


4. **Volume charge density**

   It is the charge per unit volume

$$\boxed{\frac{dQ}{dV} = \rho_V} \text{ C/m}^3$$

$$\boxed{Q = \iiint_V \rho_V dV} \text{ C}$$



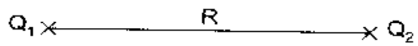
Charge in terms of Various Charge Distributions

$$\boxed{Q = \int_L \rho_L dL = \int_S \rho_S dS = \int_V \rho_V dV} \text{ C}$$

where, ρ_L, ρ_S and ρ_V are line charge, surface charge and volume charge density respectively.

Coulomb's Law

The force acting between two point charges is directly proportional to the products of charges and inversely proportional to the square of the distance between them

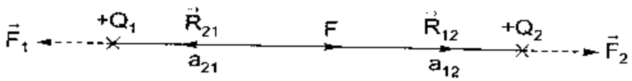


$$\boxed{F \propto \frac{Q_1 Q_2}{R^2}} \text{ Newton}$$

$$\boxed{F = K \cdot \frac{Q_1 Q_2}{R^2} = \frac{Q_1 Q_2}{4\pi \epsilon_0 R^2}}$$

Where, $K = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9$ in S.I. system

- If two same positive charge are placed then they experience a repulsive force.



$$a_{12} = \frac{\vec{R}_{12}}{|R_{12}|} \quad \text{and} \quad a_{21} = \frac{\vec{R}_{21}}{|R_{21}|}$$

$$\vec{R}_{12} = -\vec{R}_{21} \quad \text{and} \quad a_{12} = -a_{21}$$

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi \epsilon_0 R_{21}^2} a_{21}, \quad \vec{F}_2 = \frac{Q_1 Q_2}{4\pi \epsilon_0 R_{12}^2} a_{12}$$

$$\boxed{\vec{F}_1 = -\vec{F}_2}$$

Note:

- Linear forces are another property of Coulomb force i.e. $nF \propto nQ_1 Q_2$
- Coulomb's force Obeys law of superposition.
- Coulomb's forces are called mutual and linear force.

Electric Field Intensity

The electric field intensity or electric field strength (E) is the force per unit charge when placed in an electric field

$$\boxed{E = \lim_{Q \to 0} \frac{\vec{F}}{Q}} \text{ Newton/C}$$

Electric field due to continuous charge distribution

(i) Line charge

$$\boxed{E = \int_L \frac{\rho_L dL}{4\pi \epsilon_0 R^2} a_r}$$

(ii) Surface charge

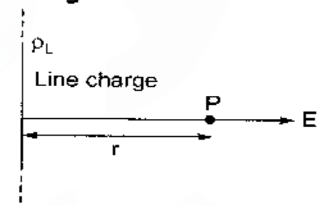
$$\boxed{E = \int_S \frac{\rho_S dS}{4\pi \epsilon_0 R^2} a_r}$$

(iii) Volume charge

$$\boxed{E = \int_V \frac{\rho_V dV}{4\pi \epsilon_0 R^2} a_r}$$

Electric field intensity due to infinitely long line charge

$$\boxed{E = \frac{\rho_L}{2\pi \epsilon_o r} a_r}$$



where, ρ_L = Charge per unit length = Line charge density C/m

Electric field intensity due to infinite charge sheet

$$\boxed{E = \frac{\rho_s}{2 \epsilon_o} i_n}$$

where, ρ_s = Surface charge density, C/m^2

If sheet is on xy plane $z = z_o$

$$E = \begin{cases} \frac{\rho_s}{2 \epsilon_o} i_z & \text{for } z > z_o \\ -\frac{\rho_s}{2 \epsilon_o} i_z & \text{for } z < z_o \end{cases}$$

Remember:

- In a parallel plate capacitor, the electric field existing between the two plates having equal and opposite charges

$$\boxed{E = \frac{\rho_s}{\epsilon_0} i_n}$$

Electric field intensity due to uniformly charged sphere

$$E = \begin{cases} \dfrac{r}{3\epsilon_o}\rho_o \, i_r \,; & 0 < r \le a \\ \dfrac{a^3}{3\epsilon_o r^2}\rho_o \, i_r \,; & r \ge a \end{cases}$$

Where ρ_o = Volume charge density

a = Radius of sphere

Electric/Displacement Flux Density

The electric flux density is always tangential to electric flux lines. Electric flux lines originates from a positive charge and ends on a negative charge.

$$D = \frac{\text{Flux}}{\text{Unit area}}$$

$$\vec{D} = \frac{Q}{4\pi r^2} i_r \quad \text{and} \quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} i_r$$

$$\vec{D} = \epsilon_0 \vec{E}$$

where, $\vec{D}$ = Electric flux density C/m²

$\vec{E}$ = Electric field intensity V/m

ϵ = Electrical permittivity of medium

$$\epsilon = \epsilon_0 \epsilon_r$$

ϵ_0 = Free space permittivity

$$= 8.854 \times 10^{-12} \text{ F/m} = \left(\frac{1}{36\pi}\right) \times 10^{-9} \text{ F/m}$$

ϵ_r = Relative permittivity or Dielectric constant of medium

ϵ_r = 1 (For air (or) free space)

ϵ_r > 1 (For rest of the material)

Note:

Electric field $\vec{E}$ depends upon the medium where as $\vec{D}$ is independent of the medium.

Gauss's Law

Gauss law states that flux leaving any closed surface is equal to the charge enclosed by that surface.

$$\psi = \oint_S \vec{D} \cdot d\vec{S} = Q = \int_V \rho_V \cdot dV$$ .... Gauss law in integral form

$$\rho_V = \nabla \cdot \vec{D}$$ ... Gauss law in differential or point form Maxwell's first equation.

Note:

- The Gauss law is applicable for time varying as well as static fields.
- The equation is valid irrespective of the shape of closed surface area 'S'.

Electrical Energy Density

It is total electrical per unit volume

$$W_e = \frac{1}{2}\epsilon \vec{E}^2 = \frac{1}{2}\vec{D} \cdot \vec{E} \text{ J/m}^3$$

Total electrical energy stored

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy Density in Electrostatic Field

Electrostatic energy density

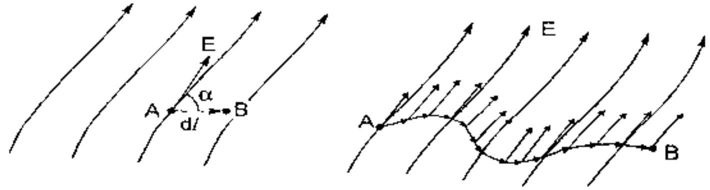
$$W_E = \frac{dW_E}{dv} = \frac{1}{2}\vec{D} \cdot \vec{E} = \frac{1}{2}\epsilon_0 E^2 = \frac{D^2}{2\epsilon_0}$$

Total electrostatic energy

$$W_E = \frac{1}{2}\int_v \vec{D} \cdot \vec{E} \, dV = \frac{1}{2}\int_v \epsilon_0 E_2 \, dV$$

Electric Potential

Potential difference



(a) (b)
Movement of a test charge in an electric field

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad ; \quad V_{AB} = \frac{W_{AB}}{q} = -\int_A^B \vec{E}\cdot d\vec{l}$$

where, V_{AB} = Potential difference between the points A & B

W_{AB} = Work done by the field in moving a test charge q from A to B

dl = Infinitesimal length of segments

Remember:

- The negative sign in above equation indicates that the work is being done by an external agent.
- If V_{AB} is negative, there is a loss in potential energy in moving charge Q from A to B i.e. the field does the work.

Potential at a point P due to point charge

$$V(r) = \int_r^\infty \vec{E}\cdot d\vec{l} = -\int_\infty^r \vec{E}\cdot d\vec{l} \text{ Volt}$$

where, $V(r)$ = Potential at a distance r from the point charge

r = distance of point P from point charge

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

For static electric field

$$\oint E\cdot dl = 0 \quad ; \quad \nabla\times E = 0$$

Remember:

The potential at any point is the potential difference between that point and a chosen point (or reference point) at which the potential is zero.

Relation between electric field intensity vector and potential at a point

$$E = -\nabla V$$

Poisson's and Laplace Equation

$$\nabla^2 V = \frac{-\rho_V}{\epsilon} \quad \text{....Poisson's equation}$$

For charge free region $\rho_v = 0$

$$\nabla^2 V = 0 \quad \text{....Laplace equation}$$

- In Poisson's equation the potential or electric field can be found due to specified volume charge distribution in the given region.
- In Laplace equation the potential or electric field can be found in the charge free region.

Remember:

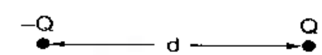
- Both Poisson's and Laplace equation are second order three dimensional non linear differential equations.
- Poisson's equation is valid in the region where some charge is present, where as Laplace equation is valid for charge free region.
- At least two boundary conditions must be known to calculate two arbitrary constants of integration when the two equations are solved.

Electric Dipole

An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by small distance

Dipole moment

$$\vec{p} = Q\vec{d}$$

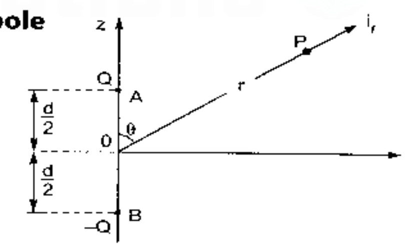


Direction of dipole moment is from negative charge to positive charge.

Electric field intensity due to electric dipole

$$E = \frac{Qd}{4\pi\epsilon_0 r^3}(2\cos\theta\, i_r + \sin\theta\, i_\theta)$$

$$E = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\, i_r + \sin\theta\, i_\theta)$$



Conductor

A perfect conductor ($\sigma = \infty$) cannot contain an electrostatic field within it.

Inside a conductor

$$\boxed{E = 0,\ \rho_v = 0,\ V_{ab} = 0}$$

where, V_{ab} = Potential difference between points a and b in the conductor.

Power

Joule's law

$$\boxed{P = \int_v \vec{E} \cdot \vec{J}\, dV}$$

where, J = Current density, A/m^2

Continuity Equation

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

Note: ....

- For steady current $\frac{\partial \rho_0}{\partial t} = 0$.
- For lossless region $\rho_v = 0$; $\nabla \cdot J = 0$.

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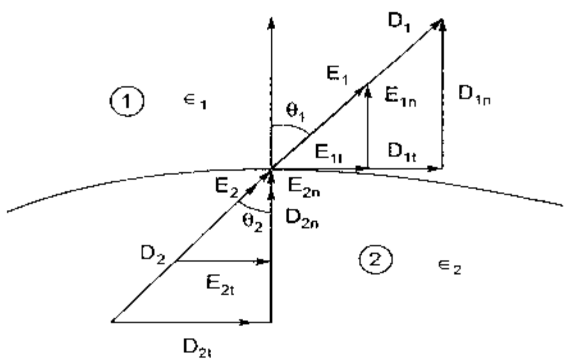
Boundary Conditions

Dielectric-Dielectric Boundary Condition



$$\boxed{\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}} \quad \text{and} \quad \boxed{\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}}$$

where, $\vec{E}_1, \vec{E}_2$ = Fields in media 1 and 2 respectively

$\vec{E}_t, \vec{E}_n$ = Tangential and normal components of E

Tangential component relation

$$\boxed{\vec{E}_{1t} = \vec{E}_{2t}} \quad \text{or} \quad \boxed{\frac{\vec{D}_{1t}}{\epsilon_1} = \frac{\vec{D}_{2t}}{\epsilon_2}}$$

Normal component relation

$$\boxed{\vec{D}_{1n} - \vec{D}_{2n} = \rho_s}$$

where, ϵ_1 & ϵ_2 = Permittivity of dielectric 1 and 2

ρ_s = Free charge density placed deliberately at the boundary

Remember: ....

- The tangential component of $\vec{E}$ is continuous while that of $\vec{D}$ is discontinuous at boundary.
- The normal component of $\vec{D}$ is continuous while that of $\vec{E}$ is discontinuous at boundary.

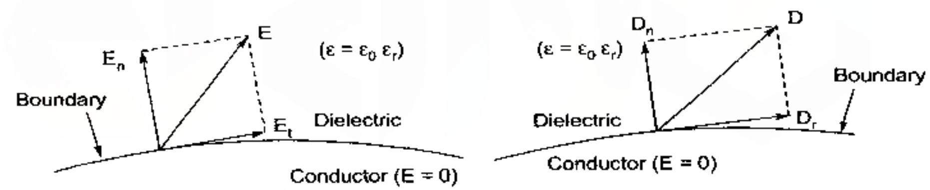
....

If no free charges exists at the interface

$$\boxed{\vec{D}_{1n} = \vec{D}_{2n}} \quad \text{or} \quad \boxed{\epsilon_1 \vec{E}_{1n} = \epsilon_2 \vec{E}_{2n}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

Conductor-Dielectric Boundary Conditions.



$$\boxed{\vec{D}_t = \epsilon \vec{E}_t = 0} \quad \text{and} \quad \boxed{\vec{D}_n = \epsilon \vec{E}_n = \rho_s}$$

Remember: ....

- Since $\vec{E} = -\nabla V = 0$, there can be no potential difference between any two points in the conductor (i.e. a conductor is an equipotential body).
- An electric field E must be external to the conductor and must be normal to its surface.

$$\boxed{\vec{D}_t = \epsilon \vec{E}_t = 0} \quad \text{and} \quad \boxed{\vec{D}_n = \epsilon \vec{E}_n = \rho_s}$$

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