Ex 19.1

Indefinite Integrals Ex 19.1 Q1

(i)

$$\int x^4 dx = \frac{x^{4+1}}{4+1} + c$$
$$= \frac{x^5}{5} + c$$

(ii)

$$\int x^{\frac{5}{4}} dx = \frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1} + c$$
$$= \frac{x^{\frac{5+4}{4}}}{\frac{5+4}{4}} + c$$
$$= \frac{4x^{\frac{9}{4}}}{9} + c$$

(iii)

$$\int \frac{1}{x^5} dx = \int x^{-5} dx$$
$$= \frac{x^{-5+1}}{-5+1} + c$$
$$= \frac{x^{-4}}{-4} + c$$
$$= \frac{-1}{4x^4} + c$$

(iv)

$$\int \frac{1}{x^{\frac{3}{2}}} dx = \int x^{\frac{-3}{2}} dx$$
$$= \int x^{\frac{-3}{2}} dx$$
$$= \frac{x^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + c$$
$$= \frac{x^{\frac{-1}{2}}}{\frac{-1}{2}} + c$$
$$= -2 \times \frac{1}{\sqrt{x}} + c$$
$$= \frac{-2}{\sqrt{x}} + c$$

(v)

$$\int 3^x dx = \frac{3^x}{\log 3} + c \qquad \left[\because \int a^x dx = \frac{a^x}{\log a} + c\right]$$

(vi)

$$\int \frac{1}{\sqrt[3]{x^2}}dx = \int \frac{1}{x^{\frac{2}{3}}}dx$$
$$= \int x^{\frac{-2}{3}}dx$$
$$= \frac{d^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + c$$
$$= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$
$$= 3\sqrt[3]{x} + c$$

(vii)

$$\int 3^{2\log_3 x}dx = \int 3^{\log_3 x^2}dx$$
$$= \int x^2 dx \qquad \left[\because a^{\log_a x} = x\right]$$
$$= \frac{x^3}{3} + c$$

(viii)

$$\int \log_x x\,dx = \int 1dx$$
$$= x + c.$$

Indefinite Integrals Ex 19.1 Q2

(i)

$$\int \sqrt{\frac{1+\cos 2x}{2}}dx = \int \sqrt{\frac{2\cos^2 x}{2}}dx \qquad \left[\because \cos 2x = 2\cos^2 - 1\right]$$
$$= \int \cos x dx$$
$$= \sin x + c$$

(ii)

$$\int \sqrt{\frac{1-\cos 2x}{2}}dx = \int \sqrt{\frac{2\sin^2 x}{2}}dx$$
$$= \int \sin x dx$$
$$= -\cos x + c$$

Indefinite Integrals Ex 19.1 Q3

Evaluate the integral as follows

$$\int \frac{e^{6\log_e x} - e^{5\log_e x}}{e^{4\log_e x} - e^{3\log_e x}}dx = \int \frac{x^6 - x^5}{x^4 - x^3}dx$$
$$= \int \frac{x^5(x-1)}{x^3(x-1)}dx$$
$$= \int x^2 dx$$
$$= \frac{x^3}{3} + C$$

Indefinite Integrals Ex 19.1 Q4

$$\int \frac{1}{a^x b^x}dx = \int a^{-x}b^{-x}dx$$
$$= \int (ab)^{-x}dx$$
$$= \frac{(ab)^{-x}}{\log_e (ab)^{-1}} + c$$
$$= \frac{(ab)^{-x}}{-\log_e (ab)} + c$$
$$= \frac{a^{-x}b^{-x}}{-\log_e (ab)} + c$$

Indefinite Integrals Ex 19.1 Q5

(i) $\int \frac{\cos 2x + 2\sin^2 x}{\sin^2 x}dx$

$$= \int \frac{\cos 2x + 2\sin^2 x}{\sin^2 x}dx$$
$$= \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\sin^2 x}dx$$
$$= \int \frac{1}{\sin^2 x}dx$$
$$= \int cosec^2 x dx$$
$$= -\cot x + c$$

(ii) $\int \frac{2\cos^2 x - \cos 2x}{\cos^2 x}dx$

$$= \int \frac{2\cos^2 x - \left(2\cos^2 x - 1\right)}{\cos^2 x}dx$$
$$= \int \frac{2\cos^2 x - 2\cos^2 x + 1}{\cos^2 x}dx$$
$$= \int \frac{1}{\cos^2 x}dx$$
$$= \int \sec^2 x dx$$
$$= \tan x + c$$

Indefinite Integrals Ex 19.1 Q6

$$\int \frac{e^{\log \sqrt{x}}}{x}dx = \int \frac{\sqrt{x}}{x}dx$$
$$= \int x^{\frac{1}{2}} \times x^{-1} dx$$
$$= \int x^{\frac{1}{2}-1} dx$$
$$= \int x^{\frac{-1}{2}} dx$$
$$= \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$$
$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$
$$= 2\sqrt{x} + c$$

Ex 19.2

Indefinite Integrals Ex 19.2 Q1

$$\int \left(3x\sqrt{5} + 4\sqrt{x} + 5\right) dx$$

$$= \int 3x\sqrt{5}dx + \int 4\sqrt{x}dx + \int 5dx$$

$$= \int 3x^{\frac{3}{2}}dx + 4\int x^{\frac{1}{2}}dx + 5\int dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5x + c$$

$$= \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 5x + c$$

Indefinite Integrals Ex 19.2 Q2

$$\int \left(2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}}\right) dx$$

$$= \int 2^x dx + 5\int \frac{1}{x}dx - \int \frac{1}{x^{\frac{1}{3}}}dx$$

$$= \frac{2^x}{\log 2} + 5\log x - \frac{3}{2}x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q3

$$\int \left\{\sqrt{x}\left(ax^2 + bx + c\right)\right\} dx$$

$$= \int \sqrt{x} \times ax^2dx + \int \sqrt{x} \times bxdx + \int c\sqrt{x}dx$$

$$= \int ax^{\frac{5}{2}}dx + \int bx^{\frac{3}{2}}dx + \int cx^{\frac{1}{2}}dx$$

$$= \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + d$$

$$= \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{3}{2}}}{3} + d$$

Indefinite Integrals Ex 19.2 Q4

$$\int (2-3x)(3+2x)(1-2x)\, dx$$

$$= \int \left(6 + 4x - 9x - 6x^2\right)(1-2x)dx$$

$$= \int \left(-6x^2 - 5x + 6\right)(1-2x)dx$$

$$= \int \left(-6x^2 + 12x^3 - 5x + 10x^2 + 6 - 12x\right) dx$$

$$= \int \left(4x^2 + 12x^3 - 17x + 6\right)dx$$

$$= \int \left(12x^3 + 4x^2 - 17x + 6\right)dx$$

$$= \frac{12}{4}x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

$$= 3x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

Indefinite Integrals Ex 19.2 Q5

$$\int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx\right)dx$$

$$= m\int \frac{1}{x}dx + \frac{1}{m}\int xdx + \int m^x dx \int x^m dx + m\int xdx$$

$$= m\log|x| + \frac{x^2}{2m} + \frac{m^x}{\log m} + \frac{x^{m+1}}{m+1} + \frac{mx^2}{2} + c$$

Indefinite Integrals Ex 19.2 Q6

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^2}{2}+\log|x|-2x+C$$

Indefinite Integrals Ex 19.2 Q7

$$\int\frac{(1+x)^3}{\sqrt{x}}dx$$

$$=\int\frac{1+x^3+3x^2+3x}{5x}dx$$

$$=\int\frac{1}{\sqrt{x}}dx+\int\frac{x^3}{\sqrt{x}}dx+\int\frac{3x^2}{\sqrt{x}}dx+\int\frac{3x}{\sqrt{x}}dx$$

$$=\int x^{\frac{-1}{2}}dx+\int x^{\frac{5}{2}}dx+3\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$=\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+\frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$$

$$=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3x^{\frac{5}{2}}}{\frac{5}{2}}+3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+\frac{6}{3}x^{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

$$\therefore \quad \int\frac{(1+x)^3}{\sqrt{x}}dx=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\cdot\frac{6}{5}\;x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

Indefinite Integrals Ex 19.2 Q8

$$\int\left\{x^2+e^{\log x}+\left(\frac{e}{2}\right)^x\right\}dx$$

$$=\int x^2dx+\int e^{\log x}dx+\int\left(\frac{e}{2}\right)^x dx$$

$$=\frac{x^3}{3}+\int x dx+\int\left(\frac{e}{2}\right)dx$$

$$=\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{\log\left(\frac{e}{2}\right)}\times\left(\frac{e}{2}\right)^x+c$$

Indefinite Integrals Ex 19.2 Q9

$$\int\left(x^e+e^x+e^e\right)dx$$

$$=\int x^e dx+\int e^x dx+\int e^e dx$$

$$=\frac{x^{e+1}}{e+1}+e^x+e^e x+c \qquad [\because e \text{ is constant}]$$

$$\therefore \quad \int\left(x^e+e^x+e^e\right)dx=\frac{x^{e+1}}{e+1}+e^x+e^e x+c$$

Indefinite Integrals Ex 19.2 Q10

$$\int \sqrt{x}\left(x^3 - \frac{2}{x}\right)dx = \int x^{\frac{7}{2}}dx - 2\int x^{-\frac{1}{2}}dx$$

$$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$$

$$= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - \frac{2x^{\frac{-1}{2}}}{\frac{-1}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - 4x^{\frac{-1}{2}} + c$$

$$\therefore \quad \int \sqrt{x}\left(x^3 - \frac{2}{x}\right) = \frac{2}{9}x^{\frac{9}{2}} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q11

$$\int \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)dx$$

$$= \int\left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x} \times x}\right)dx$$

$$= \int x^{\frac{-1}{2}} + \int x^{\frac{-3}{2}}dx$$

$$= 2x^{\frac{1}{2}} - 2x^{\frac{-1}{2}} + c$$

$$= 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

$$\therefore \quad \int \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)dx = 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

Indefinite Integrals Ex 19.2 Q12

$$\int \frac{x^6 + 1}{x^2 + 1}dx$$

$$= \int \frac{\left(x^2\right)^3 + (1)^3}{x^2 + 1}dx$$

$$= \int \frac{\left(x^2 + 1\right)\left(x^4 + 1 - x^2\right)}{x^2 + 1}dx$$

$$= \int \left(x^4 - x^2 + 1\right)dx$$

$$= \int x^4 dx - \int x^2 dx + \int 1 dx$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + x + c$$

Indefinite Integrals Ex 19.2 Q13

$$\int \frac{x^{\frac{-1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}}dx$$

$$= \int \frac{x^{\frac{-1}{3}}dx}{x^{\frac{1}{3}}} + \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}dx + \int \frac{2}{x^{\frac{1}{3}}}dx$$

$$= \int x^{\frac{-2}{3}}dx + \int x^{\frac{1}{6}}dx + 2\int x^{\frac{-1}{3}}dx$$

$$= 3x^{\frac{1}{3}} + \frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q14

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx$$

$$= \int \frac{1+x+2\sqrt{x}}{x^{\frac{1}{2}}}dx$$

$$= \int x^{\frac{-1}{2}} + \int x^{\frac{1}{2}}dx + 2\int dx$$

$$= 2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + 2x + c$$

$$\therefore \quad \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx = 2\sqrt{x} + 2x + \frac{2}{3}x^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.2 Q15

$$\int \sqrt{x}\,(3-5x)dx$$

$$= 3\int \sqrt{x}dx - 5\int x^{\frac{3}{2}}dx$$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.2 Q16

$$\int \frac{(x+1)(x-2)}{\sqrt{x}}dx$$

$$= \int \frac{x^2-2x+x-2}{x^{\frac{1}{2}}}dx$$

$$= \int \frac{x^2-x-2}{x^{\frac{1}{2}}}dx$$

$$= \int \frac{x^2}{x^{\frac{1}{2}}}dx - \int x^{\frac{1}{2}}dx - 2\int x^{\frac{-1}{2}}dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} - 4x^{\frac{1}{2}} + c$$

$$\therefore \quad \int \frac{(x+1)(x-2)}{\sqrt{x}}dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{3} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q17

$$\int \frac{x^5+x^{-2}+2}{x^2}dx$$

$$= \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2}\right)dx$$

$$= \int x^3dx + \int x^{-4} + 2\int x^{-2}dx$$

$$= \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c$$

$$= \frac{x^4}{4} - \frac{x^{-3}}{3} - \frac{2}{x} + c$$

Indefinite Integrals Ex 19.2 Q18

$\int(3x+4)^2dx$

$$= \int\left(9x^2+16+24x\right)dx$$
$$= 9\int x^2dx + 16\int dx + 24\int xdx$$
$$= 9\frac{x^3}{3}+16x+24\frac{x^2}{2}+c$$
$$= 3x^3 + 16x + 12x^2 + c$$

$$\therefore \quad \int(3x+4)^2 = 3x^3+12x^2+16x+c$$

Indefinite Integrals Ex 19.2 Q19

$\int\frac{2x^4+7x^3+6x^2}{x^2+2x}dx$

$$= \int\frac{x\left(2x^3+7x^2+6x\right)}{x(x+2)}dx$$
$$= \int\frac{2x^3+7x^2+6c}{x+2}dx$$
$$= \int\frac{2x^3+4x^2+3x^2+6x}{(x+2)}dx$$
$$= \int\frac{2x^2(x+2)+3x(x+2)}{(x+2)}dx$$
$$= \int\frac{(x+2)\left(2x^2+3x\right)}{x+2}dx$$
$$= \int\left(2x^2+3x\right)dx$$
$$= \int 2x^2dx + \int 3xdx$$

$$= \frac{2}{3}x^3+\frac{3}{2}x^2+c$$

Indefinite Integrals Ex 19.2 Q20

$$\int\frac{5x^4+12x^3+7x^2}{x^2+x}dx$$
$$= \int\frac{5x^4+7x^3+5x^3+7x^2}{x^2+x}dx$$
$$= \int\frac{5x^3+7x^2+5x^2+7x}{x+1}dx$$
$$= \int\frac{5x^2(x+1)+7x(x+1)}{x+1}dx$$
$$= \int\left(5x^2+7x\right)dx$$
$$= \frac{5x^3}{3}+\frac{7x^2}{2}+C$$

Indefinite Integrals Ex 19.2 Q21

$\int\frac{\sin^2x}{1+\cos x}dx$

$$= \int\frac{1-\cos^2x}{1+\cos x}dx$$
$$= \int\frac{(1-\cos x)(1+\cos x)}{(1+\cos x)}dx$$
$$= \int(1-\cos x)dx$$
$$= x-\sin x+c$$

Indefinite Integrals Ex 19.2 Q22

$\int\left(\sec^2 x + \cos ec^2 x\right)dx$

$$= \int \sec^2 x dx + \int \cos ec^2 x dx$$
$$= \tan x - \cot x + c$$

$\therefore \quad \int\left(\sec^2 x + \cos ec^2 x\right)dx = \tan x - \cot x + c$

Indefinite Integrals Ex 19.2 Q23

Evaluate the integral as follows

$$\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x}dx = \int\left(\frac{\sin^3 x}{\sin x^2 \cos^2 x} - \frac{\cos^3 x}{\sin x^2 \cos^2 x}\right)dx$$
$$= \int\left(\sin x \sec^2 x - \cos x \cos ec^2 x\right)dx$$
$$= \int\left(\tan x \sec x - \cot x \cos ec x\right)dx$$
$$= \sec x + \cos ec x + C$$

Indefinite Integrals Ex 19.2 Q24

$\mathrm{I} \int \frac{5\cos^3 x + 6\sin^3 x}{2\sin^2 x \cos^2 x}dx$

Now,

$$\mathrm{I} = \int \frac{5\cos^3 x + \sin^3 x}{2\sin^2 x \cos^2 x}dx$$
$$= \int \frac{5\cos^3 x}{2\sin^2 x \cos^2 x}dx + \int \frac{6\sin^3 x}{2\sin^2 x \cos^2 x}dx$$
$$= \frac{5}{2}\int \frac{\cos x}{\sin^2 x}dx + 3\int \frac{\sin x}{\cos^2 x}dx$$
$$= \frac{5}{2}\int \cot x \cos ec x dx + 3\int \sec x \tan x dx$$
$$= +\frac{-5}{2}\cos ec x + 3\sec x + c$$

$\therefore \quad \mathrm{I} = \frac{-5}{2}\cos\sec x + 3\sec x + c$

Indefinite Integrals Ex 19.2 Q25

$\int(\tan x + \cot x)^2 dx$

$$= \int\left(\tan^2 x + \cot^2 x + 2\tan x \cot x\right)dx$$
$$= \int\left(\sec^2 x - 1 + \cos ec^2 x - 1 + \frac{2\times 1}{\cot x}\cot x\right)dx$$
$$= \int\left(\sec^2 x + \cos ec^2 x\right)dx$$
$$= \int \sec^2 x dx + \int \cos ec^2 x$$
$$= \tan x - \cot x + c$$

$\therefore \quad \int(\tan x + \cot x)^2 = \tan x - \cot x + c$

Indefinite Integrals Ex 19.2 Q26

$\int \frac{1 - \cos 2x}{1 + \cos 2x}dx$

$$= \int \frac{2\sin^2 x}{2\cos^2 x}dx$$
$$= \int \tan^2 x dx$$
$$= \int\left(\sec^2 x - 1\right)dx$$
$$= \int \sec^2 x dx - \int dx$$
$$= \tan x - x + c$$

$\therefore \quad \int \frac{1 - \cos 2x}{1 + \cos 2x}dx = \tan x - x + c$

Indefinite Integrals Ex 19.2 Q27

$$\int \frac{\cos x}{1-\cos x}dx$$

$$= \int \frac{\cos x\left(1+\cos x\right)}{\left(1-\cos x\right)\left(1+\cos x\right)}dx$$
$$= \int \frac{\cos x+\cos^2 x}{1-\cos^2 x}dx$$
$$= \int \frac{\cos x+\cos^2 x}{\sin^2 x}dx$$
$$= \int \frac{\cos x}{\sin^2 x}dx + \int \frac{\cos^2 x}{\sin^2 x}dx$$
$$= \int \cot x \times \operatorname{cosec} x dx + \int \left(\operatorname{cosec}^2 x - 1\right)dx$$
$$= -\operatorname{cosec} x - \cot x - x + c$$

$$\therefore \quad \int \frac{\cos x}{1-\cos x} \times dx = -\operatorname{cosec} x - \cot x - x + c$$

Indefinite Integrals Ex 19.2 Q28

$$\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}}dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2\cos^2 2x}}dx$$
$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos 2x}dx$$
$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x}dx \qquad \left[\because \cos 2x = \cos^2 x - \sin^2 x\right]$$
$$= \frac{1}{\sqrt{2}}\int 1 \times dx$$
$$= \frac{x}{\sqrt{2}} + c$$

$$\therefore \quad \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}} \times dx = \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.2 Q29

$$\int \frac{1}{1-\cos x}dx$$

$$= \int \frac{1}{1-\cos x} \times \frac{1+\cos x}{1+\cos x} \times dx$$
$$= \int \frac{1+\cos x}{1-\cos^2 x} \times dx$$
$$= \int \frac{1+\cos x}{\sin^2 x} \times dx$$
$$= \int \frac{1}{\sin^2 x}dx + \int \frac{\cos x}{\sin^2 x}dx$$
$$= \int \operatorname{cosec}^2 x dx + \int \cot x \times \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{1}{1-\cos x}dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q30

$\int \frac{1}{1-\sin x}dx$

$$= \int \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} \times dx$$
$$= \int \frac{1+\sin x}{1-\sin^2 x} \times dx$$
$$= \int \frac{1+\sin x}{\cos^2 x} \times dx$$
$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) \times dx$$
$$=$$
$$= \int \frac{1}{\cos^2 x}dx + \int \frac{\sin x}{\cos^2 x} \times dx$$
$$= \int \sec^2 x dx + \int \tan x \sec x dx$$
$$= \tan x + \sec x + c$$

$$\therefore \quad \int \frac{1}{1-\sin x} \times dx = \tan x + \sec x + c.$$

Indefinite Integrals Ex 19.2 Q31

$\int \frac{\tan x}{\sec x + \tan x} \times dx$

$$= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \times dx$$
$$= \int \frac{\tan x (\sec x - \tan x)}{\sec^2 x - \tan^2 x} \times dx$$
$$= \int \left(\tan x \sec x - \tan^2 x\right) dx$$
$$= \int \sec \tan x dx - \int \left(\sec^2 x - 1\right) dx$$
$$= \int \sec x \tan x dx - \int \sec^2 x dx + \int dx$$
$$= \sec x - \tan x + x + c$$

$$\therefore \quad \int \frac{\tan x}{\sec x + \tan x} \times dx = \sec x - \tan x + x + c.$$

Indefinite Integrals Ex 19.2 Q32

$\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx$

$$= \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times \frac{\operatorname{co sec} x + \cot x}{\operatorname{co sec} x + \cot x} \times dx$$
$$= \int \frac{\operatorname{cosec} x (\operatorname{cosec} x + \cot x)}{\operatorname{cosec}^2 x - \cot^{2x}} \times dx$$
$$= \int \left(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x\right) dx$$
$$= \int \operatorname{cos ec}^2 x dx + \int \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q33

$\int \frac{1}{1+\cos 2x} \times dx$

$$= \int \frac{1}{2\cos^2 x} \times dx$$
$$= \frac{1}{2}\int \sec^2 x \times dx$$
$$= \frac{1}{2} \times \tan x + c$$
$$= \frac{\tan x}{2} + c$$

$$\therefore \quad \int \frac{1}{1+\cos 2x} \times dx = \frac{1}{2}\tan x + c.$$

Indefinite Integrals Ex 19.2 Q34

$\int \frac{1}{1-\cos 2x}dx$

$$= \int \frac{1}{2\sin^2 x} \times dx$$
$$= \frac{1}{2}\int cosec^2 x \times dx$$
$$= \frac{-1}{2} \times \cot x + c$$
$$= \frac{-1\cot x}{2} + c$$

$\therefore \quad \int \frac{1}{1-\cos 2x} = \frac{-1}{2}\cot x + c.$

Indefinite Integrals Ex 19.2 Q35

$\int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right]dx$

$$= \int \tan^{-1}\left[\frac{2\sin x\cos x}{2\cos^2 x}\right]dx$$
$$= \int \tan^{-1}\left[\frac{\sin x}{\cos x}\right]dx$$
$$= \int \tan^{-1}(\tan x)\,dx$$
$$= \int x\,dx$$
$$= \frac{x^2}{2} + c$$

$\therefore \quad \int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right]dx = \frac{x^2}{2} + c.$

Indefinite Integrals Ex 19.2 Q36

$\int \cos^{-1}(\sin x)dx$

$$= \int \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right]dx$$
$$= \int \left(\frac{\pi}{2} - x\right)dx$$
$$= \frac{\pi}{2}\int dx - \int x\,dx$$
$$= \frac{\pi}{2} \times x - \frac{x^2}{2} + c$$

$\therefore \quad \int \cos^{-1}(\sin x)dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.$

Indefinite Integrals Ex 19.2 Q37

$\int \cos^{-1}(\sin x)dx$

$$= \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx$$
$$= \int \cot^{-1}\left(\frac{\cos x}{\sin x}\right)dx$$
$$= \int \cot^{-1}(\cot x)dx$$
$$= \int x\,dx$$
$$= \frac{x^2}{2} + c$$

$\therefore \quad \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx = \frac{x^2}{2} + c.$

Indefinite Integrals Ex 19.2 Q38

$\int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right)dx$

$= \int \sin^{-1}(\sin 2x)dx$ $\left[\because \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$

$= \int 2x dx$

$= 2\int x\, dx$

$= \frac{2x^2}{2} + c$

$= x^2 + c$

$\therefore \int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right) = x^2 + c.$

Indefinite Integrals Ex 19.2 Q39

$\int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx$

$= \int \frac{(x+2)\left(x^2-2x+4\right)(x-1)}{x^2-2x+4}dx$

$= \int (x+2)(x-1)\, dx$

$= \int \left(x^2 - x + 2x - 2\right)dx$

$= \int \left(x^2 + x - 2\right)dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

$\therefore \int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c.$

Indefinite Integrals Ex 19.2 Q40

$\int (a\tan x + b\cot x)^2 dx$

$= \int \left(a^2\tan^2 x + b^2\cot^2 x + 2ab\tan x\cot x\right)dx$

$= \int \left[a^2\left(\sec^2 x - 1\right) + b^2\left(\operatorname{cosec}^2 x - 1\right) + 2ab\right]dx$

$= \int \left[a^2\sec^2 x - a^2 + b^2\operatorname{cosec}^2 x - b^2 + 2ab\right]dx$

$= a^2\tan x - a^2 x - b^2\cot x - b^2 x + 2abx + c$

$= a^2\tan x + -b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c$

$\therefore \int (a\tan x + b\cot x)^2 dx = a^2\tan x - b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c.$

Indefinite Integrals Ex 19.2 Q41

$\int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx$

$= \frac{1}{2}\int \frac{x^3}{x^2}dx - \frac{3}{2}\int \frac{x^2}{x^2}dx + \frac{5}{2}\int x\frac{x}{x^2}dx - \frac{7}{2}\int x^{-2}dx + \frac{1}{2}\int \frac{x^2 a^x}{x^2}dx$

$= \frac{1}{2}\times\frac{x^2}{2} - \frac{3}{2}x + \frac{5}{2}\log x - \frac{7}{2}x^{-1} + \frac{1}{2}\frac{a^x}{\log a} + c$

$= \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$

$\therefore \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx = \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$

Indefinite Integrals Ex 19.2 Q42

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2}-1\right]$$

$$= \frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$= \frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$= x-\tan\frac{x}{2}+C$$

Indefinite Integrals Ex 19.2 Q43

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$

$$= \tan^2\frac{x}{2}$$

$$= \left(\sec^2\frac{x}{2}-1\right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x}dx = \int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C$$

$$= 2\tan\frac{x}{2}-x+C$$

Indefinite Integrals Ex 19.2 Q44

$$\int\left\{3\sin x - 4\cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x\right\}dx$$

$$= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 dx - 6\int cosec^2 x + \int \tan^2 x dx - \int \cot^2 x dx$$
$$= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 x dx - 6\int cosec^2 x + \int\left(\sec^2 x - 1\right)dx - \int\left(cosec^2 x - 1\right)dx$$
$$= 3\int \sin x dx - 4\int \cos x dx + 6\int \sec^2 x dx - 7\int cosec^2 x dx$$

$$= -3\cos x - 4\sin x + 6\tan x + 7\cot x + c$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \quad \int f'(x) = \int\left(x - \frac{1}{x^2}\right)dx$$

$$\Rightarrow \quad f(x) = \int x dx - \int \frac{1}{x^2}dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + \frac{1}{x} + c \qquad \text{---(i)}$$

Now,

$$f(1) = \frac{1}{2} \qquad [\text{given}]$$

$$\Rightarrow \quad \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow \quad c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$$\therefore \quad \int f'(x) = \int (x+b)dx$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + bx + c \qquad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \quad \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \quad \frac{1}{2} + b + c = 5$$

$$\Rightarrow \quad b + c = \frac{9}{2} \qquad \text{---(ii)}$$

and, $f(2) = 13$

$$\Rightarrow \quad \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow \quad 2 + 2b + c = 13$$

$$\Rightarrow \quad 2b + c = 11 \qquad \text{---(iii)}$$

Subtracting equation (ii) from equatio n(iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow \quad b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow \quad c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow \quad c = \frac{9-13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{x} + \frac{13}{2}x - 2$$

$$\therefore \quad f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

Indefinite Integrals Ex 19.2 Q47

We have,

$$f'(x) = 8x^3 - 2x$$

$$\Rightarrow \quad f(x) = \int f'(x)dx = \int \left(8x^3 - 2x\right)dx$$

$$\Rightarrow \quad f(x) = \int \left(8x^3 - 2x\right)dx$$

$$= \int 8x^3 dx - \int 2x dx$$

$$= \frac{8x^4}{4} - \frac{2x^2}{2} + c$$

$$= 2x^4 - x^2 + c$$

$$\Rightarrow \quad f(x) = 2x^4 - x^2 + c \qquad \text{---(i)}$$

Since, $f(2) = 8$

$$\therefore \quad f(2) = 2(2)^4 - (2)^2 + c = 8$$

$$\Rightarrow \quad 32 - 4 + c = 8$$

$$\Rightarrow \quad 28 + c = 8$$

$$\Rightarrow \quad c = -20$$

Putting $c = -20$ in equation (i), we get

$$f(x) = 2x^4 - x^2 - 20$$

Hence, $f(x) = 2x^4 - x^2 - 20$.

Indefinite Integrals Ex 19.2 Q48

We have,

$$f(x) = \int f'(x)\,dx$$

$$\Rightarrow \quad f(x) = \int (a\sin x + b\cos x)\,dx$$

$$= -a\cos x + b\sin x + c$$

$$\therefore \quad f(x) = -a\cos x + b\sin x + c \qquad \text{---(i)}$$

Since,

$$f'(0) = 4$$

$$\therefore \quad f'(0) = a\sin 0 + b\cos 0 = 4$$

$$\Rightarrow \quad a \times 0 + b \times 1 = 4$$

$$\Rightarrow \quad b = 4$$

Now,

$$f(0) = 3$$

$$\therefore \quad f(0) = -a\cos 0 + b\sin 0 + c = 3$$

$$\Rightarrow \quad -a + 0 + c = 3$$

$$\Rightarrow \quad c - a = 3 \qquad \text{---(ii)}$$

and, $f\left(\frac{\pi}{2}\right) = 5$

$$\therefore \quad f\left(\frac{\pi}{2}\right) = -a\cos\left(\frac{\pi}{2}\right) + b\sin\left(\frac{\pi}{2}\right) + c = 5$$

$$\Rightarrow \quad -a \times 0 + b \times 1 + c = 5$$

$$\Rightarrow \quad b + c = 5$$

$$\Rightarrow \quad 4 + c = 5 \qquad [\because b = 4]$$

$$\Rightarrow \quad c = 5 - 4$$

$$\Rightarrow \quad c = 1$$

Putting $c = 1$ in equation (ii), we get

$$1 - a = 3$$

$$\Rightarrow \quad -a = 3 - 1$$

$$\Rightarrow \quad -a = 2$$

$$\Rightarrow \quad a = -2$$

Putting $a = -2$, $b = 4$ and $c = 1$ in equatio (i), we get

$$f(x) = -(-2)\cos x + 4\sin x + 1$$

$$\Rightarrow \quad f(x) = 2\cos x + 4\sin x + 1$$

Hence, $f(x) = 2\cos x + 4\sin x + 1$

Indefinite Integrals Ex 19.2 Q49

We have,

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\Rightarrow \quad \int f(x) = \int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx + \int x^{\frac{-1}{2}}dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

Hence, the primitive or anti-derivative of $f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.

Ex 19.3

Indefinite Integrals Ex 19.3 Q1

Let $I = \int [2x-3]^5 + \sqrt{3x+2}\,dx$. Then,

$$I = \int (2x-3)^5\,dx + \int (3x+2)^{\frac{1}{2}}dx$$
$$= \frac{(2x-3)^6}{2\times 6} + \frac{(3x+2)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$
$$= \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.3 Q2

Let $I = \int \left[\frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}}\right]dx$. Then,

$$I = \int (7x-5)^{-3}dx + \int (5x-4)^{\frac{-1}{2}}dx$$
$$= \frac{(7x-5)^{-2}}{7\times(-2)} + \frac{(5x-4)^{\frac{1}{2}}}{5\times\frac{1}{2}} + c$$
$$= -\frac{(7x-5)^{6-2}}{14} + \frac{2}{5}\sqrt{(5x-4)} + c$$

$$\therefore \quad I = \frac{-1}{14}(7x-5)^{-2} + \frac{2}{5}\times\sqrt{5x-4} + c.$$

Indefinite Integrals Ex 19.3 Q3

Let $I = \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}}dx$. Then,

$$I = \int \frac{1}{2-3x}dx + \int \frac{1}{\sqrt{3x-2}}dx$$
$$= \frac{\log|2-3x|}{-3} + \frac{2}{3}(3x-2)^{\frac{1}{2}}c$$
$$= \frac{-1}{3}\times\log|2x-3| + \frac{2}{3}\times\sqrt{3x-2} + c$$

Indefinite Integrals Ex 19.3 Q4

Let $I = \int \frac{x+3}{(x+1)^4}dx$. Then,

$$I = \int \frac{x+1+2}{(x+1)^4}dx$$
$$= \int \frac{x+1}{(x+1)^4}\times dx + 2\int \frac{1}{(x+1)^4}\times dx$$
$$= \int \frac{1}{(x+1)^3}\times dx + 2\int \frac{1}{(x+1)^4}\times dx$$
$$= \int (x+1)^{-3}\times dx + 2\int (x-1)^{-4}dx$$
$$= \frac{(x+1)^{-2}}{-2} + 2\frac{(x+1)^{-3}}{-3} + c$$
$$= -\frac{1}{2}\times\frac{1}{(x+1)^2} - \frac{2}{3}\times\frac{1}{(x+1)^3} + c$$

$$\therefore \quad I = \frac{-1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + c$$

Indefinite Integrals Ex 19.3 Q5

Let $I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \times \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{\left(\sqrt{x+1}\right)^2-\left(\sqrt{x}\right)^2} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} \times dx$$
$$= \int \left(\sqrt{x+1}-\sqrt{x}\right) \times dx$$
$$= \int (x+1)^{\frac{1}{2}} dx - \int x^{\frac{1}{2}} dx$$
$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q6

Let $I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} dx$. Then,

$$I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}} \times dx$$
$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\left(\sqrt{2x+3}\right)^2-\left(\sqrt{2x-3}\right)^2} \times dx$$
$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{2x+3-2x+3} \times dx$$
$$= \frac{1}{6}\int (2x+3)^{\frac{1}{2}} dx - \frac{1}{6}\int (2x-3)^{\frac{1}{2}} dx$$
$$= \frac{1}{6} \times \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \frac{1}{6}\frac{(2x-3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + c$$
$$= \frac{1}{18} \times (2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \frac{2x}{(2x+1)^2} dx$. Then,

$$I = \int \frac{2x+1-1}{(2x+1)^2} \times dx$$
$$= \int \frac{2x+1}{(2x+1)^2} \times dx - \int \frac{1}{(2x+1)^2} \times dx$$
$$= \int \frac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx$$
$$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-1}}{-1\times 2} + c$$
$$= \frac{1}{2}\log|2x+1| + \frac{1}{2} \times \frac{1}{2x+1} + c$$

$$\therefore \quad I = \frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + c.$$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b} \times dx$$
$$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$$
$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c.$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin \sqrt{1+\cos 2x}\ dx$

$$I = \int \sin x \times \sqrt{2\cos^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \cos x \times dx$$
$$= \sqrt{2}\int \sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int \sin 2x dx$$
$$= \frac{\sqrt{2}}{2} \times \frac{-\cos 2x}{2} + c$$
$$= \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

$$\therefore \quad I = \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \frac{1+\cos x}{1-\cos x} dx$. Then,

$$I = \int \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} \times dx$$
$$= \int \frac{\cos^2 \frac{x}{2}}{\sin^2 \frac{x}{2}} \times dx$$
$$= \int \cot^2 \frac{x}{2} \times dx$$
$$= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1\right) dx$$
$$= \frac{-\cot \frac{x}{2}}{\frac{1}{2}} - x + c$$
$$= -2\cot \frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q11

Let $I = \int \frac{1-\cos x}{1+\cos x} \times dx$. Then,

$$I = \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \times dx$$

$$= \int \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}} \times dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1\right) dx$$

$$= \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= 2\tan \frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \frac{1}{1-\sin \frac{x}{2}}$. Then,

$$I = \int \frac{1}{1-\sin \frac{x}{2}} \times \frac{1+\sin \frac{x}{2}}{1+\sin \frac{x}{2}} dx$$

$$= \int \frac{1+\sin \frac{x}{2}}{1-\sin^2 \frac{x}{2}} \times dx$$

$$= \int \frac{1+\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} \times dx$$

$$= \int \frac{1}{\cos^2 \frac{x}{2}} dx + \int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx$$

$$= \int \sec^2 \frac{x}{2} dx + \int \sec \frac{x}{2} \tan \frac{x}{2} dx$$

$$= \frac{\tan \frac{x}{2}}{\frac{1}{2} 2} + + \frac{\sec \frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan \frac{x}{2} + 2\sec \frac{x}{2} + c$$

$$\therefore \quad I = 2\left(\tan \frac{x}{2} + \sec \frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \frac{1}{1+\cos 3x} \times dx$. Then,

$$I = \int \frac{1}{1+\cos 3x} \times \frac{1-\cos 3x}{1-\cos 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{1-\cos^2 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{\sin^2 3x} \times dx$$

$$= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x} \right) dx$$

$$= \int \left(\operatorname{cosec}^2 3x - \operatorname{cosec} 3x \cot 3x \right) dx$$

$$= \frac{-\cot 3x}{3} + \frac{\operatorname{cosec} 3x}{3} + c$$

$$= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + c$$

$$= \frac{1-\cos 3x}{3 \sin 3x} + c$$

$$\therefore \quad I = \frac{1-\cos 3x}{3 \sin 3x} + c.$$

Indefinite Integrals Ex 19.3 Q14

Consider $I = \int (e^x + 1)^2 e^x dx$

let $(e^x + 1) = t \rightarrow e^x dx = dt$

$$I = \int (e^x + 1)^2 e^x dx$$

$$= \int (t)^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(e^x + 1)^3}{3} + C$$

Indefinite Integrals Ex 19.3 Q15

Let $I = \int \left(e^x + \frac{1}{e^x} \right)^2 dx$. Then,

$$I = \int \left(e^x + \frac{1}{e^x} \right)^2 \times dx$$

$$= \int \left(e^{2x} + \frac{1}{e^{2x}} + 2 \right) dx$$

$$= \frac{e^{2x}}{2} - \frac{1}{2} e^{-2x} + 2x + c$$

$$\therefore \quad I = \frac{1}{2} \times e^{2x} + 2x - \frac{1}{2} \times e^{-2x} + c$$

Indefinite Integrals Ex 19.3 Q16

Let $I = \int \frac{1+\cos 4x}{\cot x - \tan x} \times dx$. Then,

$$I = \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx$$

$$= \int \frac{2\cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} dx$$

$$= \int \frac{\cos^2 2x \times \sin 2x}{\cos^2 2x} dx$$

$$= \int \cos 2x \times \sin 2x \times dx$$

$$= \frac{1}{2} \int 2 \sin 2x \cos 2x dx$$

$$= \frac{1}{2} \int \left[\sin(2x+2x) + \sin(2x-2x)\right] dx$$

$$= \frac{1}{2} \int (\sin 4x + \sin 0) dx$$

$$= \frac{1}{2} \int (\sin 4x + 0) dx$$

$$= \frac{1}{2} \int \sin 4x$$

$$= -\frac{1}{2} \times \frac{\cos 4x}{4} + c$$

$$= -\frac{1}{8} \times \cos 4x + c$$

Indefinite Integrals Ex 19.3 Q17

Let $I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx$$

$$= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{x+3-x-2} dx$$

$$= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}}\right] dx$$

$$= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3} \times (x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

$$= \frac{2}{3}\left\{(x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}}\right\} + c$$

$$\therefore \quad I = \frac{2}{3}\left\{(x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}}\right\} + c$$

Indefinite Integrals Ex 19.3 Q18

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let $2x - 3 = t$

$\Rightarrow 2dx = dt$

$$\Rightarrow \int \tan^2(2x-3) dx = \int \left[\left(\sec^2(2x-3)\right) - 1\right] dx$$

$$= \frac{1}{2} \int \left(\sec^2 t\right) dt - \int 1 dx$$

$$= \frac{1}{2} \int \sec^2 t \, dt - \int 1 dx$$

$$= \frac{1}{2} \tan t - x + C$$

$$= \frac{1}{2} \tan(2x-3) - x + C$$

Indefinite Integrals Ex 19.3 Q19

Consider $I = \int \frac{1}{\cos^2 x\left(1-\tan x\right)^2}dx$

$$= \int \frac{1}{\cos^2 x\left(1-\frac{\sin x}{\cos x}\right)^2}dx$$

$$= \int \frac{1}{\left(\cos x-\sin x\right)^2}dx$$

$$= \int \frac{1}{1-\sin 2x}dx$$

$$= \int \frac{1}{1+\cos\left(\frac{\pi}{2}+2x\right)}dx$$

$$= \int \frac{1}{2\cos^2\left(\frac{\pi}{4}+x\right)}dx$$

$$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4}+x\right)dx$$

Ex 19.4

Indefinite Integrals Ex 19.4 Q1

Let $I = \int \frac{x^2 + 5x + 2}{x + 2} dx$

Using long division method, we have

$$\frac{x^2 + 5x + 2}{x + 2} = x + 3 - \frac{4}{x + 2}$$

$$\therefore \quad I = \int \frac{x^2 + 5x + 2}{x + 2} = \int \left(x + 3 - \frac{4}{x + 2} \right) dx$$

$$\Rightarrow \quad I = \int x dx + 3\int dx - 4\int \frac{1}{x + 2} dx$$

$$= \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

Indefinite Integrals Ex 19.4 Q2

Let $I = \int \frac{x^3}{x - 2} dx$

Using long division method, we have

$$\frac{x^3}{x - 2} = x^2 + 2x + 4 + \frac{8}{x - 2}$$

$$\therefore \quad I = \int \left(x^2 + 2x + 4 + \frac{8}{x - 2} \right) dx$$

$$= \int x^2 dx + 2\int x dx + 4\int dx + 8\int \frac{1}{x - 2} dx$$

$$= \frac{x^3}{3} + \frac{2x^2}{2} + 4x + 8\log|x - 2| + c$$

$$= \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

$$\therefore \quad I = \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

Indefinite Integrals Ex 19.4 Q3

Let $I = \int \frac{x^2 + x + 5}{3x + 2} dx$

Using long division method, we have

$$\frac{x^2 + x + 5}{3x + 2} = \frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2}$$

$$\therefore \quad I = \int \left(\frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2} \right) dx$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{9 \times 3} |3x + 2| + c$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log|3x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log|3x + 2| + c$$

Indefinite Integrals Ex 19.4 Q4

Let $I = \int \frac{2x+3}{(x-1)^2} dx$. Then,

$$I = \int \frac{2x+2-2+3}{(x-1)^2} \times dx$$

$$= \int \frac{2x-2+5}{(x-1)^2} \times dx$$

$$= 2\int \frac{(x-1)}{(x-1)^2} \times dx + 5\int \frac{1}{(x-1)^2} dx$$

$$= 2\int \frac{1}{x-1} \times dx + 5\int (x-1)^{-2} \times dx$$

$$= 2\log|x-1| + 5 \times \frac{(x-1)^{-1}}{-1} + c$$

$$= 2\log|x-1| - \frac{5}{x-1} + c$$

$\therefore \quad I = 2\log|x-1| - \frac{5}{x-1} + c.$

Indefinite Integrals Ex 19.4 Q5

Let $I = \int \frac{x^2+3x-1}{(x+1)^2} dx$. Then,

$$I = \int \frac{x^2+x+2x-1}{(x+1)^2} dx$$

$$= \int \frac{x(x+1)+2x-1}{(x+1)^2} dx$$

$$= \int \frac{x(x+1)}{(x+1)^2} dx + \int \frac{2x-1}{(x+1)^2} dx$$

$$= \int \frac{x}{x+1} dx + \int \frac{\sqrt{2x+2-2-1}}{(x+1)^2} dx$$

$$= \int \frac{x+1-1}{x+1} dx + \int \frac{2(x+1)-3}{(x+1)^2} dx$$

$$= \int \frac{x+1}{x+1} dx - \int \frac{1}{x+1} dx + \int \frac{2(x+1)}{(x+1)^2} dx - 3\int \frac{1}{(x+1)^2} dx$$

$$= \int dx - \int \frac{1}{x+1} dx + 2\int \frac{1}{x+1} dx - 3\int (x+1)^{-2} dx$$

$$= x - \log|x+1| + 2\log|x+1| + \frac{3}{x+1} + c$$

$$= x + \log|x+1| + \frac{3}{x+1} + c$$

$\therefore \quad I = x + \log|x+1| + \frac{3}{x+1} + c$

Indefinite Integrals Ex 19.4 Q6

Let $I = \int \frac{2x-1}{(x-1)^2} dx$. Then,

$$I = \int \frac{2x-1+2-2}{(x-1)^2} dx$$

$$= \int \frac{2x-2+1}{(x-1)^2} dx$$

$$= \int \frac{2(x-1)}{(x-1)^2} dx + 1\int \frac{1}{(x-1)^2} dx$$

$$= 2\int \frac{1}{x-1} dx + \int (x-1)^2 dx$$

$$= 2\log|x-1| - (x-1)^{-1} + c$$

$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$\therefore \quad I = \frac{-1}{x-1} + 2\log|x-1| + c.$

Ex 19.5

Indefinite Integrals Ex 19.5 Q1

Let $I = \int \frac{x+1}{\sqrt{2x+3}} dx$

Let $x + 1 = \lambda(2x+3) + \mu$

On equating the coefficients of like powers of x on bothy sides, we get

$$I = 2\lambda, \ 3\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } \mu = \frac{-1}{2}$$

Replacing $x+1$ by $\lambda(2x+3)+\mu$ in the given integral, we get

$$I = \int \frac{\lambda(2x+3)+\mu}{\sqrt{2x+3}} dx$$

$$= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}} dx + \mu \int \frac{1}{\sqrt{2x+3}} dx$$

$$= \lambda \int (2x+3)^{\frac{1}{2}} dx + \mu \int (2x+3)^{\frac{-1}{2}} dx$$

$$= \lambda \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + \mu \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c$$

$$= \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right) \times (2x+3)^{\frac{1}{2}} + c \qquad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2}\right]$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

$$\therefore \quad I = \frac{1}{6} \times (2x+3)^{\frac{3}{2}} - \frac{1}{2}(2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2}dx$. Then,

$$I = \int \{(x+2) - 2\}x + 2dx \qquad [\because x = (x+2) - 2]$$

$$\Rightarrow \quad I = f\left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\} dx$$

$$\Rightarrow \quad I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}} dx$. Then,

$$I = \int \frac{x+4-4-1}{\sqrt{x+4}}$$

$$= \int \frac{x+4-5}{\sqrt{x+4}} dx$$

$$= \int \frac{x+4}{\sqrt{x+4}} dx - 5\int \frac{1}{\sqrt{x+4}} dx$$

$$= \int (x+4)^{\frac{1}{2}} dx - 5\int (x+4)^{\frac{-1}{2}} dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.5 Q4

Let $I = \int (x+2)\sqrt{3x+5}dx$

Let $x+2 = \lambda(3x+5)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \quad \text{and} \quad 5\lambda + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad 5\times\frac{1}{3}+\mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3}$$

$$\therefore \quad I = \int\left\{\lambda(3x+5)+\mu\right\}\sqrt{3x+5}dx$$

$$= \lambda\int(3x+5)\sqrt{3x+5}dx + \mu\int\sqrt{3x+5}$$

$$= \lambda\int(3x+5)^{\frac{3}{2}}dx + \mu\int(3x+5)^{\frac{1}{2}}dx$$

$$= \lambda\times\frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$

$$= \frac{1}{3}\times\frac{2}{15}\times(3x+5)^{\frac{5}{2}} + \frac{1}{3}\times\frac{2}{9}(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{45}\times(3x+5)^{\frac{5}{2}} + \frac{2}{27}\times(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{1}{5}\times(3x+5)^{1}+\frac{1}{3}\right]+c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{3(3x+5)+5}{15}\right]+c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\frac{(9x+15+5)}{15}+c$$

$$= \frac{2}{135}\times(3x+5)^{\frac{3}{2}}(9x+20)+c$$

$$\therefore \quad I = \frac{2}{135}\times(9x+20)(3x+5)^{\frac{3}{2}}+c.$$

Indefinite Integrals Ex 19.5 Q5

Let $I = \int \frac{2x+1}{\sqrt{3x+2}}dx$

Let $2x+1 = \lambda(3x+2)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \text{ and } 2\lambda + \mu = 1$$

$$\Rightarrow \lambda = \frac{2}{3} \text{ and } 2 \times \frac{2}{3} + \mu = 1$$

$$\Rightarrow \lambda = \frac{2}{3} \text{ and } \mu = \frac{-1}{3}$$

$$\therefore I = \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$$

$$= \lambda\int \frac{3x+2}{\sqrt{3x+2}}dx + \mu\int \frac{1}{\sqrt{3x+2}}dx$$

$$= \lambda\int (3x+2)^{\frac{1}{2}}dx + \mu\int (3x+2)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times 3} + \mu\frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times 3} + c$$

$$= \frac{2}{3}\times\frac{2}{9}\times(3x+2)^{\frac{3}{2}} - \frac{1}{3}\times\frac{2}{3}(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{4}{27}\times(3x+2)^{\frac{3}{2}} - \frac{2}{9}\times(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{2}{3}\times(3x+2) - 1\right] + c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{6x+4-3}{3}\right] + c$$

$$= \frac{2}{27}\times\sqrt{3x+2}\,(6x+1) + c$$

$$\therefore I = \frac{2}{27}\times(6x+1)\sqrt{3x+2} + c.$$

Indefinite Integrals Ex 19.5 Q6

Let $I = \int \frac{3x+5}{\sqrt{7x+9}}dx$

Let $3x+5 = \lambda(7x+9)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$7\lambda = 3 \text{ and } 9\lambda + \mu = 5$$

$$\Rightarrow \lambda = \frac{3}{7} \text{ and } 9 \times \frac{3}{7} + \mu = 5$$

$$\Rightarrow \lambda = \frac{3}{7} \text{ and } \mu = \frac{8}{7}$$

$$\therefore I = \int \frac{\lambda(7x+9)+\mu}{\sqrt{7x+9}}dx$$

$$= \lambda\int \frac{7x+9}{\sqrt{7x+9}}dx + \mu\int \frac{1}{\sqrt{7x+9}}dx$$

$$= \lambda\int (7x+9)^{\frac{1}{2}}dx + \mu\int (7x+9)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2}\times 7} + \mu\frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2}\times 7} + c$$

$$= \frac{3}{7}\times\frac{2}{21}\times(7x+9)^{\frac{3}{2}} + \frac{8}{7}\times\frac{2}{7}(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{3}{2}} + \frac{16}{49}\times(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+9+8] + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+17] + c$$

$$= \frac{2}{49}\times(7x+17)\sqrt{7x+9} + c$$

Indefinite Integrals Ex 19.5 Q7

Let $I = \int \frac{x}{\sqrt{x+4}} dx$. Then,

$$I = \int \frac{x+4-4}{\sqrt{x+4}} dx$$
$$= \int \frac{x+4}{\sqrt{x+4}} dx - 4\int \frac{1}{\sqrt{x+4}} dx$$
$$= \int (x+4)^{\frac{1}{2}} dx - 4\int (x+4)^{\frac{-1}{2}} dx$$
$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$
$$= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + c$$
$$= 2(x+4)^{\frac{1}{2}}\left[\frac{1}{3}(x+4) - 4\right] + c$$
$$= 2(x+4)^{\frac{1}{2}}\left[\frac{(x+4)-12}{3}\right] + c$$
$$= \frac{2}{3}(x+4)^{\frac{1}{2}}\left[x-8\right] + c$$

$$\therefore \quad I = \frac{2}{3} \times (x-8)\sqrt{x+4} + c.$$

Indefinite Integrals Ex 19.5 Q8

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$I = \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx$$
$$= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx$$
$$= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$
$$= -1\int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$
$$= -1\int (1+3x)^{\frac{1}{2}} dx + 3\int (1+3x)^{\frac{-1}{2}} dx$$
$$= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c$$
$$= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[-\frac{1}{9}(1+3x)^{1} + 1\right] + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[\frac{-1-3x+9}{9}\right] + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[\frac{8-3x}{9}\right] + c$$
$$= \frac{2}{9}\sqrt{1+3x}\,(8-3x) + c$$

$$\therefore \quad I = \frac{2}{9}(8-9x)\sqrt{1+3x} + c$$

Indefinite Integrals Ex 19.5 Q9

Let $I = \int 5x + 3\sqrt{2x-1}\,dx$

Let $5x + 3 = \lambda(2x-1) + \mu$ comparing both sides, we get

$$2\lambda = 5 \quad \text{and} \quad -\lambda + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \frac{-5}{2} + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}$$

$$\therefore \quad I = \int \left\{\lambda(2x-1) + \mu\right\}\sqrt{2x-1}dx$$

$$= \lambda\int (2x-1)\sqrt{2x-1}dx + \mu\int \sqrt{2x-1}dx$$

$$= \lambda\int (2x-1)^{\frac{3}{2}}dx + \mu\int (2x-1)^{\frac{1}{2}}dx$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{\frac{5}{2}\times 2} + \mu\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{5} + \mu\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{5}{2}\times\frac{(2x-1)^{\frac{5}{2}}}{5} + \frac{11}{2}\times\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{(2x-1)^{\frac{5}{2}}}{2} + \frac{11}{6}\times(2x-1)^{\frac{3}{2}} + c$$

$$= \frac{1}{2}(2x-1)^{\frac{3}{2}}\left[(2x-1) + \frac{11}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\left[\frac{6x+8}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\times 2\frac{(3x+4)}{3} + c$$

$$= (2x-1)^{\frac{3}{2}}\times\frac{(3x+4)}{3} + c$$

$$= \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c.$$

Ex 19.6

Indefinite Integrals Ex 19.6 Q1

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}\,dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

Indefinite Integrals Ex 19.6 Q2

We need to evaluate $\int \sin^3(2x+1)\,dx$

by using the formula $\rightarrow \sin 3\theta = -4\sin^3\theta + 3\sin\theta$

$$\therefore \sin^3(2x+1) = \frac{3\sin(2x+1) - \sin 3(2x+1)}{4}$$

$$\int \sin^3(2x+1)\,dx$$

$$= \int \frac{3\sin(2x+1) - \sin 3(2x+1)}{4}\,dx$$

$$= \int \frac{3\sin(2x+1) - \sin 3(2x+1)}{4}\,dx$$

$$= -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos 3(2x+1) + C$$

Indefinite Integrals Ex 19.6 Q3

Evaluate the integral as follows

$$\text{I} \quad \int \cos^4 2x\,dx = \int (\cos^2 2x)^2\,dx$$

$$= \int \left(\frac{1}{2}(\cos 4x+1)\right)^2 dx$$

$$= \int \left(\frac{1}{4}(\cos^2 4x) + \frac{1}{4} + \frac{\cos 4x}{2}\right) dx$$

$$= \int \left(\frac{1}{4}\left(\frac{1}{2}(\cos 8x+1)\right) + \frac{1}{4} + \frac{\cos 4x}{2}\right) dx$$

$$= \int \frac{1}{8}\left(\cos 8x + \frac{3}{8} + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{1}{64}\sin 8x + \frac{3}{8}x + \frac{1}{8}\sin 4x + C$$

Indefinite Integrals Ex 19.6 Q4

Let $I = \int \sin^2 bx dx$. Then,

$$I = \int \frac{1-\cos 2bx}{2} \times dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2bx dx$$

$$= \frac{1}{2}x - \frac{1}{2}\frac{\sin(2bx)}{2b} + c$$

$$\therefore \quad I = \frac{x}{2} - \frac{\sin 2bx}{4b} + c$$

Indefinite Integrals Ex 19.6 Q5

Let $I = \int \sin^2 \frac{x}{2} dx$. Then,

$$I = \frac{1}{2}\int 2\sin^2 \frac{x}{2} dx$$
$$= \frac{1}{2}\int (1 - \cos x) dx \qquad \left[\because \cos 2x = 1 - 2\sin^2 x\right]$$
$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx$$
$$= \frac{1}{2} \times x - \frac{1}{2} \times \sin x + c$$
$$= \frac{1}{2}(x - \sin x) + c$$

$$\therefore \qquad I = \frac{1}{2}(x - \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q6

We have,

$$\int \cos^2 \frac{x}{2} dx = \frac{1}{2}\int 2\cos^2 \frac{x}{2} dx$$
$$= \frac{1}{2}\int 1 + \cos x dx$$
$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos x dx$$
$$= \frac{1}{2} \times x + \frac{1}{2}\sin x + c$$
$$= \frac{1}{2}(x + \sin x) + c$$

$$\therefore \qquad \int \cos^2 \frac{x}{2} = \frac{1}{2}(x + \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q7

Let $I = \int \cos^2 nx dx$. Then,

$$I = \frac{1}{2}\int 2\cos^2 nx dx$$
$$= \frac{1}{2}\int [1 + \cos 2nx] dx$$
$$= \frac{1}{2}\int \left[x + \frac{\sin 2nx}{2n}\right] + c$$
$$= \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c$$

$$\therefore \qquad I = \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c.$$

Indefinite Integrals Ex 19.6 Q8

Let $I = \int \sin\sqrt{1-\cos 2x}\ dx$. Then,

$$I = \int \sin x \times \sqrt{2\sin^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \sin x dx$$
$$= \sqrt{2}\int \sin^2 x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin^2 x \times dx$$
$$= \frac{\sqrt{2}}{2}\left[x - \frac{\sin 2x}{2}\right] + c$$
$$= \frac{\sqrt{2}x}{2} - \frac{\sqrt{2}}{4} \times \sin 2x + c$$
$$= \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c$$

$$\therefore \quad I = \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c.$$

Ex 19.7

Indefinite Integrals Ex 19.7 Q1

Let $I = \int \sin 4x \cos 7x dx$. Then,

$$I = \frac{1}{2}\int 2\sin 4x \times \cos 7x dx$$
$$= \frac{1}{2}\int \left(\sin 11x + \sin(-3x)\right)dx$$
$$= \frac{1}{2}\int \sin 11x dx - \frac{1}{2}\int \sin 3x dx$$
$$= \frac{-1}{2\times 11}\times \cos 11x + \frac{1}{2\times 3}\cos 3x + c$$
$$= -\frac{1}{22}\times \cos 11x + \frac{1}{6}\times \cos 3x + c$$

$$\therefore \quad I = \frac{-1}{22}\times \cos 11x + \frac{1}{6}\times \cos 3x + c.$$

Indefinite Integrals Ex 19.7 Q2

Let $I = \int \cos 3x \cos 4x dx$. Then,

$$I = \frac{1}{2}\int (2\cos 3x \cos 4x)\times dx$$
$$= \frac{1}{2}\int \left(\cos 7x + \cos(-x)\right)dx$$
$$= \frac{1}{2}\int \cos 7x + \frac{1}{2}\int \cos dx \qquad [\because \cos(-0) = \cos 0]$$
$$= \frac{\sin 7x}{2\times 7} + \frac{\sin x}{2} + c$$
$$= \frac{1}{14}\times \sin 7x + \frac{1}{2}\sin x + c$$

$$\therefore \quad I = \frac{1}{14}\times \sin 7x + \frac{1}{2}\times \sin x + c.$$

Indefinite Integrals Ex 19.7 Q3

Let $I = \int \cos mx \cos nx \; dx \;\; m \neq n$. Then,

$$I = \frac{1}{2}\int 2\cos mx \cos nx \; dx$$
$$= \frac{1}{2}\int \left[\cos(m+n)x + \cos(m-n)x\right]dx$$
$$= \frac{1}{2}\times \frac{\sin(m+n)x}{m+n} + \frac{1}{2}\times \frac{\sin(m-n)x}{m-n} + c$$

$$\therefore \quad I = \frac{1}{2}\left[\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n}\right] + c.$$

Indefinite Integrals Ex 19.7 Q4

We have,

$$\int \sin mx \cos nx \; dx, \; m \neq n$$
$$= \frac{1}{2}\int 2\sin mx \cos nx \; dx$$
$$= \frac{1}{2}\int \left[\sin(m+n)x + \sin(m-n)x\right]dx$$
$$= \frac{1}{2}\times\left[\frac{-\cos(m+n)}{m+n} - \frac{\cos(m-n)}{m-n}\right] + c$$

$$\therefore \quad \int \sin mx \cos nx = \frac{1}{2}\left[\frac{-\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n}\right] + c.$$

Ex 19.8

Indefinite Integrals Ex 19.8 Q1

We have,

$$\int \frac{1}{\sqrt{1-\cos 2x}}dx = \int \frac{1}{\sqrt{2\sin^2 x}}dx$$

$$= \int \frac{1}{\sqrt{2}\sin x}dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec} x\, dx$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1-\cos 2x}}dx = \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q2

We have,

$$\int \frac{1}{\sqrt{1+\cos x}}dx = \int \frac{1}{\sqrt{2\cos^2\frac{x}{2}}}dx$$

$$= \int \frac{1}{\sqrt{2}\cos\frac{x}{2}}dx$$

$$= \frac{1}{\sqrt{2}}\int \sec\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(\frac{\pi}{2}+\frac{x}{2}\right)dx$$

$$= \frac{2}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1+\cos x}}dx = \sqrt{2}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q3

Let $I = \int \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}dx$ then,

$$I = \int \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}dx$$

$$= \int \sqrt{\cot^2 x}\,dx$$

$$= \int \cot x\,dx$$

$$= \log|\sin x| + c \qquad \left[\because \quad \int \cot x = \log|\sin x| + c\right]$$

$$I = \log|\sin x| + c$$

Indefinite Integrals Ex 19.8 Q4

Let $I = \int \sqrt{\frac{1-\cos x}{1+\cos x}}dx$ then,

$$I = \int \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}dx$$

$$= \int \sqrt{\tan^2\frac{x}{2}}dx$$

$$= \int \tan\frac{x}{2}dx$$

$$= -2\log\left|\cos\frac{x}{2}\right| + c \qquad \left[\because \quad \int \tan x\,dx = \log|\cos x| + c\right]$$

$$\because \quad I = -2\log\left|\cos\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q5

Let $I = \int \frac{\sec x}{\sec 2x}\,dx$, then,

$$I = \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos 2x}}\,dx$$

$$= \int \frac{\cos 2x}{\cos x}\,dx$$

$$= \int \frac{2\cos^2 x - 1}{\cos x}\,dx$$

$$= \int 2\cos x\,dx - \int \frac{1}{\cos x}\,dx$$

$$= 2\int \cos dx - \int \sec x\,dx$$

$$= 2\sin x - \log|\sec x + \tan x| + c$$

$$\therefore \quad I = 2\sin x - \log|\sec x + \tan x| + c$$

Indefinite Integrals Ex 19.8 Q6

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x\cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\,dx = \int \frac{\cos 2x}{(1 + \sin 2x)}\,dx$$

Let $1 + \sin 2x = t$

$\Rightarrow 2\cos 2x\,dx = dt$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}\,dx = \frac{1}{2}\int \frac{1}{t}\,dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log|1 + \sin 2x| + C$$

$$= \frac{1}{2}\log\left|(\sin x + \cos x)^2\right| + C$$

$$= \log|\sin x + \cos x| + C$$

Indefinite Integrals Ex 19.8 Q7

Let $I = \int \frac{\sin(x-a)}{\sin(x-b)}\,dx$ then

$$I = \int \frac{\sin(x - a + b - b)}{\sin(x-b)}\,dx$$

$$= \int \frac{\sin(x - b + b - a)}{\sin(x-b)}\,dx$$

$$= \int \frac{\sin(x-b)\cos(b-a) + \cos(x-b)\sin(b-a)}{\sin(x-b)}\,dx$$

$$= \int \left(\cos(b-a) + \cot(x-b)\sin(b-a)\right)dx$$

$$= \cos(b-a)\int dx + \sin(b-a)\int \cot(x-b)\,dx$$

$$= x\cos(b-a) + \sin(b-a)\log|\sin(x-b)| + c$$

$$\therefore \quad I = x\cos(b-a) + \sin(b-a)\log|\sin(x-b)| + c$$

Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$ then,

$$I = \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha)\cos 2\alpha - \cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}dx$$
$$= \int \left[\frac{\sin(x+\alpha)\cos 2\alpha}{\sin(x+\alpha)} - \frac{\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}\right]dx$$
$$= \int (\cos 2\alpha - \cot(x+\alpha)\sin 2\alpha)\,dx$$
$$= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha)\,dx$$
$$= x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + c$$

$\therefore \quad I = x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + c$

Indefinite Integrals Ex 19.8 Q9

Let $I = \int \frac{1+\tan x}{1-\tan x}dx$

$$I = \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}dx$$

$\Rightarrow \quad I = \int \frac{\cos x + \sin x}{\cos x - \sin x}dx \text{ - - - - - (i)}$

Let $\cos x - \sin x = t$ then,

$d(\cos x - \sin x) = dt$

$\Rightarrow \quad (-\sin x - \cos x)dx = dt$

$\Rightarrow \quad -(\sin x + \cos x)dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x + \cos x}$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x + \cos x}$ in equation (i), we get

$$I = \int \frac{\cos x + \sin x}{t} \times \frac{-dt}{\sin x + \cos x}$$
$$= -\int \frac{dt}{t}$$
$$= -\log|t| + c$$
$$= -\log|\cos x - \sin x| + c$$

$\therefore \quad I = -\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.8 Q10

Let $I = \int \frac{\cos x}{\cos(x-a)} dx$ then,

$$I = \int \frac{\cos(x+a-a)}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a+a)}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a)\cos a}{\cos(x-a)} dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)} dx$$
$$= \cos a \int dx - \sin a \int \tan(x-a) dx$$

$$= x\cos a - \sin a \log\left|\sec(x-a)\right| + c$$

Indefinite Integrals Ex 19.8 Q11

Let $I = \int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} dx$ then,

$$I = \int \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{1+\cos\left(\frac{\pi}{2}-2x\right)}} dx$$
$$= \int \sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\cos^2\left(\frac{\pi}{4}-x\right)}} dx$$
$$= \int \sqrt{\tan^2\left(\frac{\pi}{4}-x\right)} dx$$
$$= \int \tan\left(\frac{\pi}{4}-x\right) dx$$
$$= \log\left|\cos\left(\frac{\pi}{4}-x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q12

Let $I = \int \frac{e^{3x}}{e^{3x}+1} dx$ -----(i)

Let $e^{3x}+1 = t$, then,
$d\left(e^{3x}+1\right) = dt$

$\Rightarrow$ $3e^{3x} dx = dt$
$\Rightarrow$ $dx = \frac{dt}{3e^{3x}}$

Putting $e^{3x}+1 = t$ and $dx = \frac{dt}{3e^{3x}}$ in equation (i), we get

$$I = \int \frac{e^{3x}}{t} \times \frac{dt}{3e^{3x}}$$
$$= \frac{1}{3}\int \frac{dt}{t}$$
$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|3e^{3x}+1\right| + c$$

$\therefore$ $= \frac{1}{3}\log\left|3e^{3x}+1\right| + c$

Indefinite Integrals Ex 19.8 Q13

Let $I = \int \frac{\sec x \tan x}{3\sec x + 5} dx$ ----- (i)

Let $3\sec x + 5 = t$, then,

$\Rightarrow \quad d(3\sec x + 5) = dt$

$\Rightarrow \quad 3\sec x \tan x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\sec x \tan x}$

Putting $3\sec x \tan x dx = t$ and $dx = \frac{dt}{3\sec x \tan x}$ in equation (i), we get

$$I = \int \frac{\sec x \tan x}{t} \times \frac{dt}{3\sec x \tan x}$$

$$= \frac{1}{3}\int \frac{1}{t} dt$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|3\sec x + 5| + c$$

Indefinite Integrals Ex 19.8 Q14

Let $I = \int \frac{1 - \cot x}{1 + \cot x} dx$ then,

$$I = \int \frac{1 - \frac{\cos x}{\sin x}}{1 + \frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x + \cos x}{\sin x}} dx$$

$\Rightarrow \quad I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ ---- (i)

Let $\sin x + \cos x = t$. then,

$d(\sin x + \cos x) = dt$

$\Rightarrow \quad (\cos x - \sin x) dx = dt$

$\Rightarrow \quad -(\sin x - \cos x) dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x - \cos x}$

Putting $\sin x + \cos x = t$ and $dx = -\frac{dt}{\sin x - \cos x}$ in equation (i), we get,

$$I = \int \frac{\sin x - \cos x}{t} \times \frac{-dt}{\sin x - \cos x}$$

$$= \int \frac{-dt}{t}$$

$$= -\log|t| + c$$

$$= -\log|\sin x + \cos x| + c$$

Indefinite Integrals Ex 19.8 Q15

Let $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ then,

Let $\log(\tan x) = t$ then,

$d[\log(\tan x)] = dt$

$\Rightarrow \quad \sec x \operatorname{cosec} x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log \tan x) = \sec x \operatorname{cosec} x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x \operatorname{cosec} x}$

Putting $\log(\tan x) = t$ and $dx = \frac{dt}{\sec x \operatorname{cosec} x}$ in equation (i), we get,

$I = \int \frac{\sec x \operatorname{cosec} x}{t} \times \frac{dt}{\sec x \operatorname{cosec} x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\log \tan x| + c$

Indefinite Integrals Ex 19.8 Q16

Let $I = \int \frac{1}{x(3+\log x)} dx$ - - - - - (i)

Let $3 + \log x = t$ then,

$d(3 + \log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x \, dt$

Putting $3 + \log x = t$ and $dx = x \, dt$ in equation (i), we get,

$I = \int \frac{1}{x \times t} \times x dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|(3 + \log x)| + c$

$\therefore \quad I = \log|(3 + \log x)| + c$

Indefinite Integrals Ex 19.8 Q17

$$\text{Let } I = \int \frac{e^x + 1}{e^x + x}\,dx \text{ - - - - - } (i)$$

$$\text{Let } e^x + x = t \quad \text{then,}$$

$$d\left(e^x + x\right) = dt$$

$$\Rightarrow \quad \left(e^x + x\right)dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{e^x + 1}$$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x + 1}$ in equation (i) ,we get,

$$I = \int \frac{e^x + 1}{t} \times \frac{dt}{e^x + 1}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|e^x + x\right| + c$$

$$\therefore \quad I = \log\left|e^x + x\right| + c$$

Indefinite Integrals Ex 19.8 Q18

$$\text{Let } I = \int \frac{1}{x \log x}\,dx \text{ - - - - - } (i)$$

$$\text{Let } \log x = t \quad \text{then,}$$

$$d\left(\log x\right) = dt$$

$$\Rightarrow \quad \frac{1}{x}\,dx = dt$$

$$\Rightarrow \quad dx = x\,dt$$

Putting $\log x = t$ and $dx = x\,dt$ in equation (i) ,we get,

$$I = \int \frac{1}{x \times t} \times x\,dt$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|\left(\log x\right)\right| + c$$

$$\therefore \quad I = \log\left|\left(\log x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q19

Let $I = \int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx$ - - - - - (i)

Let $a\cos^2 x + b\sin^2 x = t$ then,

$d\left(a\cos^2 x + b\sin^2 x\right) = dt$

$\left[a\left(2\cos x\left(-\sin x\right)\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \left[-a\left(2\sin x\cos x\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \left[-a\sin 2x + b\sin 2x\right]dx = dt$

$\Rightarrow \sin 2x\left(b-a\right)dx = dt$

$\Rightarrow dx = \frac{dt}{\left(b-a\right)\sin 2x}$

Putting $a\cos^2 x + b\sin^2 x = t$ and $dx = \frac{dt}{\left(b-a\right)\sin 2x}$ in equation (i), we get,

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{\left(b-a\right)\sin 2x}$

$= \frac{1}{b-a}\int \frac{dt}{t}$

$= \frac{1}{b-a}\log|t| + c$

$= \frac{1}{b-a}\log\left|a\cos^2 x + b\sin^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q20

Let $I = \int \frac{\cos x}{2 + 3\sin x} dx$ - - - - - (i)

Let $2 + 3\sin x = t$ then,

$d\left(2 + 3\sin x\right) = dt$

$\Rightarrow 3\cos x\, dx = dt$

$\Rightarrow dx = \frac{dt}{3\cos x}$

Putting $2 + 3\sin x = t$ and $dx = \frac{dt}{3\cos x}$ in equation (i), we get,

$I = \int \frac{\cos x}{t} \times \frac{dt}{3\cos x}$

$= \frac{1}{3}\int \frac{dt}{t}$

$= \frac{1}{3}\log|t| + c$

$= \frac{1}{3}\log\left|2 + 3\sin x\right| + c$

Indefinite Integrals Ex 19.8 Q21

Let $I = \int \frac{1-\sin x}{x+\cos x} dx$ - - - - - (i)

Let $x + \cos x = t$ then,

$d(x+\cos x) = dt$

$\Rightarrow \quad (1-\sin x)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-\sin x}$

Putting $x + \cos x = t$ and $dx = \frac{dt}{1-\sin x}$ in equation (i), we get,

$$I = \int \frac{1-\sin x}{t} \times \frac{dt}{1-\sin x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|x + \cos x| + c$$

$\therefore \quad I = \log|x+\cos x| + c$

Indefinite Integrals Ex 19.8 Q22

Let $I = \int \frac{a}{b+ce^x} dx$ then,

$$I = \int \frac{a}{e^x\left[\frac{b}{e^x}+c\right]} dx$$

$\Rightarrow \quad I = \int \frac{a}{e^x\left[be^{-x}+c\right]} dx$ - - - - - - - (i)

Let $be^{-x} + c = t$ then,

$d(be^{-x}+c) = dt$

$\Rightarrow \quad -be^{-x}dx = dt$

$\Rightarrow \quad dx = \frac{-dt}{be^{-x}}$

$= -\frac{e^x dt}{b}$

Putting $be^{-x} + c = t$ and $dx = \frac{-e^x\,dt}{b}$ in equation (i), we get,

$$I = \int \frac{a}{e^x \times t} \times \frac{-e^x\,dt}{b}$$
$$= -\frac{a}{b}\int \frac{dt}{t}$$
$$= -\frac{a}{b}\log|t| + c$$

$$= -\frac{a}{b}\log|be^{-x}+c| + c$$

Indefinite Integrals Ex 19.8 Q23

Let $I = \int \frac{1}{e^x + 1} dx$ then,

$$I = \int \frac{1}{e^x \left[1 + \frac{1}{e^x}\right]} dx$$

$$\Rightarrow \quad I = \int \frac{1}{e^x \left[1 + e^{-x}\right]} dx \text{ - - - - - - - } (i)$$

Let $1 + e^{-x} = t$ then,

$$d\left(1 + e^{-x}\right) = dt$$

$$\Rightarrow \quad -e^{-x} dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{e^{-x}}$$

$$dx = -dt \times e^x$$

Putting $1 + e^{-x} = t$ and $dx = -e^x dt$ in equation (i), we get,

$$I = \int \frac{1}{e^x \times t} \times -e^x \, dt$$

$$= -\int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log\left|1 + e^{-x}\right| + c$$

$$\therefore \quad = -\log\left|1 + e^{-x}\right| + c$$

Indefinite Integrals Ex 19.8 Q24

Let $I = \int \frac{\cot x}{\log \sin x} dx \text{ - - - - - - - } (i)$

Let $\log \sin x = t$ then,

$$d\left(\log \sin x\right) = dt$$

$$\Rightarrow \quad \frac{\cos x}{\sin x} dx = dt$$

$$\Rightarrow \quad \cot x \, dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cot x}$$

Putting $\log \sin x = t$ and $dx = \frac{dt}{\cot x}$ in equation (i), we get,

$$I = \int \frac{\cot x}{t} \times \frac{dt}{\cot x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|\log \sin x\right| + c$$

Indefinite Integrals Ex 19.8 Q25

Let I $\quad I = \int \frac{e^{2x}}{e^{2x}-2}dx$ - - - - - - - (i)

Let $e^{2x} - 2 = t$ then,

$d\left(e^{2x}-2\right) = dt$

$\Rightarrow \quad 2e^{2x}dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2e^{2x}}$

Putting $e^{2x} - 2 = t$ and $dx = \frac{dt}{2e^{2x}}$ in equation (i), we get,

$$I = \int \frac{2e^{2x}}{t} \times \frac{dt}{2e^{2x}}$$
$$= \frac{1}{2}\int \frac{dt}{t}$$
$$= \frac{1}{2}\log|t| + c$$
$$= \frac{1}{2}\log\left|e^{2x}-2\right| + c$$

$\therefore \quad = \frac{1}{2}\log\left|e^{2x}-2\right| + c$

Indefinite Integrals Ex 19.8 Q26

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$(-3\sin x + 2\cos x)dx = dt$

$$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \int \frac{dt}{2t}$$
$$= \frac{1}{2}\int \frac{1}{t}dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$$

Indefinite Integrals Ex 19.8 Q27

Let $I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ - - - - - - - (i)

Let $x^2 + \sin 2x + 2x = t$ then,

$d\left(x^2 + \sin 2x + 2x\right) = dt$

$\Rightarrow \left(2x + 2\cos 2x + 2\right) dx = dt$

$\Rightarrow 2\left(\cos 2x + x + 1\right) dx = dt$

$\Rightarrow dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$

Putting $x^2 + \sin 2x + 2x = t$ and $dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$ in equation (i), we get,

$I = \int \frac{\cos 2x + x + 1}{t} \times \frac{dt}{2\left(\cos 2x + x + 1\right)}$

$= \frac{1}{2} \int \frac{dt}{t}$

$= \frac{1}{2} \log|t| + c$

$= \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$

$\therefore I = \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$

Indefinite Integrals Ex 19.8 Q29

Let $I = \int \frac{-\sin x + 2\cos x}{2\sin x + \cos x} dx$ - - - - - - - (i)

Let $2\sin x + \cos x = t$ then,

$d\left(2\sin x + \cos x\right) = dt$

$\Rightarrow \left(2\cos x - \sin x\right) dx = dt$

$\Rightarrow dx = \frac{dt}{-\sin x + 2\cos x}$

Putting $2\sin x + \cos x = t$ and $dx = \frac{dt}{-\sin x + 2\cos x}$ in equation (i), we get,

$I = \int \frac{-\sin x + 2\cos x}{t} \times \frac{dt}{-\sin x + 2\cos x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|2\sin x + \cos x| + c$

$\therefore I = \log|2\sin x + \cos x| + c$

Indefinite Integrals Ex 19.8 Q30

$$\int \frac{\cos 4x - \cos 2x}{\sin 4x - \sin 2x} dx$$

$$= -\int \frac{2\sin 3x \sin x}{2\cos 3x \sin x} dx$$
$$= -\int \frac{\sin 3x}{\cos 3x} dx$$

Putting $\cos 3x = t$, and $-3\sin 3x dx = dt$

$$= \frac{1}{3}\int \frac{dt}{t}$$
$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|\cos 3x| + C$$

Indefinite Integrals Ex 19.8 Q31

Let $\quad I = \int \frac{\sec x}{\log(\sec x + \tan x)} dx$ - - - - - - - (i)

Let $\log(\sec x + \tan x) = t$ then,
$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \quad \sec x\, dx = dt \quad \left[\because \quad \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x}$

Putting $\log(\sec x + \tan x) = t$ and $dx = \frac{dt}{\sec x}$ in equation (i), we get,

$$I = \int \frac{\sec x}{t} \times \frac{dt}{\sec x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|\log(\sec x + \tan x)| + c$$

$\therefore \quad I = \log|\log(\sec x + \tan x)| + c$

Indefinite Integrals Ex 19.8 Q32

Let $I = \int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx$ - - - - - - - (i)

Let $\log \tan \frac{x}{2} = t$ then,

$d\left[\log \tan \frac{x}{2}\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\operatorname{cosec} x}$

Putting $\log \tan \frac{x}{2} = t$ and $dx = \frac{dt}{\operatorname{cosec} x}$ in equation (i), we get,

$I = \int \frac{\operatorname{cosec} x}{t} \times \frac{dt}{\operatorname{cosec} x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|\log \tan \frac{x}{2}\right| + c$

$\therefore \quad I = \log\left|\log \tan \frac{x}{2}\right| + c$

Indefinite Integrals Ex 19.8 Q33

Let $I = \int \frac{1}{x \log x \log(\log x)} dx$ - - - - - - - (i)

Let $\log(\log x) = t$ then,

$d[\log(\log x)] = dt$

$\Rightarrow \quad \frac{1}{x} \times \frac{1}{\log x} dx = dt$

$\Rightarrow \quad dx = x \log x \, dt$

Putting $\log(\log x) = t$ and $dx = x \log x \, dt$ in equation (i), we get,

$I = \int \frac{1}{x \log x \, t} \times x \log x \, dt$

$= \int \frac{1}{t} dt$

$= \log|t| + c$

$= \log|\log(\log x)| + c$

$\therefore \quad I = \log|\log(\log x)| + c$

Indefinite Integrals Ex 19.8 Q34

Let $I = \int \frac{\operatorname{cosec}^2 x}{1+\cot x} dx \text{-------} (i)$

Let $1+\cot x = t$ then,

$d[1+\cot x] = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x \, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $1+\cot x = t$ and $dx = \frac{-dt}{\operatorname{cosec}^2 x}$ in equation (i), we get,

$I = \int \frac{\operatorname{cosec}^2 x}{t} \times -\frac{dt}{\operatorname{cosec}^2 x}$

$= -\int \frac{1}{t} dt$

$= -\log|t| + c$

$= -\log|1+\cot x| + c$

$\therefore \quad I = -\log|1+\cot x| + c$

Indefinite Integrals Ex 19.8 Q35

Let $I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx \text{-------} (i)$

Let $10^x + x^{10} = t$ then,

$d\left(10^x + x^{10}\right) = dt$

$\Rightarrow \quad \left(10^x \log_e 10 + 10x^9\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{10x^9 + 10^x \log_e 10}$

Putting $10^x + x^{10} = t$ and $dx = \frac{dt}{10x^9 + 10^x \log_e 10}$ in equation (i), we get,

$I = \int \frac{10x^9 + 10^x \log_e 10}{t} \times \frac{dt}{10x^9 + 10^x \log_e 10}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|10^x + x^{10}\right| + c$

$\therefore \quad I = \log\left|10^x + x^{10}\right| + c$

Indefinite Integrals Ex 19.8 Q36

Let $\quad I = \int \frac{1 - \sin 2x}{x + \cos^2 x} dx \text{ - - - - - (i)}$

Let $x + \cos^2 x = t$ then,

$d\left(x + \cos^2 x\right) = dt$

$\Rightarrow \quad \left(1 - 2\cos x \sin x\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1 - 2\cos x \sin x}$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1 - 2\cos x \sin x}$ in equation (i), we get

$$I = \int \frac{1 - \sin 2x}{t} \times \frac{dt}{1 - 2\cos x \sin x}$$
$$= \int \frac{1 - \sin 2x}{t} \times \frac{dt}{1 - \sin 2x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log\left|x + \cos^2 x\right| + c$$

$\therefore \quad I = \log\left|x + \cos^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q37

Let $\quad I = \int \frac{1 + \tan x}{x + \log \sec x} dx \text{ - - - - - - - (i)}$

Let $x + \log \sec x = t$ then,

$d\left(x + \log \sec x\right) = dt$

$\Rightarrow \quad \left(1 + \tan x\right) dx = dt \quad \left[\because \quad \frac{d}{dx}\left(\log \sec x\right) = \tan x\right]$

$\Rightarrow \quad dx = \frac{dt}{1 + \tan x}$

Putting $x + \log \sec x = t$ and $dx = \frac{dt}{1 + \tan x}$ in equation (i), we get,

$$I = \int \frac{1 + \tan x}{t} \times \frac{dt}{1 + \tan x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$\Rightarrow \quad I = \log\left|x + \log \sec x\right| + c$

Indefinite Integrals Ex 19.8 Q38

Let $I = \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$ - - - - - - - (i)

Let $a^2 + b^2 \sin^2 x = t$ then,

$d\left(a^2 + b^2 \sin^2 x\right) = dt$

$\Rightarrow$ $b^2 (2\sin x \cos x) dx = dt$

$\Rightarrow$ $dx = \frac{dt}{b^2 (2\sin x \cos x)}$

$= \frac{dt}{b^2 \sin 2x}$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i), we get,

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x}$

$= \frac{1}{b^2} \int \frac{dt}{t}$

$= \frac{1}{b^2} \log|t| + c$

$= \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

$\Rightarrow$ $I = \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q39

Let $I = \int \frac{x+1}{x(x + \log x)} dx$ - - - - - - - (i)

Let $(x + \log x) = t$ then,

$d(x + \log x) = dt$

$\Rightarrow$ $\left(1 + \frac{1}{x}\right) dx = dt$

$\Rightarrow$ $\left(\frac{x+1}{x}\right) dx = dt$

$\Rightarrow$ $dx = \frac{x}{x+1} dt$

Putting $(x + \log x) = t$ and $dx = \frac{x}{x+1}$ in equation (i), we get,

$I = \int \frac{x+1}{x \times t} \times \frac{x}{x+1} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|x + \log x| + c$

$\Rightarrow$ $I = \log|x + \log x| + c$

Indefinite Integrals Ex 19.8 Q40

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(2+3\sin^{-1}x\right)}dx$ ------- (i)

Let $2+3\sin^{-1}x = t$ then,

$d\left(2+3\sin^{-1}x\right) = dt$

$\Rightarrow$ $3 \times \frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow$ $dx = \frac{\sqrt{1-x^2}}{3}dt$

Putting $2+3\sin^{-1}x = t$ and $dx = \frac{\sqrt{1-x^2}}{3}$ in equation (i), we get,

$I = \int \frac{\sqrt{1-x^2}}{3} \times \frac{1}{\sqrt{1-x^2}\,t}dt$

$= \frac{1}{3}\int \frac{dt}{t}$

$= \frac{1}{3}\log|t| + c$

$= \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$

$\Rightarrow$ $I = \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$

Indefinite Integrals Ex 19.8 Q41

Let $I = \int \frac{\sec^2 x}{\tan x + 2}dx$ ------- (i)

Let $\tan x + 2 = t$ then,

$d\left(\tan x + 2\right) = dt$

$\Rightarrow$ $\sec^2 x dx = dt$

$\Rightarrow$ $dx = \frac{1}{\sec^2 x}dt$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get,

$I = \int \frac{\sec^2 x}{t} \times \frac{1}{\sec^2 x}dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\tan x + 2| + c$

$\Rightarrow$ $I = \log|\tan x + 2| + c$

Indefinite Integrals Ex 19.8 Q42

Let $I = \int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx$ - - - - - - - (i)

Let $\sin 2x + \tan x - 5 = t$ then,

$d\left(\sin 2x + \tan x - 5\right) = dt$

$\Rightarrow \left(2\cos 2x + \sec^2 x\right) dx = dt$

$\Rightarrow dx = \frac{1}{2\cos 2x + \sec^2 x} dt$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2\cos 2x + \sec^2 x}$ in equation (i), we get,

$I = \int \frac{2\cos 2x + \sec^2 x}{t} \times \frac{1}{2\cos 2x + \sec^2 x} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\sin 2x + \tan x - 5| + c$

$\therefore I = \log|\sin 2x + \tan x - 5| + c$

Indefinite Integrals Ex 19.8 Q43

Let $I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx$ then,

$I = \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx$

$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$

$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$

$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$

$= \int \cot 3x \, dx - \int \cot 5x \, dx$

$= \frac{1}{3}\log|\sin 3x| - \frac{1}{5}\log|\sin 5x| + c$

$\therefore I = \frac{1}{3}\log|\sin 3x| - \frac{1}{5}\log|\sin 5x| + c$

Indefinite Integrals Ex 19.8 Q44

Let $I = \int \frac{1 + \cot x}{x + \log \sin x} dx$ - - - - - (i)

Let $x + \log \sin x = t$ then,

$d\left(x + \log \sin x\right) = dt$

$\Rightarrow (1 + \cot x) dx = dt$ $\left[\because \frac{d}{dx}(\log \sin x) = \cot x\right]$

$\Rightarrow dx = \frac{dt}{1 + \cot x}$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$I = \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|x + \log \sin x| + c$

$\therefore I = \log|x + \log \sin x| + c$

Indefinite Integrals Ex 19.8 Q45

Let $I = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$ - - - - - (i)

Let $\sqrt{x}+1 = t$ then,

$d\left(\sqrt{x}+1\right) = dt$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow dx = 2\sqrt{x}\,dt$

Putting $\sqrt{x}+1 = t$ and $dx = 2\sqrt{x}\,dt$ in equation (i), we get

$I = \int \frac{1}{\sqrt{x}\,t} \times 2\sqrt{x}\,dt$

$= 2\int \frac{dt}{t}$

$= 2\log|t| + c$

$= 2\log\left|\sqrt{x}+1\right| + c$

$\therefore \quad I = 2\log\left|\sqrt{x}+1\right| + c$

Indefinite Integrals Ex 19.8 Q46

Let $I = \int \tan 2x \tan 3x \tan 5x dx$ - - - - - - - - - (i)

Now,

$\tan(5x) = \tan(2x+3x)$

$= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$\Rightarrow \tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$\Rightarrow \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$

$\Rightarrow \tan 5x - \tan 2x - \tan 3x = \tan 2x \tan 3x \tan 5x$ - - - - - (ii)

Using equation (i) and equation (ii), we get

$I = \int\left[\tan 5x - \tan 2x - \tan 3x\right]dx$

$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

$\therefore \quad I = \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

Indefinite Integrals Ex 19.8 Q47

Since,

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\therefore \quad \tan(x+\theta-x) = \frac{\tan(x+\theta) - \tan x}{1 + \tan(x+\theta)\tan x}$

$\Rightarrow \quad 1 + \tan(x+\theta)\tan x = \frac{\tan(x+\theta) - \tan x}{\tan\theta}$

$\Rightarrow \quad \int 1 + \tan(x+\theta)\tan x dx$

$= \frac{1}{\tan\theta}\left[\int \tan(x+\theta)dx - \int \tan x dx\right]$

$= \frac{1}{\tan\theta}\left[-\log|\cos(x+\theta)| + \log|\cos x|\right] + C$

$= \frac{1}{\tan\theta}\left[\log|\cos x| - \log|\cos(x+\theta)|\right] + C$

$= \frac{1}{\tan\theta}\log\left|\frac{\cos x}{\cos(x+\theta)}\right| + C$

Indefinite Integrals Ex 19.8 Q48

$$\text{Consider } I = \int \left(\frac{\sin 2x}{\sin\left(x-\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4}\sin^2 x - \frac{1}{4}\cos^2 x\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4}(1-\cos^2 x) - \frac{1}{4}\cos^2 x\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4} - \cos^2 x\right)} \right) dx$$

$$let\ \cos^2 x = t \rightarrow \sin 2x dx = -dt$$

$$I = \int \left(\frac{-dt}{\left(\frac{3}{4} - t\right)} \right)$$

$$I = \log\left|\sin^2 x - \frac{1}{4}\right| + C$$

Indefinite Integrals Ex 19.8 Q49

$$\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$$

$$= \frac{1}{e} \int \frac{e^x + ex^{e-1}}{e^x + x^e} dx$$

$$\text{Let } e^x + x^e = u$$

$$\Rightarrow \quad \left(e^x + ex^{e-1}\right) dx = du$$

$$= \frac{1}{e}\int \frac{1}{4} du = \frac{1}{e}\log|u| + C$$

$$= \frac{1}{e}\log\left|e^x + x^e\right| + C$$

Indefinite Integrals Ex 19.8 Q50

$$\text{Let } I = \int \frac{1}{\sin x \cos^2 x} dx, \quad \text{then,}$$

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \sec x \tan x \, dx + \int \operatorname{cosec} x \, dx$$

$$= \sec x + \log\left|\tan\frac{x}{2}\right| + c$$

$$\therefore \quad I = \sec x + \log\left|\tan\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q51

Let $I = \int \frac{1}{\cos 3x - \cos x} dx$, then,

$$I = \int \frac{\sin^2 x + \cos^2 x}{-2\sin 2x \sin x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{-4\sin^2 x \cos x} dx$$

$$= -\frac{1}{4}\int \left[\frac{\sin^2 x}{\sin^2 x \cos x} + \frac{\cos^2 x}{\sin^2 x \cos x}\right] dx$$

$$= -\frac{1}{4}\int \left[\sec x + \operatorname{cosec} x \cot x\right] dx$$

$$= -\frac{1}{4}\left[\log\left|\sec x + \tan x\right| - \operatorname{cosec} x\right] + c$$

$$\therefore \quad I = \frac{1}{4}\left[\operatorname{cosec} x - \log\left|\sec x + \tan x\right|\right] + c$$

EX 19.9

Indefinite Integrals Ex 19.9 Q1

Let $I = \int \frac{\log x}{x} dx$

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow dx = x\,dt$

Putting $\log x = t$ and $dx = x\,dt$, we get

$$I = \int \frac{t}{x} \times x\,dt$$

$$= \int t\,dt$$

$$= \frac{t^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

$\therefore \quad I = \frac{(\log x)^2}{2} + c$

Indefinite Integrals Ex 19.9 Q2

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x\left(1+x\right)} dx \text{ - - - - - - (i)}$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2\left(x+1\right)} dx = -dt$$

$$\Rightarrow \quad \frac{dx}{x\left(x+1\right)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x\left(x+1\right)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x\left(1+x\right)} dx \text{ - - - - - - (i)}$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2\left(x+1\right)} dx = dt$$

$$\Rightarrow \quad \frac{dx}{x\left(x+1\right)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x\left(x+1\right)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q3

Let $I = \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}} dx$

Let $\left(1+\sqrt{x}\right) = t$ then,

$d\left(1+\sqrt{x}\right) = dt$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow dx = dt \times 2\sqrt{x}$

Putting $\left(1+\sqrt{x}\right) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$I = \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x}$

$= 2\int t^2 dt$

$= 2 \times \frac{t^3}{3} + c$

$= \frac{2}{3}\left[1+\sqrt{x}\right]^3 + c$

$\therefore \quad I = \frac{2}{3}\left(1+\sqrt{x}\right)^3 + c$

Indefinite Integrals Ex 19.9 Q4

Let $I = \int \sqrt{1+e^x}\, e^x dx$ ------ (i)

Let $1 + e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow e^x dx = dt$

$\Rightarrow dx = \frac{dt}{e^x}$

Putting $1 + e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$I = \int \sqrt{t}\, e^x \frac{dt}{e^x}$

$= \int t^{\frac{1}{2}} dt$

$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{} + c$

$= \frac{2}{3}\left(1+e^x\right)^{\frac{3}{2}} + c$

Indefinite Integrals Ex 19.9 Q5

Let $I = \int \sqrt[3]{\cos^2 x}\, \sin x\, dx$ - - - - - (i)

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow$ $-\sin x\, dx = dt$

$\Rightarrow$ $dx = -\dfrac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\dfrac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2}\, \sin x \times \frac{-dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} \sin x \frac{dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}}\, dt$$

$$= -\frac{3}{5} \times \frac{t^{\frac{5}{3}}}{} + c$$

$$= -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$$

$\therefore$ $I = -\dfrac{3}{5}(\cos x)^{\frac{5}{3}} + c$

Indefinite Integrals Ex 19.9 Q6

Let $I = \int \dfrac{e^x}{(1+e^x)^2} dx$ - - - - - (i)

Let $1 + e^x = t$ then,

$d(1+e^x) = dt$

$\Rightarrow$ $e^x\, dx = dt$

$\Rightarrow$ $dx = \dfrac{dt}{e^x}$

Putting $1 + e^x = t$ and $dx = \dfrac{dt}{e^x}$ in equation (i), we get

$$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2}\, dt$$

$$= -t^{-1} + c$$

$$= -\frac{1}{t} + c$$

$$= -\frac{1}{1+e^x} + c$$

$\therefore$ $I = -\dfrac{1}{1+e^x} + c$

Indefinite Integrals Ex 19.9 Q7

Let $I = \int \cot^3 x \operatorname{cosec}^2 x \, dx$ - - - - - (i)

Let $\cot x = t$ then,

$d(\cot x) = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $\cot x = t$ and $dx = -\frac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$
$$= -\int t^3 dt$$
$$= -\frac{t^4}{4} + c$$
$$= -\frac{\cot^4 x}{4} + c$$

$\therefore \quad I = -\frac{\cot^4 x}{4} + c$

Indefinite Integrals Ex 19.9 Q8

Let $I = \int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1 - x^2}} dx$ - - - - - (i)

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1 - x^2}} dx = dt$

$\Rightarrow \quad dx = \sqrt{1 - x^2}\, dt$

Putting $\sin^{-1} x = t$ and $dx = \sqrt{1 - x^2} dt$ in equation (i), we get

$$I = \int \frac{\left(e^t\right)^2}{\sqrt{1 - x^2}} \times \sqrt{1 - x^2} dt$$
$$= \int e^{2t} dt$$
$$= \frac{e^{2t}}{2} + c$$
$$= \frac{e^{2\sin^{-1} x}}{2} + c$$

$\therefore \quad I = \frac{\left\{e^{\sin^{-1} x}\right\}^2}{2} + c$

Indefinite Integrals Ex 19.9 Q9

Let $I = \int \frac{1+\sin x}{\sqrt{x-\cos x}}dx$ ----- (i)

Let $x - \cos x = t$ then,

$d(x - \cos x) = dt$

$\Rightarrow$ $[1 - (-\sin x)]dx = dt$

$\Rightarrow$ $(1 + \sin x)dx = dt$

Putting $x - \cos x = t$ and $(1+\sin x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{\frac{-1}{2}}dt$$

$$= 2t^{\frac{1}{2}} + c$$

$$= 2(x - \cos x)^{\frac{1}{2}} + c$$

$\therefore$ $I = 2\sqrt{x - \cos x} + c$

Indefinite Integrals Ex 19.9 Q10

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1}x\right)^2}dx$ ----- (i)

Let $\sin^{-1}x = t$ then,

$d\left(\sin^{-1}x\right) = dt$

$\Rightarrow$ $\frac{1}{\sqrt{1-x^2}}dx = dt$

Putting $\sin^{-1}x = t$ and $\frac{1}{\sqrt{1-x^2}}dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^2}$$

$$= \int t^{-2}dt$$

$$= -1t^{-1} + c$$

$$= \frac{-1}{t} + c$$

$$= \frac{-1}{\sin^{-1}x} + c$$

$\therefore$ $I = \frac{-1}{\sin^{-1}x} + c$

Indefinite Integrals Ex 19.9 Q11

Let $\quad I = \int \frac{\cot x}{\sqrt{\sin x}} dx - - - - - (i)$

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Now, $\quad I = \int \frac{\cot x}{\sqrt{\sin x}} dx$

$= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx$

$= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$

$\Rightarrow \quad = \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx - - - - - - (ii)$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (ii), we get

$I = \int \frac{dt}{t^{\frac{3}{2}}}$

$= \int t^{-\frac{3}{2}} dt$

$= -2t^{-\frac{1}{2}} + c$

$= \frac{-2}{\sqrt{t}} + c$

$= \frac{-2}{\sqrt{\sin x}} + c$

$\therefore \quad I = \frac{-2}{\sqrt{\sin x}} + c$

Indefinite Integrals Ex 19.9 Q12

Let $I = \int \frac{\tan x}{\sqrt{\cos x}} dx$

$\therefore$ $I = \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx$

$= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$

$\Rightarrow$ $I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$ ------ (i)

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow$ $-\sin x\, dx = dt$

$\Rightarrow$ $\sin x\, dx = -dt$

Putting $\cos x = t$ and $\sin x\, dx = -dt$ in equation (i), we get

$I = \int \frac{-dt}{t^{\frac{3}{2}}}$

$= -\int t^{-\frac{3}{2}} dt$

$= -\left[-2t^{-\frac{1}{2}}\right] + c$

$= \frac{2}{t^{\frac{1}{2}}} + c$

$= \frac{2}{\sqrt{\cos x}} + c$

$\therefore$ $I = \frac{2}{\sqrt{\cos x}} + c$

Indefinite Integrals Ex 19.9 Q13

Let $I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

$\therefore$ $I = \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx$

$= \int \frac{\left(1 - \sin^2 x\right) \cos x}{\sqrt{\sin x}} dx$

$\therefore$ $I = \int \frac{\left(1 - \sin^2 x\right)}{\sqrt{\sin x}} \cos x \, dx$ ------ (i)

Let $\sin x = t$ then,

$d\left(\sin x\right) = dt$

$\Rightarrow$ $\cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$I = \int \frac{1 - t^2}{\sqrt{t}} dt$

$= \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}}\right) dt$

$= \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}}\right) dt$

$= 2t^{\frac{1}{2}} - \frac{2}{5} t^{\frac{5}{2}} + c$

$\Rightarrow$ $I = 2\left(\sin x\right)^{\frac{1}{2}} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

$\therefore$ $I = 2\sqrt{\sin x} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q14

Let $I = \int \frac{\sin^3 x}{\sqrt{\cos x}}\, dx$

$\therefore \quad I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}}\, dx$

$\Rightarrow \quad I = \int \frac{\left(1-\cos^2 x\right)}{\sqrt{\cos x}} \sin x\, dx \text{ ------ (i)}$

Let $\cos x = t$ then,
$d\left(\cos x\right) = dt$

$\Rightarrow \quad -\sin x\, dx = dt$
$\Rightarrow \quad \sin x\, dx = -dt$

Putting $\cos x = t$ and $\sin x\, dx = -dt$ in equation (i), we get

$$I = \int \frac{\left(1-t^2\right)}{\sqrt{t}} \times -dt$$
$$= \int \frac{t^2-1}{\sqrt{t}} dt$$
$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}} \right) dx$$
$$= \int \left(t^{2-\frac{1}{2}} - t^{\frac{-1}{2}} \right) dt$$
$$= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\cos^{\frac{1}{2}} x + c$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$

Indefinite Integrals Ex 19.9 Q15

Let $I = \int \frac{1}{\sqrt{\tan^{-1} x}\left(1+x^2\right)} dx \text{ ----- (i)}$

Let $\tan^{-1} x = t$ then,
$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{1}{1+x^2} dx = dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{t}} dt$$
$$= \int t^{-\frac{1}{2}} dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2\sqrt{\tan^{-1} x} + c$$

$\therefore \quad I = 2\sqrt{\tan^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q16

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$
$$\text{Let } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$
$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Indefinite Integrals Ex 19.9 Q17

Let $I = \int \frac{1}{x} (\log x)^2 dx \text{ - - - - - (i)}$

Let $\log x = t$ then,
$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{(\log x)^3}{3} + c$$

$\therefore \quad I = \frac{1}{3} (\log x)^3 + c$

Indefinite Integrals Ex 19.9 Q18

Let $I = \int \sin^5 x \cos x \, dx \text{ - - - - - (i)}$

Let $\sin x = t$ then,
$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$

$\therefore \quad I = \frac{1}{6} \sin^6 x + c$

Indefinite Integrals Ex 19.9 Q19

Let $I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx$ - - - - - (i)

Let $\tan x = t$ then,

$d(\tan x) = dt$

$\Rightarrow \sec^2 x \, dx = dt$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} + c$$
$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$\therefore \quad I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$

Indefinite Integrals Ex 19.9 Q20

Let $I = \int \frac{x^3}{\left(x^2+1\right)^3} x\,dx$ - - - - - (i)

Let $1 + x^2 = t$ then,

$d\left(1+x^2\right) = dt$

$\Rightarrow$ $2x\,dx = dt$

$\Rightarrow$ $x\,dx = \frac{dt}{2}$

Putting $1 + x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \frac{x^2}{t^3} \times \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{(t-1)}{t^3} dt \qquad \left[\because \quad 1 + x^2 = t\right]$$

$$= \frac{1}{2}\int \left[\left(\frac{t}{t^3} - \frac{1}{t^3}\right) dt\right]$$

$$= \frac{1}{2}\int \left(t^{-2} - t^{-3}\right) dt$$

$$= \frac{1}{2}\left[-1t^{-1} - \frac{t^{-2}}{-2}\right] + c$$

$$= \frac{1}{2}\left[-\frac{1}{t} + \frac{1}{2t^2}\right] + c$$

$$= -\frac{1}{2t} + \frac{1}{4t^2} + c$$

$$= -\frac{1}{2\left(1+x^2\right)} + \frac{1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2\left(1+x^2\right)+1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2 - 2x^2 + 1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2x^2 - 1}{4\left(1+x^2\right)^2} + c$$

$$= -\frac{\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c$$

$\therefore$ $I = \frac{-\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c$

Indefinite Integrals Ex 19.9 Q21

Let $x^2+x+1=t$

$(2x+1)dx = dt$

$$\int(4x+2)\sqrt{x^2+x+1}\,dx$$

$$= \int 2\sqrt{t}\,dt$$

$$= 2\int\sqrt{t}\,dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

Indefinite Integrals Ex 19.9 Q22

Let $I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}}dx$ - - - - - (i)

Let $2x^3+3x+1 = t$ then,

$d\left(2x^3+3x+1\right) = dt$

$\Rightarrow$ $(4x+3)dx = dt$

Putting $2x^3+3x+1 = t$ and $(4x+3)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{\frac{-1}{2}}dt$$

$$= 2t^{\frac{1}{2}}+c$$

$$= 2\sqrt{t}+c$$

$\therefore$ $I = 2\sqrt{2x^3+3x+1}+c$

Indefinite Integrals Ex 19.9 Q23

Let $I = \int \frac{1}{1+\sqrt{x}} dx - - - - - (i)$

Let $x = t^2$ then,

$dx = d\left(t^2\right)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{2t}{1+\sqrt{t^2}} dt$$
$$= \int \frac{2t}{1+t} dt$$
$$= 2\int \frac{t}{1+t} dt$$
$$= 2\int \frac{1+t-1}{1+t} dt$$
$$= 2\int \left[\frac{1+t}{1+t} - \frac{1}{1+t}\right] dt$$
$$= 2\int dt - 2\int \frac{1}{1+t} dt$$
$$= 2t - 2\log\left(1+t\right) + c$$
$$= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$$

$\therefore \quad I = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$

Indefinite Integrals Ex 19.9 Q24

Let $I = \int e^{\cos^2 x} \sin 2x\, dx - - - - (i)$

Let $\cos^2 x = t$ then,

$d\left(\cos^2 x\right) = dt$

$\Rightarrow \quad -2\cos x \sin x\, dx = dt$

$\Rightarrow \quad -\sin 2x\, dx = dt$

$\Rightarrow \quad \sin 2x\, dx = -dt$

Putting $\cos^2 x = t$ and $\sin 2x\, dx = -dt$ in equation (i),
we get

$$I = \int e^t \left(-dt\right)$$
$$= -e^t + c$$
$$= -e^{\cos^2 x} + c$$

$\therefore \quad I = -e^{\cos^2 x} + c$

Indefinite Integrals Ex 19.9 Q25

Let $\quad I = \int \frac{1+\cos x}{(x+\sin x)^3} dx -----(i)$

Let $\quad x + \sin x = t \quad$ then,

$d(x+\sin x) = dt$

$\Rightarrow \quad (1+\cos x)\,dx = dt$

Putting $x + \sin x = t$ and $(1+\cos x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^3}$$
$$= \int t^{-3} dt$$
$$= \frac{t^{-2}}{-2} + c$$
$$= -\frac{1}{2t^2} + c$$
$$= \frac{-1}{2(x+\sin x)^2} + c$$

$$\therefore \quad I = \frac{-1}{2(x+\sin x)^2} + c$$

Indefinite Integrals Ex 19.9 Q26

$$\frac{\cos x - \sin x}{1+\sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1; \;\; \sin 2x = 2\sin x \cos x\right]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$\therefore (\cos x - \sin x)\,dx = dt$

$$\Rightarrow \int \frac{\cos x - \sin x}{1+\sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$
$$= \int \frac{dt}{t^2}$$
$$= \int t^{-2} dt$$
$$= -t^{-1} + C$$
$$= -\frac{1}{t} + C$$
$$= \frac{-1}{\sin x + \cos x} + C$$

Indefinite Integrals Ex 19.9 Q27

Let $I = \int \frac{\sin 2x}{(a+b\cos 2x)^2} dx -----(i)$

Let $a + b\cos 2x = t$ then,

$d(a+b\cos 2x) = dt$

$\Rightarrow \quad b(-2\sin 2x)\,dx = dt$

$\Rightarrow \quad \sin 2x\,dx = -\frac{dt}{2b}$

Putting $a + b\cos 2x = t$ and $\sin 2x\,dx = -\frac{dt}{2b}$ in equation (i), we get

$$I = \int \frac{1}{t^2} \times \frac{-dt}{2b}$$
$$= \frac{-1}{2b}\int t^{-2}dt$$
$$= -\frac{1}{2b}\left(-1t^{-1}\right) + c$$
$$= \frac{1}{2bt} + c$$
$$= \frac{1}{2b(a+b\cos 2x)} + c$$

$\therefore \quad I = \frac{1}{2b(a+b\cos 2x)} + c$

Indefinite Integrals Ex 19.9 Q28

Let $I = \int \frac{\log x^2}{x} dx -----(i)$

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad \frac{dx}{x} = dt$

Now, $I = \int \frac{\log x^2}{x}dx$

$$= \int \frac{2\log x}{x}dx$$
$$= 2\int \frac{\log x}{x}dx ----(ii)$$

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$$I = 2\int t\,dt$$
$$= \frac{2t^2}{2} + c$$
$$= t^2 + c$$

$\therefore \quad I = (\log x)^2 + c$

Indefinite Integrals Ex 19.9 Q29

Let $I = \int \frac{\sin x}{(1+\cos x)^2}\, dx$ - - - - - (i)

Let $1+\cos x = t$ then,
$d(1+\cos x) = dt$

$\Rightarrow$ $-\sin x\, dx = dt$
$\Rightarrow$ $\sin x\, dx = -dt$

Putting $1+\cos x = t$ and $\sin dx = -dt$ in equation (ii), we get

$$I = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= -\left(-1t^{-1}\right) + c$$
$$= \frac{1}{t} + c$$
$$= \frac{1}{1+\cos x} + c$$

$\therefore$ $I = \frac{1}{1+\cos x} + c$

Indefinite Integrals Ex 19.9 Q30

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x\ dx = dt$$
$$\therefore \cot x\ dx = dt$$

$$\Rightarrow \int \cot x\ \log \sin x\ dx = \int t\, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Indefinite Integrals Ex 19.9 Q31

Let $I = \int \sec x \cdot \log(\sec x + \tan x)\, dx$ - - - - - (i)

Let $\log(\sec x + \tan x) = t$ then,
$d\left[\log(\sec x + \tan x)\right] = dt$

$\Rightarrow$ $\sec x\, dx = dt$ $\left[\because \quad \frac{d}{dx}\left(\log(\sec x + \tan x)\right) = \sec x\right]$

Putting $\log(\sec x + \tan x) = t$ and $\sec x\, dx = dt$ in equation (i), we get

$$I = \int t\, dt$$
$$= \frac{t^2}{2} + c$$
$$= \frac{1}{2}\left[\log(\sec x + \tan x)\right]^2 + c$$

$\therefore$ $I = \frac{1}{2}\left[\log(\sec x + \tan x)\right]^2 + c$

Indefinite Integrals Ex 19.9 Q32

Let $I = \int \operatorname{cosec} x \log\left(\operatorname{cosec} x - \cot x\right) dx$ - - - - - (i)

Let $\log\left(\operatorname{cosec} x - \cot x\right) = t$ then,

$dx\left[\log\left(\operatorname{cosec} x - \cot x\right)\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x \, dx = dt \quad \left[\because \quad \frac{d}{dx}\left(\log\left(\operatorname{cosec} x - \cot x\right)\right) = \operatorname{cosec} x\right]$

Putting $\log\left(\operatorname{cosec} x - \cot x\right) = t$ and $\operatorname{cosec} x \, dx = dt$ in equation (i), we get

$$I = \int t \, dt$$
$$= \frac{t^2}{2} + c$$

$$\therefore \quad I = \frac{1}{2}\left[\log\left(\operatorname{cosec} x - \cot x\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q33

Let $I = \int x^3 \cos x^4 dx$ - - - - - (i)

Let $x^4 = t$ then,

$dx\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{4}$$
$$= \frac{1}{4}\sin t + c$$

$$\therefore \quad I = \frac{1}{4}\sin x^4 + c$$

Indefinite Integrals Ex 19.9 Q34

Let $I = \int x^3 \sin x^4 dx$ - - - - - (i)

Let $x^4 = t$ then,

$d\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= \frac{1}{4}\int \sin t \, dt$$
$$= -\frac{1}{4}\cos t + c$$
$$= -\frac{1}{4}\cos x^4 + c$$

$$\therefore \quad I = -\frac{1}{4}\cos x^4 + c$$

Indefinite Integrals Ex 19.9 Q35

Let $I = \int \frac{x \sin^{-1} x^2}{\sqrt{1 - x^4}} dx \text{ ----(i)}$

Let $\sin^{-1} x^2 = t$ then,

$d\left(\sin^{-1} x^2\right) = dt$

$\Rightarrow \quad 2x \times \frac{1}{\sqrt{1 - x^4}} dx = dt$

$\Rightarrow \quad \frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int t \frac{dt}{2}$$

$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$= \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

$$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

Indefinite Integrals Ex 19.9 Q36

Let $I = \int x^3 \sin\left(x^4 + 1\right) dx \text{ -----(i)}$

Let $x^4 + 1 = t$ then,

$d\left(x^4 + 1\right) = dt$

$\Rightarrow \quad x^4 dx = dt$

$\Rightarrow \quad x^3 dx = \frac{dt}{4}$

Putting $x^4 + 1 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$

$$= -\frac{1}{4} \cos t + c$$

$$= -\frac{1}{4} \cos\left(x^4 + 1\right) + c$$

$$\therefore \quad I = -\frac{1}{4} \cos\left(x^4 + 1\right) + c$$

Indefinite Integrals Ex 19.9 Q37

Let $I = \int \frac{(x+1)e^x}{\cos^2(xe^x)}dx$ - - - - - (i)

Let $xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow \quad (e^x + xe^x)dx = dt$

$\Rightarrow \quad (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x\,dx = dt$ in equation (i), we get

$I = \int \frac{dt}{\cos^2 t}$

$= \int \sec^2 t\,dt$

$= \tan t + c$

$= \tan(xe^x) + c$

$\therefore \quad I = \tan(xe^x) + c$

Indefinite Integrals Ex 19.9 Q38

Let $I = \int x^2 e^{x^3} \cos(e^{x^3})dx$ - - - - - (i)

Let $e^{x^3} = t$ then,

$d(e^{x^3}) = dt$

$\Rightarrow \quad 3x^2 e^{x^3}\,dx = dt$

$\Rightarrow \quad x^2 e^{x^3} dx = \frac{dt}{3}$

Putting $e^{x^3} = t$ and $x^2 e^{x^3}\,dx = \frac{dt}{3}$ in equation (i), we get

$I = \int \cos t \frac{dt}{3}$

$= \frac{\sin t}{3} + c$

$= \frac{\sin(e^{x^3})}{3} + c$

$\therefore \quad I = \frac{1}{3}\sin(e^{x^3}) + c$

Indefinite Integrals Ex 19.9 Q39

Let $\quad I = \int 2x \sec^3\left(x^2+3\right)\tan\left(x^2+3\right)dx$ - - - - - (i)

Let $\quad \sec\left(x^2+3\right) = t$ $\quad$ then,

$$d\left[\sec\left(x^2+3\right)\right] = dt$$

$\Rightarrow \quad 2x\sec\left(x^2+3\right)\tan\left(x^2+3\right)dx = dt$

Putting $\sec\left(x^2+3\right) = t$ and $2x\sec\left(x^2+3\right)\tan\left(x^2+3\right)dx = dt$ in equation (i),

we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$$

$\therefore \quad I = \frac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\Rightarrow \left(1+\frac{1}{x}\right)dx = dt$$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{1}{3}(x+\log x)^3 + C$$

Indefinite Integrals Ex 19.9 Q41

Let $I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x}\, dx$ - - - - - (i)

Let $1 - \tan^2 x = t$ then,

$d\left(1 - \tan^2 x\right) = dt$

$\Rightarrow \quad -2 \tan x \sec^2 x\, dx = dt$

$\Rightarrow \quad \tan x \sec^2 x dx = \frac{-dt}{2}$

Putting $1 - \tan^2 x = t$ and $\tan x \sec^2 x\, dx = -\frac{dt}{2}$ in equation (i), we get

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$
$$= -\frac{1}{2}\int t^{\frac{1}{2}} dt$$
$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$
$$= -\frac{1}{3} t^{\frac{3}{2}} + c$$

$$\therefore \quad I = -\frac{1}{3}\left[1 - \tan^2 x\right]^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.9 Q42

Let $I = \int \log x \frac{\sin\left(1 + (\log x)^2\right)}{x} dx$ - - - - - (i)

Let $1 + (\log x)^2 = t$ then,

$d\left(1 + (\log x)^2\right) = dt$

$\Rightarrow \quad 2 \log x \frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{\log x}{x} dx = \frac{dt}{2}$

Putting $1 + (\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \sin t \times \frac{dt}{2}$$
$$= \frac{1}{2}\int \sin t\, dt$$
$$\therefore \quad I = -\frac{1}{2}\cos t + c$$
$$= -\frac{1}{2}\cos\left[1 + (\log x)^2\right] + c$$

$$\therefore \quad I = -\frac{1}{2}\cos\left[1 + (\log x)^2\right] + c$$

Indefinite Integrals Ex 19.9 Q43

Let $I = \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx$ - - - - - (i)

Let $\frac{1}{x} = t$ then,

$d\left(\frac{1}{x}\right) = dt$

$\Rightarrow \quad \frac{-1}{x^2} dx = dt$

$\Rightarrow \quad \frac{1}{x^2} dx = -dt$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2} dx = -dt$ in equation (i),

we get

$$I = \int \cos^2 t \, (-dt)$$
$$= -\int \cos^2 t \, dt$$
$$= -\int \frac{\cos^2 2t + 1}{2} dt$$
$$= -\frac{1}{2} \int \cos 2t \, dt - \frac{1}{2} \int dt$$
$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2} t + c$$
$$\therefore \quad I = -\frac{1}{4} \sin 2t - \frac{1}{2} t + c$$
$$= -\frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$$\therefore \quad I = -\frac{1}{4} \sin\left(\frac{2}{x}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.9 Q44

Let $I = \int \sec^4 x \tan x \, dx$ - - - - (i)

Let $\tan x = t$ then,

$d(\tan x) = dt$

$\Rightarrow \quad \sec^2 x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i),

we get

$$I = \int \sec^4 x \tan x \frac{dt}{\sec^2 x}$$
$$= \int \sec^2 x \, t dt$$
$$= \int \left(1 + \tan^2 x\right) t \, dt$$
$$= \int \left(1 + t^2\right) t \, dt$$
$$= \int \left(t + t^3\right) dt$$
$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$
$$= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$$

$$\therefore \quad I = \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + c$$

Indefinite Integrals Ex 19.9 Q45

Let $\quad I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx - - - - - (i)$

Let $\quad e^{\sqrt{x}} = t \quad$ then,

$d\left(e^{\sqrt{x}}\right) = dt$

$\Rightarrow \quad e^{\sqrt{x}} \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i),

we get

$$I = \int \cos t \times 2dt$$
$$= 2\int \cos t \, dt$$
$$= 2 \sin t + c$$
$$= 2 \sin\left(e^{\sqrt{x}}\right) + c$$

$\therefore \quad I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Indefinite Integrals Ex 19.9 Q46

Let $\quad I = \int \frac{\cos^5 x}{\sin x} dx - - - - (i)$

Let $\quad \sin x = t \quad$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\cos x}$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i),

we get

$$I = \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x}$$
$$= \int \frac{\cos^4 x}{t} dt$$
$$= \int \frac{\left(1 - \sin^2 x\right)^2}{t} dt$$
$$= \int \frac{\left(1 - t^2\right)^2}{t} dt$$
$$= \int \frac{1 + t^4 - 2t^2}{t} dt$$
$$= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2\int \frac{t^2}{t} dt$$
$$= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c$$
$$= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c$$

$\therefore \quad I = \frac{1}{4} \sin^4 x - \sin^2 x + \log|\sin x| + c$

Indefinite Integrals Ex 19.9 Q47

Let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

Therefore,

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

$= 2\int \sin t\, dt$

$= -2\cos t + C$

$= -2\cos\sqrt{x} + C$

Indefinite Integrals Ex 19.9 Q48

Let $\quad I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$ ----- (i)

Let $\quad xe^x = t \quad$ then,

$d\left(xe^x\right) = dt$

$\Rightarrow \quad \left(xe^x + e^x\right) dx = dt$

$\Rightarrow \quad (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sin^2 t}$$

$$= \int \operatorname{cosec}^2 t\, dt$$

$$= -\cot t + c$$

$$= -\cot\left(xe^x\right) + c$$

$\therefore \quad I = -\cot\left(xe^x\right) + c$

Indefinite Integrals Ex 19.9 Q49

Let $\quad I = \int 5^{x+\tan^{-1}x}\left(\frac{x^2+2}{x^2+1}\right)dx$ - - - - - (i)

Let $\quad x + \tan^{-1}x = t$ then,

$d\left(x + \tan^{-1}x\right) = dt$

$\Rightarrow \quad \left(1 + \frac{1}{1+x^2}\right)dx = dt$

$\Rightarrow \quad \left(\frac{1+x^2+1}{1+x^2}\right)dx = dt$

$\Rightarrow \quad \frac{\left(x^2+2\right)}{\left(x^2+1\right)}dx = dt$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1}\right)dx = dt$ in equation (i),

we get

$I = \int 5^t\,dt$

$= \frac{5^t}{\log 5} + c$

$= \frac{5^{x+\tan^{-1}x}}{\log 5} + c$

$\therefore \quad I = \frac{5^{x+\tan^{-1}x}}{\log 5} + c$

Indefinite Integrals Ex 19.9 Q50

Let $\quad I = \int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx$ - - - - - (i)

Let $\quad m\sin^{-1}x = t$ then,

$d\left(m\sin^{-1}x\right) = dt$

$\Rightarrow \quad m\frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$

Putting $m\sin^{-1}x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$I = \int e^t \frac{dt}{m}$

$= \frac{1}{m}e^t + c$

$= \frac{1}{m}e^{m\sin^{-1}x} + c$

$\therefore \quad I = \frac{1}{m}e^{m\sin^{-1}x} + c$

Indefinite Integrals Ex 19.9 Q51

Let $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \cos t \, dt$$

$$= 2\sin t + C$$

$$= 2\sin\sqrt{x} + C$$

Indefinite Integrals Ex 19.9 Q52

Let $\quad I = \int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2} dx \text{ - - - - - (i)}$

Let $\quad \tan^{-1} x = t \quad$ then,

$d\left(\tan^{-1} x\right) = dt$

$$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i),

we get

$$I = \int \sin t \, dt$$

$$= -\cos t + c$$

$$= -\cos\left(\tan^{-1} x\right) + c$$

$$\therefore \quad I = -\cos\left(\tan^{-1} x\right) + c$$

Indefinite Integrals Ex 19.9 Q53

Let $\quad I = \int \frac{\sin(\log x)}{x} dx \text{ - - - - - (i)}$

Let $\quad \log x = t \quad$ then,

$d(\log x) = dt$

$$\Rightarrow \quad \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i),

we get

$$I = \int \sin t \, dt$$

$$= -\cos t + c$$

$$= -\cos(\log x) + c$$

$$\therefore \quad I = -\cos(\log x) + c$$

Indefinite Integrals Ex 19.9 Q54

Let $\tan^{-1}x = t$

Differentiating the above function with respect to, w, we have,

$$\frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \int e^{mt} \times dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{mt}}{m}$$

Resubstituting the value of t in the above solution, we have,

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{m\tan^{-1}x}}{m} + C$$

Indefinite Integrals Ex 19.9 Q55

Let $I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}dx$

$$\therefore \quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}} \times \frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}dx$$

$$= \int \frac{x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)}{x^2+a^2-x^2+a^2}dx$$

$$= \int \frac{x}{2a^2}\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx$$

$$\therefore \quad I = \frac{1}{2a^2}\int x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx \; ------ (i)$$

Let $x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \frac{1}{2a^2}\int\left(\sqrt{t+a^2}-\sqrt{t-a^2}\right)\frac{dx}{2}$$

$$= \frac{1}{4a^2}\left[\frac{2}{3}\left(t+a^2\right)^{\frac{3}{2}} - \frac{2}{3}\left(t-a^2\right)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \frac{1}{4a^2}\left[\frac{2}{3}\left(x^2+a^2\right)^{\frac{3}{2}} - \frac{2}{3}\left(x^2-a^2\right)^{\frac{3}{2}}\right] + c$$

$$= \frac{1}{6a^2}\left[\left(x^2+a^2\right)^{\frac{3}{2}} - \left(x^2-a^2\right)^{\frac{3}{2}}\right] + c$$

Indefinite Integrals Ex 19.9 Q56

Let $I = \int x \frac{\tan^{-1} x^2}{1+x^4} dx$ ----- (i)

Let $\tan^{-1} x^2 = t$ then,

$d\left(\tan^{-1} x^2\right) = dt$

$\Rightarrow \frac{1 \times 2x}{1+\left(x^2\right)^2} dx = dt$

$\Rightarrow \frac{1 \times x}{1+x^4} dx = \frac{dt}{2}$

Putting $\tan^{-1} x^2 = t$ and $\frac{x}{1+x^4} dx = \frac{dt}{2}$ in equation (i), we get

$I = \int t \frac{dx}{2}$

$= \frac{1}{2} \int t \, dt$

$= \frac{1}{2} \times \frac{t^2}{2} + c$

$\therefore I = \frac{t^2}{4} + c$

$= \frac{\left(\tan^{-1} x^2\right)^2}{4} + c$

$\therefore I = \frac{1}{4}\left(\tan^{-1} x^2\right)^2 + c$

Indefinite Integrals Ex 19.9 Q57

Let $I = \int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} dx$ ---- (i)

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$I = \int t^3 dt$

$= \frac{t^4}{4} + c$

$\therefore I = \frac{1}{4}\left(\sin^{-1} x\right)^4 + c$

Indefinite Integrals Ex 19.9 Q58

Let $I = \int \frac{\sin(2+3\log x)}{x} dx ----(i)$

Let $2+3\log x = t$ then,

$d(2+3\log x) = dt$

$\Rightarrow$ $3\frac{1}{x}dx = dt$

$\Rightarrow$ $\frac{dx}{x} = \frac{dt}{3}$

Putting $2+3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i),

we get

$I = \int \sin t \frac{dt}{3}$

$= \frac{1}{3}(-\cos t) + c$

$= -\frac{1}{3}\cos(2+3\log x) + c$

$\therefore$ $I = -\frac{1}{3}\cos(2+3\log x) + c$

Indefinite Integrals Ex 19.9 Q59

Let $I = \int x e^{x^2} dx ----(i)$

Let $x^2 = t$ then,

$d(x^2) = dt$

$\Rightarrow$ $2x\, dx = dt$

$\Rightarrow$ $x\, dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\, dx = \frac{dt}{2}$ in equation (i),

we get

$I = \int e^t \frac{dt}{2}$

$= \frac{1}{2}e^t + c$

$= \frac{1}{2}e^{x^2} + c$

$\therefore$ $I = \frac{1}{2}e^{x^2} + c$

Indefinite Integrals Ex 19.9 Q60

Let $\quad I = \int \frac{e^{2x}}{1+e^x} dx \quad ----(i)$

Let $\quad 1+e^x = t \qquad$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),

we get

$$I = \int \frac{e^{2x}}{t} \times \frac{dt}{e^x}$$
$$= \int \frac{e^x}{t} dt$$
$$= \int \frac{t-1}{t} dt$$
$$= \int \left(\frac{t}{t} - \frac{1}{t}\right) dt$$
$$= t - \log|t| + c$$
$$= \left(1+e^x\right) - \log\left|1+e^x\right| + c$$

$\therefore \quad I = 1+e^x - \log\left|1+e^x\right| + c$

Indefinite Integrals Ex 19.9 Q61

Let $\quad I = \int \frac{\sec^2\sqrt{x}}{\sqrt{x}} dx \quad ----(i)$

Let $\quad \sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\,dt$

$\Rightarrow \quad dx = 2t\,dt \qquad \left[\because \quad \sqrt{x} = t\right]$

Putting $\sqrt{x} = t$ and $dx = 2t\,dt$ in equation (i),

we get

$$I = \int \frac{\sec^2 t}{t} \times 2t\,dt$$
$$= 2\int \sec^2 t\,dt$$
$$= 2\tan t + c$$
$$= 2\tan\sqrt{x} + c$$

$\therefore \quad I = 2\tan\sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x \, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

Indefinite Integrals Ex 19.9 Q63

Let $\quad I = \int \frac{x + \sqrt{x+1}}{x+2} dx - - - - - (i)$

Let $\quad x + 1 = t^2$ then,

$d(x+1) = d(t^2)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x + 1 = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{x + \sqrt{t^2}}{x+2} 2t\,dt$$
$$= 2\int \frac{(t^2 - 1) + t}{(t^2 - 1) + 2} t\,dt \quad [\because \quad x + 1 = t^2]$$
$$= 2\int \frac{t^2 + t - 1}{t^2 + 1} t\,dt$$
$$= 2\int \frac{t^3 + t^2 - t}{t^2 + 1} dt$$
$$= 2\left[\int \frac{t^3}{t^2+1} dt + \int \frac{t^2}{t^2+1} dt - \int \frac{t}{t^2+1} dt\right]$$

$\therefore \quad I = 2\left[\int \frac{t^3}{t^2+1} dt + \int \frac{t^2}{t^2+1} dt - \int \frac{t}{t^2+1} dt\right] - - - - (ii)$

Let $I_1 = \int \frac{t^3}{t^2+1} dt$

$I_2 = \int \frac{t^2}{t^2+1} dt$

and $\quad I_3 = \int \frac{t}{t^2+1} dt$

Now, $\quad I_1 = \int \frac{t^3}{t^2+1} dt$

$$= \int \left(t - \frac{t}{t^2+1}\right) dt$$
$$= \frac{t^2}{2} - \frac{1}{2}\log(t^2 + 1)$$

$\therefore \quad I = \frac{t^2}{2} - \frac{1}{2}\log(t^2 + 1) + c_1 - - - - (iii)$

Since, $\quad I_2 = \int \frac{t^2}{t^2+1} dt$

$$= \int \frac{t^2 + 1 - 1}{t^2+1} dt$$
$$= \int \frac{t^2+1}{t^2+1} dt - \int \frac{1}{t^2+1} dt$$
$$= \int dt - \int \frac{1}{t^2+1} dt$$

$\Rightarrow \quad I_2 = t - \tan^{-1}(t^2) + c_2 - - - - (iv)$

and, $\quad I_3 = \int \frac{t}{t^2+1}dt$

$= \frac{1}{2}\log\left(1+t^2\right)+c_3 ------(v)$

Using equations (ii), (iii), (iv) and (v), we get

$$I = 2\left[\frac{t^2}{2} - \frac{1}{2}\log\left(t^2+1\right)+c_1+t-\tan^{-1}\left(t^2\right)+c_2-\frac{1}{2}\log\left(1+t^2\right)+c_3\right]$$

$$= 2\left[\frac{t^2}{2}+t-\tan^{-1}\left(t^2\right)-\log\left(1+t^2\right)+c_1+c_2+c_3\right]$$

$$= 2\left[\frac{t^2}{2}+t-\tan^{-1}\left(t^2\right)-\log\left(1+t^2\right)+c_4\right] \qquad \left[\text{Putting } c_1+c_2+c_3=c_4\right]$$

$$= t^2+2t-2\tan^{-1}\left(t^2\right)-2\log\left(1+t^2\right)+2c_4$$

$$= \left(x+1\right)+2\sqrt{x+1}-2\tan^{-1}\left(\sqrt{x+1}\right)-2\log\left(1+x+1\right)+2c_4$$

$$= \left(x+1\right)+2\sqrt{x+1}-2\tan^{-1}\left(\sqrt{x+1}\right)-2\log\left(x+2\right)+c \qquad \left[\text{Putting } 2c_4=c\right]$$

$\therefore \qquad I = \left(x+1\right)+2\sqrt{x+1}-2\tan^{-1}\left(\sqrt{x+1}\right)-2\log\left(x+2\right)+c$

Indefinite Integrals Ex 19.9 Q64

Let $\quad I = \int 5^{5^{5^x}}5^{5^x}5^x dx -----(i)$

Let $\quad 5^{5^{5^x}} = t$ then,

$d\left(5^{5^{5^x}}\right) = dt$

$\Rightarrow \quad 5^{5^{5^x}}5^{5^x}5^x\left(\log 5\right)^3 dx = dt$

$\Rightarrow \quad 5^{5^{5^x}}5^{5^{5^x}}5^x dx = \frac{dt}{\left(\log 5\right)^3}$

Putting $5^{5^{5^x}} = t$ and $5^{5^{5^x}}5^{5^x}5^x dx = \frac{dt}{\left(\log 5\right)^3}$ in equation (i), we get

$$I = \int \frac{dt}{\left(\log 5\right)^3}$$

$$= \frac{1}{\left(\log 5\right)^3}\int dt$$

$$= \frac{t}{\left(\log 5\right)^3}+c$$

$\therefore \qquad I = \frac{5^{5^{5^x}}}{\left(\log 5\right)^3}+c$

Indefinite Integrals Ex 19.9 Q65

Let $I = \int \frac{1}{x\sqrt{x^4 - 1}} dx$ - - - - - (i)

Let $x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2x}$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i), we get

$$I = \int \frac{1}{x\sqrt{t^2 - 1}} \times \frac{dt}{2x}$$
$$= \frac{1}{2}\int \frac{1}{x^2\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2}\int \frac{1}{t\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2}\sec^{-1} t + c$$
$$= \frac{1}{2}\sec^{-1} x^2 + c$$

$\therefore \quad I = \frac{1}{2}\sec^{-1}\left(x^2\right) + c$

Indefinite Integrals Ex 19.9 Q66

Let $I = \int \sqrt{e^x - 1}\, dx$ - - - - - (i)

Let $e^x - 1 = t^2$ then,

$d\left(e^x - 1\right) = dt\left(t^2\right)$

$\Rightarrow \quad e^x\, dx = 2t\, dt$

$\Rightarrow \quad dx = \frac{2t}{e^x} dt$

$\Rightarrow \quad dx = \frac{2t}{t^2 + 1} dt \qquad \left[\because \quad e^x - 1 = t^2\right]$

Putting $e^x - 1 = t^2$ and $dx = \frac{2t\,dt}{t^2 + 1}$ in equation (i), we get

$$I = \int \sqrt{t^2} \times \frac{2t\,dt}{t^2 + 1}$$
$$= 2\int \frac{t \times t}{t^2 + 1} dt$$
$$= 2\int \frac{t^2}{t^2 + 1} dt$$
$$= 2\int \frac{t^2 + 1 - 1}{t^2 + 1} dt$$
$$= 2\int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1}\right] dt$$
$$= 2\int dt - 2\int \frac{1}{t^2 + 1} dt$$
$$= 2t - 2\tan^{-1}(t) + c$$
$$= 2\sqrt{\left(e^x - 1\right)} - 2\tan^{-1}\left(\sqrt{e^x - 1}\right) + c$$

$\therefore \quad I = 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Indefinite Integrals Ex 19.9 Q67

$$I = \int \frac{1}{(x+1)(x^2+2x+2)} dx$$

$$= \int \frac{1}{(x+1)((x+1)^2+1)} dx$$

Let $x+1 = \tan u$

$\Rightarrow \quad dx = \sec^2 u \, du$

$$\therefore \quad I = \int \frac{\sec^2 u}{\tan u (\tan^2 u + 1)} du$$

$$= \int \frac{\cos u}{\sin u} du$$

$$= \log|\sin u| + C$$

$$= \log\left|\frac{\tan u}{\sec^2 u}\right| + C$$

$$= \log\left|\frac{x+1}{\sqrt{x^2+2x+2}}\right| + C$$

Indefinite Integrals Ex 19.9 Q68

Let $\quad I = \int \frac{x^5}{\sqrt{1+x^3}} dx \text{ - - - - (i)}$

Let $\quad 1+x^3 = t^2 \quad$ then,

$d(1+x^3) = d(t^2)$

$\Rightarrow \quad 3x^2 dx = dt \, 2t$

$\Rightarrow \quad dx = \frac{dt}{3x^2} 2t$

Putting $1+x^3 = t^2$ and $dx = \frac{2t}{3x^2} dt$ in equation (i),

we get

$$I = \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt$$

$$= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt$$

$$= \frac{2}{3} \int x^3 dt$$

$$= \frac{2}{3} \int (t^2 - 1) dt$$

$$= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + c$$

$$\therefore \quad I = \frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}\sqrt{1+x^3} + c$$

Indefinite Integrals Ex 19.9 Q69

Let $I = \int 4x^3\sqrt{5-x^2}\,dx$ ----(i)

Let $5 - x^2 = t^2$ then,

$d\left(5-x^2\right) = t^2$

$\Rightarrow$ $-2x\,dx = 2t\,dt$

$\Rightarrow$ $dx = \frac{-t}{x}dt$

Putting $5 - x^2 = t^2$ and $dx = \frac{-t}{x}dt$ in equation (i),
we get

$$I = \int 4x^3\sqrt{t^2} \times \frac{-t}{x}dt$$
$$= -4\int x^2 t \times t\,dt$$
$$= -4\int\left(5-t^2\right)t^2 dt \qquad \left[\because \quad 5 - x^2 = t^2\right]$$
$$= -4\int\left(5t^2 - t^4\right)dt$$
$$= -20 \times \frac{t^3}{3} + 4\frac{t^5}{5} + c$$
$$= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + c$$
$$= \frac{-20}{3} \times \left(5-x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5-x^2\right)^{\frac{5}{2}} + c$$

$\therefore$ $I = \frac{-20}{3} \times \left(5-x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5-x^2\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q70

Let $I = \int \frac{1}{\sqrt{x}+x}dx$ ----(i)

Let $\sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow$ $\frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow$ $dx = 2\sqrt{x}\,dt$

Putting $\sqrt{x} = t$ and $2\sqrt{x}\,dt = dx$ in equation (i),
we get

$$I = \int \frac{1}{t+t^2}2t \times dt \qquad \begin{bmatrix}\because & \sqrt{x} = t \\ \Rightarrow & x = t^2\end{bmatrix}$$
$$= \int \frac{2t}{t\left(1+t\right)}dt$$
$$= 2\int \frac{t}{\left(1+t\right)}dt$$
$$= 2\log\left|1+t\right| + c$$
$$= 2\log\left|1+\sqrt{x}\right| + c$$

$\therefore$ $I = 2\log\left|1+\sqrt{x}\right| + c$

Indefinite Integrals Ex 19.9 Q71

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{-\frac{3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

Indefinite Integrals Ex 19.9 Q72

Let $I = \int \frac{\sin^5 x}{\cos^4 x} dx$ ----(i)

Let $\cos x = t$ then,
$d(\cos x) = dt$

$\Rightarrow$ $-\sin x \, dx = dt$

$\Rightarrow$ $dx = -\frac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i),
we get

$$I = \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x}$$
$$= -\int \frac{\sin^4 x}{t^4} dt$$
$$= -\int \frac{\left(1-\cos^2 x\right)^2}{t^4} dt$$
$$= -\int \frac{\left(1-t^2\right)^2}{t^4} dt$$
$$= -\int \frac{1+t^4-2t^2}{t^4} dt$$
$$= -\int \left(\frac{1}{t^4}+\frac{t^4}{t^4}-\frac{2t^2}{t^4}\right) dt$$
$$= -\int \left(t^{-4}+1-2t^{-2}\right) dt$$
$$= -\left[\frac{t^{-3}}{-3}+t-2\frac{t^{-1}}{-1}\right]+c$$
$$= -\left[-\frac{1}{3}\times\frac{1}{t^3}+t+\frac{2}{t}\right]+c$$
$$= \frac{1}{3}\times\frac{1}{t^3}-t-\frac{2}{t}+c$$
$$= \frac{1}{3}\times\frac{1}{\cos^3 x}-\cos x-\frac{2}{\cos x}+c$$

$\therefore$ $I = -\cos x - \frac{2}{\cos x} + \frac{1}{3\cos^3 x} + c$

Ex 19.10

Indefinite Integrals Ex 19.10 Q1

Let $I = \int x^2\sqrt{x+2}dx$

Substituting $x+2 = t$ and $dx = dt$ we get,

$$I = \int (t-2)^2\sqrt{t}dt$$
$$= \int \left(t^2+4-4t\right)\sqrt{t}dt$$
$$= \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}}\right)dt$$
$$= \frac{2}{7}t^{\frac{7}{2}} - \frac{8}{5}t^{\frac{5}{2}} + \frac{8}{3}t^{\frac{3}{2}} + c$$
$$= \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$
$$\therefore \quad I = \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q2

Let $I = \int \frac{x^2}{\sqrt{x-1}}dx$

Substituting $x-1 = t$ and $dx = dt$ we get,

$$I = \int \frac{(t+1)^2}{\sqrt{t}}dx$$
$$= \int \frac{t^2+1+2t}{\sqrt{t}}dt$$
$$= \int \left(t^{\frac{3}{2}} + t^{\frac{-1}{2}} + 2t^{\frac{-1}{2}}\right)dt$$
$$= \frac{2}{5}t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3}t^{\frac{3}{2}} + c$$
$$= \frac{6t^{\frac{5}{2}} + 30t^{\frac{1}{2}} + 20t^{\frac{3}{2}}}{15} + c$$
$$= \frac{2}{15}t^{\frac{1}{2}}\left(3t^2+15+10t\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3(x-1)^2+15+10(x-1)\right) + x$$
$$= \frac{2}{15}\sqrt{x-1}\left(3\left(x^2+1-2x\right)+15+10x-10\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+3-6x+15+10x-10\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right) + c$$
$$\therefore \quad I = \frac{2}{15}\left(3x^2+4x+8\right)\sqrt{x-1} + c$$

Indefinite Integrals Ex 19.10 Q3

Let $I = \int \frac{x^2}{\sqrt{3x+4}} dx$

Substituting $3x + 4 = t$ and $dx = \frac{dt}{3}$ we get,

$$I = \int \frac{\left(\frac{t-4}{3}\right)^2}{\sqrt{t}} \times \frac{dt}{3} \qquad \left[\because x = \frac{t-4}{3}\right]$$

$$= \int \frac{(t-4)^2}{9\sqrt{t}x^3} dt$$

$$= \frac{1}{27} \int \frac{t^2 + 16 - 8t}{\sqrt{t}} dt$$

$$= \frac{1}{27} \int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{\frac{1}{2}} 16t^{\frac{-1}{2}} \right) dt$$

$$= \frac{1}{27} \left[\frac{2}{5} t^{\frac{5}{2}} - \frac{16}{3} t^{\frac{3}{2}} + 32t^{\frac{1}{1}} \right] + c$$

$$= \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c$$

$$\therefore \qquad I = \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.10 Q4

Let $I = \int \frac{2x-1}{(x-1)^2} dx$

Substituting $x - 1 = t$ and $dx = dt$, we get

$$I = \int \frac{2(t+1)}{t^2} dt$$

$$= \int \frac{2t + 2 - 1}{t^2} dt$$

$$= \int \frac{2t+1}{t^2} dt$$

$$= \int \left(\frac{2t}{t^2} + \frac{1}{t^2} \right) dt$$

$$= 2\int \frac{1}{t} dt + \int t^{-2} dt$$

$$= 2\log|t| - t^{-1} + c$$

$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$$\int \frac{2x-1}{(x-1)^2} dx = 2\log|x-1| - \frac{1}{x-1} + c$$

Indefinite Integrals Ex 19.10 Q5

Let $I = \int (2x^2 + 3)\sqrt{x+2} dx$

Substituting $x + 2 = t$ and $dx = dt$, we get

$$I = \int \left[2(t-2)^2 + 3 \right] \sqrt{t} dt$$

$$= \int \left[2(t^2 + 4 - 4t) + 3 \right] \sqrt{t} dt$$

$$= \int \left[2t^2 + 8 - 8t + 3 \right] \sqrt{t} dt$$

$$= \int \left(2t^{\frac{5}{2}} + 11t^{-\frac{1}{2}} - 8t^{\frac{3}{2}} \right) dt$$

$$= \frac{4}{7} t^{\frac{7}{2}} + \frac{22}{3} t^{\frac{3}{2}} - \frac{16}{5} t^{\frac{5}{2}} + c$$

$$= \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+1)\frac{5}{2} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

$$\therefore \qquad I = \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+2)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q6

Let $I = \int \frac{x^2+3x+1}{(x+1)^2} dx$

Substituting $x+1 = t$ and $dx = dt$, we get

$$I = \int \frac{(t-1)^2 + 3(t-1)+1}{t^2} dt$$
$$= \int \frac{t^2+1-2t+3t-3+1}{t^2} dt$$
$$= \int \frac{t^2+t-1}{t^2} dt$$
$$= \int \left(\frac{t^2}{t^2} + \frac{t}{t^2} - \frac{1}{t^2}\right) dt$$
$$= \int \left(1 + \frac{1}{t} - r^{-2}\right) dt$$
$$= t + \log|t| + t^{-1} + c$$
$$= t + \log|t| + \frac{1}{t} + c$$
$$= (x+1) + \log|x+1| + \frac{1}{x+1} + c$$

Indefinite Integrals Ex 19.10 Q7

Let $I = \int \frac{x^2}{\sqrt{1-x}} dx$

Substituting $1-x = t$ and $dx = -dt$, we get

$$I = \int \frac{(1-t)^2}{\sqrt{t}} \times -dt$$
$$= -\int \frac{1+t^2-2t}{\sqrt{t}} \times dt$$
$$= -\int \left(t^{\frac{-1}{2}} + t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right) dt$$
$$= -\left[2t^{\frac{1}{2}} + \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}}\right] + c$$
$$= -\left[\frac{30t^{\frac{1}{2}} + 6t^{\frac{5}{2}} - 20t^{\frac{3}{2}}}{15}\right] + c$$
$$= -\frac{2t^{\frac{1}{2}}}{15}\left[15 + 3t^2 - 10t\right] + c$$
$$= -\frac{2}{15}\sqrt{1-x}\left[15 + 3(1-x)^2 - 10(1-x)\right] + c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(15 + 3(1+x^2-2x) - 10 + 10x\right) + c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(5 + 3 + 3x^2 - 6x + 10x\right) + c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(3x^2 + 4x + 8\right) + c$$
$$= -\frac{2}{15}\left(3x^2 + 4x + 8\right)\sqrt{1-x} + c$$

$$\therefore \quad I = -\frac{2}{15}\left(3x^2 + 4x + 8\right)\sqrt{1-x} + c$$

Indefinite Integrals Ex 19.10 Q8

Let $I = \int x\left(1-x\right)^{23} dx$

Substituting $1 - x = t$ and $dx = -dt$, we get

$$I = -\int \left(1-t\right) t^{23} dt$$

$$= -\int \left(t^{23} - t^{24}\right) dt$$

$$= -\int \left(\frac{t^{24}}{24} - \frac{t^{25}}{25}\right) + c$$

$$= \frac{t^{25}}{25} - \frac{t^{24}}{24} + c$$

$$= \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c$$

$$\therefore \quad I = \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c.$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[24\left(1-x\right) - 25\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[24 - 24x - 25\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[-1 - 24x\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24} \times -\left[1 + 24x\right] + c$$

$$= -\frac{1}{600}\left(1-x\right)^{24}\left(1 + 24x\right) + c$$

Ex 19.11

Indefinite Integrals Ex 19.11 Q1

Let $I = \int \tan^3 x \sec^2 x dx$ ---(i)

Let $\tan x = t$. Then

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Puting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 dt$$

$$= \frac{t^{3+1}}{3+1} + c$$

$$= \frac{t^4}{4} + c$$

$$= \frac{(\tan x)^4}{4} + c$$

$$\therefore \quad I = \frac{(\tan x)^4}{4} + c$$

$$= \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q2

Let $I = \int \tan x \sec^4 x dx$. Then

$$I = \int \tan x \sec^2 x \sec^2 x dx$$

$$= \int \tan x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$\Rightarrow \quad I = \int \left(\tan x + \tan^3 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t + t^3\right) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4}{4} + c$$

$$\therefore \quad I = \frac{1}{2} \times \tan^2 x + \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q3

Let $I = \int \tan^5 x \sec^4 x dx$. Then

$$I = \int \tan^4 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^5 x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$= \int \left(\tan^5 x + \tan^7 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^5 + t^7\right) dt$$

$$= \frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= \frac{(\tan x)^6}{6} + \frac{(\tan x)^8}{8} + c$$

$$\therefore \quad I = \frac{1}{6} \times \tan^6 x + \frac{1}{8} \times \tan^8 x + c.$$

Indefinite Integrals Ex 19.11 Q4

Let $I = \int \sec^6 x \tan x dx$. Then

$$I = \int \sec^5 x \left(\sec x \tan x\right) dx$$

Substituting $\sec x = t$ and $\sec x \tan x = dt$, we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\left(\sec x\right)^6}{6} + c$$

$\therefore \quad I = \frac{1}{6}\sec^6 x + c$

Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$I = \int \tan^2 x \tan^3 x dx$$
$$= \int \left(\sec^2 x - 1\right) \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \left(\sec^2 x - 1\right) \tan x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$I = \int t^3 dt - \int t dt + \int \tan x dx$$
$$= \frac{t^4}{4} - \frac{t^2}{2} + \log\left|\sec x\right| + c$$
$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$$

$\therefore \quad I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$I = \int \sqrt{\tan x} \sec^2 x \sec^2 x dx$$
$$= \int \sqrt{\tan x} \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$= \int \tan x^{\frac{1}{2}} \left(1 + \tan^2 x\right) \sec^2 x dx$$

$\Rightarrow \quad I = \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}}\right) \sec^2 x dx$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}}\right) dt$$
$$= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + c$$
$$= \frac{2}{3}\left(\tan x\right)^{\frac{3}{2}} + \frac{2}{7}\left(\tan x\right)^{\frac{7}{2}} + c$$

$\therefore \quad I = \frac{2}{3}\tan^{\frac{3}{2}} x + \frac{2}{7}\tan^{\frac{7}{2}} x + c$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$I = \int \sec^2 2x \sec^2 2x dx$$
$$= \int \left(1 + \tan^2 2x\right) \sec^2 2x dx$$
$$= \int \left(\sec^2 2x + \sec^2 2x \tan^2 2x\right) dx$$

$\Rightarrow \quad I = \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx$

$\Rightarrow \quad I = \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$I = \int t^2 \frac{dt}{2} + \int \sec^2 2x dx$$
$$= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c$$

$\Rightarrow \quad I = \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c$

$\therefore \quad I = \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int \operatorname{cosec}^4 3x dx$. Then

$$I = \int \operatorname{cosec}^2 3x \operatorname{cosec}^2 3x dx$$
$$= \int \left(1 + \cot^2 3x\right) \operatorname{cosec}^2 3x dx$$
$$= \int \left(\operatorname{cosec}^2 3x + \cot^2 3x \operatorname{cosec}^2 3x\right) dx$$

$\Rightarrow \quad I = \int \operatorname{cosec}^2 3x dx + \int \cot^2 3x \operatorname{cosec}^2 3x dx$

Substituting $\cot 3x = t$ and $\operatorname{cosec}^2 3x dx = -dt$ in 2nd integral, we get

$$I = \int \operatorname{cosec}^2 3x dx - \int t^2 \frac{dt}{3}$$
$$= \frac{-1}{3} \cot 3x - \frac{t^3}{9} + c$$
$$= \frac{-1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c$$

$\therefore \quad I = \frac{-1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c$

Indefinite Integrals Ex 19.11 Q9

Let $I = \int \cot^n x \operatorname{cosec}^2 x dx$. $n \neq -1$ ---(i)

Let $\cot x = t$. Then

$$d(\cot x) = dt$$

$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$

$\Rightarrow \quad \operatorname{cosec}^2 x dx = -dt$

Putting $\cot x = t$ and $\operatorname{cosec}^2 x dx = -dt$ in equation (i), we get

$$I = \int t^n \times (-dt)$$
$$= -\frac{t^{n+1}}{n+1} + c$$

$\Rightarrow \quad = -\frac{(\cot x)^{n+1}}{n+1} + c$

Indefinite Integrals Ex 19.11 Q10

Let $I = \int \cot^5 x \operatorname{cosec}^4 x dx$. Then,

$$I = \int \cot^5 x \operatorname{cosec}^2 x \operatorname{cosec}^2 x dx$$
$$= \int \cot^5 x \left(1 + \cot^2 x\right) \operatorname{cosec}^2 x dx$$
$$\Rightarrow \quad I = \int \left(\cot^5 x + \cot^7 x\right) \operatorname{cosec}^2 x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x\, dx = dt$, we get

$$I = \int \left(t^5 + t^7\right)(-dt)$$
$$= -\frac{t^6}{6} - \frac{t^8}{8} + c$$
$$= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$
$$\therefore \quad I = -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

Indefinite Integrals Ex 19.11 Q11

Let $I = \int \cot^5 x dx$. Then,

$$I = \int \cot^3 x \times \cot^2 x dx$$
$$= \int \cot^3 x \times \left(\operatorname{cosec}^2 x - 1\right) dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \cot^3 x dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \left(\operatorname{cosec}^2 x - 1\right) \cot dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$
$$\Rightarrow \quad I = \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x\, dx = dt$ in first two integral, we get

$$I = \int t^3 (-dt) - \int t \times (-dt) + \int \cot x dx$$
$$= -\frac{t^4}{4} + \frac{t^2}{2} + \log|\sin x| + c$$
$$= -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$
$$\therefore \quad I = -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.11 Q12

Let $I = \int \cot^6 x dx$. Then,

$$I = \int \cot^2 x \times \cot^4 x dx$$
$$= \int \left(\operatorname{cosec}^2 x - 1\right) \times \cot^4 x dx$$
$$= \int \left(\operatorname{cosec}^2 x \cot^4 x - \cot^4 x\right) dx$$
$$= \int \operatorname{cosec}^2 x \times \cot^4 x dx - \int \cot^4 dx$$
$$= \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \left(\operatorname{cosec}^2 - 1\right) dx$$
$$= \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \operatorname{cosec}^2 x dx + \int \cot^2 x dx$$
$$\Rightarrow \quad I = \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \operatorname{cosec}^2 x dx + \int \left(\operatorname{cosec}^2 x - 1\right) dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x dx = dt$ in first two integral, we get

$$I = \int t^4 (-dt) - \int t^2 (-dt) + \int \operatorname{cosec}^2 x dx - \int dx$$
$$= -\frac{t^5}{5} + \frac{t^3}{3} - \cot x - x + c$$
$$= -\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3} - \cot x - x + c$$
$$\therefore \quad I = -\frac{1}{5} \times \cot^5 x + \frac{1}{3} \times \cot^3 x - \cot x - x + c$$

Ex 19.12

Indefinite Integrals Ex 19.12 Q1

Let $I = \int \sin^4 x \cos^3 x dx$

Here, power of $\cos x$ is odd, so we substitute

$$\sin x = t$$

$$\Rightarrow \quad \cos x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cos x}$$

$$\therefore \quad I = \int t^4 \cos^3 x \frac{dt}{\cos x}$$

$$= \int t^4 \cos^2 x dt$$

$$= \int t^4 \left(1 - \sin^2 x\right) dt$$

$$= \int t^4 \left(1 - t^2\right) dt$$

$$= \int \left(t^4 - t^6\right) dt$$

$$= \frac{t^5}{5} - \frac{t^7}{7} + c$$

$$\therefore \quad I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q2

Let $I = \int \sin^5 x dx$. Then

$$I = \int \sin^3 x \sin^2 x dx$$

$$= \int \sin^3 x \left(1 - \cos^2 x\right) dx$$

$$= \int \left(\sin^3 x - \sin^3 x \cos^2 x\right) dx$$

$$= \int \left[\sin x \left(1 - \cos^2 x\right) - \sin^3 x \cos^2 x\right] dx$$

$$= \int \left(\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x\right) dx$$

$$\Rightarrow \quad I = \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt$$

$$= \int \sin x dx + \int t^2 dt + \int \left(1 - \cos^2 x\right) t^2 dt$$

$$= \int \sin x dx + \int t^2 dt + \int \left(1 - t^2\right) t^2 dt$$

$$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$$

$$= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + c$$

$$= -\cos x + \frac{2}{3}\left(\cos^3 x\right) - \frac{1}{5}\left(\cos^5 x\right) + c$$

$$\therefore \quad I = -\left[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right] + c$$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$I = \int \cos^2 x \cos^3 x dx$$
$$= \int \left(1 - \sin^2 x\right) \cos^3 x dx$$
$$= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx$$
$$= \int \cos^2 x \cos x dx - \int \sin^2 x \left(1 - \sin^2 x\right) \cos x dx$$
$$= \int \left(\cos x - \sin^2 x \cos x\right) dx - \int \left(\sin^2 x \cos x - \sin^4 x \cos x\right) dx$$

$\Rightarrow \quad I = \int \cos x dx - 2\int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \cos x dx - 2\int t^2 dt + \int t^4 dt$$
$$= \sin x - \frac{2}{3}t^3 + \frac{t^5}{5} + c$$
$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

$\therefore \quad I = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

Indefinite Integrals Ex 19.12 Q4

Let $I = \int \sin^5 x \cos x \, dx$ ---(i)

Let $\sin x = t$. Then,

$$d\left(\sin x\right) = dt$$

$\Rightarrow \quad \cos x dx = dt$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$

$\therefore \quad I = \frac{1}{6}\sin^6 x + c$

Indefinite Integrals Ex 19.12 Q5

Let $I = \int \sin^3 x \cos^6 x dx$

Here, power of $\sin x$ is odd, so we substitute

$$\cos x = t$$

$\Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \sin^2 x t^6 \left(-dt\right)$

$$= -\int \left(1 - \cos^2 x\right) t^6 dt$$
$$= -\int \left(t^6 - t^8\right) dt$$
$$= -\frac{t^7}{7} + \frac{t^9}{9} + c$$
$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + c$$

$\therefore \quad I = -\frac{1}{7}\cos^7 x + \frac{1}{9}\cos^9 x + c$

Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$I = \int \cos^6 x \cos x dx$$
$$= \int \left(\cos^2 x\right)^3 \cos x dx$$
$$= \int \left(1 - \sin^2 x\right)^3 \cos x dx$$
$$= \int \left[1 - \sin^6 x - 3\sin^2 x + 3\sin^4 x\right] \cos x dx$$
$$= \int \left[\cos x - \sin^6 x \cos x - 3\sin^2 x \cos x + 3\sin^4 x \cos x\right] dx + c$$

$\Rightarrow \quad I = \int \cos x dx - \int \sin^6 x \cos x dx - 3\int \sin^2 x \cos x dx + 3\int \sin^4 x \cos x dx$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$I = \int \cos x dx - \int t^6 dt - 3\int t^2 dt + 3\int t^4 dt$$
$$= \sin x - \frac{t^7}{7} - \frac{3}{3}t^3 + \frac{3}{5}t^5 + c$$
$$= \sin x - \frac{1}{7}\sin^7 x - \sin^3 x + \frac{3}{5}\sin^5 x + c$$

$\therefore \quad I = \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

Indefinite Integrals Ex 19.12 Q7

Let $I = \int x \cos^3 x^2 \sin x^2 dx$

Let $\cos x^2 = t$. Then

$$d\left(\cos x^2\right) = dt$$

$\Rightarrow \quad -2x \sin x^2 x = dt$

$\Rightarrow \quad x \sin x^2 dx = -\frac{dt}{2}$

$\therefore \quad I = \int t^3 \times \frac{-dt}{2}$

$$= -\frac{t^4}{8} + c$$
$$= -\frac{1}{8}\cos^4 x^2 + c$$

$\therefore \quad I = -\frac{1}{8}\cos^4 x^2 + c$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$I = \int \sin^6 x \sin x dx$$
$$= \int \left(\sin^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x\right) \sin x dx$$

$\Rightarrow \quad I = \int \sin x dx - \int \cos^6 x \sin x dx + 3\int \cos^4 x \sin x dx - 3\int \cos^2 x \sin x dx$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2ne, 3rd and 4th integral, we get

$$I = \int \sin x dx - \int t^6 (-dt) + 3\int t^4 (-dt) - 3\int t^2 (-dt)$$
$$= -\cos x + \frac{t^7}{7} - \frac{3}{5}t^5 + \frac{3}{3}t^3 + c$$
$$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5}\cos^5 x + \cos^3 x + c$$

$\therefore \quad I = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$d(\cos x) = dt$$

$$\Rightarrow \quad -\sin x dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{\sin x}$$

$$\therefore \quad I = \int \sin^3 x t^5 \frac{-dt}{\sin x}$$

$$= -\int \sin^2 x t^5 dt$$

$$= -\int \left(1 - \cos^2 x\right) t^5 dt$$

$$= -\int \left(1 - t^2\right) t^5 dt$$

$$= -\int \left(t^5 - t^7\right) dt$$

$$= -\frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

$$\therefore \quad I = \frac{-1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

Indefinite Integrals Ex 19.12 Q10

Let $I = \int \frac{1}{\sin^4 x \cos^2 x} dx$ ---(i)

Then, $I = \int \sin^{-4} x \cos^{-2} x dx$

Since $-4 - 2 = -6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx$$

$$= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^6 x}{\tan^4 x} dx$$

$$= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(\sec^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(1 + \tan^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$\Rightarrow \quad I = \int \frac{\left(1 + \tan^4 x + 2\tan^2 x\right) \times \sec^2 x}{\tan^4 x} dx \qquad \text{---(ii)}$$

Let $\tan x = t$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int \frac{\left(1 + t^4 + 2t^2\right)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int \left(t^{-4} + 1 + 2t^{-2}\right) dt$$

$$= -\frac{t^{-3}}{3} + t - 2t^{-1} + c$$

$$= -\frac{1}{3t^3} + t - \frac{2}{t} + c$$

$$= -\frac{1}{3\tan^3 x} + \tan x - \frac{2}{\tan x} + c$$

$$= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + c$$

$$\therefore \quad I = \frac{-1}{3} \times \cot^3 x - 2\cot x + \tan x + c$$

Indefinite Integrals Ex 19.12 Q11

Let $I = \int \frac{1}{\sin^3 x \cos^5 x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-5} x dx$

Since $-3-5=-8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^6 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^3}{\tan^3 x} \times \sec^2 x dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^6 x + 3\tan^4 x + 3\tan^2 x\right) \times \sec^2 x}{\tan^3 x} dx \quad \text{---(ii)}$$

Let $t = \tan x$. Then,

$$d\left(\tan x\right) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\therefore \quad I = \int \frac{\left(1+t^6+3t^4+3t^2\right)}{t^3} dt$$

$$= \int \left(t^{-3}+t^3+3t+3t^{-1}\right) dt$$

$$= -\frac{t^{-2}}{2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2t^2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2} \times \frac{1}{\tan^2 x} + \frac{\tan^4 x}{4} + \frac{3}{2} \times \tan^2 x + 3\log\left|\tan x\right| + c$$

$$\therefore \quad I = \frac{-1}{2\tan^2 x} + 3\log\left|\tan x\right| + \frac{3}{2}\tan^2 x + \frac{1}{4} \times \tan^4 x + c$$

Indefinite Integrals Ex 19.12 Q12

Let $I = \int \frac{1}{\sin^3 x \cos x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-1} x dx$

Since $-3-1=-4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)}{\tan^3 x} \times \sec^2 x dx$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \frac{1+t^2}{t^3} dt$$

$$= \int \left(t^{-3} + \frac{1}{t}\right) dt$$

$$= -\frac{t^{-2}}{2} + \log\left|t\right| + c$$

$$= -\frac{1}{2t^2} + \log\left|t\right| + c$$

$$= -\frac{1}{2\tan^2 x} + \log\left|\tan x\right| + c$$

Indefinite Integrals Ex 19.12 Q13

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$
$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$
$$=\tan x\sec^2 x+\frac{\frac{1\cos^2 x}{\sin x\cos x}}{\cos^2 x}$$
$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

$$\therefore\ \int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 x\ dx+\int\frac{\sec^2 x}{\tan x}\ dx$$

Let $\tan x=t\Rightarrow\sec^2 x\ dx=dt$

$$\Rightarrow\int\frac{1}{\sin x\cos^3 x}dx=\int t\,dt+\int\frac{1}{t}dt$$
$$=\frac{t^2}{2}+\log|t|+C$$
$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

Ex 19.13

Indefinite Integrals Ex 19.13 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$

$= \frac{1}{2} \log \left| t + \sqrt{t^2 + (a^2)^2} \right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]$

$I = \frac{1}{2} \log \left| x^2 + \sqrt{(x^2)^2 + (a^2)^2} \right| + c$

$I = \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + a^4} \right| + c$

Indefinite Integrals Ex 19.13 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$

$= \log \left| t + \sqrt{t^2 + 4} \right| + C$

$= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C$

Indefinite Integrals Ex 19.13 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c \right]$

$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$

Indefinite Integrals Ex 19.13 Q4

Let $I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$

$= \log \left| t + \sqrt{(2)^2 + t^2} \right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + c \right]$

$I = \log \left| \sin x + \sqrt{4 + \sin^2 x} \right| + c$

Indefinite Integrals Ex 19.13 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}}dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x\, dx = dt$

$\Rightarrow \quad \sin x\, dx = -\frac{dt}{2}$

$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$

$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}}dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$

Indefinite Integrals Ex 19.13 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}}dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}}dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x}dx = dt$

$\Rightarrow \quad \frac{1}{x}dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{3}\sin^{-1}\left(\frac{3\log x}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}}dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\, dx = dt$

$\Rightarrow \quad \sin 8x\, dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2 + t^2}}$

$= \frac{1}{4}\log\left|t + \sqrt{(3)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}}dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \frac{1}{4}\log\left|\sin^2 4x + \sqrt{9 + \sin^4 4x}\right| + c$

Indefinite Integrals Ex 19.13 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\, dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2 + \left(2\sqrt{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|t + \sqrt{t^2 + \left(2\sqrt{2}\right)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = \frac{1}{2}\log\left|\sin 2x + \sqrt{\sin^2 2x + 8}\right| + c$$

Indefinite Integrals Ex 19.13 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4\sin^2 x - 2}} dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x \cos x\, dx = dt$

$\Rightarrow \quad \sin 2x\, dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2 + 4t - 2}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} dx$$

$$= \int \frac{dt}{\sqrt{(t+2)^2 - 6}}$$

Let $t + 2 = u$

$dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\sqrt{6}\right)^2}}$$

$$= \log\left|u + \sqrt{u^2 - 6}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|t + 2 + \sqrt{(t+2)^2 - 6}\right| + c$$

$$I = \log\left|\sin^2 x + 2 + \sqrt{\sin^4 x + 4\sin^2 x - 2}\right| + c$$

Indefinite Integrals Ex 19.13 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

$let\ t = \cos^2 x \rightarrow -dt = 2\cos x \sin x dx$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.13 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}} \sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3} - 1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$

$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since} \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$

Indefinite Integrals Ex 19.13 Q14

Let $I = \int \frac{1}{\sqrt{(1 - x^2)\left[9 + \left(\sin^{-1} x\right)^2\right]}} dx$

Let $\sin^{-1} x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1 - x^2}} dx = dt$

$I = \int \frac{dt}{\sqrt{(3)^2 + t^2}}$

$= \log\left|t + \sqrt{9 + t^2}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \log\left|\sin^{-1} x + \sqrt{9 + \left(\sin^{-1} x\right)^2}\right| + c$

Indefinite Integrals Ex 19.13 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$

$= \int \frac{dt}{\sqrt{t^2 - 2t + (1)^2 - (1)^2 - 3}}$

$= \int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$

Let $t - 1 = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{du}{\sqrt{u^2 - (2)^2}}$

$= \log\left|u + \sqrt{u^2 - 4}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$= \log\left|t - 1 + \sqrt{(t-1)^2 + 4}\right| + c$

$I = \log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right| + c$

Indefinite Integrals Ex 19.13 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1}\, dx$

$= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx$

$= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x\,(1 + \sin x)}} dx$

$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx$

$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2 + t}}$

$= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$== \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

Let, $t + \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}$

$== \log\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$= \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c$

$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$

Indefinite Integrals Ex 19.13 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx=\int \frac{\sin x-\cos x}{\sqrt{(\sin x+\cos x)^2-1}}dx$$

Let $\sin x+\cos x=t$ therefore $(\cos x-\sin x)dx=dt$

Now

$$\begin{aligned}\int \frac{\sin x-\cos x}{\sqrt{(\sin x+\cos x)^2-1}}dx &= -\int \frac{dt}{\sqrt{t^2-1}} \\ &= -\ln\left|t+\sqrt{t^2-1}\right|+c \\ &= -\ln\left|\sin x+\cos x+\sqrt{\sin 2x}\right|+c\end{aligned}$$

Ex 19.14

Indefinite Integrals Ex 19.14 Q1

$$\text{Let } I = \int \frac{1}{a^2 - b^2x^2} dx$$

$$= \frac{1}{b^2} \int \frac{1}{\frac{a^2}{b^2} - x^2} dx$$

$$= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx$$

$$I = \frac{1}{b^2} \times \frac{1}{2 \times \left(\frac{a}{b}\right)} \log \left| \frac{\frac{a}{b} + x}{\frac{a}{b} - x} \right| + c \qquad \left[\text{Since } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x + a}{x - a} \right| + c\right]$$

$$I = \frac{1}{2ab} \log \left| \frac{a + bx}{a - bx} \right| + c$$

Indefinite Integrals Ex 19.14 Q2

$$\text{Let } I = \int \frac{1}{a^2x^2 - b^2} dx$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 - \frac{b^2}{a^2}} dx$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2} dx$$

$$I = \frac{1}{a^2} \times \frac{1}{2 \times \left(\frac{b}{a}\right)} \times \log \left| \frac{x - \frac{b}{a}}{x + \frac{b}{a}} \right| + c \qquad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c\right]$$

$$I = \frac{1}{2ab} \log \left| \frac{ax - b}{ax + b} \right| + c$$

Indefinite Integrals Ex 19.14 Q3

$$\text{Let } I = \int \frac{1}{a^2x^2 + b^2} dx$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \frac{b^2}{a^2}} dx$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2} dx$$

$$I = \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1} \left(\frac{x}{\frac{b}{a}} \right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{ab} \tan^{-1} \left(\frac{ax}{b}\right) + c$$

Indefinite Integrals Ex 19.14 Q4

Let $I = \int \frac{x^2-1}{x^2-4}dx$

Let $I = \int \frac{x^2-1}{x^2+4}dx$

$= \int \frac{\left(x^2+4\right)-4-1}{x^2+4}dx$

$= \int \frac{x^2+4}{x^2+4}dx - \int \frac{5}{x^2+4}dx$

$= \int dx - 5\int \frac{1}{x^2+(2)^2}dx$

$I = x - 5 \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$ $\left[\text{Since } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$

$I = x - \frac{5}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$

Indefinite Integrals Ex 19.14 Q5

Let $2x = t$

$\Rightarrow 2dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$

$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right]+C$ $\left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$

$= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right|+C$

Indefinite Integrals Ex 19.14 Q6

Let $I = \int \frac{1}{\sqrt{a^2+b^2x^2}}dx$

Let $bx = t$

$\Rightarrow$ $bdx = dt$

$dx = \frac{dt}{b}$

$I = \frac{1}{b}\int \frac{1}{\sqrt{a^2+t^2}}dt$

$I = \frac{1}{b}\log\left|t+\sqrt{a^2+t^2}\right|+c$ $\left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \frac{1}{b}\log\left|bx+\sqrt{a^2+b^2x^2}\right|+c$ $[\text{since } t = bx]$

Indefinite Integrals Ex 19.14 Q7

Let $I = \int \frac{1}{\sqrt{a^2-b^2x^2}}dx$

Let $bx = t$

$\Rightarrow$ $bdx = dt$

$dx = \frac{dt}{b}$

so, $I = \frac{1}{b}\int \frac{1}{\sqrt{a^2-t^2}}dt$

$I = \frac{1}{b}\sin^{-1}\left(\frac{t}{a}\right)+c$ $\left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)+c$ $[\text{since } bx = t]$

Indefinite Integrals Ex 19.14 Q8

Let $I = \int \frac{1}{\sqrt{(2-x)^2+1}}dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $I = -\int \frac{1}{\sqrt{t^2+(1)^2}}dt$

$I = -\log\left|t + \sqrt{t^2+1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = -\log\left|(2-x) + \sqrt{(2-x)^2+1}\right| + c \qquad \left[\text{since } t = (2-x)\right]$

Indefinite Integrals Ex 19.14 Q9

Let $I = \int \frac{1}{\sqrt{(2-x)^2-1}}dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $I = -\int \frac{1}{\sqrt{t^2-(1)^2}}dt$

$I = -\log\left|t + \sqrt{t^2-(1)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = -\log\left|(2-x) + \sqrt{(2-x)^2-1}\right| + c \qquad \left[\text{since } t = (2-x)\right]$

Indefinite Integrals Ex 19.14 Q10

Let $I = \int \frac{x^4+1}{x^2+1}dx$

$I = \int \frac{(x^2+1)^2 - 2x^2}{x^2+1}dx \qquad \left[a^2+b^2 = (a+b)^2 - 2ab\right]$

$I = \int \frac{(x^2+1)^2}{x^2+1}dx - \int \frac{2x^2}{(x^2+1)}dx$

$I = \int (x^2+1)dx - \int \frac{2x^2+2-2}{(x^2+1)}dx$

$I = \int (x^2+1)dx - \int \frac{2(x^2+1)}{(x^2+1)}dx + 2\int \frac{1}{x^2+1}dx$

$I = \int (x^2+1)dx - \int 2dx + 2\int \frac{1}{x^2+1}dx$

$I = \frac{x^3}{3} + x - 2x + 2\,x\tan^{-1}(x) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+1}}dx = \tan^{-1}(x)+c\right]$

$I = \frac{x^3}{3} - x + 2\tan^{-1}(x) + c$

Ex 19.15

Indefinite Integrals Ex 19.15 Q1

Let $I = \int \frac{1}{4x^2+12x+5}dx$

$$= \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{4}}dx$$

$$= \frac{1}{4}\int \frac{1}{x^2+2\times x\times\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{5}{4}}dx$$

$$I = \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2-1}dx$$

Let $\left(x+\frac{3}{2}\right) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \frac{1}{4}\int \frac{1}{t^2-(1)^2}dt$$

$$I = \frac{1}{4}\times\frac{1}{2\times(1)}\log\left|\frac{t-1}{t+1}\right|+c \quad \left[\text{Since, } \int\frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{8}\log\left|\frac{x+\frac{3}{2}-1}{x+\frac{3}{2}+1}\right|+c \quad [\text{using (i)}]$$

$$I = \frac{1}{8}\log\left|\frac{2x+1}{2x+5}\right|+c$$

Indefinite Integrals Ex 19.15 Q2

Let $I = \int \frac{1}{x^2-10x+34}dx$

$$= \int \frac{1}{x^2-2x\times 5+(5)^2-(5)^2+34}dx$$

$$= \int \frac{1}{(x-5)^2+9}dx$$

Let $(x-1) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \int \frac{1}{t^2+(3)^2}dt$$

$$I = \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c \quad \left[\text{Since, } \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{2}\right)+c\right]$$

$$I = \frac{1}{3}\tan^{-1}\left(\frac{x-5}{3}\right)+c \quad [\text{using (i)}]$$

Indefinite Integrals Ex 19.15 Q3

$$\int \frac{1}{1+x-x^2}dx = \int \frac{1}{-\left(x^2-x-1\right)}dx$$

adding and subtracting $\frac{1}{4}$ in the denominator to make it a perfect square

$$= \int \frac{1}{-\left(x^2-x+\frac{1}{4}-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x^2-x+\frac{1}{4}\right]-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-1-\frac{1}{4}\right)}dx = \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-\frac{5}{4}\right)}dx$$

$$= \int \frac{1}{\left(\left(\frac{\sqrt{5}}{2}\right)^2-\left[x-\frac{1}{2}\right]^2\right)}d$$

$$= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)}\log\left|\frac{\frac{\sqrt{5}}{2}+\left(x-\frac{1}{2}\right)}{\frac{\sqrt{5}}{2}-\left(x-\frac{1}{2}\right)}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right|$$

Indefinite Integrals Ex 19.15 Q4

$$\text{Let } I = \int \frac{1}{2x^2-x-1}dx$$

$$= \frac{1}{2}\int \frac{1}{x^2-\frac{x}{2}-\frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{x^2-2x\times\frac{1}{4}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-\frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{4}\right)^2-\frac{9}{16}}dx$$

$$\text{Let } x-\frac{1}{4} = t$$

$$\Rightarrow \quad dx = dt$$

$$I = \frac{1}{2}\int \frac{1}{t^2-\left(\frac{3}{4}\right)^2}dt$$

$$I = \frac{1}{2}\times\frac{1}{2\times\left(\frac{3}{4}\right)}\log\left|\frac{t-\frac{3}{4}}{t+\frac{3}{4}}\right|+c \quad \left[\text{Since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{3}\log\left|\frac{x-\frac{1}{4}-\frac{3}{4}}{x-\frac{1}{4}+\frac{3}{4}}\right|+c$$

$$I = \frac{1}{3}\log\left|\frac{x-1}{2x+1}\right|+c$$

Indefinite Integrals Ex 19.15 Q5

We have $x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13 = (x + 3)^2 + 4$

Sol, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{1}{(x + 3)^2 + 2^2} dx$

Let $x + 3 = t$. Then $dx = dt$

Therefore, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dt}{t^2 + 2^2} = \frac{1}{2} \tan^{-1} \frac{t}{2} + C$ [by 7.4 (3)]

$= \frac{1}{2} \tan^{-1} \frac{x + 3}{2} + C$

Ex 19.16

Indefinite Integrals Ex 19.16 Q1

Let $I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x \, dx = dt$

$I = \int \frac{dt}{(1)^2 - t^2}$

$= \frac{1}{2(1)} \log\left|\frac{1+t}{1-t}\right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c\right]$

$I = \frac{1}{2} \log\left|\frac{1 + \tan x}{1 - \tan x}\right| + c$

Indefinite Integrals Ex 19.16 Q2

Let $I = \int \frac{e^x}{1 + e^{2x}} dx$

Let $\tan e^x = t$

$\Rightarrow \quad e^x \, dx = dt$

so, $\quad I = \int \frac{dt}{1 + t^2}$

$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(e^x) + c$

Indefinite Integrals Ex 19.16 Q3

Let $I = \int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

so, $\quad I = \int \frac{dx}{t^2 + 4t + 5}$

$= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5}$

$= \int \frac{dt}{(t+2)^2 + 1}$

Again, Let $(t+2) = u$

$dt = du$

$I = \int \frac{dt}{u^2 + 1}$

$= \tan^{-1}(u) + c \quad \left[\text{Since, } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(t + 2) + c$

$I = \tan^{-1}(\sin x + 2) + c$

Indefinite Integrals Ex 19.16 Q4

Let $I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

so, $\quad I = \int \frac{dt}{t^2 + 5t + 6}$

$= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6}$

$= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}}$

Put $\left(t + \frac{5}{2}\right) = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{dt}{u^2 - \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{u - \frac{1}{2}}{u + \frac{1}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x - a}{x + a}\right| + c\right]$

$I = \log\left|\frac{2u - 1}{2u + 1}\right| + c$

$I = \log\left|\frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1}\right| + c$

$I = \log\left|\frac{e^x + 2}{e^x + 3}\right| + c$

Indefinite Integrals Ex 19.16 Q5

Let $I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$

Let $e^{3x} = t$

$\Rightarrow \quad 3e^{3x}\, dx = dt$

$\Rightarrow \quad e^{3x} dx = \frac{dt}{3}$

$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$

$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$

$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$

$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log\left|\frac{t - \frac{3}{2}}{t + \frac{3}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x - a}{x + a}\right| + c\right]$

$I = \frac{1}{36} \log\left|\frac{2t - 3}{2t + 3}\right| + c$

$I = \frac{1}{36} \log\left|\frac{2e^{3x} - 3}{2e^{3x} + 3}\right| + c$

Indefinite Integrals Ex 19.16 Q6

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}}$$

$$= \int \frac{dx}{e^x + \frac{1}{e^x}}$$

$$= \int \frac{e^x dx}{\left(e^x\right)^2 + 1}$$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$$I = \int \frac{dt}{t^2 + 1}$$

$$I = \tan^{-1} t + c \qquad \left[\text{Since } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c\right]$$

$$I = \tan^{-1}\left(e^x\right) + c$$

Indefinite Integrals Ex 19.16 Q7

$$\text{Let } I = \int \frac{x}{x^4 + 2x^2 + 3} dx$$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$$I = \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2}$$

put $t + 1 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$$

Indefinite Integrals Ex 19.16 Q8

$$\text{Let } I = \int \frac{3x^5}{1 + x^{12}} dx$$

$$= \int \frac{3x^5}{1 + \left(x^6\right)^2} dx$$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\,dx = dt$

$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$

$$I = \frac{3}{6} \int \frac{dt}{1 + t^2}$$

$$= \frac{1}{2} \tan^{-1}(t) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c\right]$$

$$I = \frac{1}{2} \tan^{-1}\left(x^6\right) + c$$

Indefinite Integrals Ex 19.16 Q9

Let $I = \int \frac{x^2}{x^6 - a^6} dx$

$= \int \frac{x^2}{\left(x^3\right)^2 - \left(a^3\right)^2} dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 - \left(a^3\right)^2}$

$= \frac{1}{3} \times \frac{1}{2a^3} \log \left| \frac{t - a^3}{t + a^3} \right| \qquad \left[\text{Since} \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$

$I = \frac{1}{6a^3} \log \left| \frac{x^3 - a^3}{x^3 + a^3} \right| + c$

Indefinite Integrals Ex 19.16 Q10

Let $I = \int \frac{x^2}{x^6 + \left(a^3\right)^2} dx$

$= \int \frac{x^2}{\left(x^3\right)^2 + \left(a^3\right)^2} dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 + \left(a^3\right)^2}$

$= \frac{1}{3} \times \frac{1}{\left(a^3\right)} \tan^{-1} \left(\frac{t}{a^3} \right) + c \qquad \left[\text{Since} \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$

$I = \frac{1}{3a^3} \tan^{-1} \left(\frac{x^3}{a^3} \right) + c$

Indefinite Integrals Ex 19.16 Q11

Let $I = \int \frac{1}{x\left(x^6+1\right)}dx$

$= \int \frac{x^5}{x^6\left(x^6+1\right)}dx$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\,dx = dt$

$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$

$I = \frac{1}{6}\int \frac{dt}{t\left(t+1\right)}$

$= \frac{1}{6}\int \frac{dt}{t^2+t}$

$= \frac{1}{6}\int \frac{dt}{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{6}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

Let $t+\frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$I = \frac{1}{6}\int \frac{du}{u^2-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{6}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right|+c \qquad \left[\text{Since } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{6}\log\left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right|+c$

$I = \frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right|+c$

Indefinite Integrals Ex 19.16 Q12

Let $I = \int \frac{x}{x^4 - x^2 + 1} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

so, $\quad I = \frac{1}{2}\int \frac{dt}{t^2 - t + 1}$

$$= \frac{1}{2}\int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)}$$

Let $\quad t - \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2}\int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{u}{\frac{\sqrt{3}}{2}}\right) + c \qquad \left[\text{Since} \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.16 Q13

Let $I = \int \frac{x}{3x^4 - 18x^2 + 11}dx$

$= \frac{1}{3}\int \frac{x}{x^4 - 6x^2 + \frac{11}{3}}dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{3} \times \frac{1}{2}\int \frac{dt}{t^2 - 6t + \frac{11}{3}}$

$= \frac{1}{6}\int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$

$= \frac{1}{6}\int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}$

Let t-3 = u

$\Rightarrow \quad dt = du$

$I = \frac{1}{6}\int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2}$

$= \frac{1}{6} \times \frac{1}{2\left(\frac{4}{\sqrt{3}}\right)} \log\left|\frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}}\right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$

$I = \frac{\sqrt{3}}{48}\log\left|\frac{t - 3 - \frac{4}{\sqrt{3}}}{t - 3 + \frac{4}{\sqrt{3}}}\right| + c$

$I = \frac{\sqrt{3}}{48}\log\left|\frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}}\right| + c$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\int \frac{e^x}{(1+e^x)(2+e^x)}dx = \int \frac{dt}{(1+t)(2+t)}$$

$$= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)}$$

$$= \ln|1+t| - \ln|2+t| + c$$

$$= \ln\left|\frac{1+t}{2+t}\right| + c$$

$$= \ln\left|\frac{1+e^x}{2+e^x}\right| + c$$

Ex 19.17

Indefinite Integrals Ex 19.17 Q1

Let $I = \int \frac{1}{\sqrt{2x - x^2}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x(1) + 1^2 - 1^2\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[(x-1)^2 - 1\right]}} dx$

$= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx$

Let $(x - 1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \int \frac{1}{\sqrt{1-t^2}} dt$

$= \sin^{-1} t + c \quad \left[\text{Since } \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c\right]$

$I = \sin^{-1}(x-1) + c$

Indefinite Integrals Ex 19.17 Q2

$8 + 3x - x^2$ can be written as $8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$.

Therefore,

$8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$

$= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$

$\Rightarrow \int \frac{1}{\sqrt{8 + 3x - x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx$

Let $x - \frac{3}{2} = t$

$\therefore \ dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt$

$= \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{2x - 3}{\sqrt{41}}\right) + C$

Indefinite Integrals Ex 19.17 Q3

$$\text{Let } I = \int \frac{1}{\sqrt{5 - 4x - 2x^2}} dx$$
$$= \int \frac{1}{\sqrt{-2\left[x^2 + 2x - \frac{5}{2}\right]}} dx$$
$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2 + 2x + (1)^2 - (1)^2 - \frac{5}{2}\right]}} dx$$
$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[(x+1)^2 - \frac{7}{2}\right]}} dx$$
$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2} - (x+1)^2}} dx$$

Let $(x+1) = t$

$\Rightarrow \quad dx = dt$

$$\text{so, } \quad I = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - t^2}} dt$$
$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\sqrt{\frac{7}{2}}}\right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\sqrt{\frac{2}{7}} \times (x+1)\right) + c$$

Indefinite Integrals Ex 19.17 Q4

$$\text{Let } I = \int \frac{1}{\sqrt{3x^2 + 5x + 7}} dx$$
$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{5}{3}x + \frac{7}{3}}} dx$$
$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + 2x\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{7}{3}}} dx$$
$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{\left(x + \frac{5}{6}\right)^2 - \frac{59}{36}}} dx$$

Let $\left(x + \frac{5}{6}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)^2}} dt$$
$$= \frac{1}{\sqrt{3}} \log\left|t + \sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$
$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6}\right)^2 - \left(\frac{\sqrt{59}}{6}\right)^2}\right| + c$$

$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}}\right| + c$$

Indefinite Integrals Ex 19.17 Q5

Let $I = \int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx, (\text{as } \beta > \alpha)$

$$= \int \frac{1}{\sqrt{-x^2 - x(\alpha+\beta) - \alpha\beta}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}}dx$$

$$= \int \frac{1}{\sqrt{\left[\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2\right]}}dx, \quad [\therefore \quad \beta > \alpha]$$

Let $\left(x - \frac{\alpha+\beta}{2}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\beta-\alpha}{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta-\alpha}\right) + c$$

$$I = \sin^{-1}\left(\frac{2x - \alpha - \beta}{\beta - \alpha}\right) + c$$

Indefinite Integrals Ex 19.17 Q6

Let $I = \int \frac{1}{\sqrt{7-3x-2x^2}}dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2+\frac{3}{2}x-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[x^2+2x\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{3}{4}\right)^2-\frac{65}{16}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-\left(x+\frac{3}{4}\right)^2}}dx$$

Let $\left(x+\frac{3}{4}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-t^2}}dt$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4\left(x+\frac{3}{4}\right)}{\sqrt{65}}\right)+c$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x+3}{\sqrt{65}}\right)+c$$

Indefinite Integrals Ex 19.17 Q7

Let $I = \int \frac{1}{\sqrt{16-6x-x^2}}dx$

$$= \int \frac{1}{\sqrt{-\left[x^2+6x-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2+2x(3)+(3)^2-(3)^2-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[(x+3)^2-25\right]}}dx$$

$$= \int \frac{1}{\sqrt{25-(x+3)^2}}dx$$

Let $(x+3) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{5^2-t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{5}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \sin^{-1}\left(\frac{x+3}{5}\right)+c$$

Indefinite Integrals Ex 19.17 Q8

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-(x+3)^2$$
$$=(4)^2-(x+3)^2$$

$$\therefore \int \frac{1}{\sqrt{7-6x-x^2}}dx = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx = \int \frac{1}{\sqrt{(4)^2-(t)^2}}dt$$
$$= \sin^{-1}\left(\frac{t}{4}\right)+C$$
$$= \sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Indefinite Integrals Ex 19.17 Q9

We have $\int \frac{dx}{\sqrt{5x^2-2x}} = \int \frac{dx}{\sqrt{5\left(x^2-\frac{2x}{5}\right)}}$

$$= \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2}} \quad \text{(completing the square)}$$

Put $x-\frac{1}{5}=t$. Then $dx = dt$.

Therefore, $\int \frac{dx}{\sqrt{5x^2-2x}} = \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{t^2-\left(\frac{1}{5}\right)^2}}$

$$= \frac{1}{\sqrt{5}} \log\left|t+\sqrt{t^2-\left(\frac{1}{5}\right)^2}\right| + C \qquad [\text{by } 7.4\ (4)]$$

$$= \frac{1}{\sqrt{5}} \log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right| + C$$

Ex 19.18

Indefinite Integrals Ex 19.18 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$

$= \frac{1}{2} \log\left|t + \sqrt{t^2 + (a^2)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{(x^2)^2 + (a^2)^2}\right| + c$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{x^4 + a^4}\right| + c$

Indefinite Integrals Ex 19.18 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$

$= \log\left|t + \sqrt{t^2 + 4}\right| + C$

$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$

Indefinite Integrals Ex 19.18 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$

Indefinite Integrals Ex 19.18 Q4

Let $I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$

$= \log\left|t + \sqrt{(2)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \log\left|\sin x + \sqrt{4 + \sin^2 x}\right| + c$

Indefinite Integrals Ex 19.18 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}}dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x\,dx = dt$

$\Rightarrow \quad \sin x\,dx = -\frac{dt}{2}$

$$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}}dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$$

Indefinite Integrals Ex 19.18 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}}dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$$

Indefinite Integrals Ex 19.18 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}}dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x}dx = dt$

$\Rightarrow \quad \frac{1}{x}dx = \frac{dt}{3}$

$$I = \frac{1}{3}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{3}\sin^{-1}\left(\frac{3\log x}{2}\right) + c$$

Indefinite Integrals Ex 19.18 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9+(\sin 4x)^4}}dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\,dx = dt$

$\Rightarrow \quad \sin 8x\,dx = \frac{dt}{4}$

$$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2+t^2}}$$

$$= \frac{1}{4}\log\left|t+\sqrt{(3)^2+t^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$$

$$I = \frac{1}{4}\log\left|\sin^2 4x+\sqrt{9+\sin^4 4x}\right|+c$$

Indefinite Integrals Ex 19.18 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x+8}}dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\,dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2+\left(2\sqrt{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|t+\sqrt{t^2+\left(2\sqrt{2}\right)^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$$

$$I = \frac{1}{2}\log\left|\sin 2x+\sqrt{\sin^2 2x+8}\right|+c$$

Indefinite Integrals Ex 19.18 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x+4\sin^2 x-2}}dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x\cos x\,dx = dt$

$\Rightarrow \quad \sin 2x\,dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2+4t-2}}$$

$$= \int \frac{dt}{\sqrt{t^2+2t(2)+(2)^2-(2)^2-2}}dx$$

$$= \int \frac{dt}{\sqrt{(t+2)^2-6}}$$

Let $t+2 = u$

$dt = du$

$$= \int \frac{du}{\sqrt{u^2-\left(\sqrt{6}\right)^2}}$$

$$= \log\left|u+\sqrt{u^2-6}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$$

$$= \log\left|t+2+\sqrt{(t+2)^2-6}\right|+c$$

$$I = \log\left|\sin^2 x+2+\sqrt{\sin^4 x+4\sin^2 x-2}\right|+c$$

Indefinite Integrals Ex 19.18 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

let $t = \cos^2 x \to -dt = 2\cos x \sin x dx$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.18 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Indefinite Integrals Ex 19.18 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3}-1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$$

$$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$$

Indefinite Integrals Ex 19.18 Q14

Let $I = \int \frac{1}{\sqrt{(1-x^2)\left[9+(\sin^{-1}x)^2\right]}}dx$

Let $\sin^{-1}x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$I = \int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \log\left|t+\sqrt{9+t^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \log\left|\sin^{-1}x+\sqrt{9+(\sin^{-1}x)^2}\right|+c$

Indefinite Integrals Ex 19.18 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2-2t-3}}$

$= \int \frac{dt}{\sqrt{t^2-2t+(1)^2-(1)^2-3}}$

$= \int \frac{dt}{\sqrt{(t-1)^2-(2)^2}}$

Let $t-1 = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{du}{\sqrt{u^2-(2)^2}}$

$= \log\left|u+\sqrt{u^2-4}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$= \log\left|t-1+\sqrt{(t-1)^2+4}\right|+c$

$I = \log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right|+c$

Indefinite Integrals Ex 19.18 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1}\, dx$

$$= \int \sqrt{\frac{1 - \sin x}{\sin x}}\, dx$$

$$= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x\,(1 + \sin x)}}\, dx$$

$$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}}\, dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}}\, dx$$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$$= \int \frac{dt}{\sqrt{t^2 + t}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

Let, $t + \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \log\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c$$

$$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$$

Indefinite Integrals Ex 19.18 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}}\, dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\, dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x)\, dx = dt$

Now

$$\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\, dx = -\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\ln\left|t + \sqrt{t^2 - 1}\right| + c$$

$$= -\ln\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + c$$

Ex 19.19

Indefinite Integrals Ex 19.19 Q1

Let $I = \int \frac{x}{x^2+3x+2}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+3x+2\right)+\mu$

$= \lambda\left(2x+3\right)+\mu$

$x = \left(2\lambda\right)x+\left(3\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$3\lambda+\mu = 0 \quad \Rightarrow \quad 3\left(\frac{1}{2}\right)+\mu = 0$

$\mu = -\frac{3}{2}$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+3\right)-\frac{3}{2}}{x^2+3x+2}dx$

$I = \frac{1}{2}\int \frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int \frac{1}{x^2+3x+2}dx$

$= \frac{1}{2}\int \frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2}dx$

$I = \frac{1}{2}\int \frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$= \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right|+c \qquad \left[\text{since, } \int\frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right|+c$

Indefinite Integrals Ex 19.19 Q2

Let $I = \int \frac{x+1}{x^2+x+3}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+x+3\right)+\mu$

$x+1 = \lambda\left(2x+1\right)+\mu$

$x+1 = \left(2\lambda\right)x+\left(\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \qquad \lambda = \frac{1}{2}$

$\lambda+\mu = 1 \qquad \Rightarrow \qquad \left(\frac{1}{2}\right)+\mu = 0$

$\mu = \frac{1}{2}$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+1\right)+\frac{1}{2}}{x^2+x+3}dx$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int \frac{1}{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+3}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{11}{4}\right)}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}dx$$

$$= \frac{1}{2}\log\left|x^2+x+3\right| + \frac{1}{2}\times\frac{1}{\left(\frac{\sqrt{11}}{2}\right)}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+x+3\right| + \frac{1}{\sqrt{11}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)+c$$

Indefinite Integrals Ex 19.19 Q3

Let $I = \int \frac{x-3}{x^2+2x-4}dx$

Let $x-3 = \lambda \frac{d}{dx}\left(x^2+2x-4\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$x-3 = \left(2\lambda\right)x+\left(2\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \qquad \lambda = \frac{1}{2}$

$2\lambda+\mu = -3 \qquad \Rightarrow \qquad 2\left(\frac{1}{2}\right)+\mu = -3$

$\mu = -4$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+2\right)-4}{x^2+2x-4}dx$

$$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{x^2+2x+\left(1\right)^2-\left(1\right)^2-4}dx$$

$$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{\left(x+1\right)^2-\left(\sqrt{5}\right)}dx$$

$$I = \frac{1}{2}\log\left|x^2+2x-4\right| - 4\times\frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+2x-4\right| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$$

Indefinite Integrals Ex 19.19 Q4

Let $I = \int \frac{2x-3}{x^2+6x+13}dx$

Let $2x-3 = \lambda \frac{d}{dx}\left(x^2+6x+13\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$2x-3 = \left(2\lambda\right)x+\left(6\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \Rightarrow \quad \lambda = 1$

$6\lambda+\mu = -3 \quad \Rightarrow \quad 6\left(1\right)+\mu = -3$

$\mu = -9$

so, $\quad I = \int \frac{1\left(2x+6\right)-9}{x^2+6x+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{\left(x+3\right)^2+\left(2\right)^2}dx$

$= \log\left|x^2+6x+13\right| - 9\times\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c \quad \left[\text{since, } \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \log\left|x^2+6x+13\right| - \frac{9}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$

Indefinite Integrals Ex 19.19 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$= \int\left\{1-\frac{7x+10}{x^2+7x+10}\right\}dx$

$I = x - \int\frac{7x+10}{x^2+7x+10}dx + c_1 ----\left(\text{i}\right)$

Let $I_1 = \int\frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda\frac{d}{dx}\left(x^2+7x+10\right)+\mu$

$= \lambda\left(2x+7\right)+\mu$

$7x+10 = \left(2\lambda\right)x+7\lambda+\mu$

Comparing the coefficients of like powers of x,

$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$

$7\lambda+\mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right)+\mu = 10$

$\mu = -\frac{29}{2}$

so, $\quad I = \int\frac{\frac{1}{6}\left(6x-4\right)-\frac{1}{3}}{3x^2-4x+3}dx$

$I = \frac{1}{6}\int\frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int\frac{1}{x^2-\frac{4}{3}x+1}dx$

$I = \frac{1}{6}\int\frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int\frac{1}{x^2-2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\left(2\right)^2}dx$

$I = \frac{1}{6}\int\frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int\frac{1}{\left(x-\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}dx$

$= \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{1}{9}\times\frac{1}{\left(\frac{\sqrt{5}}{3}\right)}\tan^{-1}\left(\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c \quad \left[\text{since, } \int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{\sqrt{5}}{15}\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)+c$

Indefinite Integrals Ex 19.19 Q6

We need to evaluate the integral $\int \frac{2x}{2+x-x^2}dx$

write the numerator in the following form

$$2x = \lambda\left\{\frac{d}{dx}\left(2+x-x^2\right)\right\}+\mu$$

i.e. $2x = \lambda\{-2x+1\}+\mu$

Equating the coefficients will give the values of λ,μ

$\lambda = -1, \mu = 1$

$$\int \frac{2x}{2+x-x^2}dx = \int \frac{\lambda\{-2x+1\}+\mu}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}+1}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}}{2+x-x^2}dx + \frac{1}{2+x-x^2}dx$$

$$= -1\log\left|2+x-x^2\right| + \int \frac{1}{2+x-x^2}dx$$

$$= -1\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x-2\right)}dx$$

$$= -1\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)}dx$$

$$= -1\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)}dx$$

$$= -1\log\left|2+x-x^2\right| - \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

$$= -1\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{\left(x-\frac{1}{2}\right)-\left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right)+\left(\frac{3}{2}\right)}\right| + C$$

$$= -1\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{(x-2)}{(x+1)}\right| + C$$

Indefinite Integrals Ex 19.19 Q7

Let $I = \int \frac{1-3x}{3x^2+4x+2}dx$

Let $1\text{-}3x = \lambda \frac{d}{dx}\left(3x^2+4x+2\right)+\mu$

$= \lambda\left(6x+4\right)+\mu$

$1-3x = \left(6\lambda\right)x+\left(4\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$4\lambda+\mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = 3$

so, $I = \int \frac{-\frac{1}{2}\left(6x+4\right)+3}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + 3\int \frac{1}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \frac{3}{3}\int \frac{1}{x^2+\frac{4}{3}x+\frac{2}{3}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{x^2+2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\frac{2}{3}}dx$

$= -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\frac{2}{9}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}dx$

$= -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+c$

Indefinite Integrals Ex 19.19 Q8

Let $I = \int \frac{2x+5}{x^2-x-2}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2-x-2\right)+\mu$

$= \lambda\left(2x-1\right)+\mu$

$2x+5 = \left(2\lambda\right)x-\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$-\lambda+\mu = 5 \quad \Rightarrow \quad -1+\mu = 5$

$\mu = 6$

so, $I = \int \frac{\left(2x-1\right)+6}{x^2-x-2}dx$

$I = \int \frac{\left(2x-1\right)}{x^2-x-2}dx + 6\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}dx$

$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{9}{4}}dx$

$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$

$I = \log\left|x^2-x-2\right| + \frac{6}{2\left(\frac{3}{2}\right)}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \log\left|x^2-x-2\right| + 2\log\left|\frac{x-2}{x+1}\right|+c$

Indefinite Integrals Ex 19.19 Q9

Let $I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$

Let $ax^3 + bx = \lambda \frac{d}{dx}\left(x^4 + c^2\right) + \mu$

$ax^3 + bx = \lambda\left(4x^3\right) + \mu$

Comparing the coefficients of like powers c

$4\lambda = a \quad \Rightarrow \quad \lambda = \frac{a}{4}$

$\mu = 0 \quad \Rightarrow \quad \mu = 0$

so, $I = \int \frac{\frac{a}{4}\left(4x^3\right) + bx}{x^4 + c^2} dx$

$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + b \int \frac{x}{\left(x^2\right)^2 + c^2} dx$

$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2} \int \frac{2x}{\left(x^2\right)^2 + c^2} dx$

$= \frac{a}{4} \log\left|x^4 + c^2\right| + \frac{b}{2} I_1 - - - - (i)$

Now,

$I_1 = \int \frac{2x}{\left(x^2\right)^2 + c^2} dx$

Put $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$I_1 = \int \frac{1}{(t)^2 + c^2} dx$

$= \frac{1}{c} \tan^{-1}\left(\frac{t}{c}\right) + c_1$

$I_1 = \frac{1}{c} \tan^{-1}\left(\frac{x^2}{c}\right) + c_1 - - - - - (ii)$

Using equation (ii) in equation (i),

$I = \frac{a}{4} \log\left|x^4 + c^2\right| + \frac{b}{2c} \tan^{-1}\left(\frac{x^2}{c}\right) + k$

K = Integration constant

Indefinite Integrals Ex 19.19 Q10

Let $I = \int \frac{x+2}{2x^2+6x+5}dx$

Let $x + 2 = \lambda \frac{d}{dx}\left(2x^2+6x+5\right) + \mu$

$= \lambda\left(4x+6\right) + \mu$

$x + 2 = \left(4\lambda\right)x + \left(6\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$6\lambda + \mu = 2 \quad \Rightarrow \quad 6\left(\frac{1}{4}\right) + \mu = 2$

$\mu = \frac{1}{2}$

so, $I = \int \frac{\frac{1}{4}\left(4x+6\right)+\frac{1}{2}}{2x^2+6x+5}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{1}{2x^2+6x+5}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{2}}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^3+\frac{5}{2}}dx$

$= \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{1}{4}}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right) + c\,dx$ $\quad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}\tan^{-1}\left(2x+3\right) + c$

Indefinite Integrals Ex 19.19 Q11

Let $I=\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

$\therefore I=\int \frac{(3\sin x-2)\cos x}{5-(1-\sin^2 x)-4\sin x}dx$

$\Rightarrow I=\int \frac{(3\sin x-2)\cos x}{5-1+\sin^2 x-4\sin x}dx$

Substitute sinx=t

⇒ cosx dx = dt

Thus,

$I=\int \frac{(3t-2)}{4+t^2-4t}dt$

$\Rightarrow I=\int \frac{(3t-2)}{t^2-4t+4}dt$

$\Rightarrow I=\int \frac{(3t-2)}{(t-2)^2}dt$

Now let us separate the integrand into the simplest form using partial fractions.

$\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$

$=\frac{A(t-2)+B}{(t-2)^2}$

$=\frac{At-2A+B}{(t-2)^2}$

$\Rightarrow 3t-2=At-2A+B$

Comparing the coefficients, we have,

A=3

and

$-2A+B=-2$

Susbtituting the value of A=3 in the above equation, we have,

$\Rightarrow -2\times 3+B=-2$

$\Rightarrow -6+B=-2$

$\Rightarrow B=6-2$

$\Rightarrow B=4$

Thus, $I=\int \frac{(3t-2)}{(t-2)^2}dt$ becomes,

$I=\int \frac{3}{(t-2)}dt+\int \frac{4}{(t-2)^2}dt$

$=3\log|t-2|-4\left(\frac{1}{t-2}\right)+C$

$=3\log|2-t|+4\left(\frac{1}{2-t}\right)+C$

Now substituting t=sinx, we have,

$I=3\log|2-\sin x|+4\left(\frac{1}{2-\sin x}\right)+C$

Indefinite Integrals Ex 19.19 Q12

Let I=$\int \frac{5x-2}{1+2x+3x^2}dx$

Rewriting the numerator we have,

$5x-2 = A\frac{d}{dx}\left(1+2x+3x^2\right)+B$

$\Rightarrow 5x-2 = A\left(2+6x\right)+B$

$\Rightarrow 5x-2 = 6xA+2A+B$

Comparing the coefficients, we have,

6A=5 and $2A+B=-2$

$\Rightarrow A=\frac{5}{6}$

Substituting the value of A in $2A+B=-2, we have,$

$2\times\frac{5}{6}+B=-2$

$\Rightarrow \frac{10}{6}+B=-2$

$\Rightarrow B=-2-\frac{10}{6}$

$\Rightarrow B=\frac{-12-10}{6}$

$\Rightarrow B=\frac{-22}{6}$

$\Rightarrow B=\frac{-11}{3}$

$5x-2=\frac{5}{6}(2+6x)-\frac{11}{3}$

Thus, I=$\int \frac{5x-2}{1+2x+3x^2}dx$ becomes,

$$I=\int \frac{\left[\frac{5}{6}(2+6x)-\frac{11}{3}\right]}{3x^2+2x+1}dx$$

$$=\frac{5}{6}\int \frac{(2+6x)}{3x^2+2x+1}dx-\frac{11}{3}\int \frac{dx}{3x^2+2x+1}$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\times 3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int \frac{dx}{x^2+\frac{2}{3}x+\left(\frac{4}{3}\right)^2+\frac{1}{3}-\left(\frac{4}{3}\right)^2}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left[\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{3}{\sqrt{2}}\tan^{-1}\left[\frac{\left(\frac{3x+1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\sqrt{2}}\tan^{-1}\left[\frac{3x+1}{\sqrt{2}}\right]+C$$

Ex 19.20

Indefinite Integrals Ex 19.20 Q1

Let $I = \int \frac{x^2+x+1}{x^2-x}dx$

$= \int\left[1+\frac{2x+1}{x^2-x}\right]dx$

$= x + \int \frac{2x+1}{x^2-x}dx + c_1 ----(i)$

$I_1 = \int \frac{2x+1}{x^2-x}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2-x\right)+\mu$

$= \lambda\left(2x-1\right)+\mu$

$2x+1 = \left(2\lambda\right)x - \lambda + \mu$

Comparing the coefficients of like powers of x,

$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 2$

so, $I_1 = \int \frac{\left(2x-1\right)+2}{x^2-x}dx$

$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2-x}dx$

$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$= \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$I = \log\left|x^2-x\right| + 2\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x-\frac{1}{2}-\frac{1}{2}}{x-\frac{1}{2}+\frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int\frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I_1 = \log\left|x^2-x\right| + 2\log\left|\frac{x-1}{x}\right| + c_2 ----(ii)$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

$x \quad -x$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 - x\right) + \mu$

$= \lambda\left(2x - 1\right) + \mu$

$2x + 1 = \left(2\lambda\right)x - \lambda + \mu$

Comparing the coefficients of like powers of x,

$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 2$

so, $I_1 = \int \frac{\left(2x-1\right)+2}{x^2 - x}dx$

$$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2-x}dx$$

$$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$= \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$I = \log\left|x^2 - x\right| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c_2 ----(ii)$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

Indefinite Integrals Ex 19.20 Q2

Let $I = \int \frac{x^2 + x - 1}{x^2 + x - 6}dx$

$$= \int\left[1 + \frac{5}{x^2 + x - 6}\right]dx$$

$$I = x + \int \frac{5}{x^2 + x - 6}dx + c_1 ----(i)$$

Let $I_1 = 5\int \frac{1}{x^2 + x - 6}dx$

$$= 5\int \frac{1}{x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}dx$$

$$= 5\int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}dx$$

$$= 5\frac{1}{2\left(\frac{5}{2}\right)}\log\left|\frac{x + \frac{1}{2} - \frac{5}{2}}{x + \frac{1}{2} + \frac{5}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|\frac{x-2}{x+3}\right| + c_2 ----(ii)$$

Using equation (i) and (ii)

$$I = x + \log\left|\frac{x-2}{x+3}\right| + c$$

Indefinite Integrals Ex 19.20 Q3

Let $I = \int \frac{1-x^2}{x(1-2x)}dx$

$$= \int \frac{1-x^2}{x-2x^2}dx$$

$$= \int \frac{x^2-1}{2x^2-x}dx$$

$$= \int \left[\frac{1}{2} + \frac{\frac{x}{2}-1}{2x^2-x}\right]dx$$

$$I = \frac{1}{2}x + \int \frac{\frac{x}{2}-1}{2x^2-x}dx + c_1 \text{ ----(i)}$$

Let $I_1 = \int \frac{\frac{x}{2}-1}{2x^2-x}dx$

Let $\frac{x}{2} - 1 = \lambda \frac{d}{dx}\left(2x^2 - x\right) + \mu$

$$= \lambda(4x-1) + \mu$$

$$\frac{x}{2} - 1 = (4\lambda)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$\frac{1}{2} = 4\lambda \quad \Rightarrow \quad \lambda = \frac{1}{8}$$

$$-\lambda + \mu = -1 \quad \Rightarrow \quad -\left(\frac{1}{8}\right) + \mu = -1$$

$$\mu = -\frac{7}{8}$$

so, $I_1 = \int \frac{\frac{1}{8}(4x-1) - \frac{7}{8}}{2x^2-x}dx$

$$I = \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{8}\int \frac{1}{2\left(x^2 - \frac{x}{2}\right)}dx$$

$$I = \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{16}\int \frac{1}{x^2 - 2x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2}dx$$

$$= \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{16}\int \frac{1}{\left(x-\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2}dx$$

$$I_1 = \frac{1}{8}\log\left|2x^2 - x\right| - \frac{7}{16} \times \frac{1}{2\left(\frac{1}{4}\right)}\log\left|\frac{x - \frac{1}{4} - \frac{1}{4}}{x - \frac{1}{4} + \frac{1}{4}}\right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I = \frac{1}{8}\log|x| + \frac{1}{8}\log|2x-1| - \frac{7}{8}\log|1-2x| + \frac{7}{8}\log 2 + \frac{7}{8}\log|x| + c_2$$

$$I_1 = \log|x| - \frac{3}{4}\log|1-2x| + c_3 \text{ ----(ii)} \quad \left[\text{say, } c_3 = c_2 + \frac{7}{8}\log 2\right]$$

Using equation (i) and (ii)

$$I = \frac{1}{2}x + \log|x| - \frac{3}{4}\log|1-2x| + c$$

Indefinite Integrals Ex 19.20 Q4

Here the integrand $\frac{x^2+1}{x^2-5x+6}$ isnot proper rational function, so we divide x^2+1 by x^2-5x+6 and find tha

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

Let $\frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

So that

Equating the coefficients of x and constant terms on both sides, we get A + B = 5 and 3A + 2B = 5. Solving th we get A = −5 and B = 10

Thus, $\frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$

Therefore, $\int \frac{x^2+1}{(x+1)^2(x+3)}dx = \int dx - 5\int \frac{1}{x-2}dx + 10\int \frac{x}{x-3}$

$$= x - 5\log|x-2| + 10\log|x-3| + C.$$

Indefinite Integrals Ex 19.20 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$$= \int \left\{1 - \frac{7x+10}{x^2+7x+10}\right\}dx$$

$$I = x - \int \frac{7x+10}{x^2+7x+10}dx + c_1 ----(i)$$

Let $I_1 = \int \frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda \frac{d}{dx}\left(x^2+7x+10\right) + \mu$

$$= \lambda\left(2x+7\right) + \mu$$

$$7x+10 = \left(2\lambda\right)x + 7\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$7 = 2\lambda \qquad \Rightarrow \qquad \lambda = \frac{7}{2}$$

$$7\lambda + \mu = 10 \qquad \Rightarrow \qquad 7\left(\frac{7}{2}\right) + \mu = 10$$

$$\mu = -\frac{29}{2}$$

so, $I_1 = \int \frac{\frac{7}{2}\left(2x+7\right) - \frac{29}{2}}{x^2+7x+10}dx$

$$= \frac{7}{2}\int \frac{\left(2x+7\right)}{x^2+7x+10}dx - \frac{29}{2}\int \frac{1}{x^2+2x\left(\frac{7}{2}\right)+\left(\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+10}dx$$

$$I_1 = \frac{7}{2}\int \frac{2x+7}{x^2+7x+10}dx - \frac{29}{2}\int \frac{1}{\left(x+\frac{7}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$$

$$I_1 = \frac{2}{7}\log\left|x^2+7x+10\right| - \frac{29}{2}\times\frac{1}{2\left(\frac{3}{2}\right)}\log\left|\frac{x+\frac{7}{2}-\frac{3}{2}}{x+\frac{7}{2}+\frac{3}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$= \frac{7}{2}\log\left|x^2+7x+10\right| - \frac{29}{6}\log\left|\frac{x+2}{x+5}\right| + c_2 ------(ii)$$

Using equation (i) and (ii)

$$I = x - \frac{7}{2}\log\left|x^2+7x+10\right| + \frac{29}{6}\log\left|\frac{x+2}{x+5}\right| + c$$

Indefinite Integrals Ex 19.20 Q6

Let $I = \int \frac{x^2 + x + 1}{x^2 - x + 1} dx$

$= \int \left[1 + \frac{2x}{x^2 - x + 1}\right] dx$

$I = x + \int \frac{2x}{x^2 - x + 1} dx + c_1$ - - - - (i)

Let $I = \int \frac{2x}{x^2 - x + 1} dx$

Let $2x = \lambda \frac{d}{dx}\left(x^2 - x + 1\right) + \mu$

$= \lambda\left(2x - 1\right) + \mu$

$2x = \left(2\lambda\right)x - \lambda + \mu$

Comparing the coefficients of like powers of x,

$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$

$-\lambda + \mu = 0 \quad \Rightarrow \quad -1 + \mu = 0$

$\mu = 1$

so, $I_1 = \int \frac{\left(2x - 1\right) + 1}{x^2 - x + 1} dx$

$= \int \frac{\left(2x - 1\right)}{x^2 - x + 1} dx + \int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} dx$

$I_1 = \int \frac{2x - 1}{x^2 - x + 1} dx + \int \frac{1}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$

$= \log\left|x^2 - x + 1\right| + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$= \log\left|x^2 - x + 1\right| + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) + c_2$ - - - - - - (ii)

Using equation (i) and (ii)

$I = x + \log\left|x^2 - x + 1\right| + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) + c$

Indefinite Integrals Ex 19.20 Q7

Let $I = \int \frac{(x-1)^2}{x^2+2x+2}dx$

$$= \int \frac{x^2-2x+1}{x^2+2x+2}dx$$

$$= \int \left[1 - \frac{4x+1}{x^2+2x+2}\right]dx$$

$$I = x - \int \frac{4x+1}{x^2+2x+2}dx + c_1 ----(i)$$

Let $I = \int \frac{4x+1}{x^2+2x+2}dx$

Let $4x+1 = \lambda \frac{d}{dx}(x^2+2x+2) + \mu$

$$= \lambda(2x+2) + \mu$$

$$= (2\lambda)x + (2\lambda + \mu)$$

Comparing the coefficients of like powers of x,

$$4 = 2\lambda \quad \Rightarrow \quad \lambda = 2$$

$$2\lambda + \mu = 1 \quad \Rightarrow \quad 2(2) + \mu = 1$$

$$\mu = -3$$

so, $I_1 = \int \frac{2(2x+2)-3}{x^2+2x+2}dx$

$$= 2\int \frac{(2x+2)}{x^2+2x+2}dx - 3\int \frac{1}{x^2-2x+(1)^2-(1)^2+2}dx$$

$$I_1 = 2\int \frac{2x+2}{x^2+2x+2}dx - 3\int \frac{1}{(x+1)^2+(1)^2}dx$$

$$I_1 = 2\log\left|x^2+2x+2\right| - 3\tan^{-1}(x+1) + c_2 ----(ii) \quad \left[\text{since, } \int \frac{1}{x^2+1}dx = \tan^{-1}x + c\right]$$

Using equation (i) and (ii)

$$I = x - 2\log\left|x^2+2x+2\right| + 3\tan^{-1}(x+1) + c$$

Indefinite Integrals Ex 19.20 Q8

Let $I = \int \frac{x^3+x^2+2x+1}{x^2-x+1}dx$

$$= \int \left[x+2+\frac{3x-1}{x^2-x+1}\right]dx$$

$$I = \frac{x^2}{2}+2x+\int \frac{3x-1}{x^2-x+1}dx + c_1 - - - - (i)$$

Let $I_1 = \int \frac{3x-1}{x^2-x+1}dx$

Let $3x-1 = \lambda \frac{d}{dx}\left(x^2-x+1\right)+\mu$

$$= \lambda\left(2x-1\right)+\mu$$

$$3x-1 = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$3 = 2\lambda \qquad \Rightarrow \qquad \lambda = \frac{3}{2}$$

$$-\lambda+\mu = -1 \qquad \Rightarrow \qquad -\left(\frac{3}{2}\right)+\mu = -1$$

$$\mu = \frac{1}{2}$$

so, $I_1 = \int \frac{\frac{3}{2}\left(2x-1\right)+\frac{1}{2}}{x^2-x+1}dx$

$$= \frac{3}{2}\int \frac{\left(2x-1\right)}{x^2-x+1}dx + \frac{1}{2}\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$$

$$I_1 = \frac{3}{2}\int \frac{2x-1}{x^2-x+1}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$I_1 = \frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c_2 \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I_1 = \frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c_2 - - - (ii)$$

Using equation (i) and (ii)

$$I = \frac{x^2}{2}+2x+\frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$$

Indefinite Integrals Ex 19.20 Q9

Let $I = \int \frac{x^2\left(x^4+4\right)}{\left(x^2+4\right)}dx$

$$= \int \frac{x^6+4x^2}{\left(x^2+4\right)}dx$$

$$= \int \left[x^4-4x^2+20-\frac{80}{x^2+4}\right]dx$$

$$I = \frac{x^5}{5}-\frac{4x^3}{3}+20x-80\int \frac{1}{x^2+4}dx + c_1 - - - - (i)$$

Let $I_1 = \int \frac{1}{x^2+4}dx$

$$= \int \frac{1}{x^2+\left(2\right)^2}dx$$

$$I_1 = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c_2 - - - (ii) \qquad \left[\text{Since } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

Using equation (i) and (ii)

$$I = \frac{x^5}{5}-\frac{4x^3}{3}+20x-\frac{80}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$I = \frac{x^5}{5}-\frac{4x^3}{3}+20x-40\tan^{-1}\left(\frac{x}{2}\right)+c$$

Indefinite Integrals Ex 19.20 Q10

Let $I = \int \frac{x^2}{x^2+6x+12}dx$

$= \int\left[1 - \frac{6x+12}{x^2+6x+12}\right]dx$

$= x - \int \frac{6x+12}{x^2+6x+12}dx + c_1$ - - - - (i)

Let $I_1 = \int \frac{6x+12}{x^2+6x+12}dx$

Let $6x+12 = \lambda \frac{d}{dx}\left(x^2+6x+12\right) + \mu$

$= \lambda\left(2x+6\right) + \mu$

$6x+12 = \left(2\lambda\right)x + 6\lambda + \mu$

Comparing the coefficients of like powers of x,

$6 = 2\lambda \quad \Rightarrow \quad \lambda = 3$

$6\lambda + \mu = 12 \quad \Rightarrow \quad 6\left(3\right) + \mu = 12$

$\mu = -6$

so, $I_1 = \int \frac{3\left(2x+6\right) - 6}{x^2+6x+12}dx$

$= 3\int \frac{\left(2x+6\right)}{x^2+6x+12}dx - 6\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+12}dx$

$I_1 = 3\int \frac{2x+6}{x^2+6x+12}dx + 6\int \frac{1}{\left(x+3\right)^2+\left(\sqrt{3}\right)^2}dx$

$I_1 = 3\log\left|x^2+6x+12\right| + \frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2$ $\quad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I_1 = 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2$ - - - (ii)

Using equation (i) and (ii)

$I = x - 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$

Ex 19.21

Indefinite Integrals Ex 19.21 Q1

Let $I = \int \frac{x}{\sqrt{x^2+6x+10}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+6x+10\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$x = \left(2\lambda\right)x + 6\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$6\lambda + \mu = 0 \quad \Rightarrow \quad 6\left(\frac{1}{2}\right)+\mu = 0$

$\mu = -3$

so, $I_1 = \int \frac{\frac{1}{2}\left(2x+6\right)-3}{\sqrt{x^2+6x+10}}dx$

$= \frac{1}{2}\int \frac{\left(2x+6\right)}{\sqrt{x^2+6x+10}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+10}}dx$

$I_1 = \frac{1}{2}\int \frac{2x+6}{\sqrt{x^2+6x+10}}dx - 3\int \frac{1}{\sqrt{\left(x+3\right)^2+\left(1\right)^2}}dx$

$I_1 = \frac{1}{2}\left(2\sqrt{x^2+6x+10}\right) - 3\log\left|x+3+\sqrt{\left(x+3\right)^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = \sqrt{x^2+6x+10} - 3\log\left|x+3+\sqrt{x^2+6x+10}\right| + c$

Indefinite Integrals Ex 19.21 Q2

Let $I = \int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+2x-1\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$2x+1 = \left(2\lambda\right)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda + \mu = 1 \quad \Rightarrow \quad 2\left(1\right)+\mu = 1$

$\mu = -1$

so, $I = \int \frac{\left(2x+2\right)-1}{\sqrt{x^2+2x-1}}dx$

$= \int \frac{\left(2x+2\right)}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2-1}}dx$

$I = \int \frac{2x+2}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \left(2\sqrt{x^2+2x-1}\right) - \log\left|\left(x+1\right)+\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = 2\sqrt{x^2+2x-1} - \log\left|x+1+\sqrt{x^2+2x-1}\right| + c$

Indefinite Integrals Ex 19.21 Q3

Let $I = \int \frac{x+1}{\sqrt{4+5x-x^2}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(4+5x-x^2\right)+\mu$

$= \lambda\left(5-2x\right)+\mu$

$x = \left(-2\lambda\right)x + 5\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \qquad \Rightarrow \qquad \lambda = -\frac{1}{2}$

$5\lambda + \mu = 1 \qquad \Rightarrow \qquad 5\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = \frac{7}{2}$

so, $\quad I = \int \frac{-\frac{1}{2}\left(5-2x\right)+\frac{7}{2}}{\sqrt{4+5x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(5-2x\right)}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-5x-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{5}{2}\right)^2-\left(\frac{\sqrt{41}}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{5}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\left(2\sqrt{4+5x-x^2}\right)+\frac{7}{2}\sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{\sqrt{41}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\sqrt{4+5x-x^2}+\frac{7}{2}\sin^{-1}\left(\frac{2x-5}{\sqrt{41}}\right)+c$

Indefinite Integrals Ex 19.21 Q4

Let $I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}}dx$

Let $3x^2-5x+1 = t$

$\left(6x-5\right)dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t}+c$

$I = 2\sqrt{3x^2-5x+1}+c$

Indefinite Integrals Ex 19.21 Q5

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$= \lambda\left(-2-2x\right)+\mu$

$3x+1 = \left(-2\lambda\right)x - 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu = 1$

$\mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left[\left(\sqrt{6}\right)^2-\left(x+1\right)^2\right]}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

Indefinite Integrals Ex 19.21 Q6

Let $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

Let $x = \lambda \frac{d}{dx}\left(8+x-x^2\right)+\mu$

$= \lambda\left(1-2x\right)+\mu$

$x = \left(-2\lambda\right)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(-\frac{1}{2}\right)+\mu = 0$

$\mu = \frac{1}{2}$

so, $I = \int \frac{-\frac{1}{2}\left(1-2x\right)+\frac{1}{2}}{\sqrt{8+x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-x-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{2}\right)^2-\left(\frac{33}{4}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{33}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\times 2\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{33}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right)+c$

Indefinite Integrals Ex 19.21 Q7

Let $I = \int \frac{x+2}{\sqrt{x^2+2x-1}}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+2x-1\right) + \mu$

$x+2 = \lambda(2x+2) + \mu$

$x+2 = (2\lambda)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda + \mu = 2 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right) + \mu = 2$

$\Rightarrow \quad \mu = 1$

so, $I_1 = \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x-1}}dx$

$= \frac{1}{2}\int \frac{(2x+2)}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2-1}}dx$

$I = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{(x+1)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \frac{1}{2}2\sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{(x+1)^2-\left(\sqrt{2}\right)^2}\right| + c$ $\quad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x},\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{x^2+2x-1}\right| + c$

Indefinite Integrals Ex 19.21 Q8

$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \quad \ldots(1)$

$\Rightarrow x+2 = A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \frac{1}{2}$

$B = 2$

From (1), we obtain

$(x+2) = \frac{1}{2}(2x)+2$

$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \quad \ldots(2)$

$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2xdx = dt$

$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2xdx = dt$

$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\left[2\sqrt{t}\right]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$

From equation (2), we obtain

$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + C$

Indefinite Integrals Ex 19.21 Q9

$$\int \frac{x-1}{\sqrt{x^2+1}}dx =$$

$$\int \frac{x-1}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}dx - \int \frac{1}{\sqrt{x^2+1}}dx$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}(2\sqrt{t}) - \int \frac{1}{\sqrt{x^2+1}}dx = \sqrt{t} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

$$= \sqrt{x^2+1} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

Indefinite Integrals Ex 19.21 Q10

Let $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x+1\right) + \mu$

$= \lambda(2x+1) + \mu$

$x = (2\lambda)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(\frac{1}{2}\right) + \mu = 0$

$\Rightarrow \quad \mu = -\frac{1}{2}$

so, $I = \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{(2x+1)}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}}dx$

$I = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + c$

Indefinite Integrals Ex 19.21 Q11

Let $I = \int \frac{x+1}{\sqrt{x^2+1}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+1\right) + \mu$

$x+1 = \lambda(2x) + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}(2x)+1}{\sqrt{x^2+1}}dx$

$= \frac{1}{2}\int \frac{(2x)}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+1}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c$

Indefinite Integrals Ex 19.21 Q12

Let $I = \int \frac{2x+5}{\sqrt{x^2+2x+5}}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2+2x+5\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$2x+5 = \left(2\lambda\right)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda+\mu = 5 \quad \Rightarrow \quad 2\left(1\right)+\mu = 5$

$\Rightarrow \quad \mu = 3$

so, $I = \int \frac{\left(2x+2\right)+3}{\sqrt{x^2+2x+5}}dx$

$= \int \frac{\left(2x+3\right)}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2+5}}dx$

$I = \int \frac{2x+3}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{\left(x+1\right)^2+\left(2\right)^2}}dx$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{\left(x+1\right)^2+\left(2\right)^2}\right|+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$

Indefinite Integrals Ex 19.21 Q13

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$= \lambda\left(-2-2x\right)+\mu$

$3x+1 = \left(-2\lambda\right)x-2\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda+\mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu = 1$

$\Rightarrow \quad \mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2+5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2-\left(x+1\right)^2}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

Indefinite Integrals Ex 19.21 Q14

Let $I = \int \sqrt{\frac{1-x}{1+x}}dx$

$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$

$= \int \frac{1-x}{\sqrt{1-x^2}}dx$

Let $1-x = \lambda \frac{d}{dx}\left(1-x^2\right) + \mu$

$= \lambda\left(-2x\right) + \mu$

$1-x = \left(-2\lambda\right)x + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = -1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(-2x\right)+1}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{\left(-2x\right)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$

$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}\left(x\right) + c\right]$

$I = \sqrt{1-x^2} + \sin^{-1}x + c$

Indefinite Integrals Ex 19.21 Q15

Let $I = \int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+4x+3\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+1 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(1\right) + \mu = 1$

$\Rightarrow \quad \mu = -3$

so, $I = \int \frac{\left(2x+4\right)-3}{\sqrt{x^2+4x+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{\left(x+2\right)^2-1}}dx$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{\left(x+2\right)^2-1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{x^2+4x+3}\right| + c$

Indefinite Integrals Ex 19.21 Q16

Let $I = \int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

Let $2x+3 = \lambda \frac{d}{dx}\left(x^2+4x+5\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+3 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 3 \quad \Rightarrow \quad 4\left(1\right) + \mu = 3$

$\Rightarrow \quad \mu = -1$

so, $I = \int \frac{\left(2x+4\right)-1}{\sqrt{x^2+4x+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{\left(x+2\right)^2+\left(1\right)^2}}dx$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{\left(x+2\right)^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{x^2+4x+5}\right| + c$

Indefinite Integrals Ex 19.21 Q17

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$\rightarrow let\ 5x+3 = \lambda(2x+4)+\mu$

$\lambda = \frac{5}{2}, \mu = -7$

$$\int \frac{\lambda(2x+4)+\mu}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}dt}{\sqrt{t}} - \int \frac{7}{\sqrt{(x+2)^2+6}}dx$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$

Indefinite Integrals Ex 19.21 Q18

Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}}$

$x+2 = A\frac{d}{dx}\left[x^2+2x+3\right]+B$

$\Rightarrow x+2 = 2Ax+2A+B$

Comparing the coefficients, we have,

$2A=1$ and $2A+B=2$

$\Rightarrow A = \frac{1}{2}$

Substituting the value of A in $2A+B=2$, we have,

$2\times\frac{1}{2}+B=2$

$\Rightarrow 1+B=2$

$\Rightarrow B = 2-1$

$\Rightarrow B = 1$

Thus we have,

$x+2 = \frac{1}{2}[2x+2]+1$

Hence,

$$I = \int \frac{x+2}{\sqrt{x^2+2x+3}}dx$$

$$= \int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^2+2x+3}}dx$$

$$= \int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \frac{1}{2}\int \frac{[2x+2]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

Substituting $t=x^2+2x+3$ and $dt=2x+2$ in the first integrand, we have,

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \frac{1}{2}\times 2\sqrt{t} + \int \frac{dx}{\sqrt{x^2+2x+1+2}} + C$$

$$= \sqrt{t} + \int \frac{dx}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}} + C$$

$$I = \sqrt{x^2+2x+3} + \log\left[|x+1| + \sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right] + C$$

$$\Rightarrow I = \sqrt{x^2+2x+3} + \log\left[|x+1| + \sqrt{x^2+2x+3}\right] + C$$

Ex 19.22

Indefinite Integrals Ex 19.22 Q1

Let $I = \int \frac{1}{4\cos^2 x + 9\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$= \int \frac{\frac{1}{\cos^2 x}}{4 + 9\tan^2 x} dx$$

$$I = \int \frac{\sec^2 x}{4 + 9\tan^2 x} dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4 + (3t)^2}$$

Let $3t = u$

$3dt = du$

$$I = \frac{1}{3}\int \frac{du}{(2)^2 + (u)^2}$$

$$= \frac{1}{3} \times \frac{1}{2} \times \tan^{-1}\left(\frac{u}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3t}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right) + c$$

Indefinite Integrals Ex 19.22 Q2

Let $I = \int \frac{1}{4\sin^2 x + 5\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{4\tan^2 x + 5} dx$$

$$= \int \frac{\sec^2 x}{4\tan^2 x + 5} dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4t^2 + 5}$$

Let $2t = u$

$2dt = du$

$$I = \frac{1}{2}\int \frac{du}{(4)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{5}} \times \tan^{-1}\left(\frac{u}{\sqrt{5}}\right) + c$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$$

$$I = \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.22 Q3

Let $I = \int \frac{2}{2+\sin 2x}dx$

$= \int \frac{2}{2+2\sin x\cos x}dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}}dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + \tan x}dx$$

$$I = \int \frac{\sec^2 x}{1+\tan^2 x + \tan x}dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{t^2+t+1}$$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$I = \int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right) + c$$

$$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.22 Q4

Let $I = \int \frac{\cos x}{\cos 3x} dx$

$= \int \frac{\cos x}{4\cos^3 x - 3\cos x} dx$

Diving numerator and denominator by $\cos^3 x$

$I = \int \frac{\frac{\cos x}{\cos^3 x}}{\frac{4\cos^3 x}{\cos^3 x} + \frac{3\cos x}{\cos^3 x}} dx$

$= \int \frac{\sec^2 x}{4 - 3\sec^2 x} dx$

$= \int \frac{\sec^2 x}{4 - 3\left(1 + \tan^2 x\right)} dx$

$= \int \frac{\sec^2 x}{4 - 3 - 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{1 - 3\tan^2 x} dx$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$I = \int \frac{dt}{1 - 3t^2}$

$= \int \frac{dt}{1 - \left(\sqrt{3}t\right)^2}$

Let $\sqrt{3}t = u$

$\sqrt{3}dt = du$

$= \int \frac{du}{(1)^2 - (4)^2}$

$= \frac{1}{2\sqrt{3}} \log\left|\frac{u + 1}{1 - u}\right| + c$

$= \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3}t + 1}{1 - \sqrt{3}t}\right| + c$

$I = \frac{1}{2\sqrt{3}} \log\left|\frac{1 + \sqrt{3}\tan x}{1 - \sqrt{3}\tan x}\right| + c$

Indefinite Integrals Ex 19.22 Q5

Let $I = \int \frac{1}{1 + 3\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3\sin^2 x}{\cos^2 x}} dx$

$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{1 + \tan^2 x + 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{1 + 4\tan^2 x} dx$

$= \int \frac{\sec^2 x}{1 + \left(2\tan x\right)^2} dx$

Let $2\tan x = t$

$2\sec^2 x \, dx = dt$

$I = \frac{1}{2} \int \frac{dt}{1 + t^2}$

$= \frac{1}{2} \tan^{-1} t + c$

$I = \frac{1}{2} \tan^{-1}\left(2\tan x\right) + c$

Indefinite Integrals Ex 19.22 Q6

Let $I = \int \frac{1}{3+2\cos^2 x}dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x}+\frac{2\cos^2 x}{\cos^2 x}}dx$$

$$= \int \frac{\sec^2 x}{3\sec^2 x+2}dx$$

$$= \int \frac{\sec^2 x}{3\left(1+\tan^2 x\right)+2}dx$$

$$= \int \frac{\sec^2 x}{3+3\tan^2 x+2}dx$$

$$= \int \frac{\sec^2 x}{5+3\tan^2 x}dx$$

Let $\sqrt{3}\tan x = t$

$\sqrt{3}\sec^2 x\, dx = dt$

$$I = \frac{1}{\sqrt{3}}\int \frac{dt}{\left(\sqrt{5}\right)^2+t^2}$$

$$= \frac{1}{\sqrt{3}\times\sqrt{5}}\tan^{-1}\left(\frac{t}{\sqrt{5}}\right)+c$$

$$I = \frac{1}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right)+c$$

Indefinite Integrals Ex 19.22 Q7

Let $I = \int \frac{1}{\left(\sin x-2\cos x\right)\left(2\sin x+\cos x\right)}dx$

$$= \int \frac{1}{2\sin^2 x+\sin x\cos x-4\sin x\cos x-2\cos^2 x}dx$$

$$= \int \frac{1}{2\sin^2 x-3\sin x\cos x-2\cos^2 x}dx$$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\sec^2 x}{2\tan^2 x-3\tan x-2}dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{2t^2-3t-2}$$

$$= \frac{1}{2}\int \frac{dt}{t^2-\frac{3}{2}t-1}$$

$$= \frac{1}{2}\int \frac{dt}{t^2-2t\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-1}$$

$$I = \frac{1}{2}\int \frac{dt}{\left(t-\frac{3}{4}\right)^2-\left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{2}\times\frac{1}{2\left(\frac{5}{4}\right)}\log\left|\frac{t-\frac{3}{4}-\frac{5}{4}}{t-\frac{3}{4}+\frac{5}{4}}\right|+c$$

$$= \frac{1}{5}\log\left|\frac{t-2}{2t+1}\right|+c$$

$$I = \frac{1}{5}\log\left|\frac{\tan x-2}{2\tan x+1}\right|+c$$

Indefinite Integrals Ex 19.22 Q8

Let $I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

Diving numerator and denominator by $\cos^4 x$

$I = \int \frac{2 \tan x \sec^2 x}{\tan^4 x + 1} dx$

Let $\tan^2 x = t$

$2 \tan x \sec^2 x \, dx = dt$

$I = \int \frac{dt}{t^2 + 1}$

$= \tan^{-1} t + c$

$I = \tan^{-1}\left(\tan^2 x\right) + c$

Indefinite Integrals Ex 19.22 Q9

Let $I = \int \frac{1}{\cos x \left(\sin x + 2 \cos x\right)} dx$

$= \int \frac{1}{\sin x \cos x + 2 \cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$,

$I = \int \frac{\sec^2 x}{\tan x + 2} dx$

Let $2 + \tan x = t$

$\sec^2 x \, dx = dt$

$I = \int \frac{dt}{t}$

$= \log|t| + c$

$I = \log|2 + \tan x| + c$

Indefinite Integrals Ex 19.22 Q10

Let $I = \int \frac{1}{\sin^2 x + \sin 2x} dx$

$= \int \frac{1}{\sin^2 x + 2 \sin x \cos x} dx$

Diving numerator and denominator by $\cos^2 x$,

$I = \int \frac{\sec^2 x}{\tan^2 x + 2 \tan x} dx$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$= \int \frac{dt}{t^2 + 2t + (1)^2 - (1)^2}$

$= \int \frac{dt}{(t+1)^2 - (1)^2}$

$= \frac{1}{2} \log\left|\frac{t+1-1}{t+1+1}\right| + c$

$= \frac{1}{2} \log\left|\frac{t}{t+2}\right| + c$

$I = \frac{1}{2} \log\left|\frac{\tan x}{\tan x + 2}\right| + c$

Indefinite Integrals Ex 19.22 Q11

Let $\quad I = \int \frac{1}{\cos 2x + 3\sin^2 x} dx$

$$= \int \frac{1}{2\cos^2 x - 1 + 3\sin^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{2 - \sec^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{2 - \left(1 + \tan^2 x\right)^2 + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{2 - 1 - \tan^2 x + 3\tan^2 x} dx$$

$$= \int \frac{dt}{1 + 2\tan^2 x}$$

Let $\quad \sqrt{2}\tan x = t$

$$\sqrt{2}\sec^2 x\, dx = dt$$

$$I = \frac{1}{\sqrt{2}} \int \frac{1}{1 + t^2}$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} t + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2}\tan x\right) + c$$

Ex 19.23

Indefinite Integrals Ex 19.23 Q1

Let $I = \int \frac{1}{5+4\cos x}\,dx$

Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+4-4\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

$$= \int \frac{2dt}{(3)^2+t^2}$$

$$= 2\times\frac{1}{3}\tan^{-1}(t)+c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$$

Indefinite Integrals Ex 19.23 Q2

Let $I = \int \frac{1}{5 - 4\sin x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$= \int \frac{1}{5 - 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{1 + \tan^2\frac{x}{2}}{5\left(1 + \tan^2\frac{x}{2}\right) - 4\left(2\tan\frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5 + 5\tan^2\frac{x}{2} - 8\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{5t^2 - 8t + 5}$$

$$= \frac{2}{5}\int \frac{dt}{t^2 - \frac{8}{5}t + 1}$$

$$= \frac{2}{5}\int \frac{dt}{t^2 - 2t\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 - \left(\frac{4}{5}\right)^2 + 1}$$

$$I = \frac{2}{5}\int \frac{dt}{\left(t - \frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}$$

$$= \frac{2}{5} \times \frac{1}{\frac{3}{5}} \tan^{-1}\left(\frac{t - \frac{4}{5}}{\frac{3}{5}}\right) + c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5t - 4}{3}\right) + c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2} - 4}{3}\right) + c$$

Indefinite Integrals Ex 19.23 Q3

Let $\quad I = \int \frac{1}{1 - 2\sin x} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{1 - 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{1 + \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2} - 4\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1} dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 - 4t + 1}$$

$$= \int \frac{2dt}{t^2 - 2t(2) + (2)^2 - (2)^2 + 1}$$

$$I = 2\int \frac{dt}{(t - 2)^2 + 3}$$

$$= 2\int \frac{dt}{(t - 2)^2 + \left(\sqrt{3}\right)^2}$$

$$= 2 \times \frac{1}{2\sqrt{3}} \log\left|\frac{t - 2 - \sqrt{3}}{t - 2 + \sqrt{3}}\right| + c$$

$$I = \frac{1}{\sqrt{3}} \log\left|\frac{\tan\frac{x}{2} - 2 - \sqrt{3}}{\tan\frac{x}{2} - 2 + \sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.23 Q4

Let $I = \int \frac{1}{4\cos x - 1} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - 1} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{4\left(1 - \tan^2 \frac{x}{2}\right) - \left(1 + \tan^2 \frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4\tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5\tan^2 \frac{x}{2}} dx$$

Let $\sqrt{5} \tan \frac{x}{2} = t$

$$\frac{\sqrt{5}}{2} \sec^2 \frac{x}{2} dt = dt$$

$$I = \int \frac{dt}{\left(\sqrt{3}\right)^2 - t^2}$$

$$I = \frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + c$$

Indefinite Integrals Ex 19.23 Q5

Let $I = \int \frac{1}{1 - \sin x + \cos x} dx$

Put $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{1 - \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 2\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 - 2\tan \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \frac{2}{2} \int \frac{dt}{1 - t}$$

$$= -\log |1 - t| + c$$

$$I = -\log \left| 1 - \tan \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.23 Q6

Let $I = \int \frac{1}{3 + 2\sin x + \cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$, $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{3 + 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{3\left(1 + \tan^2\frac{x}{2}\right) + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{3 + 3\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$I = \int \frac{dt}{(t+1)^2 + (1)^2}$$

$$= \tan^{-1}(t+1) + c$$

$$I = \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + c$$

Indefinite Integrals Ex 19.23 Q7

Let $\quad I = \int \frac{1}{13 + 3\cos x + 4\sin x} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{13 + 3\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{13\left(1 + \tan^2\frac{x}{2}\right) + 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{16 + 13\tan^2\frac{x}{2} - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{16 + 10t^2 + 8t}$$

$$= \frac{2}{10}\int \frac{dt}{t^2 + \frac{4}{5}t + \frac{8}{5}}$$

$$I = \frac{1}{5}\int \frac{dt}{t^2 + 2t\left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{8}{5}}$$

$$= \frac{1}{5}\int \frac{dt}{\left(t + \frac{2}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

6

$$= \frac{1}{5} \times \frac{1}{\left(\frac{6}{5}\right)} \tan^{-1}\left(\frac{t + \frac{2}{5}}{\frac{6}{5}}\right) + c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{5t + 2}{6}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{5\tan\frac{x}{2} + 2}{6}\right) + c$$

Indefinite Integrals Ex 19.23 Q8

Let $I = \int \frac{1}{\cos x - \sin x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - \left(\frac{2\tan\frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{1 - \tan^2 \frac{x}{2} - 2\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 2\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{1 - t^2 - 2t}$$

$$= -\int \frac{2dt}{t^2 + 2t - 1}$$

$$I = -\int \frac{2dt}{t^2 + 2t + 1 - 1 - 1}$$

$$I = -\int \frac{2dt}{(t+1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \int \frac{2dt}{\left(\sqrt{2}\right)^2 - (t+1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log\left|\frac{\sqrt{2} + t + 1}{\sqrt{2} - t - 1}\right| + c$$

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2} + \tan\frac{x}{2} + 1}{\sqrt{2} - \tan\frac{x}{2} - 1}\right| + c$$

Indefinite Integrals Ex 19.23 Q9

Let $I = \int \frac{1}{\sin x + \cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t + 1 - t^2}$$

$$= -2\int \frac{dt}{t^2 - 2t - 1}$$

$$I = -2\int \frac{dt}{t^2 - 2t + 1 - 1 - 1}$$

$$I = -2\int \frac{dt}{(t-1)^2 - (\sqrt{2})^2}$$

$$= 2\int \frac{2dt}{(\sqrt{2})^2 - (t-1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log\left|\frac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1}\right| + c$$

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2} + \tan\frac{x}{2} - 1}{\sqrt{2} - \tan\frac{x}{2} + 1}\right| + c$$

Indefinite Integrals Ex 19.23 Q10

Let $I = \int \frac{1}{5 - 4\cos x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{5 - 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{5 + 5\tan^2 \frac{x}{2} - 4 + 4\tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9\tan^2 \frac{x}{2} + 1} dx$$

Let $3\tan \frac{x}{2} = t$

$$\frac{3}{2}\sec^2 \frac{x}{2} dx = dt$$

$$I = \frac{2}{3}\int \frac{dt}{t^2 + 1}$$

$$= \frac{2}{3}\tan^{-1}(t) + c$$

$$I = \frac{2}{3}\tan^{-1}\left(3\tan \frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.23 Q11

Let $I = \int \frac{1}{2 + \sin x + \cos x} dx$

Put $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{2 + \left(\frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{2 + 2\tan^2 \frac{x}{2} + 2\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{\tan^2 \frac{x}{2} + 2\tan \frac{x}{2} + 3} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 + 2t + 3}$$

$$= 2\int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$I = 2\int \frac{dt}{(t+1)^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$$

$$I = \sqrt{2}\tan^{-1}\left(\frac{\tan \frac{x}{2} + 1}{\sqrt{2}}\right) + c$$

Indefinite Integrals Ex 19.23 Q12

Let $I = \int \frac{1}{\sin x + \sqrt{3}\cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\sqrt{3}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2}+\sqrt{3}-\sqrt{3}\tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+\sqrt{3}-\sqrt{3}\tan^2\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \int \frac{2dt}{2t+\sqrt{3}-\sqrt{3}t^2}$$

$$= -\frac{2}{\sqrt{3}}\int \frac{dt}{t^2-\frac{2}{\sqrt{3}}t+\left(\frac{1}{\sqrt{3}}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2-1}$$

$$I = -\frac{2}{\sqrt{3}}\int \frac{dt}{\left(t-\frac{1}{\sqrt{3}}\right)^2-\left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}}\int \frac{dt}{\left(\frac{2}{\sqrt{3}}\right)^2-\left(t-\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2\left(\frac{2}{\sqrt{3}}\right)}\log\left|\frac{\frac{2}{\sqrt{3}}+t+\frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}}-t+\frac{1}{\sqrt{3}}}\right|+c$$

$$I = \frac{1}{2}\log\left|\frac{\sqrt{3}t+1}{3-\sqrt{3}t}\right|+c$$

$$I = \frac{1}{2}\log\left|\frac{1+\sqrt{3}\tan\frac{x}{2}}{3-\sqrt{3}\tan\frac{x}{2}}\right|+c$$

Indefinite Integrals Ex 19.23 Q13

Let $I = \int \frac{1}{\sqrt{3}\sin x + \cos x} dx$

Let $\sqrt{3} = r\cos\theta$, and $1 = r\sin\theta$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

$$r = \sqrt{3+1} = 2$$

$$I = \int \frac{1}{r\cos\theta\sin x + r\sin\theta\cos x} dx$$

$$= \frac{1}{r}\int \frac{1}{\sin(x+\theta)} dx$$

$$= \frac{1}{2}\int \operatorname{cosec}(x+\theta)\, dx$$

$$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\theta}{2}\right)\right|+c$$

$$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right|+c$$

Indefinite Integrals Ex 19.23 Q14

Let $I = \int \frac{1}{\sin x - \sqrt{3}\cos x}\,dx$

Let $1 = r\cos\theta,$ and $\sqrt{3} = r\sin\theta$

$r = \sqrt{3+1} = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$I = \int \frac{1}{r\cos\theta\sin x - r\sin\theta\cos x}\,dx$

$= \frac{1}{2}\int \frac{1}{\sin(x-\theta)}\,dx$

$= \frac{1}{2}\int \operatorname{cosec}(x-\theta)\,dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2} - \frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q15

Let $I = \int \frac{1}{5 + 7\cos x + \sin x}\,dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Now,

$I = \int \frac{1}{5 + \frac{7\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} + \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}\,dx$

$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right) + 7 - 7\tan^2\frac{x}{2} + 2\tan\frac{x}{2}}\,dx$

$= \int \frac{\sec^2\frac{x}{2}}{-2\tan^2\frac{x}{2} + 12 + 2\tan\frac{x}{2}}\,dx$

Let $\tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$

$I = \int \frac{2dt}{-2t^2 + 12 + 2t}$

$= -\int \frac{dt}{t^2 - t - 6}$

$= -\int \frac{dt}{t^2 - 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}$

$= -\int \frac{dt}{\left(t - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}$

$= -\frac{1}{2\left(\frac{5}{2}\right)}\log\left|\frac{t - \frac{1}{2} - \frac{5}{2}}{t - \frac{1}{2} + \frac{5}{2}}\right| + c$

$= -\frac{1}{5}\log\left|\frac{t-3}{t+2}\right| + c$

$I = \frac{1}{5}\log\left|\frac{\tan\frac{x}{2} + 2}{\tan\frac{x}{2} - 3}\right| + c$

Ex 19.24

Indefinite Integrals Ex 19.24 Q1

Let $I = \int \frac{1}{1 - \cot x} dx$

$= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx$

$= \int \frac{\sin x}{\sin x - \cos x} dx$

Let $\sin x = \lambda \frac{d}{dx}(\sin x - \cos x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \lambda \frac{d}{dx}(\cos x + \sin x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \cos(\lambda - \mu) + \sin x(\lambda + \mu) + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$\lambda + \mu = 1 \quad \ldots(1)$

$\lambda - \mu = 1 \quad \ldots(2)$

$\nu = 0 \quad \ldots(3)$

Equation $(1),(2),(3)$ gives

$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx$

$= \frac{1}{2} \int \frac{(\cos x + \sin x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int dx$

$I = \frac{1}{2} \log|\sin x - \cos x| + \frac{1}{2} x + c$

Indefinite Integrals Ex 19.24 Q2

Let $I = \int \frac{1}{1 - \tan x} dx$

$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx$

$= \int \frac{\cos x}{\cos x - \sin x} dx$

Let $\cos x = \lambda \frac{d}{dx}(\cos x - \sin x) + \mu(\cos x - \sin x) + \nu$

$= \lambda \frac{d}{dx}(-\sin x - \cos x) + \mu(\cos x - \sin x) + \nu$

$\cos x = \sin x(-\lambda - \mu) + \cos x(-\lambda + \mu) + \nu$

Camparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$-\lambda - \mu = 0 ------(1)$

$-\lambda + \mu = 1 ------(2)$

$\nu = 0 ---------(3)$

Equation $(1),(2),(3)$ gives

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)} dx$

$= \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\cos x - \sin x)} dx + \frac{1}{2} \int dx$

$= -\frac{1}{2} \log|\cos x - \sin x| + \frac{1}{2} x + c$

$I = \frac{1}{2} x - \frac{1}{2} \log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.24 Q3

Let $I = \int \frac{3 + 2\cos x + 4\sin x}{2\sin x + \cos x + 3}dx$

Let $3 + 2\cos x + 4\sin x = \lambda \frac{d}{dx}(2\sin x + \cos x + 3) + \mu(2\sin x + \cos x + 3) + \nu$

$3 + 2\cos x + 4\sin x = \lambda(2\cos x - \sin x) + \mu(2\sin x + \cos x + 3) + \nu$

$3 + 2\cos x + 4\sin x = (-\lambda + 2\mu)\sin x + (2\lambda + \mu)\cos x + 3\mu + \nu$

Camparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$-\lambda + 2\mu = 4 ------(1)$

$2\lambda + \mu = 2 ------(2)$

$2\mu + \nu = 3 ---------(3)$

Solving equation $(1), (2)$ and (3), we get

$\lambda = 0, \mu = 2, \nu = -3$

$I = \int \frac{2(2\sin x + \cos x + 3) - 3}{(2\sin x + \cos x + 3)}dx$

$= 2\int dx - 3\int \frac{1}{2\sin x + \cos x + 3}dx$

$I = 2x - 3I_1 + C_1 -------(4)$

Let $I_1 = \int \frac{1}{2\sin x + \cos x + 3}dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}$

$I_1 = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 3}dx$

$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3\left(1 + \tan^2\frac{x}{2}\right)}dx$

$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4}dx$

Let $\tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2} = dt$

$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$

$= \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$

$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$

$= \int \frac{dt}{(t + 1)^2 + 1}$

$= \tan^{-1}(t + 1) + C_2$

$= \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C_2$

Now, using equation (1),

$I = 2x - 3\tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C$

Indefinite Integrals Ex 19.24 Q4

Let $I = \int \frac{1}{p + q\tan x}dx$

$= \int \frac{1}{p + q\left(\frac{\sin x}{\cos x}\right)}dx$

$= \int \frac{\cos x}{p\cos x + q\sin x}dx$

Let $\cos x = \lambda \frac{d}{dx}(p\cos x + q\sin x) + \mu(p\cos x + q\sin x) + \nu$

$\cos x = \lambda(-p\sin x + q\cos x) + \mu(p\cos x + q\sin x) + \nu$

$\cos x = (-p\lambda + q\mu)\sin x + (q\lambda + p\mu)\cos x + \nu$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$-p\lambda + q\mu = 0$ ------ (1)

$q\lambda + p\mu = 1$ ------- (2)

$\nu = 0$ ---------- (3)

Solving equation (1), (2) and (3),

$\lambda = \frac{q}{(p^2 + q^2)}$

$\mu = \frac{p}{(p^2 + q^2)}$

$\nu = 0$

Now,

$$I = \int \frac{q}{(p^2+q^2)} \frac{(-p\sin x + q\cos x)}{(p\cos x + q\sin x)}dx + \int \frac{p}{(p^2+q^2)} \frac{(p\cos x + q\sin x)}{(p\cos x + q\sin x)}dx$$

$$I = \frac{q}{(p^2+q^2)}\left(\log|p\cos x + q\sin x|\right) + \frac{p}{(p^2+q^2)}x + c$$

Indefinite Integrals Ex 19.24 Q5

Let $I = \int \frac{5\cos x + 6}{2\cos x + \sin x + 3}dx$

Let $(5\cos x + 6) = \lambda \frac{d}{dx}(2\cos x + \sin x + 3) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = \lambda(-2\sin x + \cos x) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = (-2\lambda + \mu)\sin x + (\lambda + 2\mu)\cos x + (3\mu + \nu)$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-2\lambda + \mu = 0$ ------ (1)

$\lambda + 2\mu = 5$ ------- (2)

$3\mu + \nu = 6$ ------- (3)

Solving equation (1), (2) and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

Now,

$$I = \int \frac{(-2\sin x + \cos x)}{(2\cos x + \sin x + 3)}dx + 2\int dx$$

$$I = \log|2\cos x + \sin x + 3| + 2x + c$$

Indefinite Integrals Ex 19.24 Q6

Let $I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $(2\sin x + 3\cos x) = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$(2\sin x + 3\cos x) = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$(2\sin x + 3\cos x) = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------ (1)$

$-4\lambda + 3\mu = 2 ----- (2)$

$\nu = 0 ------- (3)$

Solving the equation $(1), (2)$ and (3),

$\lambda = \frac{1}{25}$

$\mu = \frac{18}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{18}{25}\int dx$

$I = \frac{1}{25}\log|3\sin x + 4\cos x| + \frac{18}{25}x + c$

Indefinite Integrals Ex 19.24 Q7

Let $I = \int \frac{1}{3 + 4\cot x} dx$

$= \int \frac{\sin x}{3\sin x + 4\cos x} dx$

Let $\sin x = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$\sin x = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$\sin x = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 0 ------ (1)$

$-4\lambda + 3\mu = 1 ----- (2)$

$\nu = 0 ------- (3)$

Solving the equation $(1), (2)$ and (3), we get

$\lambda = -\frac{4}{25}$

$\mu = \frac{3}{25}$

$\nu = 0$

$I = -\frac{4}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{3}{25}\int dx$

$I = -\frac{4}{25}\log|3\sin x + 4\cos x| + \frac{3}{25}x + c$

Indefinite Integrals Ex 19.24 Q8

Let $I = \int \frac{2\tan x + 3}{3\tan x + 4} dx$

$= \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $2\sin x + 3\cos x = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------ (1)$

$-4\lambda + 3\mu = 2 ----- (2)$

$\nu = 0$

Solving the equation $(1),(2)$ and (3),

$\mu = \frac{18}{25}$

$\lambda = \frac{1}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{18}{25}\int dx$

$I = \frac{18}{25}x + \frac{1}{25}\log|3\sin x + 4\cos x| + c$

Indefinite Integrals Ex 19.24 Q9

Let $I = \int \frac{1}{4 + 3\tan x} dx$

$I = \int \frac{\cos x}{4\cos x + 3\sin x} dx$

Let $\cos x = \lambda \frac{d}{dx}(4\cos x + 3\sin x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = \lambda(-4\sin x + 3\cos x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = (-4\lambda + 3\mu)\sin x + (3\lambda + 4\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 3\mu = 0 ----- (1)$

$3\lambda + 4\mu = 1 ------ (2)$

$\nu = 0 ------- (3)$

Solving the equation $(1),(2)$ and (3),

$\lambda = \frac{3}{25}$

$\mu = \frac{4}{25}$

$\nu = 0$

$I = \int \frac{3}{25} \frac{(-4\sin x + 3\cos x)}{(4\cos x + 3\sin x)} dx + \frac{4}{25}\int dx$

$I = \frac{3}{25}\log|4\cos x + 3\sin x| + \frac{4}{25}x + c$

Indefinite Integrals Ex 19.24 Q10

Let $I = \int \frac{8\cot x + 1}{3\cot x + 2} dx$

$I = \int \frac{8\cos x + \sin x}{3\cos x + 2\sin x} dx$

Let $8\cos x + \sin x = \lambda \frac{d}{dx}(3\cos x + 2\sin x) + \mu(3\cos x + 2\sin x) + \nu$

$8\cos x + \sin x = \lambda(-3\sin x + 2\cos x) + \mu(3\cos x + 2\sin x) + \nu$

$8\cos x + \sin x = (-3\lambda + 2\mu)\sin x + (2\lambda + 3\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$2\lambda + 3\mu = 8 - - - - - (1)$

$-3\lambda + 2\mu = 1 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation (1), (2) and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

$I = \int \frac{(-3\sin x + 2\cos x)}{(3\cos x + 2\sin x)} dx + 2\int dx$

$I = \log|3\cos x + 2\sin x| + 2x + c$

Indefinite Integrals Ex 19.24 Q11

Let $I = \int \frac{4\sin x + 5\cos x}{5\sin x + 4\cos x} dx$

Let $4\sin x + 5\cos x = \lambda \frac{d}{dx}(5\sin x + 4\cos x) + \mu(5\sin x + 4\cos x) + \nu$

$4\sin x + 5\cos x = \lambda(5\cos x - 4\sin x) + \mu(5\sin x + 4\cos x) + \nu$

$4\sin x + 5\cos x = (5\lambda + 4\mu)\cos x + (-4\lambda + 5\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 5\mu = 4 - - - - - (1)$

$5\lambda + 4\mu = 5 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation (1), (2) and (3),

$\lambda = \frac{9}{41}$

$\mu = \frac{40}{41}$

$\nu = 0$

Now,

$I = \frac{40}{41}\int dx + \frac{9}{41}\int \frac{(5\cos x - 4\sin x)}{(5\sin x + 4\cos x)} dx$

$I = \frac{40}{41}x + \frac{9}{41}\log|5\sin x + 4\cos x| + c$

Ex 19.25

Indefinite Integrals Ex 19.25 Q1

Let $I = \int x \cos x \, dx$

Using integration by parts,

$$I = x\int \cos x \, dx - \int\left(1 \times \int \cos x \, dx\right)dx + c$$
$$= x \sin x - \int \sin x \, dx + c$$

$$I = x \sin x + \cos x + c$$

Indefinite Integrals Ex 19.25 Q2

Let $I = \int \log(x+1) dx$

$= \int 1 \times \log(x+1) dx$

Using integration by parts,

$$I = \log(x+1) \int 1 dx - \int\left(\frac{1}{x+1} \times \int 1 dx\right)dx + c$$
$$= x \log(x+1) - \int\left(\frac{x}{x+1}\right)dx + c$$
$$= x \log(x+1) - \int\left(1 - \frac{1}{x+1}\right)dx + c$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

Indefinite Integrals Ex 19.25 Q3

Let $I = \int x^3 \log x \, dx$

Using integration by parts,

$$I = \log x \int x^3 dx - \int\left(\frac{1}{x} \times \int x^3 dx\right)dx + c$$
$$= \frac{x^4}{4}\log x - \int \frac{x^4}{4x} dx + c$$
$$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3 dx + c$$
$$= \frac{x^4}{4}\log x - \frac{1}{4}\int \frac{x^4}{4} dx + c$$

$$I = \frac{x^4}{4}\log x - \frac{1}{16}x^4 + c$$

Indefinite Integrals Ex 19.25 Q4

Take first function as x and second function as e^x. The integral of the second function is e^x.
Therefore, $\int xe^x \, dx = xe^x - \int 1.\, e^x \, dx = xe^x - e^x + C.$

Indefinite Integrals Ex 19.25 Q5

Let $I = \int xe^{2x} dx$

Using integration by parts,

$$I = x\int e^{2x} dx - \int\left(1 \times \int e^{2x} dx\right)dx + c$$
$$= \frac{xe^{2x}}{2} - \int\left(\frac{e^{2x}}{2}\right)dx + c$$
$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$I = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + c$$

Indefinite Integrals Ex 19.25 Q6

Let $I = \int x^2 e^{-x} dx$

Using integration by parts,

$$I = x^2 \int e^{-x}\, dx - \int \left(2x \int e^{-x}\, dx\right)$$
$$= -x^2 e^{-x} - \int (2x)\left(-e^{-x}\right)$$
$$= -x^2 e^{-x} + 2\int x\, e^{-x} dx$$
$$= -x^2 e^{-x} + 2\left[x \int e^{-x} dx - \int \left(1 \times \int e^{-x} dx\right) dx\right]$$
$$= -x^2 e^{-x} + 2\left[x\left(-e^{-x}\right) - \int \left(-e^{-x}\right) dx\right]$$
$$= -x^2 e^{-x} - 2x e^{-x} + 2\int e^{-x} dx$$
$$I = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$$
$$I = -e^{-x}\left(x^2 + 2x + 2\right) + c$$

Indefinite Integrals Ex 19.25 Q7

Let $I = \int x^2 \cos x\, dx$

Using integration by parts,

$$I = x^2 \int \cos x\, dx - \int \left(2x \int \cos x\, dx\right) dx$$
$$= x^2 \sin x - 2\int (x)(\sin x) dx$$
$$= x^2 \sin x - 2\left[x \int \sin x\, dx - \int \left(1 \times \int \sin x\, dx\right) dx\right]$$
$$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$$
$$= x^2 \sin x + 2x \cos x - 2\int (\cos x) dx$$

$$I = x^2 \sin x + 2x \cos x - 2\sin x + c$$

Indefinite Integrals Ex 19.25 Q8

Let $I = \int x^2 \cos 2x\, dx$

Using integration by parts,

$$I = x^2 \int \cos 2x\, dx - \int \left(2x \int \cos 2x\, dx\right) dx$$
$$= x^2 \frac{\sin 2x}{2} - 2\int x\left(\frac{\sin 2x}{2}\right) dx$$
$$= \frac{1}{2}x^2 \sin 2x - \int x \sin 2x\, dx$$
$$= \frac{1}{2}x^2 \sin 2x - \left[x \int \sin 2x\, dx - \int \left(1 \int \sin 2x\, dx\right) dx\right]$$
$$= \frac{1}{2}x^2 \sin 2x - \left[x\left(\frac{-\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$
$$= \frac{1}{2}x^2 \sin 2x + \frac{x}{2}\cos 2x - \frac{1}{2}\int (\cos 2x) dx$$

$$I = \frac{1}{2}x^2 \sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + c$$

Indefinite Integrals Ex 19.25 Q9

Let $I = \int x \sin 2x\, dx$

Using integration by parts,

$$I = x \int \sin 2x\, dx - \int \left(1 \int \sin 2x\, dx\right) dx$$
$$= x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx$$
$$= -\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x\, dx$$
$$= -\frac{x}{2}\cos 2x + \frac{1}{2}\frac{\sin 2x}{2} + c$$
$$I = -\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + c$$

Indefinite Integrals Ex 19.25 Q10

Let $I = \int \frac{\log(\log x)}{x} dx$

$= \int \left(\frac{1}{x}\right)(\log(\log x)) dx$

Using integration by parts,

$I = \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx\right) dx$

$= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x\right) dx$

$= \log x \times \log(\log x) - \int \frac{1}{x} dx$

$= \log x \times \log(\log x) - \log x + c$

$I = \log x (\log\log x - 1) + c$

Indefinite Integrals Ex 19.25 Q11

Let $I = \int x^2 \cos x \, dx$

Using integration by parts,

$I = x^2 \int \cos x \, dx - \int (2x \int \cos x \, dx) dx$

$= x^2 \sin x - 2\int x \sin x \, dx$

$= x^2 \sin x - 2\left[x \int \sin x \, dx - \int (1 \int \sin x \, dx) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$

$= x^2 \sin x + 2x \cos x - 2\int (\cos x) dx$

$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$

Indefinite Integrals Ex 19.25 Q12

Let $I = \int x \operatorname{cosec}^2 x \, dx$

Using integration by parts,

$I = x \int \operatorname{cosec}^2 x \, dx - \int \left(\int \operatorname{cosec}^2 x \, dx\right) dx$

$= -x \cot x + \int \cot x \, dx$

$= -x \cot x + \log|\sin x| + c$

Indefinite Integrals Ex 19.25 Q13

Let $I = \int x \cos^2 x \, dx$

Using integration by parts,

$I = x \int \cos^2 x \, dx - \int \left(1 \int \cos^2 x \, dx\right) dx$

$= x \int \left(\frac{\cos 2x + 1}{2}\right) dx - \int \left(\int \left(\frac{1 + \cos 2x}{2}\right) dx\right) dx$

$= \frac{x}{2}\left[\frac{\sin 2x}{2} + x\right] - \frac{1}{2}\int \left(x + \frac{\sin 2x}{2}\right) dx$

$= \frac{x}{4} \sin 2x + \frac{x^2}{2} - \frac{1}{2} \times \frac{x^2}{2} - \frac{1}{4}\left(-\frac{\cos 2x}{2}\right) + c$

$I = \frac{x}{4} \sin 2x + \frac{x^2}{4} + \frac{1}{8} \cos 2x + c$

Indefinite Integrals Ex 19.25 Q14

Let $I = \int x^n \log x \, dx$

Using integration by parts,

$$I = \log x \int x^2 \, dx - \int \left(\frac{1}{x} \int x^2 \, dx \right) dx$$

$$= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{1}{x} \times \frac{x^{n+1}}{n+1} \right) dx$$

$$= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{x^n}{n+1} \right) dx$$

$$I = \frac{x^{n+1}}{n+1} \log x - \frac{1}{(n+1)^2} x^{n+1} + c$$

Indefinite Integrals Ex 19.25 Q15

$$\int \frac{\log x}{x^n} dx = \int (\log x) \left(\frac{1}{x^n} \right) dx$$

by integration by parts

$$\int (\log x) \left(\frac{1}{x^n} \right) dx = \log x \int \left(\frac{1}{x^n} \right) dx - \int \left(\frac{d(\log x)}{dx} \right) \left(\int \left(\frac{1}{x^n} \right) dx \right) dx$$

$$= \log x \left(\frac{x^{1-n}}{1-n} \right) - \int \frac{1}{x} \left(\frac{x^{1-n}}{1-n} \right) dx = \log x \left(\frac{x^{1-n}}{1-n} \right) - \int \left(\frac{x^{-n}}{1-n} \right) dx$$

$$= \log x \left(\frac{x^{1-n}}{1-n} \right) - \left(\frac{1}{1-n} \right) \left(\frac{x^{1-n}}{1-n} \right) = \log x \left(\frac{x^{1-n}}{1-n} \right) - \left(\frac{x^{1-n}}{[1-n]^2} \right) + C$$

Indefinite Integrals Ex 19.25 Q16

Let $I = \int x^2 \sin^2 x \, dx$

$$= \int x^2 \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \int \frac{x^2}{2} dx - \int \left(\frac{x^2 \cos 2x}{2} \right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2} \left[\int x^2 \cos 2x \, dx \right]$$

$$= \frac{x^3}{6} - \frac{1}{2} \left[x^2 \int \cos 2x \, dx - \int \left(2x \int \cos 2x \, dx \right) dx \right]$$

$$= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \frac{\sin 2x}{2} \right) + \frac{1}{2} \times 2 \int \left(x \ \frac{\sin 2x}{2} \right) dx$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx \right) dx \right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[x \left(-\frac{\cos 2x}{2} \right) - \int \left(-\frac{\cos 2x}{2} \right) dx \right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \frac{\sin 2x}{2} + c$$

$$I = \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q17

Let $I = \int 2x^3 e^{x^2} x \, dx$

Let $x^2 = t$

$2x \, dx = dt$

$I = \int t \times e^t dt$

Using integration by parts,

$$= t \int e^t dt - \int \left(1 \times \int e^t dt \right) dt$$

$$= te^t - \int e^t dt$$

$$= te^t - e^t + c$$

$$= e^t (t - 1) + c$$

$$I = e^{x^2} (x^2 - 1) + c$$

Indefinite Integrals Ex 19.25 Q18

Let $\quad I = \int x^3 \cos x^2\,dx$

Let $\quad x^2 = t$

$2x\,dx = dt$

$I = \frac{1}{2}\int t \cos t\,dt$

Using integration by parts,

$= \frac{1}{2}\left[t\int \cos t\,dt - \int\left(1\times\int \cos t\,dt\right)dt\right]$

$= \frac{1}{2}\left[t \times \sin t - \int \sin t\,dt\right]$

$= \frac{1}{2}\left[t \sin t + \cos t\right] + c$

$I = \frac{1}{2}\left[x^2 \sin x^2 + \cos x^2\right] + c$

Indefinite Integrals Ex 19.25 Q19

Let $\quad I = \int x \sin x \cos x\,dx$

$= \int \frac{x}{2}\left(2\sin x \cos x\right)dx$

$= \frac{1}{2}\int x \sin 2x\,dx$

Using integration by parts,

$= \frac{1}{2}\left[x\int \sin 2x\,dx - \int\left(1\times\int \sin 2x\,dx\right)dx\right]$

$= \frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right) - \int\left(\frac{-\cos 2x}{2}\right)dx\right]$

$= -\frac{1}{4}x\cos 2x + \frac{1}{4}\int \cos 2x\,dx$

$I = -\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + c$

Indefinite Integrals Ex 19.25 Q20

Let $\quad I = \int \sin x\left(\log \cos x\right)dx$

Let $\quad \cos x = t$

$-\sin x\,dx = dt$

$I = -\int \log t\,dt$

$= -\int 1\times\log t\,dt$

Using integration by parts,

$= -\left[\log t\int dt - \int\left(\frac{1}{t}\times\int dt\right)dt\right]$

$= -\left[t\log t - \int\frac{1}{t}\times t\,dt\right]$

$= -\left[t\log t - \int dt\right]$

$= -\left[t\log t - t + c_1\right]$

$= t\left(1 - t\log t\right) + c$

$I = \cos x\left(1 - \log\cos x\right) + c$

Indefinite Integrals Ex 19.25 Q21

Let $I = \int (\log x)^2 \, x \, dx$

Using integration by parts,

$$= (\log x)^2 \int x\,dx - \int\left(2(\log x)\left(\frac{1}{x}\right)\int x\,dx\right)dx$$

$$= \frac{x^2}{2}(\log x)^2 - 2\int(\log x)\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right)dx$$

$$= \frac{x^2}{2}(\log x)^2 - \int x(\log x)\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\log x\int x\,dx - \int\left(\frac{1}{x}\int x\,dx\right)dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2}\log x - \int\left(\frac{1}{x}\times\frac{x^2}{2}\right)dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{4}x^2 + c$$

$$I = \frac{x^2}{2}\left[(\log x)^2 - \log x + \frac{1}{2}\right] + c$$

Indefinite Integrals Ex 19.25 Q22

Let $I = \int e^{\sqrt{x}}\,dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$I = 2\int e^t t\,dt$

$I = 2\left[t\int e^t dt - \int\left(1\int e^t dt\right)dt\right]$

$I = 2\left[te^t - \int e^t dt\right]$

$= 2\left[te^t - e^t\right] + c$

$= 2e^t(t-1) + c$

$$I = 2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$$

Indefinite Integrals Ex 19.25 Q23

Let $I = \int \frac{\log(x+2)}{(x+2)^2}dx$

Let $\frac{1}{x+2} = t$

$-\frac{1}{(x+2)^2}dx = dt$

$I = -\int \log\left(\frac{1}{t}\right)dt$

$= -\int \log t^{-1}\,dt$

$= -\int 1\times\log t\,dt$

Using integration by parts,

$I = \log t\int dt - \int\left(\frac{1}{t}\int dt\right)dt$

$= t\log t - \int\left(\frac{1}{t}\times t\right)dt$

$= t\log t - \int dt$

$= t\log t - t + c$

$= \frac{1}{x+2}\left(\log(x+2)^{-1} - 1\right) + c$

$$I = \frac{-1}{x+2} - \frac{\log(x+2)}{x+2} + c$$

Indefinite Integrals Ex 19.25 Q24

Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \frac{1}{2}\int x \sec^2 \frac{x}{2} dx + \int \tan\frac{x}{2} dx$$

Using integration by parts,

$$= \frac{1}{2}\left[x\int \sec^2\frac{x}{2} dx - \int\left(1\int \sec^2\frac{x}{2} dx\right)dx\right] + \int \tan\frac{x}{2} dx$$

$$= \frac{1}{2}\left[2x\tan\frac{x}{2} - 2\int \tan\frac{x}{2} dx\right] + \int \tan\frac{x}{2} dx + c$$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx + c$$

$$I = x\tan\frac{x}{2} + c$$

Indefinite Integrals Ex 19.25 Q25

Let $I = \int \log_{10} x\, dx$

$$= \int \frac{\log x}{\log 10} dx$$

$$= \frac{1}{\log 10}\int 1 \times \log x\, dx$$

Using integration by parts,

$$= \frac{1}{\log 10}\left[\log x \int dx - \int\left(\frac{1}{x}\int dx\right)dx\right]$$

$$= \frac{1}{\log 10}\left[x\log x - \int\left(\frac{x}{x}\right)dx\right]$$

$$= \frac{1}{\log 10}\left[x\log x - x\right]$$

$$I = \frac{x}{\log 10}(\log x - 1)$$

Indefinite Integrals Ex 19.25 Q26

Let $I = \int \cos\sqrt{x}\, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$= \int 2t\cos t\, dt$

$I = 2\int t\cos t\, dt$

$I = 2\left[t\int \cos t\, dt - \int\left(1\int \cos t\, dt\right)dt\right]$

$= 2\left[t\sin t - \int \sin t\, dt\right]$

$= 2\left[t\sin t + \cos t\right] + c$

$$I = 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$$

Indefinite Integrals Ex 19.25 Q27

Let $I=\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

Let us substitute, $t=\cos^{-1}x$

$\Rightarrow dt=\frac{-1}{\sqrt{1-x^2}}dx$

Also, cost=x

Thus,

$I=-\int t\cos t\,dt$

Now let us solve this by the 'by parts' method.

Let u=t; du=dt

$\int \cos t\,dt=\int dv$

$\Rightarrow \sin t=v$

Thus, $I=-\left[t\sin t-\int \sin t\,dt\right]$

$\Rightarrow I=-\left[t\sin t+\cos t\right]+C$

Substituting the value $t=\cos^{-1}x$, we have,

$I=-\left[\cos^{-1}x\sin t+x\right]+C$

$\Rightarrow I=-\left[\cos^{-1}x\sqrt{1-x^2}+x\right]+C$

Indefinite Integrals Ex 19.25 Q29

Let $I=\int \operatorname{cosec}^3 dx$

$=\int \operatorname{cosec}x-\operatorname{cosec}^2 x\,dx$

Using integration by parts,

$=\operatorname{cosec}x\times\int \operatorname{cosec}^2 x\,dx+\int\left(\operatorname{cosec}x\cot x\int \operatorname{cosec}^2 x\,dx\right)dx$

$=\operatorname{cosec}x\times(-\cot x)+\int \operatorname{cosec}x\cot x\,(-\cot x)\,dx$

$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}x\cot^2 x\,dx$

$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}x\left(\operatorname{cosec}^2 x-1\right)dx$

$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}^3 x\,dx+\int \operatorname{cosec}x\,dx$

$I=-\operatorname{cosec}x\cot x-I+\log\left|\tan\frac{x}{2}\right|+c_1$

$2I=-\operatorname{cosec}x\cot x+\log\left|\tan\frac{x}{2}\right|+c_1$

$I=-\frac{1}{2}\operatorname{cosec}x\cot x+\frac{1}{2}\log\left|\tan\frac{x}{2}\right|+c$

Indefinite Integrals Ex 19.25 Q30

Let $I=\int \sec^{-1}\sqrt{x}\,dx$

Let $\sqrt{x}=t$

$x=t^2$

$dx=2t\,dt$

$I=\int 2t\sec^{-1}t\,dt$

$=2\left[\sec^{-1}t\int t\,dt-\int\left(\frac{1}{t\sqrt{t^2-1}}\int t\,dt\right)dt\right]$

$=2\left[\frac{t^2}{2}\sec^{-1}-\int\left(\frac{t}{2t\sqrt{t^2-1}}\right)dt\right]$

$=t^2\sec^{-1}t-\int\frac{t}{\sqrt{t^2-1}}dt$

$=t^2\sec^{-1}t-\frac{1}{2}\int\frac{2t}{\sqrt{t^2-1}}dt$

$=t^2\sec^{-1}t-\frac{1}{2}\times 2\sqrt{t^2-1}+c$

$I=x\sec^{-1}\sqrt{x}-\sqrt{x-1}+c$

Indefinite Integrals Ex 19.25 Q31

$\int \sin^{-1}\sqrt{x}\ dx =$

let $x = t^2 \rightarrow dx = 2tdt$

$\int \sin^{-1}\sqrt{x}\ dx = \int \sin^{-1}\sqrt{t^2}\ 2tdt = \int \sin^{-1} t\ 2tdt$

$= \sin^{-1} t \int 2tdt - \left(\int \frac{d \sin^{-1} t}{dt} \left(\int 2tdt \right) dt \right)$

$= \sin^{-1} t (t^2) - \int \frac{1}{\sqrt{1-t^2}} (t^2) dt$

Lets solve $\int \frac{1}{\sqrt{1-t^2}} (t^2) dt$

$\int \frac{1}{\sqrt{1-t^2}} (t^2) dt = \int \frac{t^2 - 1 + 1}{\sqrt{1-t^2}} dt = \int \frac{t^2-1}{\sqrt{1-t^2}} dt + \int \frac{1}{\sqrt{1-t^2}} dt$

we know that, value of $\int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t$

Remaining integral to evaluate is $\int \frac{t^2-1}{\sqrt{1-t^2}} dt = \int -\sqrt{1-t^2}\, dt$

sub t=sin u, dt=cos$u\ du$

$\int -\sqrt{1-t^2}\, dt = \int -\cos^2 u du = -\int \left[\frac{1+\cos 2u}{2} \right] du$

$= -\frac{u}{2} - \frac{\sin 2u}{4}$

Substitute back u=sin^{-1}t $and\ t = \sqrt{x}$

$= -\frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$

$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$

$\sin(2\sin^{-1}\sqrt{x}) = 2\sqrt{x}\sqrt{1-x}$

$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$

Indefinite Integrals Ex 19.25 Q32

Let $I = \int x \tan^2 x\, dx$

$= \int x \left(\sec^2 x - 1 \right) dx$

$= \int x \sec^2 x\, dx - \int x\, dx$

$= \left[x \int \sec^2 x\, dx - \int \left(1 \int \sec^2 x\, dx \right) dx \right] - \frac{x^2}{2}$

$= x \tan x - \int \tan x\, dx - \frac{x^2}{2}$

$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$

Indefinite Integrals Ex 19.25 Q33

Let $I = \int x \left(\frac{\sec 2x - 1}{\sec 2x + 1} \right) dx$

$= \int x \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) dx$

$= \int x \left(\frac{\sec^2 x}{\cos^2 x} \right) dx$

$= \int x \tan^2 x dx$

$= \int x \left(\sec^2 x - 1 \right) dx$

$= \int x \sec^2 x dx - \int dx$

$= \left[x \int \sec^2 x\, dx - \int \left(1 \int \sec^2 x\, dx \right) dx \right] - \frac{x^2}{2}$

$= x \tan x - \int \tan x\, dx - \frac{x^2}{2}$

$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$

Indefinite Integrals Ex 19.25 Q34

Let $I = \int (x+1) e^x \log\left(xe^x\right) dx$

Let $xe^x = t$

$\left(1 \times e^x + xe^x\right) dx = dt$

$(x+1) e^x dx = dt$

$I = \int \log t \, dt$

$= \int 1 \times \log t \, dt$

$= \log t \int dt - \int \left(\frac{1}{t} \int dt\right) dt$

$= t \log t - \int \left(\frac{1}{t} t\right) dt$

$= t \log t - \int dt$

$= t \log t - t + c$

$= t (\log t - 1) + c$

$I = xe^x \left(\log xe^x - 1\right) + c$

Indefinite Integrals Ex 19.25 Q35

Let $I = \int \sin^{-1}\left(3x - 4x^3\right) dx$

Let $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$= \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta \, d\theta$

$= \int \sin^{-1}(\sin 3\theta) \cos\theta \, d\theta$

$= \int 3\theta \cos\theta \, d\theta$

$= 3\left[\theta \int \cos\theta \, d\theta - \int \left(1 \int \cos\theta \, d\theta\right) d\theta\right]$

$= 3\left[\theta \sin\theta - \int \sin\theta \, d\theta\right]$

$= 3\left[\theta \sin\theta + \cos\theta\right] + c$

$I = 3\left[x \sin^{-1} x + \sqrt{1 - x^2}\right] + c$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2\theta \, d\theta = 2\int \theta \cdot \sec^2\theta \, d\theta$

Integrating by parts, we obtain

$2\left[\theta \cdot \int \sec^2\theta d\theta - \int \left\{\left(\frac{d}{d\theta}\theta\right) \int \sec^2\theta d\theta\right\} d\theta\right]$

$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$

$= 2\left[\theta \tan\theta + \log|\cos\theta|\right] + C$

$= 2\left[x \tan^{-1} x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x \tan^{-1} x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$

$= 2x \tan^{-1} x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$

$= 2x \tan^{-1} x - \log\left(1+x^2\right) + C$

Indefinite Integrals Ex 19.25 Q37

Let $I = \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2 x}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 3\theta)\sec^2\theta\, d\theta$

$= \int 3\theta \sec^2\theta\, d\theta$

$= 3\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 3\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 3\left[\theta\tan\theta + \log\sec\theta\right] + c$

$= 3\left[x\tan^{-1}x - \log\sqrt{1 + x^2}\right] + c$

$I = 3x\tan^{-1}x - \frac{3}{2}\log\left|1 + x^2\right| + c$

Indefinite Integrals Ex 19.25 Q38

Let $I = \int x^2 \sin^{-1}x\, dx$

$I = \sin^{-1}x\int x^2\, dx - \int\left(\frac{1}{\sqrt{1 - x^2}}\int x^2\, dx\right)dx$

$= \frac{x^3}{3}\sin^{-1}x - \int\frac{x^3}{3\sqrt{1 - x^2}}dx$

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{3}I_1 + c_1 \text{ - - - - - - } (1)$

$I_1 = \int\frac{x^3}{\sqrt{1 - x^2}}dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$-x\, dx = t\, dt$

$I_1 = -\int\frac{(1 - t^2)\, t\, dt}{t}$

$= \int(t^2 - 1)\, dt$

$= \frac{t^3}{3} - t + c_2$

$= \frac{(1 - x^2)^{\frac{3}{2}}}{3} - (1 - x^2)^{\frac{1}{2}} + c_2$

Now,

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{9}(1 - x^2)^{\frac{3}{2}} + \frac{1}{3}(1 - x^2)^{\frac{1}{2}} + c$

Indefinite Integrals Ex 19.25 Q39

Let $I = \int \frac{\sin^{-1} x}{x^2} dx$

$= \int \left(\frac{1}{x^2}\right)\left(\sin^{-1} x\right) dx$

$I = \left[\sin^{-1} x \int \frac{1}{x^2} dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx\right) dx\right]$

$= \sin^{-1} x \left(-\frac{1}{x}\right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\frac{1}{x}\right) dx$

$I = -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x\sqrt{1-x^2}} dx$

$I = -\frac{1}{x} \sin^{-1} x + I_1 ------(1)$

Where,

$I_1 = \int \frac{1}{x\sqrt{1-x^2}} dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$I_1 = \int \frac{x}{x^2\sqrt{1-x^2}} dx$

$= -\int \frac{t\, dt}{\left(1-t^2\right)\sqrt{t}}$

$= -\int \frac{dt}{\left(1-t^2\right)}$

$= \int \frac{1}{t^2 - 1} dt$

$= \frac{1}{2} \log\left|\frac{t-1}{t+1}\right|$

$= \frac{1}{2} \log\left|\frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2} + 1}\right| + c_1$

Now,

$I = -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log\left|\left(\frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2} + 1}\right)\left(\frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2} - 1}\right)\right| + c$

$= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log\left|\frac{\left(\sqrt{1-x^2} - 1\right)^2}{1 - x^2 - 1}\right| + c$

$= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log\left|\frac{\left(\sqrt{1-x^2} - 1\right)^2}{-x^2}\right| + c$

$= -\frac{\sin^{-1} x}{x} + \log\left|\frac{\sqrt{1-x^2} - 1}{-x}\right| + c$

$I = -\frac{\sin^{-1} x}{x} + \log\left|\frac{1 - \sqrt{1-x^2}}{x}\right| + c$

Indefinite Integrals Ex 19.25 Q40

Let $I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$

Let $\tan^{-1} x = t$

$x = \tan t$

$\frac{1}{1+x^2} dx = dt$

$I = \int t \tan^2 t \, dt$

$= \int t\left(\sec^2 t - 1\right) dt$

$= \int \left(t \sec^2 t - t\right) dt$

$= \int t \sec^2 t \, dt - \int t \, dt$

$= \left[t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt\right] - \frac{t^2}{2}$

$= \left[t \tan t - \int \tan t \, dt\right] - \frac{t^2}{2}$

$= t \tan t - \log \sec t - \frac{t^2}{2} + c$

$= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$

$I = x \tan^{-1} x - \frac{1}{2} \log \left|1+x^2\right| - \frac{\tan^2 x}{2} + c$

Indefinite Integrals Ex 19.25 Q41

Let $I = \int \cos^{-1}\left(4x^3 - 3x\right) dx$

Let $x = \cos\theta$

$dx = -\sin\theta \, d\theta$

$I = -\int \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right) \sin\theta \, d\theta$

$= -\int \cos^{-1}\left(\cos 3\theta\right) \sin\theta \, d\theta$

$= -\int 3\theta \sin\theta \, d\theta$

$= -3\left[\theta \int \sin\theta \, d\theta - \int \left(1 \int \sin\theta \, d\theta\right) d\theta\right]$

$= -3\left[-\theta \cos\theta + \int \cos\theta \, d\theta\right]$

$= 3\theta \cos\theta - 3\sin\theta + c$

$I = 3x \cos^{-1} x - 3\sqrt{1-x^2} + c$

Indefinite Integrals Ex 19.25 Q42

Let $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$

Let $x = \tan t$

$dx = \sec^2 t \, dt$

$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right) \sec^2 t \, dt$

$= \int \cos^{-1}\left(\cos 2t\right) \sec^2 t \, dt$

$= \int 2t \sec^2 t \, dt$

$= 2\left[t \int sce^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt\right]$

$= 2\left[t \tan^2 t - \int \tan t \, dt\right]$

$= 2\left[t \tan^2 t - \log \sec t\right] + c$

$= 2\left[x \tan^{-1} x - \log \sqrt{1+x^2}\right] + c$

$I = 2x \tan^{-1} x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q43

Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}\left(\tan 2\theta\right)\sec^2\theta\, d\theta$

$= \int 2\theta \sec^2\theta\, d\theta$

$= 2\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 2\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 2\left[\theta\tan\theta - \log\sec\theta\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q44

Let $I = \int\left(x+1\right)\log x\, dx$

$= \log x\int\left(x+1\right)dx - \int\left(\frac{1}{x}\int\left(x+1\right)dx\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \int\frac{1}{x}\left(\frac{x^2}{2}+x\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \frac{1}{2}\int x\, dx - \int dx$

$= \left(x+\frac{x^2}{2}\right)\log x - \frac{1}{2}\times\frac{x^2}{2} - x + c$

$I = \left(x+\frac{x^2}{2}\right)\log x - \left(\frac{x^2}{4}+x\right) + c$

Indefinite Integrals Ex 19.25 Q45

Let $I = \int x^2\tan^{-1}x\, dx$

$= \tan^{-1}x\int x^2\, dx - \int\left(\frac{1}{1+x^2}\int x^2 dx\right)$

$= \tan^{-1}x\left(\frac{x^3}{3}\right) - \frac{1}{3}\int\frac{x^3}{1+x^2}dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\times\frac{x^2}{2} + \frac{1}{3}\int\frac{x}{1+x^2}dx$

$I = \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q46

Let $I = \int \left(e^{\log x} + \sin x\right) \cos x \, dx$

$= \int (x + \sin x) \cos x \, dx$

$= \int x \cos x \, dx + \int \sin x \cos x \, dx$

$= \left[x \int \cos x \, dx - \int \left(1 \int \cos x \, dx\right) dx\right] + \frac{1}{2} \int \sin 2x \, dx$

$= \left[x \sin x - \int \sin x \, dx\right] + \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$

$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$

$I = x \sin x + \cos x - \frac{1}{4}\left[1 - 2\sin^2 x\right] + c$

$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$

$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$

$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k$, *where* $k = c - \frac{1}{4}$

Indefinite Integrals Ex 19.25 Q47

Let $I = \int \frac{\left(x \tan^{-1} x\right)}{\left(1 + x^2\right)^{\frac{3}{2}}} dx$

Let $\tan^{-1} x = t$

$\frac{1}{1 + x^2} dx = dt$

$I = \int \frac{t \tan t}{\sqrt{1 + \tan^2 t}} dt$

$= \int \frac{t \tan t}{\sec t} dt$

$= \int t \frac{\sin t}{\cos t} \cos t \, dt$

$= \int t \sin t \, dt$

$= \left[t \int \sin t dt - \int \left(1 \int \sin t \, dt\right) dt\right]$

$= \left[-t \cos t + \int \cos t \, dt\right]$

$= \left[-t \cos t + \sin t\right] + c$

$I = -\frac{\tan^{-1} x}{\sqrt{1 + x^2}} + \frac{x}{\sqrt{1 + x^2}} + c$

Indefinite Integrals Ex 19.25 Q48

Let $I = \int \tan^{-1}\left(\sqrt{x}\right) dx$

Let $x = t^2$

$dx = 2t \, dt$

$I = \int 2t \tan^{-1} t \, dt$

$= 2\left[\tan^{-1} t \int t \, dt - \int \left(\frac{1}{1 + t^2} \int t \, dt\right) dt\right]$

$= 2\left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2\left(1 + t^2\right)} dt\right]$

$= t^2 \tan^{-1} t - \int \frac{t^2 + 1 - 1}{1 + t^2} dt$

$= t^2 \tan^{-1} t - \int \left(1 - \frac{1}{1 + t^2}\right) dt$

$= t^2 \tan^{-1} t - t + \tan^{-1} t + c$

$= \left(t^2 + 1\right) \tan^{-1} t - t + c$

$I = (x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$

Indefinite Integrals Ex 19.25 Q49

$$\int x^3 \tan^{-1} x \, dx =$$

$$\int x^3 \tan^{-1} x \, dx = \tan^{-1} x \int x^3 dx - \left(\int \frac{d \tan^{-1} x}{dx} \left(\int x^3 dx \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right]$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right]$$

$$\int x^3 \tan^{-1} x \, dx = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] + C$$

Indefinite Integrals Ex 19.25 Q50

Let $I = \int x \sin x \cos 2x \, dx$

$$= \frac{1}{2} \int x \left(2 \sin x \cos 2x \right) dx$$

$$= \frac{1}{2} \int x \left(\sin\left(x + 2x\right) - \sin\left(2x - x\right) \right) dx$$

$$= \frac{1}{2} \int x \left(\sin 3x - \sin x \right) dx$$

$$= \frac{1}{2} \left[x \int \left(\sin 3x - \sin x \right) dx - \int \left(1 \int \left(\sin 3x - \sin x \right) dx \right) dx \right]$$

$$= \frac{1}{2} \left[x \left(\frac{-\cos 3x}{3} + \cos x \right) - \int \left(-\frac{\cos 3x}{3} + \cos x \right) dx \right]$$

$$I = \frac{1}{2} \left[-x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + c$$

Indefinite Integrals Ex 19.25 Q51

Let $I = \int \left(\tan^{-1} x^2 \right) x dx$

Let $x^2 = t$

$2x \, dx = dt$

$$I = \frac{1}{2} \int \tan^{-1} t \, dt$$

$$= \frac{1}{2} \int 1 \tan^{-1} t \, dt$$

$$= \frac{1}{2} \left[\tan^{-1} t \int dt - \left(\int \frac{1}{1+t^2} \int dt \right) dt \right]$$

$$= \frac{1}{2} \left[t \tan^{-1} t - \int \frac{t}{1+t^2} dt \right]$$

$$= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \int \frac{2t}{1+t^2} dt$$

$$= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \log \left| 1 + t^2 \right| + c$$

$$I = \frac{1}{2} x^2 \tan^{-1} x^2 - \frac{1}{4} \log \left| 1 + x^4 \right| + c$$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second dunction be $\frac{x}{\sqrt{1-x^2}}$.

First we find the intergral of the second function, i.e., $\int \frac{x\,dx}{\sqrt{1-x^2}}$.

Put $t = 1 - x^2$. Then $dt = -2x\ dx$

Therefore, $\int \frac{x\,dx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

Hence, $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx = \left(\sin^{-1} x\right)\left(-\sqrt{1-x^2}\right) - \int \frac{1}{\sqrt{1-x^2}}\left(-\sqrt{1-x^2}\right)dx$

$$= -\sqrt{1-x^2}\sin^{-1} x + x + C = x - \sqrt{1-x^2}\sin^{-1} x + C$$

Indefinite Integrals Ex 19.25 Q53

Let $I = \int \sin^3 \sqrt{x}\,dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$I = 2\int t \sin^3 t\,dt$

$= 2\int t\left(\frac{3\sin t - \sin 3t}{4}\right)dt$

$= \frac{1}{2}\int t\left(3\sin t - \sin 3t\right)dt$

Using integration by parts,

$I = \frac{1}{2}\left[t\left(-3\cos t + \frac{1}{3}\cos 3t\right) - \int\left(-3\cos t + \frac{\cos 3t}{3}\right)dt\right]$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} - \left\{-3\sin t + \frac{\sin 3t}{9}\right\}\right] + c$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} + \frac{27\sin t - 3\sin 3t}{9}\right] + c$

$= \frac{1}{18}\left[-27t\cos t + 3t\cos 3t + 27\sin t - 3\sin 3t\right] + c$

$I = \frac{1}{18}\left[3\sqrt{x}\cos 3\sqrt{x} + 27\sin\sqrt{x} - 27\sqrt{x}\cos\sqrt{x} - 3\sin 3\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q54

Let $I = \int x\sin^3 x\,dx$

$= \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$

$= \frac{1}{4}\int x\left(3\sin x - \sin 3x\right)dx$

Using integration by parts,

$I = \frac{1}{4}\left[x\int\left(3\sin x - \sin 3x\right)dx - \int\left(1\int\left(3\sin x - \sin 3x\right)dx\right)dx\right]$

$= \frac{1}{4}\left[x\left(-3\cos x + \frac{\cos 3x}{3}\right) - \int\left(-3\cos x + \frac{\cos 3x}{3}\right)dx\right]$

$= \frac{1}{4}\left[-3x\cos x + \frac{x\cos 3x}{3} + 3\sin x - \frac{\sin 3x}{9}\right] + c$

$I = \frac{1}{36}\left[3x\cos 3x - 27x\cos x + 27\sin x - \sin 3x\right] + c$

Indefinite Integrals Ex 19.25 Q55

Let $I = \int \cos^3 \sqrt{x}\, dx$

Let $x = t^2$

$dx = 2t\,dt$

$= 2\int t \cos^3 t\, dt$

$= 2\int t\left(\frac{3\cos t + \cos 3t}{4}\right)dt$

$= \frac{1}{2}\int t\left(3\cos t + \cos 3t\right)dt$

Using integration by parts,

$I = \frac{1}{2}\left[t\left(3\sin t + \frac{1}{3}\sin 3t\right) + \int\left(1 \times 3\sin t + \frac{\sin 3t}{3}\right)dt\right]$

$= \frac{1}{2}\left[t\left(\frac{9\sin t + \sin 3t}{3}\right) + 3\cos t + \frac{\cos 3t}{9}\right] + c$

$= \frac{1}{18}\left[27t\sin t + 3t\sin 3t + 9\cos t + \cos 3t\right] + c$

$I = \frac{1}{18}\left[27\sqrt{x}\sin\sqrt{x} + 3\sqrt{x}\sin 3\sqrt{x} + 9\cos\sqrt{x} + \cos 3\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q56

Let $I = \int x\cos^3 x\, dx$

$= \int x\left(\frac{3\cos x + \cos 3x}{4}\right)dx$

$= \frac{1}{4}\int x\left(3\cos x + \cos 3x\right)dx$

Using integration by parts,

$I = \frac{1}{4}\left[x\int\left(3\cos x + \cos 3x\right)dx - \int\left(1\int\left(3\cos x + \cos 3x\right)dx\right)dx\right]$

$= \frac{1}{4}\left[x\left(3\sin x + \frac{\sin 3x}{3}\right) - \int\left(3\sin x + \frac{\sin 3x}{3}\right)dx\right]$

$= \frac{1}{4}\left[3x\sin x + \frac{x\sin 3x}{3} + 3\cos x + \frac{\cos 3x}{9}\right] + c$

$I = \frac{3x\sin x}{4} + \frac{x\sin 3x}{12} + \frac{3\cos x}{4} + \frac{\cos 3x}{36} + c$

Indefinite Integrals Ex 19.25 Q57

Let I = $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ and x = cos θ $\Rightarrow dx = -\sin\theta d\theta$

$\therefore I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)d\theta$

$= -\frac{1}{2}\int \theta\sin\theta d\theta$

$Let\,\theta = u\; and\; \sin\theta d\theta = v\; so that\; \sin\theta = \int v\, d\theta$

$Then, \int uv\,dx = u\int(v\,dx) - \left(\int\frac{du}{dx}\int v\,dx\right)dx$

$Hence, I = -\frac{1}{2}\left(-\theta\cos\theta - \int -\cos\theta d\theta\right)$

$= -\frac{1}{2}\left(-\theta\cos\theta + \sin\theta\right) + c$

$= -\frac{1}{2}\left(-\theta\cos\theta + \sqrt{1-\cos^2\theta}\right) + c$

$= -\frac{1}{2}\left(-x\cos^{-1}x + \sqrt{1-x^2}\right) + c$

Indefinite Integrals Ex 19.25 Q58

Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x = a\tan^2\theta$

$dx = 2a\tan\theta\sec^2\theta\,d\theta$

$I = \int\left(\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int\left(\sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int\sin^{-1}\left(\sin\theta\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int 2\theta a\tan\theta\sec^2\theta d\theta$

$= 2a\int\theta\left(\tan\theta\sec^2\theta\right)d\theta$

$= 2a\left[\theta\int\tan\theta\sec^2\theta\,d\theta - \int\left(\int\tan\theta\sec^2\theta\,d\theta\right)d\theta\right]$

$= 2a\left[\theta\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right]$

$= a\theta\tan^2\theta - \frac{2a}{2}\int\left(\sec^2\theta - 1\right)d\theta$

$= a\theta\tan^2\theta - a\tan\theta + a\theta + c$

$= a\left(\tan^{-1}\sqrt{\frac{x}{a}}\right)\frac{x}{a} - a\sqrt{\frac{x}{a}} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

$I = x\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

Indefinite Integrals Ex 19.25 Q59

Let $I = \int\frac{x^3\sin^{-1}x^2}{\sqrt{1-x^4}}dx$

Let $\sin^{-1}x^2 = t$

$\frac{1}{\sqrt{1-x^4}}\left(2x\right)dx = dt$

$I = \int\frac{x^2\sin^{-1}x^2}{\sqrt{1-x^2}}x\,dx$

$= \int\left(\sin t\right)t\frac{dt}{2}$

$= \frac{1}{2}\int t\sin t\,dt$

$= \frac{1}{2}\left[t\int\sin t\,dt - \int\left(1\int\sin t\,dt\right)dt\right]$

$= \frac{1}{2}\left[t\left(-\cos t\right) - \int\left(-\cos t\right)dt\right]$

$= \frac{1}{2}\left[-t\cos t + \sin t\right] + c$

$I = \frac{1}{2}\left[x^2 - \sqrt{1-x^4}\sin^{-1}x^2\right] + c$

Indefinite Integrals Ex 19.25 Q60

Let $I = \int \frac{x^2 \sin^{-1} x}{\left(1 - x^2\right)^{\frac{3}{2}}} dx$

Let $\sin^{-1} x = t$

$\frac{1}{\sqrt{1 - x^2}} dx = dt$

$I = \int \frac{\sin^2 t \times t}{\left(1 - \sin^2 t\right)} dt$

$= \int \frac{t \sin^2 t}{\cos^2 t} dt$

$= \int t \tan^2 t dt$

$= \int t \left(\sec^2 t - 1\right) dt$

$= \int t \sec^2 t \, dt - \frac{t^2}{2}$

$= t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt - \frac{t^2}{2}$

$= t \tan t - \int \tan t \, dt - \frac{t^2}{2}$

$= t \tan t - \log \sec t - \frac{t^2}{2} + c$

$I = \frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \log\left|1 - x^2\right| - \frac{1}{2}\left(\sin^{-1} x\right)^2 + c$

Ex 19.26

Indefinite Integrals Ex 19.26 Q1

Let $I = \int e^x \left(\cos x - \sin x\right) dx$

$= \int e^x \cos x dx - \int e^x \sin x dx$

Integrating by parts

$= e^x \cos x - \int e^x \left(\frac{d}{dx}\cos x\right) dx - \int e^x \sin x dx$

$= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx$

$= e^x \cos x + c$

$\therefore \quad \int e^x \left(\cos x - \sin x\right) dx = e^x \cos x + c$

Indefinite Integrals Ex 19.26 Q2

$I = \int e^x \left(x^{-2} - 2x^{-3}\right) dx$

$= \int e^x x^{-2} dx - 2\int e^x x^{-3} dx$

Integrating by parts

$= e^x x^{-2} - \int e^x \left(\frac{d}{dx}\left(x^{-2}\right)\right) dx - 2\int e^x x^{-3} dx$

$= e^x x^{-2} + 2\int e^x x^{-3} dx - 2\int e^x x^{-3} dx$

$= \frac{e^x}{x^2} + c$

$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = \frac{e^x}{x^2} + c$

Indefinite Integrals Ex 19.26 Q3

$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$

$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$

$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$

$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$

$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$

$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$

$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]$ ...(1)

Let $\tan\frac{x}{2}=f(x)\Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

From equation (1), we obtain

$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$

Indefinite Integrals Ex 19.26 Q4

Let $I=\int e^x\left(\cot x-\operatorname{cosec}^2 x\right)dx$

$=\int e^x\cot x\,dx-\int e^x\operatorname{cosec}^2 x\,dx$

Integrating by parts

$=e^x\cot x-\int e^x\left(\frac{d}{dx}\cot x\right)dx-\int e^x\operatorname{cosec}^2 x\,dx$

$=e^x\cot x+\int e^x\operatorname{cosec}^2 x\,dx-\int e^x\operatorname{cosec}^2 x\,dx$

$=e^x\cot x+c$

$\int e^x\left(\cot x-\operatorname{cosec}^2 x\right)dx=e^x\cot x+c$

Indefinite Integrals Ex 19.26 Q5

Let $I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$

Integrating by parts

$$= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx}\left(\frac{1}{2x}\right)\right) dx - \int \frac{e^x}{2x^2} dx$$
$$= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + c$$

Indefinite Integrals Ex 19.26 Q6

Let $I = \int e^x \sec x \left(1 + \tan x\right) dx$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Integrating by parts

$$= e^x \sec x - \int e^x \left(\frac{d}{dx} \sec x\right) dx + \int e^x \sec x \tan x dx$$
$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x + c$$

$$\therefore \qquad \int e^x \sec x \left(1 + \tan x\right) dx = e^x \sec x + c$$

Indefinite Integrals Ex 19.26 Q7

Let $I = \int e^x \left(\tan x - \log \cos x\right) dx$

$$= \int e^x \tan x dx - \int e^x \log \cos x dx$$

Integrating by parts

$$= \int e^x \tan x dx - \left\{e^x \log \cos x - \int e^x \left(\frac{d}{dx} \log \cos x\right) dx\right\}$$
$$= \int e^x \tan x dx - \left\{e^x \log \cos x + \int e^x \tan x dx\right\}$$
$$= \int e^x \tan x dx - e^x \log \cos x - \int e^x \tan x dx + c$$

$$= -e^x \log \cos x + c$$

$$= e^x \log \sec x + c \qquad \left[\because \qquad \log \sec x = -\log \cos x\right]$$

Indefinite Integrals Ex 19.26 Q8

Let $I = \int e^x \left[\sec x + \log\left(\sec x + \tan x\right)\right] dx$

$$= \int e^x \sec x dx + \int e^x \log\left(\sec x + \tan x\right) dx$$

Integrating by parts

$$= \int e^x \sec x dx + e^x \log\left(\sec x + \tan x\right) - \int e^x \left\{\frac{d}{dx} \log\left(\sec x + \tan x\right)\right\} dx$$
$$= \int e^x \sec x dx + e^x \log\left(\sec x + \tan x\right) - \int e^x \sec x dx$$

$$= e^x \log\left(\sec x + \tan x\right) + c$$

Indefinite Integrals Ex 19.26 Q9

Let $I = \int e^x \left(\cot x + \log \sin x\right) dx$

$$= \int e^x \cot x dx + \int e^x \log \sin x dx$$

Integrating by parts

$$= \int e^x \log \sin x dx + \int e^x \cot x dx$$

$$= \left(\log \sin x\right) e^x - \int e^x \left(\frac{d}{dx} \log \sin x\right) dx + \int e^x \cot x dx$$

$$= e^x \log \sin x - \int e^x \cot x dx + \int e^x \cot x dx$$

$$= e^x \log \sin x + c$$

Indefinite Integrals Ex 19.26 Q10

Let $I = \int e^x \frac{x + 1 - 2}{\left(x + 1\right)^3}$

$$= \int e^x \left\{\frac{1}{\left(x + 1\right)^2} + \frac{-2}{\left(x + 1\right)^3}\right\} dx$$

$$= \int e^x \frac{1}{\left(x + 1\right)^2} dx + \int e^x \frac{\left(-2\right)}{\left(x + 1\right)^3} dx$$

Integrating by parts

$$= e^x \frac{1}{\left(x + 1\right)^2} - \int e^x \left(\frac{d}{dx} \left(x + 1\right)^{-2}\right) dx + \int e^x \frac{\left(-2\right)}{\left(x + 1\right)^3} dx$$

$$= e^x \frac{1}{\left(x + 1\right)^2} - \int e^x \frac{\left(-2\right)}{\left(x + 1\right)^3} dx + \int e^x \frac{\left(-2\right)}{\left(x + 1\right)^3} dx$$

$$= e^x \frac{1}{\left(x + 1\right)^2} + c$$

Indefinite Integrals Ex 19.26 Q11

Let $I = \int e^x \left(\frac{\sin 4x - 4}{2 \sin^2 2x}\right) dx$

$$= \int e^x \left\{\frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x}\right\} dx$$

$$= \int e^x \left(\cot 2x - 2 \operatorname{cosec}^2 2x\right) dx$$

$$= \int e^x \cot 2x dx - 2 \int e^x \operatorname{cosec}^2 2x dx$$

Integrating by parts

$$= e^x \cot 2x - \int e^x \frac{d}{dx} \left(\cot 2x\right) dx - 2 \int e^x \operatorname{cosec}^2 2x dx$$

$$= e^x \cot 2x + 2 \int e^x \operatorname{cosec}^2 2x - 2 \int e^x \operatorname{cosec}^2 2x dx$$

$$= e^x \cot 2x + c$$

Indefinite Integrals Ex 19.26 Q12

Let $I = \int \frac{2-x}{(1-x)^2} e^x dx$

$$= \int e^x \left\{ \frac{(1-x)+1}{(1-x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx$$

Here, $f(x) = \frac{1}{1-x}$ and $f'(x) = \frac{1}{(1-x)^2}$

And we know that,

$$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$$

$$\therefore \quad \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx = e^x . \frac{1}{1-x} + c$$

Hence,

$$I = \frac{e^x}{1-x} + c$$

Indefinite Integrals Ex 19.26 Q13

Let $I = \int e^x \frac{1+x}{(2+x)^2} dx$

$$= \int e^x \left(\frac{x+2-1}{(2+x)^2} \right) dx$$

$$= \int e^x \left\{ \frac{1}{x+2} - \frac{1}{(x+2)^2} \right\} dx$$

$$= \int e^x \frac{1}{x+2} dx - \int e^x \frac{1}{(x+2)^2} dx$$

Integrating by parts

$$= e^x \frac{1}{x+2} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{x+2} \right) \right) dx - \int e^x \frac{1}{(x+2)^2} dx$$

$$= e^x \frac{1}{x+2} + \int e^x \frac{1}{(x+2)^2} dx - \int e^x \frac{1}{(x+2)^2} dx$$

$$= \frac{e^x}{x+2} + c$$

Indefinite Integrals Ex 19.26 Q14

Let $I = \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

Put $= \frac{x}{2} = t$

$\Rightarrow \quad x = 2t$

$dx = 2dt$

$\therefore \quad \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

$= 2\int \frac{\sqrt{1-\sin 2t}}{1+\cos 2t} e^{-t} dt \qquad \left[\because \sin^2 t + \cos^2 t = 1\right]$

$= 2\int \frac{\sqrt{\sin^2 t + \cos^2 t - 2\sin t \cos t}}{1+\cos 2t} e^{-t} dt$

$= 2\int \frac{\sqrt{(\cos t - \sin t)^2}}{2\cos^2 t} e^{-t} dt$

$= 2\int \frac{(\cos t - \sin t)}{2\cos^2 t} e^{-t} dt$

$= \int (\sec t - \tan t \sec t) e^{-t} dt$

$= \int \sec t\, e^{-t} dt - \int \tan t \sec t\, e^{-t} dt$

Integrating by parts

$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt}(\sec t) dt - \int \tan t \sec t\, e^{-t} dt$

$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t e^{-t} dt$

$= -e^{-t} \sec t + c$

Putting the value of t

$= -e^{-\frac{x}{2}} \sec\frac{x}{2} + c$

Indefinite Integrals Ex 19.26 Q15

We have,

$I = \int e^x \left(\log x + \frac{1}{x}\right) dx$

We know that

$\int e^{ax} \left(af(x) + f'(x)\right) dx = e^{ax} f(x) + c$

Here $f(x) = \log x$ and $f'(x) = \frac{1}{x}$

$\therefore \quad \int e^x \left(\log x + \frac{1}{x}\right) dx = e^x \log x + c$

Indefinite Integrals Ex 19.26 Q16

We have,

$$I = \int e^x \left(\log x + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

Integrating by parts

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \frac{d}{dx}\left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.26 Q17

We have,

$$I = \int \frac{e^x}{x}\left\{x(\log x)^2 + 2\log x\right\} dx$$

$$= \int e^x \left\{(\log x)^2 + \frac{2}{x}\log x\right\} dx$$

$$= \int e^x (\log x)^2 + 2\int \frac{e^x}{x}\log x dx$$

Integrating by parts

$$= e^x (\log x)^2 - \int e^x \frac{d}{dx}(\log x)^2 dx + 2\int e^x \frac{1}{x}\log x dx$$

$$= e^x (\log x)^2 - \int e^x \frac{2\log x}{x} dx + 2\int e^x \frac{\log x}{x} dx$$

$$= e^x (\log x)^2 + c$$

Indefinite Integrals Ex 19.26 Q18

Let $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1 - x^2}}\right) dx$

$$I = \int e^x \sin^{-1} x + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$

Integrating by parts

$$= e^x \sin^{-1} x - \int e^x \left(\frac{d}{dx}\left(\sin^{-1} x\right)\right) dx + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$

$$= e^x \sin^{-1} x - \int e^x \frac{1}{\sqrt{1 - x^2}} dx + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$

$$= e^x \sin^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q19

$$\text{Let } I = \int e^{2x}\left(-\sin x + 2\cos x\right)dx$$
$$= -\int e^{2x}\sin x dx + 2\int e^{2x}\cos x dx$$

Applying by parts in the 2nd integrand

$$\therefore \quad I = -\int e^{2x}\sin x dx + 2\left\{\frac{1}{2}e^{2x}\cos x + \int \frac{1}{2}e^{2x}\sin x dx\right\}$$
$$= -\int e^{2x}\sin x dx + e^{2x}\cos x + \int e^{2x}\sin x dx + c$$

$$= e^{2x}\cos x + c$$

Thus,

$$I = e^{2x}\cos x + c$$

Indefinite Integrals Ex 19.26 Q20

$$\text{Let } I = \int e^{x}\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

Here, $f(x) = \tan^{-1}x$ and $f'(x) = \dfrac{1}{1+x^2}$

And we know that,

$$\int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax}f(x) + c$$

$$\therefore \quad \int e^{x}\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx = e^{x}\tan^{-1}x + c$$

Thus,
$$I = e^{x}\tan^{-1}x + c$$

Indefinite Integrals Ex 19.26 Q21

$$\text{Let } I = \int e^{x}\left(\frac{\sin x\cos x - 1}{\sin^2 x}\right)dx$$
$$= \int e^{x}\left(\cot x - \operatorname{cosec}^2 x\right)dx$$
$$= \int e^{x}\left(\cot x + \left(-\operatorname{cosec}^2 x\right)\right)dx$$

$$\because \quad \int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax}f(x) + c$$

Here $f(x) = \cot x$
$$\Rightarrow \quad f'(x) = -\operatorname{cosec}^2 x$$

$$\therefore \quad \int e^{x}\left(\cot x - \operatorname{cosec}^2 x\right)dx = e^{x}\cot x + c$$

Thus,

$$I = e^{x}\cot x + c$$

Indefinite Integrals Ex 19.26 Q22

Let $I = \int \left\{\tan(\log x) + \sec^2(\log x)\right\} dx$

Let $\log x = z$

$\Rightarrow \quad x = e^z$

$\Rightarrow \quad dx = e^z dz$

$\therefore \quad I = \int \left\{\tan z + \sec^2 z\right\} e^z dz$

Here, $f(z) = \tan z$ and $f'(z) = \sec^2 z$

And we know that

$\int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax} f(x) + c$

$\therefore \quad \int e^z \left\{\tan z + \sec^2 z\right\} dz = e^z \tan z + c$

$\therefore \quad I = x \tan(\log x) + c$

Indefinite Integrals Ex 19.26 Q23

Let $I = \int \dfrac{e^x (x-4)}{(x-2)^3} dx$

$= \int e^x \left\{\dfrac{(x-2)-2}{(x-2)^3}\right\} dx$

$= \int e^x \left\{\dfrac{1}{(x-2)^2} - \dfrac{2}{(x-2)^3}\right\} dx$

Here, $f(x) = \dfrac{1}{(x-2)^2}$ and $f'(x) = \dfrac{-2}{(x-2)^3}$

And we know that,

$\int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax} f(x) + c$

$\therefore \quad \int e^x \left\{\dfrac{1}{(x-2)^2} - \dfrac{2}{(x-2)^3}\right\} dx = \dfrac{e^x}{(x-2)^2} + c$

$\therefore \quad I = \dfrac{e^x}{(x-2)^2} + c$

Indefinite Integrals Ex 19.26 Q24

Let I=$\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

We have, $\cos 2x = 1-2\sin^2 x$

I=$\int e^{2x}\left(\frac{1-\sin 2x}{1-\left(1-2\sin^2 x\right)}\right)dx$

$= \int e^{2x}\left(\frac{1-\sin 2x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \frac{2\sin x\cos x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \frac{\cos x}{\sin x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \cot x\right)dx$

$= \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx - \int e^{2x}\cot x dx$

That is

I=$I_1 + I_2$, where, $I_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$ and $I_2 = -\int e^{2x}\cot x dx$

Consider $I_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$

Take e^{2x} as the first function and $\operatorname{cosec}^2 x$ as the second function.

So, u=e^{2x}; du=$2e^{2x}dx$

and

$\int \operatorname{cosec}^2 x dx = \int dv$

$\Rightarrow v = -\cot x$

$I_1 = \frac{1}{2}\left[e^{2x}(-\cot x) - \int(-\cot x)2e^{2x}dx\right]$

$\Rightarrow I_1 = \frac{1}{2}\left[e^{2x}(-\cot x) + 2\int \cot x e^{2x}dx\right]$

$\Rightarrow I_1 = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + \int \cot x e^{2x}dx$

Thus,

$I = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + \int \cot x e^{2x}dx - \int e^{2x}\cot x dx$

$= I = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + C$

Ex 19.27

Indefinite Integrals Ex 19.27 Q1

Let $I = \int e^{ax} \cos bx dx$

Intergrating by parts,

$$I = e^{ax} \frac{\sin bx}{b} - a\int e^{ax} \frac{\sin bx}{b} fx$$
$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx$$
$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b}\left[-e^{ax} \frac{\cos bx}{b} + \int ae^{ax} \frac{\cos bx}{b} dx\right]$$
$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx$$

$$\Rightarrow \quad I = \frac{e^{ax}}{b^2}\left[b \sin bx + a \cos bx\right] - \frac{a^2}{b^2} I + c$$
$$\Rightarrow \quad I.\left\{\frac{a^2 + b^2}{b^2}\right\} = \frac{e^{ax}}{b^2}\left[b \cos bx + a \cos bx\right] + c$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2}\left[a \cos bx + a \cos bx\right] + c$$

Indefinite Integrals Ex 19.27 Q2

Let $I = \int e^{ax} \sin(bx + c) dx$

$$\Rightarrow \quad -e^{ax} \frac{\cos(bx + c)}{b} + \int ae^{ax} \frac{\cos(bx + c)}{b} dx$$
$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c) dx$$
$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b}\left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int ae^{ax} \frac{\sin(bx + c)}{b} dx\right] + c_1$$
$$= \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c) dx + c_1$$

$$\Rightarrow \quad I = \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} - \frac{a^2}{b^2} I + c_1$$
$$\Rightarrow \quad I = \left\{\frac{a^2 + b^2}{b^2}\right\} - \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} + c_1$$

$$\Rightarrow \quad I = \frac{e^{ax}}{a^2 + b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} + c_1$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad dx = xdt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int \cos(\log x)dx = \int e^t \cos t dt$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos bx + b \sin bx\} + c$$

Here $a = 1$, $b = 1$

So, $I = \frac{e^t}{2}\{\cos t + \sin t\} + c$

Hence,

$$I = \int \cos(\log x)dx = \frac{e^{\log x}}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow \quad I = \frac{x}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

Let $I = \int e^{2x} \cos(3x + 4)dx$

Integrating by parts

$$I = e^{2x}\frac{\sin(3x+4)}{3} - \int 2e^{2x}\frac{\sin(3x+4)}{3}dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\left[-e^{2x}\frac{\cos(3x+4)}{3} + \int 2e^{2x}\frac{\cos(3x+4)}{3}dx\right] + c$$

$$I = \frac{e^{2x}}{9}\{2\cos(3x+4) + 3\sin(3x+4)\} + c$$

Hence,

$$I = \frac{e^{2x}}{13}\{2\cos(3x+4) + 3\sin(3x+4)\} + c$$

Indefinite Integrals Ex 19.27 Q5

Let $I = \int e^{2x} \sin x \cos x dx$

$$= \frac{1}{2}\int e^{2x} 2 \sin x \cos x dx$$
$$= \frac{1}{2}\int e^{2x} \sin 2x dx$$

We know that

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2}\{a \sin bx - b \cos bx\} + c$$
$$\Rightarrow \quad \int e^{2x} \sin 2x dx = \frac{e^{2x}}{8}\{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore \quad I = \frac{1}{2}\cdot\frac{e^{2x}}{8}\{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore \quad I = \frac{e^{2x}}{8}\{\sin 2x - \cos 2x\} + c$$

Indefinite Integrals Ex 19.27 Q6

Let $I = \int e^{2x} \sin x\, dx \qquad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x} dx\right\} dx$$
$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x} \cos x\, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\cos x \int e^{2x} dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x} dx\right\} dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2} dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x} \sin x dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I \qquad \text{[From (1)]}$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I$$
$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4}$$
$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$
$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C$$
$$\Rightarrow I = \frac{e^{2x}}{5}\left[2\sin x - \cos x\right] + C$$

Indefinite Integrals Ex 19.27 Q8

Let $I = \int e^x \sin^2 x dx$

$$= \frac{1}{2}\int e^x 2\sin^2 x dx$$
$$= \frac{1}{2}\int e^x \left(1 - \cos 2x\right) dx$$
$$= \frac{1}{2}\int e^x dx - \frac{1}{2}\int e^x \cos 2x dx$$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\left\{a \cos bx - b \sin bx\right\} + c$

$\therefore \quad I = \frac{1}{2}\left[e^x - \frac{e^x}{5}\left\{\cos 2x + 2\sin 2x\right\}\right] + c$

$\therefore \quad I = \frac{e^x}{2} - \frac{e^x}{10}\left\{\cos 2x + 2\sin 2x\right\} + c$

Indefinite Integrals Ex 19.27 Q9

Let $I = \int \frac{1}{x^3} \sin\left(\log x\right) dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int e^{-2t} \sin t dt$

We know that,

$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}\left\{a \sin bx - b \cos bx\right\} + c$

$\therefore \quad \int e^{-2t} \sin t dt = \frac{e^{-2t}}{5}\left\{-2 \sin t - \cos t\right\} + c$

$\therefore \quad I = \frac{x^{-2}}{5}\left\{-2 \sin\left(\log x\right) - \cos\left(\log x\right)\right\} + c$

Hence,

$\int \frac{1}{x^3} \sin\left(\log x\right) dx = \frac{-1}{5x^2}\left\{2 \sin\left(\log x\right) + \cos\left(\log x\right)\right\} + c$

Indefinite Integrals Ex 19.27 Q10

Let $I = \int e^{2x} \cos^2 x dx$

$$= \frac{1}{2}\int e^{2x} 2\cos^2 x dx$$
$$= \frac{1}{2}\int e^{2x} (1+\cos 2x) dx$$
$$= \frac{1}{2}\int e^{2x} dx + \frac{1}{2}\int e^{2x} \cos 2x dx$$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2}\{a\cos bx - b\sin bx\} + c$

$\therefore \quad I = \frac{1}{4}e^{2x} + \frac{1}{2}\frac{e^{2x}}{8}\{2\cos 2x + 2\sin 2x\} + c$

Hence,

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{16}\{2\cos 2x + 2\sin 2x\} + c$$
or
$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{8}\{\cos 2x + \sin 2x\} + c$$

Indefinite Integrals Ex 19.27 Q11

Let $I = \int e^{-2x} \sin x dx$

$\because \quad \int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2}\{a\sin bx - b\cos bx\} + c$

$\therefore \quad I = \frac{e^{-2x}}{5}\{-2\sin x - \cos x\} + c$

Indefinite Integrals Ex 19.27 Q12

Let $I = \int x^2 e^{x^3} \cos x^3 dx$

Let $x^3 = t$
$\Rightarrow \quad 3x^2 dx = dt$

$\therefore \quad I = \frac{1}{3}\int e^t \cos t dt$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2}\{a\cos bx + b\sin bx\} + c$

$\therefore \quad I = \frac{1}{3}\left\{\frac{e^t}{2}(\cos t + \sin t)\right\} + c$

$\therefore \quad I = \frac{1}{3}\left\{\frac{e^{x^3}}{2}\left(\cos x^3 + \sin x^3\right)\right\} + c$

Ex 19.28

Indefinite Integrals Ex 19.28 Q1

$\int\sqrt{3+2x-x^2}\,dx = \int\sqrt{4-(x-1)^2}\,dx$

Let $x-1=t$, so that $dx = dt$

Thus, $\int\sqrt{3+2x-x^2}\,dx = \int\sqrt{4-t^2}\,dt$

$$= \frac{1}{2}t\sqrt{4-t^2} + \frac{4}{2}\sin^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$$

Indefinite Integrals Ex 19.28 Q2

Let $I = \int\sqrt{x^2+x+1}\,dx$

$$= \int\sqrt{x^2+x+\frac{1}{4}+\frac{3}{4}}\,dx$$

$$= \int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$

$$= \frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}.\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$$

$$= \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(\frac{2x+1}{2}\right) + \frac{1}{2}\sqrt{x^2+x+1}\right| + c$$

$$\therefore \quad I = \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|2x+1+\sqrt{x^2+x+1}\right| + c$$

Indefinite Integrals Ex 19.28 Q3

Let $I = \int\sqrt{x-x^2}\,dx$

$$= \int\sqrt{\frac{1}{4}-\frac{1}{4}+x-x^2}\,dx \qquad \left[\text{Add and subtract } \frac{1}{4}\right]$$

$$= \int\sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2}\,dx$$

$$= -\left(\frac{1-2x}{4}\right)\sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2} - \frac{\left(\frac{1}{2}\right)^2}{2}\sin^{-1}\left(\frac{\frac{1-2x}{2}}{\frac{1}{2}}\right) + c$$

$$\therefore \quad I = \left(\frac{2x-1}{4}\right)\sqrt{x-x^2} + \frac{1}{8}\sin^{-1}(2x-1) + c$$

Indefinite Integrals Ex 19.28 Q4

Let $I = \int \sqrt{1 + x - 2x^2}\,dx$

$$= \sqrt{2}\int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2}\,dx$$

$$= \sqrt{2}\int \sqrt{\frac{9}{16} - \left(\frac{1}{16} - \frac{x}{2} + x^2\right)}\,dx$$

$$= \sqrt{2}\int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}\,dx$$

$$= \sqrt{2}\left\{\frac{\left(x - \frac{1}{4}\right)}{2}\sqrt{\frac{1}{2} + \frac{x}{2} - x^2} + \frac{9}{32}\sin^{-1}\left(\frac{x - \frac{1}{4}}{\frac{3}{4}}\right)\right\} + c$$

$$I = \frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x - 1}{3}\right) + c$$

Indefinite Integrals Ex 19.28 Q5

Let $I = \int \cos x\sqrt{4 - \sin^2 x}\,dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$\Rightarrow \quad I = \int \sqrt{4 - t^2}\,dt$

$= \int \sqrt{2^2 - t^2}\,dt$

$= \frac{t}{2}\sqrt{2^2 - t^2} + \frac{4}{2}\sin^{-1}\frac{t}{2} + c$

$\therefore \quad I = \frac{1}{2}\sin x\sqrt{4 - \sin^2 x} + 2\sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Indefinite Integrals Ex 19.28 Q6

Let $I = \int e^x\sqrt{e^{2x} + 1}\,dx$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$\therefore \quad I = \int \sqrt{t^2 + 1^2}\,dt$

$= \frac{t}{2}\sqrt{t^2 + 1} + \frac{1}{2}\log\left|t + \sqrt{t^2 + 1}\right| + c$

$\therefore \quad I = \frac{e^x}{2}\sqrt{e^{2x} + 1} + \frac{1}{2}\log\left|e^x + \sqrt{e^{2x} + 1}\right| + c$

Indefinite Integrals Ex 19.28 Q7

Let $I = \int \sqrt{3^2 - x^2}$

We know that,

$\int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

$\therefore \quad I = \frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + c$

Indefinite Integrals Ex 19.28 Q8

Let $I = \int \sqrt{16x^2 + 25}\,dx$

$$= \int \sqrt{(4x)^2 + 5^2}\,dx$$

$$= 4\int \sqrt{x^2 + \left(\frac{5}{4}\right)^2}\,dx$$

$$= 4\left\{\frac{x}{2}\sqrt{x^2 + \left(\frac{5}{4}\right)^2} + \frac{\left(\frac{5}{4}\right)^2}{2}\log\left|x + \sqrt{x^2 + \left(\frac{5}{4}\right)^2}\right| + c\right\}$$

$$\therefore \quad I = 2x\sqrt{x^2 + \frac{25}{16}} + \frac{25}{8}\log\left|x + \sqrt{x^2 + \frac{25}{16}}\right| + c$$

Indefinite Integrals Ex 19.28 Q9

Let $I = \int \sqrt{4x^2 - 5}\,dx$

$$= 2\int \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}\,dx$$

$$= 2\left\{\frac{x}{2}\sqrt{x^2 - \frac{5}{4}} - \frac{5}{8}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c\right.$$

$$\therefore \quad I = x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$$

Indefinite Integrals Ex 19.28 Q10

Let $I = \int \sqrt{2x^2 + 3x + 4}\,dx$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + 2}\,dx$$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + \frac{9}{16} + \frac{23}{16}}\,dx$$

$$= \sqrt{2}\int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}\,dx$$

$$= \sqrt{2}\left\{\frac{\left(x + \frac{3}{4}\right)}{2}\sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c\right\}$$

$$\therefore \quad I = \frac{4x + 3}{8}\sqrt{2x^2 + 3x + 4} + \frac{23\sqrt{2}}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c$$

Indefinite Integrals Ex 19.28 Q11

Let $I = \int \sqrt{3 - 2x - 2x^2}\,dx$

$$= \sqrt{2}\int \sqrt{\frac{3}{2} - x - x^2}\,dx$$

$$= \sqrt{2}\int \sqrt{\frac{7}{4} - \left(\frac{1}{4} + x + x^2\right)}\,dx \qquad \left[\text{Adding and subtracting } \frac{1}{4}\right]$$

$$= \sqrt{2}\int \sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2}\,dx$$

$$= \sqrt{2}\left\{\frac{x + \frac{1}{2}}{2}\sqrt{\frac{3}{2} - x - x^2} + \frac{7}{8}\sin^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{7}}{2}}\right)\right\} + c$$

$$\therefore \quad I = \frac{2x+1}{4}\sqrt{3 - 2x - 2x^2} + \frac{7\sqrt{2}}{8}\sin^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) + c$$

Indefinite Integrals Ex 19.28 Q12

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$$\therefore \quad I = \frac{1}{2}\int \sqrt{t^2 + 1^2}\,dt$$

$$= \frac{1}{2}\left\{\frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\log\left|t + \sqrt{t^2+1}\right|\right\} + c$$

$$\therefore \quad I = \frac{1}{2}\left\{\frac{x^2}{2}\sqrt{x^4+1} + \frac{1}{2}\log\left|x^2 + \sqrt{x^4+1}\right|\right\} + c$$

Indefinite Integrals Ex 19.28 Q13

Let $I = \int x^2\sqrt{a^6 - x^6}\,dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2\,dx = dt$

$$\therefore \quad I = \frac{1}{3}\int \sqrt{a^6 - t^2}\,dt$$

$$= \frac{1}{3}\left\{\frac{t}{2}\sqrt{a^6 - t^2} + \frac{a^6}{2}\sin^{-1}\left(\frac{t}{a^3}\right)\right\} + c$$

$$\therefore \quad I = \frac{x^3}{6}\sqrt{a^6 - x^6} + \frac{a^6}{6}\sin^{-1}\left(\frac{x^3}{a^3}\right) + c$$

Indefinite Integrals Ex 19.28 Q14

Let $I = \int \frac{\sqrt{16 + (\log x)^2}}{x}\,dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}\,dx = dt$

$$\therefore \quad I = \int \sqrt{16 + t^2}\,dt$$

$$= \int \sqrt{4^2 + t^2}\,dt$$

$$= \frac{t}{2}\sqrt{16 + t^2} + \frac{16}{2}\log\left|t + \sqrt{16 + t^2}\right| + c$$

$$\therefore \quad I = \frac{\log x}{2}\sqrt{16 + (\log x)^2} + 8\log\left|\log x + \sqrt{16 + (\log x)^2}\right| + c$$

Indefinite Integrals Ex 19.28 Q15

Let $I = \int \sqrt{2ax - x^2}\,dx$

$= \int \sqrt{a^2 - \left(a^2 - 2ax + x^2\right)}\,dx$ [Adding and subtracting a^2]

$= \int \sqrt{a^2 - (a - x)^2}\,dx$

$= \int \sqrt{a^2 - (x - a)^2}\,dx$

$= \frac{(x - a)}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$

$\therefore \quad I = \frac{1}{2}(x - a)\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$

Indefinite Integrals Ex 19.28 Q16

Let $I = \int \sqrt{3 - x^2}\,dx$

$= \int \sqrt{\left(\sqrt{3}\right)^2 - x^2}\,dx$

$I = \frac{x}{2}\sqrt{3 - x^2} + \frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

Ex 19.29

Indefinite Integrals Ex 19.29 Q1

Let $I = \int (x+1)\sqrt{x^2 - x + 1}dx$ $\quad --- (1)$

Let $x + 1 = \lambda \frac{d}{dx}(x^2 - x + 1) + \mu$

$= \lambda(2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$-\lambda + \mu = 1$

$\Rightarrow \quad \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \quad \mu = \frac{3}{2}$

So,

$I = \int \left(\frac{1}{2}(2x - 1) + \frac{3}{2}\right)\sqrt{x^2 - x + 1}dx$

$= \frac{1}{2}\int (2x - 1)\sqrt{x^2 - x + 1}dx + \frac{3}{2}\int \sqrt{x^2 - x + 1}dx$

Let $x^2 - x + 1 = t$

$\Rightarrow \quad (2x - 1)dx = dt$

$= \frac{1}{2}\int \sqrt{t}dt + \frac{3}{2}\int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$

$= \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x + 1} + \frac{3}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right|\right.$

$\Rightarrow \quad I = \frac{1}{3}t^{\frac{3}{2}} + \frac{3}{8}(2x - 1)\sqrt{x^2 - x + 1} + \frac{9}{16}.\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Hence,

$I = \frac{1}{3}(x^2 - x + 1)^{\frac{3}{2}} + \frac{3}{8}(2x - 1)\sqrt{x^2 - x + 1} + \frac{9}{16}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Indefinite Integrals Ex 19.29 Q2

Let $I = \int (x+1)\sqrt{2x^2+3}\,dx$

Let $x + 1 = \lambda \frac{d}{dx}\left(2x^2+3\right) + \mu$

$= \lambda\left(4x\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{4}$

$\mu = 1$

$\therefore \quad I = \int \frac{1}{4}\left(4x\right)\sqrt{2x^2+3}\,dx + \int 1.\sqrt{2x^2+3}\,dx$

Let $2x^2 + 3 = t$

$\Rightarrow \quad 4x\,dx = dt$

$I = \frac{1}{4}\int \sqrt{t}\,dt + \sqrt{2}\int \sqrt{x^2+\frac{3}{2}}\,dx$

$= \frac{1}{4}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2}\left\{\frac{x}{2}\sqrt{x^2+\frac{3}{2}} + \frac{3}{4}\log\left|x + \sqrt{x^2+\frac{3}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}\left(2x^2+3\right)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log\left|x + \sqrt{x^2+\frac{3}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q3

Let $I = \int (2x-5)\sqrt{2+3x-x^2}dx$

Let $2x-5 = \lambda \frac{d}{dx}\left(2+3x-x^2\right)+\mu$

$= \lambda\left(3-2x\right)+\mu$

Equating similar terms, we get,

$-2\lambda = 2 \quad \Rightarrow \quad \lambda = -1$

$3\lambda+\mu = -5 \quad \Rightarrow \quad \mu = -5-3\lambda = -2$

$\therefore \quad \mu = -2$

So,

$I = \int\left(-1\left(3-2x\right)-2\right)\sqrt{2+3x-x^2}dx$

$= -\int\left(3-2x\right)\sqrt{2+3x-x^2}dx - 2\int\sqrt{2+3x-x^2}dx$

Let $2+3x-x^2 = t$

$\Rightarrow \quad \left(3-2x\right)dx = dt$

$I = -\int\sqrt{t}dt - 2\int\sqrt{\frac{17}{4}-\left(\frac{9}{4}-3x-x^2\right)}dx$

$= -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\int\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}dx$

$\Rightarrow \quad I = -\frac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - 2\left\{\frac{\left(x-\frac{3}{2}\right)}{2}\sqrt{2+3x-x^2}+\frac{17}{8}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right)\right.+c$

Hence,

$I = -\frac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - \frac{\left(2x-3\right)}{2}\sqrt{2+3x-x^2} - \frac{17}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{17}}\right)+c$

Indefinite Integrals Ex 19.29 Q4

Let $I = \int (x+2)\sqrt{x^2+x+1}dx$

Let $x+2 = \lambda \frac{d}{dx}(x^2+x+1) + \mu$
$= \lambda(2x+1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 2 \quad \Rightarrow \quad \mu = 2 - \lambda = \frac{3}{2}$

$\therefore \quad \mu = \frac{3}{2}$

$$\therefore \quad I = \int\left(\frac{1}{2}(2x+1) + \frac{3}{2}\right)\sqrt{x^2+x+1}dx$$
$$= \frac{1}{2}\int(2x+1)\sqrt{x^2+x+1} + \frac{3}{2}\int\sqrt{x^2+x+1}dx$$

Let $x^2+x+1 = t$
$(2x+1)dx = dt$

$$\therefore \quad I = \frac{1}{2}\int\sqrt{t}dt + \frac{3}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$\Rightarrow \quad I = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c\right.$$

Hence,

$$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{3}{8}(2x+1)\sqrt{x^2+x+1} + \frac{9}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$$

Indefinite Integrals Ex 19.29 Q5

Let $I = \int (4x+1)\sqrt{x^2 - x - 2}dx$

Let $4x + 1 = \lambda \frac{d}{dx}\left(x^2 - x - 2\right) + \mu$
$= \lambda (2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 4 \quad \Rightarrow \quad \lambda = 2$
$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 3$

So,
$I = \int \left(2(2x-1) + 3\right)\sqrt{x^2 - x - 2}dx$
$= 2\int (2x-1)\sqrt{x^2 - x - 2}dx + 3\int \sqrt{x^2 - x - 2}dx$

Let $x^2 - x - 2 = t$
$(2x-1)dx = dt$

$\therefore \quad I = 2\int \sqrt{t}dt + 3\int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2}dx$

$\Rightarrow \quad I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x - 2} - \frac{9}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c\right.$

Hence,

$I = \frac{4}{3}\left(x^2 - x - 2\right)^{\frac{3}{2}} + \frac{3}{4}(2x-1)\sqrt{x^2 - x - 2} - \frac{27}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c$

Indefinite Integrals Ex 19.29 Q6

Let $I = \int (x-2)\sqrt{2x^2 - 6x + 5}dx$

Let $x - 2 = \lambda \frac{d}{dx}\left(2x^2 - 6x + 5\right) + \mu$
$= \lambda\left(4x - 6\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$-6\lambda + \mu = -2 \quad \Rightarrow \quad \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2}$

$\therefore \quad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{4}\left(4x - 6\right) + \left(-\frac{1}{2}\right)\right)\sqrt{2x^2 - 6x + 5}dx$

$= \frac{1}{4}\int\left(4x - 6\right)\sqrt{2x^2 - 6x + 5} - \frac{1}{2}\int\sqrt{2x^2 - 6x + 5}dx$

Let $2x^2 - 6x + 5 = t$
$\left(4x - 6\right)dx = dt$

$\therefore \quad I = \frac{1}{4}\int\sqrt{t}dt - \frac{\sqrt{2}}{2}\int\sqrt{x^2 - 3x + \frac{5}{2}}dx$

$\Rightarrow \quad I = \frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}}\int\sqrt{\left(x - \frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dx$

$= \frac{1}{6}\left(2x^2 - 6x + 5\right)^{\frac{3}{2}} - \frac{1}{\sqrt{2}}\left\{\frac{\left(x - \frac{3}{2}\right)}{2}\sqrt{x^2 - 3x + \frac{5}{2}} + \frac{1}{8}\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}\left(2x^2 - 6x + 5\right)^{\frac{3}{2}} - \frac{1}{8}\left(2x - 3\right)\sqrt{2x^2 - 6x + 5} - \frac{1}{8\sqrt{2}}\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q7

Let $I = \int (x+1)\sqrt{x^2+x+1}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+x+1\right)+\mu$

$= \lambda(2x+1)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 1 \Rightarrow \quad \mu = \frac{1}{2}$

So,

$I = \int \left(\frac{1}{2}(2x+1)+\frac{1}{2}\right)\sqrt{x^2+x+1}dx$

$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx + \frac{1}{2}\int \sqrt{x^2+x+1}dx$

Let $x^2+x+1 = t$

$\Rightarrow \quad (2x+1)dx = dt$

$= \frac{1}{2}\int \sqrt{t}dt + \frac{1}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c\right.$

$\Rightarrow \quad I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{8}(2x+1)\sqrt{x^2+x+1}+\frac{3}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{8}(2x+1)\sqrt{x^2+x+1}+\frac{3}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Indefinite Integrals Ex 19.29 Q8

Let $I = \int (2x+3)\sqrt{x^2+4x+3}dx$

Let $(2x+3) = \lambda \frac{d}{dx}(x^2+4x+3) + \mu$

$= \lambda(2x+4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $4\lambda + \mu = 3$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x+4) + (-1))\sqrt{x^2+4x+3}dx$

$= \int (2x+4)\sqrt{x^2+4x+3}dx - \int \sqrt{x^2+4x+3}dx$

Let $x^2+4x+3 = t$

$\Rightarrow \quad (2x+4)dx = dt$

$\therefore \quad I = \int \sqrt{t}dt - \int \sqrt{(x+2)^2 - 1}dx$

$= \frac{3}{2}t^{\frac{3}{2}} - \frac{(x+2)}{2}\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2) + \sqrt{x^2+4x+3}\right| + c$

Hence,

$$I = \frac{2}{3}(x^2+4x+3)^{\frac{3}{2}} - \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2) + \sqrt{x^2+4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q9

Let $I = \int (2x-5)\sqrt{x^2-4x+3}dx$

Let $(2x-5) = \lambda \frac{d}{dx}(x^2-4x+3) + \mu$

$= \lambda(2x-4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $-4\lambda + \mu = -5$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x-4) - 1)\sqrt{x^2-4x+3}dx$

$= \int (2x-4)\sqrt{x^2-4x+3}dx - \int \sqrt{x^2-4x+3}dx$

Let $x^2-4x+3 = t$

$\Rightarrow \quad 2x - 4dx = dt$

$I = \int \sqrt{t}dt - \int \sqrt{(x-2)^2 - 1}dx$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2) + \sqrt{x^2-4x+3}\right| + c$

Thus,

$$I = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}} - \frac{1}{2}(x-2)\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2) + \sqrt{x^2-4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q10

Let $I = \int x\sqrt{x^2+x}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 0 \quad \Rightarrow \quad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{2}\left(2x+1\right)-\frac{1}{2}\right)\sqrt{x^2+x}dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x} - \frac{1}{2}\int\sqrt{x^2+x}dx$

Let $x^2+x = t$

$\Rightarrow \quad \left(2x+1\right)dx = dt$

So,

$I = \frac{1}{2}\int\sqrt{t}dt - \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$I = \frac{1}{2}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{8}\left(x+\frac{1}{2}\right)\sqrt{x^2+x}+\frac{1}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c$

Indefinite Integrals Ex 19.29 Q11

Consider the integral $I=\int (x-3)\sqrt{x^2+3x-18}\,dx$

Let us express $x-3=\lambda\frac{d}{dx}\left[x^2+3x-18\right]+\mu$

$\Rightarrow x-3=\lambda\left[2x+3\right]+\mu$

$\Rightarrow x-3=2\lambda x+3\lambda+\mu$

Comparing the coefficients, we have,

$2\lambda=1$ and $3\lambda+\mu=-3$

$\Rightarrow \lambda=\frac{1}{2}$ and $3\times\frac{1}{2}+\mu=-3$

$\Rightarrow \lambda=\frac{1}{2}$ and $\mu=-3-\frac{3}{2}$

$\Rightarrow \lambda=\frac{1}{2}$ and $\mu=-\frac{9}{2}$

Then

$x-3=\lambda\left[2x+3\right]+\mu$

Now the integral $I=\int (x-3)\sqrt{x^2+3x-18}\,dx$

$=\int\left[\frac{1}{2}[2x+3]-\frac{9}{2}\right]\sqrt{x^2+3x-18}\,dx$

$I=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx-\frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

$\Rightarrow I=I_1+I_2$

where, $I_1=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$ and

$I_2=-\frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

Let us consider the integral, I_1 :

$I_1=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$

Substituting, $x^2+3x-18=t$

$\Rightarrow (2x+3)dx=dt$

Thus,

$I_1=\frac{1}{2}\int\sqrt{t}\,dt$

$=\frac{1}{2}\times\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$

$$= \frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times \left(x^2 + 3x - 18\right)^{\frac{3}{2}} + C$$

Now consider the integral

$$I_2 = -\frac{9}{2}\int \sqrt{x^2 + 3x - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{x^2 + 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{4} + 18\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9 + 72}{4}\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{81}{4}\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$

We know that $\int \sqrt{x^2 - a^2}dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2 \log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I_2 = -\frac{9}{2}\left\{\frac{1}{2}\left(x + \frac{3}{2}\right)\sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} - \frac{1}{2}\left(\frac{9}{2}\right)^2 \log\left|\left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\right|\right\} + C$$

$$= -\frac{9}{4}\left\{\left(\frac{2x + 3}{2}\right)\sqrt{x^2 + 3x - 18} - \left(\frac{729}{4}\right)\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right|\right\} + C$$

$$= -\frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$$

Thus, $I = = \frac{1}{3}\left(x^2 + 3x - 18\right)^{\frac{3}{2}} - \frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$

Indefinite Integrals Ex 19.29 Q12

$$\int (x+3)\sqrt{3-4x-x^2}dx$$

$$Let\ x+3 = A\frac{d}{dx}(3-4x-x^2)+B$$

$$x+3 = A(-4-2x)+B$$
$$x+3 = -2Ax+B-4A$$
$$-2A = 1, B-4A = 3$$

$$A = -\frac{1}{2},$$

$$B = 4\times\left(-\frac{1}{2}\right)+3 = 1$$

$$\int (x+3)\sqrt{3-4x-x^2}dx$$

$$x+3 = -\frac{1}{2}(-4-2x)+1$$

$$\int\left[-\frac{1}{2}(-4-2x)+1\right]\sqrt{3-4x-x^2}dx$$

$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}dx + \int\sqrt{3-4x-x^2}dx$$

$$= I_1 + I_2 ....(i)$$

$$I_1 = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}dx$$

$$Let\ z = 3-4x-x^2$$
$$dz = -4-2x$$

$$I_1 = -\frac{1}{2}\int\sqrt{z}dz$$

$$= -\frac{1}{2}\left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$

$$= -\frac{1}{2}\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= -\left[\frac{\left(3-4x-x^2\right)^{\frac{3}{2}}}{3}\right]$$

$$I_2 = \int\sqrt{3-4x-x^2}dx$$

$$= \int\sqrt{3-(x^2+4x+4)+4}dx$$

$$= \int\sqrt{7-(x^2+4x+4)}dx$$

$$= \int\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}dx$$

$$= \frac{(x+2)\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}}{2} + \frac{1}{2}\left(\sqrt{7}\right)^2\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+C$$

$$= \frac{(x+2)\sqrt{3-4x-x^2}}{2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)$$

$$From (i),$$
$$= I_1 + I_2$$

$$= -\frac{1}{3}\left(3-4x-x^2\right)^{\frac{3}{2}} + \frac{1}{2}(x+2)\sqrt{3-4x-x^2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+C$$

Ex 19.30

Indefinite Integrals Ex 19.30 Q1

Let $\int \frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$\Rightarrow \quad 2x+1 = A(x-2) + B(x+1)$

Put $x = 2$

$\Rightarrow \quad 5 = 3B \Rightarrow \quad B = \frac{5}{3}$

Put $x = -1$

$\Rightarrow \quad -1 = -3A \quad \Rightarrow \quad A = \frac{1}{3}$

So,

$$\int \frac{2x+1}{(x+1)(x-2)} dx = \frac{1}{3}\int \frac{dx}{x+1} + \frac{5}{3}\int \frac{dx}{x-2}$$

$$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Thus,

$$I = \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Indefinite Integrals Ex 19.30 Q2

Let $\int \frac{1}{x(x-2)(x-4)}dx = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$

$\Rightarrow \quad 1 = A(x-2)(x-4) + B(x)(x-4) + Cx(x-2)$

Put $x = 0$

$\Rightarrow \quad 1 = 8A \Rightarrow \quad A = \frac{1}{8}$

Put $x = 2$

$\Rightarrow \quad 1 = -4B \quad \Rightarrow \quad B = -\frac{1}{4}$

Put $x = 4$

$\Rightarrow \quad 1 = 8C \Rightarrow \quad C = \frac{1}{8}$

So,

$$\int \frac{1}{x(x-2)(x-4)}dx = \frac{1}{8}\int \frac{dx}{x} + \left(-\frac{1}{4}\right)\int \frac{dx}{x-2} + \frac{1}{8}\int \frac{dx}{x-4}$$
$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$
$$= \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Thus,

$$I = \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Indefinite Integrals Ex 19.30 Q3

Let $I = \int \frac{x^2+x-1}{x^2+x-6}dx$

$= \int 1 + \frac{5}{x^2+x-6}dx$

$\Rightarrow \quad I = \int dx + \int \frac{5dx}{(x+3)(x-2)} \qquad ---(1)$

Let $\frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$\Rightarrow \quad 5 = A(x-2) + B(x+3)$

Put $x = 2$

$\Rightarrow \quad 5 = 5B \Rightarrow \quad B = 1$

Put $x = -3$

$\Rightarrow \quad 5 = -5A \quad \Rightarrow \quad A = -1$

$\therefore \quad I = \int dx + \int \frac{-dx}{x+3} + \int \frac{dx}{x-2}$

$= x - \log|x+3| + \log|x-2| + c$

Hence,

$I = x - \log|x+3| + \log|x-2| + c$

Indefinite Integrals Ex 19.30 Q4

Let $I = \int \frac{3 + 4x - x^2}{(x+2)(x-1)} dx$

$= \int -1 + \frac{5x + 1}{(x+2)(x-1)} dx$

$\Rightarrow \quad I = -\int dx + \int \frac{5x+1}{(x+2)(x-1)} dx \qquad ---(1)$

Let $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$

$\Rightarrow \quad 5x + 1 = A(x-1) + B(x+2)$

Put $x = 1$

$\Rightarrow \quad 6 = 3B \Rightarrow \quad B = 2$

Put $x = -2$

$\Rightarrow \quad -9 = -3A \Rightarrow \quad A = 3$

So,

$I = -\int dx + 3\int \frac{dx}{x+2} + 2\int \frac{dx}{x-1}$

$I = -x + 3\log|x+2| + 2\log|x-1| + c$

Indefinite Integrals Ex 19.30 Q5

Let $I = \int \frac{x^2+1}{x^2-1} dx$

$= \int 1 + \frac{2}{x^2-1} dx$

$= \int dx + \int \frac{2dx}{(x+1)(x-1)}$

$= \int dx + \int \frac{-1}{x+1} + \frac{1}{x-1} dx$

$= x - \log|x+1| + \log|x-1| + c$

$I = x + \log\left|\frac{x-1}{x+1}\right| + c$

Indefinite Integrals Ex 19.30 Q6

Let $I = \int \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 4 = -B \Rightarrow \quad B = -4$

Put $x = 3$

$\Rightarrow \quad 9 = 2C \Rightarrow \quad C = \frac{9}{2}$

Thus,

$I = \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{1}{2}\int \frac{dx}{x-1} - 4\int \frac{dx}{x-2} + \frac{9}{2}\int \frac{dx}{x-3}$

$= \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$

Hence,

$I = \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$

Indefinite Integrals Ex 19.30 Q7

$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \quad \text{...(1)}$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$A = \frac{5}{3}, B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$

$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$

$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$

$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$

Indefinite Integrals Ex 19.30 Q8

Let $I = \int \frac{x^2+1}{x\left(x^2-1\right)}dx = \int \frac{x^2+1}{x\left(x+1\right)\left(x-1\right)}dx$

Let $\frac{x^2+1}{x\left(x+1\right)\left(x-1\right)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$\Rightarrow \quad x^2+1 = A\left(x+1\right)\left(x-1\right) + B.x\left(x-1\right) + Cx\left(x+1\right)$

Put $x = 0$

$\Rightarrow \quad 1 = -A \Rightarrow \quad A = -1$

Put $x = -1$

$\Rightarrow \quad 2 = 2B \quad \Rightarrow \quad B = 1$

Put $x = 1$

$\Rightarrow \quad 2 = 2C \Rightarrow \quad C = 1$

Thus,

$I = -\int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$

$= -\log|x| + \log|x+1| + \log|x-1| + c$

$I = \log\left|\frac{x^2-1}{x}\right| + c$

Indefinite Integrals Ex 19.30 Q9

Let $I = \int \frac{2x-3}{\left(x^2-1\right)\left(2x+3\right)}dx = \int \frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)}dx$

Let $\frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)} = \frac{A}{\left(x+1\right)} + \frac{B}{\left(x-1\right)} + \frac{C}{\left(2x+3\right)}$

$\Rightarrow \quad 2x-3 = A\left(x-1\right)\left(2x+3\right) + B\left(x+1\right)\left(2x+3\right) + C\left(x^2-1\right)$

Put $x = -1$

$\Rightarrow \quad -5 = -2A \quad \Rightarrow \quad A = \frac{5}{2}$

Put $x = 1$

$\Rightarrow \quad -1 = 10B \quad \Rightarrow \quad B = -\frac{1}{10}$

Put $x = -\frac{3}{2}$

$\Rightarrow \quad -6 = \frac{5}{4}C \quad \Rightarrow \quad C = -\frac{24}{5}$

Thus,

$I = \frac{5}{2}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{dx}{x-1} - \frac{24}{5}\int \frac{dx}{2x+3}$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5}.\frac{1}{2}\log|2x+3| + c$

Hence,

$I = \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + c$

Indefinite Integrals Ex 19.30 Q10

Let $I = \int \frac{x^3}{(x-1)(x-2)(x-3)}dx$

$= \int 1 + \frac{6x^2 - 9x + 6}{(x-1)(x-2)(x-3)}dx$

Let $\frac{6x^2 - 11x + 6}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad 6x^2 - 11x + 6 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 8 = -B \quad \Rightarrow \quad B = -8$

Put $x = 3$

$\Rightarrow \quad 27 = 2C \quad \Rightarrow \quad C = \frac{27}{2}$

Thus,

$I = \int dx + \frac{1}{2}\int \frac{dx}{x-1} \quad -8\int \frac{dx}{x-2} + \frac{27}{2}\int \frac{dx}{x-3}$

$= x + \frac{1}{2}\log|x-1| \; -8 \log|x-2| + \frac{27}{2}\log|x-3| + c$

Hence,

$I = x + \frac{1}{2}\log|x-1| \; -8 \log|x-2| + \frac{27}{2}\log|x-3| + c$

Indefinite Integrals Ex 19.30 Q11

Let $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)}dx = \frac{A}{1+\sin x} + \frac{B}{2+\sin x}$

$\Rightarrow \quad \sin 2x = A(2+\sin x) + B(1+\sin x)$

$\Rightarrow \quad 2\sin x \cos x = (2A+B) + (A+B)\sin x$

Equating similar terms, we get,

$2A + B = 0 \quad \Rightarrow \quad B = -2A$ and

$A + B = 2\cos x \Rightarrow \quad -A = 2\cos x$

$\Rightarrow \quad A = -2\cos x$

and $B = +4\cos x$

Thus,

$I = \int -\frac{2\cos x}{1+\sin x}dx + \int \frac{4\cos x}{2+\sin x}dx$

$= -2\log|1+\sin x| + 4\log|2+\sin x| + c$

$I = \log\left|\frac{(2+\sin x)^4}{(1+\sin x)^2}\right| + c$

Indefinite Integrals Ex 19.30 Q12

Let $\int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)} dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$

$$\Rightarrow \quad 2x = \left(Ax+B\right)\left(x^2+3\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(A+C\right)x^3 + \left(B+D\right)x^2 + \left(3A+C\right)x + \left(3B+D\right)$$

Equating similar terms, we get,

$A+C=0$, $B+D=0$, $3A+C=2$ and $3B+D=0$

$$\Rightarrow \quad A=-C,\ B=D=0\ 2A=2 \quad \Rightarrow \quad A=1\ \&\ C=-1$$

Thus,

$$I = \int \frac{xdx}{x^2+1} - \int \frac{xdx}{x^2+3}$$
$$= \frac{1}{2}\log\left|x^2+1\right| - \frac{1}{2}\log\left|x^2+3\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + c$$

Indefinite Integrals Ex 19.30 Q13

Let $\int \frac{1}{x\log x\left(2+\log x\right)} = \frac{A}{x\log x} + \frac{B}{x\left(2+\log x\right)}$

$$\Rightarrow \quad 1 = A\left(2+\log x\right) + B\log x$$

Put $x=1$

$$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$$

Put $x = 10^{-2}$

$$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$$

Thus,

$$I = \frac{1}{2}\int \frac{dx}{x\log x} + \left(-\frac{1}{2}\right)\int \frac{dx}{x\left(2+\log x\right)}$$
$$= \frac{1}{2}\log\left|\log x\right| - \frac{1}{2}\log\left|2+\log x\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\log x}{2+\log x}\right| + c$$

Indefinite Integrals Ex 19.30 Q15

Let $\frac{ax^2+bx+c}{(x-a)(x-b)(x-c)} = \frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

$\Rightarrow \quad ax^2+bx+c = A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)$

Put $x = a$

$\Rightarrow \quad a^3+ba+c=(a-b)(a-c)A \Rightarrow \quad A=\frac{a^3+ba+c}{(a-b)(a-c)}$

Put $x = b$

$\Rightarrow \quad ab^2+b^2+c=(b-a)(b-c)B \Rightarrow \quad B=\frac{ab^2+b^2+c}{(b-a)(b-c)}$

Put $x = c$

$\Rightarrow \quad ac^2+bc+c=(c-a)(c-b)C \Rightarrow \quad C=\frac{ac^2+bc+c}{(c-a)(c-b)}$

Thus,

$$I=\frac{a^3+ba+c}{(a-b)(a-c)}\int\frac{dx}{x-a}+\frac{ab^2+b^2+c}{(b-a)(b-c)}\int\frac{dx}{x-b}+\frac{ac^2+bc+c}{(c-a)(c-b)}\int\frac{dx}{x-c}$$

Hence,

$$I=\frac{a^3+ba+c}{(a-b)(a-c)}\log|x-a|+\frac{ab^2+b^2+c}{(b-a)(b-c)}\log|x-b|+\frac{ac^2+bc+c}{(c-a)(c-b)}\log|x-c|+c$$

Indefinite Integrals Ex 19.30 Q16

Consider the integral

$$I=\int \frac{x}{\left(x^2+1\right)(x-1)}dx$$

Now let us separate the fraction $\frac{x}{\left(x^2+1\right)(x-1)}$

through partial fractions.

$$\frac{x}{\left(x^2+1\right)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{A\left(x^2+1\right)+(Bx+C)(x-1)}{\left(x^2+1\right)(x-1)}$$

$$\Rightarrow x=A\left(x^2+1\right)+(Bx+C)(x-1)$$

$$\Rightarrow x=Ax^2+A+Bx^2-Bx+Cx-C$$

Comparing the coefficients, we have,

A+B=0, $-B+C=1$ and A $-C$=0

Solving the equations, we get,

$$\Rightarrow A=\frac{1}{2},\ B=-\frac{1}{2} \text{ and } C=\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{1}{2}\times\frac{1}{x-1}-\frac{1}{2}\times\frac{x-1}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{1}{2(x-1)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}$$

Thus, we have,

$$I=\int \frac{x}{\left(x^2+1\right)(x-1)}dx$$

$$=\int\left[\frac{1}{2(x-1)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}\right]dx$$

$$=\int\frac{dx}{2(x-1)}-\int\frac{xdx}{2\left(x^2+1\right)}+\int\frac{dx}{2\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{(x-1)}-\frac{1}{2}\int\frac{xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{(x-1)}-\frac{1}{2}\times\frac{1}{2}\int\frac{2xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\log|x-1|-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}x+C$$

Indefinite Integrals Ex 19.30 Q17

Let $I = \int \frac{1}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$

$\Rightarrow \quad 1 = A(x+1)(x+2) + B(x-1)(x+2) + C(x^2-1)$

Put $x = 1$

$\Rightarrow \quad 1 = 6A \Rightarrow \quad A = \frac{1}{6}$

Put $x = -1$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Put $x = -2$

$\Rightarrow \quad 1 = 3C \Rightarrow \quad C = \frac{1}{3}$

So,

$I = \frac{1}{6}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{x+2}$

$I = \frac{1}{6}\log|x-1| - \frac{1}{2}\log|x+1| + \frac{1}{3}\log|x+2| + c$

Indefinite Integrals Ex 19.30 Q18

Consider the integral

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

Now let us separate the fraction $\frac{x^2}{(x^2+4)(x^2+9)}$ through partial fractions.

Substitute $x^2 = t$

$$\frac{x^2}{(x^2+4)(x^2+9)} = \frac{t}{(t+4)(t+9)}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A(t+9)+B(t+4)}{(t+4)(t+9)}$$

$$\Rightarrow t = A(t+9)+B(t+4)$$

$$\Rightarrow t = At + 9A + Bt + 4B$$

Comparing the coefficients, we have,

A+B=1 and 9A+4B=0

$$\Rightarrow A=-\frac{4}{5} \text{ and } B=\frac{9}{5}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(t+4)} + \frac{9}{5(t+9)}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$$

Thus, we have,

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

$$= \int \left[-\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}\right]dx$$

$$= -\int \frac{4dx}{5(x^2+4)} + \int \frac{9dx}{5(x^2+9)}$$

$$= -\frac{4}{5}\int \frac{dx}{(x^2+4)} + \frac{9}{5}\int \frac{dx}{(x^2+9)}$$

$$= -\frac{4}{5} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Indefinite Integrals Ex 19.30 Q19

Let $\int \frac{5x^2-1}{x(x-1)(x+1)}dx = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\Rightarrow \quad 5x^2 - 1 = A(x^2-1) + B(x+1)x + C(x-1)x$

Put $x = 0$

$\Rightarrow \quad -1 = -A \Rightarrow \quad A = 1$

Put $x = +1$

$\Rightarrow \quad 4 = 2B \Rightarrow \quad B = 2$

Put $x = -1$

$\Rightarrow \quad 4 = 2C \quad \Rightarrow \quad C = 2$

So,

$$I = \int \frac{dx}{x} + \int \frac{2dx}{x-1} + \int \frac{2dx}{x+1}$$

$$= \log|x| + 2\log|x-1| + 2\log|x+1| + c$$

$$I = \log\left|x(x^2-1)^2\right|$$

Indefinite Integrals Ex 19.30 Q20

Let $I = \int \frac{x^2+6x-8}{x^3-4x}dx$

$\Rightarrow \quad I = \int \frac{x^2+6x-8}{x(x+2)(x-2)}dx$

Now,

Let $\frac{x^2+6x-8}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$\Rightarrow \quad x^2+6x-8 = A(x^2-4) + B(x-2)x + C(x+2)x$

Put $x = 0$

$\Rightarrow \quad -8 = -4A \Rightarrow \quad A = 2$

Put $x = -2$

$\Rightarrow \quad -16 = 8B \quad \Rightarrow \quad B = -2$

Put $x = 2$

$\Rightarrow \quad 8 = 8C \Rightarrow \quad C = 1$

Thus,

$I = \int \frac{2dx}{x} - \int \frac{2dx}{x+2} + \int \frac{dx}{x-2}$

$= 2\log|x| - 2\log|x+2| + \log|x-2| + c$

$\therefore \quad I = \log\left|\frac{x^2(x-2)}{(x+2)^2}\right| + c$

Indefinite Integrals Ex 19.30 Q21

Let $\int \frac{x^2+1}{(2x+1)(x^2-1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2-1}$

$\Rightarrow \quad x^2+1 = A(x^2-1) + (Bx+C)(2x+1)$

$= (A+2B)x^2 + (B+2C)x + (-A+C)$

Equating similar terms, we get,

$A+2B = 1$, $B+2C = 0$ and $-A+C = 1$

Solving we get,

$A = -\frac{5}{3} \qquad B = \frac{4}{3} \qquad C = -\frac{2}{3}$

Thus,

$I = -\frac{5}{3}\int \frac{dx}{2x+1} + \int \frac{\frac{4}{3}x - \frac{2}{3}}{x^2-1}dx$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2xdx}{x^2-1} - \frac{2}{3}\int \frac{dx}{x^2-1}$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2x-1}{(x+1)(x-1)}dx$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\left(\int\left(\frac{\frac{3}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)}\right)dx\right)$

$I = -\frac{5}{6}\log|2x+1| + \log|x+1| + \frac{1}{3}\log|x-1| + c$

Indefinite Integrals Ex 19.30 Q22

Let $I = \int \frac{dx}{x\left[6(\log x)^2 + 7\log x + 2\right]}$

$= \int \frac{1}{x(2\log x + 1)(3\log x + 2)} dx$

Now,

Let $\frac{1}{x(2\log x + 1)(3\log x + 2)} = \frac{A}{x(2\log x + 1)} + \frac{B}{x(3\log x + 2)}$

$\Rightarrow \quad 1 = A(3\log x + 2) + B(2\log x + 1)$

Put $x = 10^{-\frac{1}{2}}$

$\Rightarrow \quad 1 = \frac{1}{2}A \Rightarrow \quad A = 2$

Put $x = 10^{-\frac{2}{3}}$

$\Rightarrow \quad 1 = -\frac{1}{3}B \quad \Rightarrow \quad B = -3$

$\therefore \quad I = \int \frac{2dx}{x(2\log x + 1)} - \int \frac{3dx}{x(3\log x + 2)}$

$= \log|2\log x + 1| - \log|3\log x + 2| + c$

$\therefore \quad I = \log\left|\frac{2\log x + 1}{3\log x + 2}\right| + c$

Indefinite Integrals Ex 19.30 Q23

$\frac{1}{x(x^n + 1)}$

Multiplying numerator and denominator by x^{n-1}, we obtain

$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$\therefore \int \frac{1}{x(x^n+1)}dx = \int \frac{x^{n-1}}{x^n(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t+1)}dt$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$1 = A(1+t) + Bt \quad \ldots(1)$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$

$\Rightarrow \int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\int\left\{\frac{1}{t} - \frac{1}{(t+1)}\right\}dx$

$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$

$= -\frac{1}{n}\left[\log|x^n| - \log|x^n+1|\right] + C$

$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

Indefinite Integrals Ex 19.30 Q24

$$\text{Let } \int \frac{x}{\left(x^2-a^2\right)\left(x^2-b^2\right)} = \frac{Ax+B}{\left(x^2-a^2\right)} + \frac{Cx+D}{\left(x^2-b^2\right)}$$

$$\Rightarrow \quad x = (Ax+B)\left(x^2-b^2\right) + (Cx+D)\left(x^2-a^2\right)$$

$$x = (A+C)x^3 + (B+D)x^2 + \left(-Ab^2 - Ca^2\right)x + \left(-Bb^2 - Da^2\right)$$

$$\Rightarrow \quad A+C=0,\ B+D=0,\ -Ab^2-Ca^2=1,\ -Bb^2-Da^2=0$$

We get $B=0,\ D=0,\ C=\frac{1}{b^2-a^2},\ A=-\frac{1}{b^2-a^2}$

Thus,

$$I = -\frac{1}{b^2-a^2}\int \frac{xdx}{x^2-a^2} + \frac{1}{b^2-a^2}\int \frac{xdx}{x^2-b^2}$$

$$I = -\frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-a^2\right| + \frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-b^2\right| + c$$

Indefinite Integrals Ex 19.30 Q25

Consider the integral

$$I=\int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}dx$$

Let $y=x^2$

Thus,

$$\frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} = \frac{y+1}{(y+4)(y+25)}$$

$$\Rightarrow \frac{y+1}{(y+4)(y+25)} = \frac{A}{y+4} + \frac{B}{y+25}$$

$$\Rightarrow \frac{y+1}{(y+4)(y+25)} = \frac{A(y+25)+B(y+4)}{(y+4)(y+25)}$$

$\Rightarrow y+1=Ay+25A+By+4B$

Comparing the coefficients, we have,

$A+B=1$ and $25A+4B=1$

Solving the above equations, we have,

$A=\frac{-1}{7}$ and $B=\frac{8}{7}$

Thus, $\int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}dx$

$$= \int \frac{\frac{-1}{7}}{x^2+4}dx + \int \frac{\frac{8}{7}}{x^2+25}dx$$

$$= \frac{-1}{7}\int \frac{1}{x^2+4}dx + \frac{8}{7}\int \frac{1}{x^2+25}dx$$

$$= \frac{-1}{7} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + \frac{8}{7} \times \frac{1}{5}\tan^{-1}\frac{x}{5} + C$$

$$= \frac{-1}{14}\tan^{-1}\frac{x}{2} + \frac{8}{35}\tan^{-1}\frac{x}{5} + C$$

Indefinite Integrals Ex 19.30 Q26

Let $I = \int \frac{x^3 + x + 1}{x^2 - 1} dx$

$= \int \left(x + \frac{2x + 1}{x^2 - 1} \right) dx$

Now,

Let $\frac{2x + 1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$

$\Rightarrow \quad 2x + 1 = A(x - 1) + B(x + 1)$

Put $x = 1$

$\Rightarrow \quad 3 = 2B \Rightarrow \quad B = \frac{3}{2}$

Put $x = -1$

$\Rightarrow \quad -1 = -2A \Rightarrow \quad A = \frac{1}{2}$

$\therefore \quad I = \int x dx + \frac{1}{2} \int \frac{dx}{x + 1} + \frac{3}{2} \int \frac{dx}{x - 1}$

$I = \frac{x^2}{2} + \frac{1}{2} \log|x + 1| + \frac{3}{2} \log|x - 1| + c$

Indefinite Integrals Ex 19.30 Q27

Let $\frac{3x - 2}{(x + 1)^2 (x + 3)} = \frac{A}{(x + 1)} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 3)}$

$\Rightarrow \quad 3x - 2 = A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2$

$= (A + C)x^2 + (4A + B + 2C)x + (3A + 3B + C)$

Equating similar terms, we get,

$A + C = 0 \quad \Rightarrow \quad A = -C$

$4A + B + 2C = 3 \quad \Rightarrow \quad B = -2C = 3$

$3A + 3B + C = -2 \Rightarrow \quad 3B - 2C = -2$

Solving, we get, $B = -\frac{5}{2}$, $C = -\frac{11}{4}$ & $A = \frac{11}{4}$

Thus,

$I = \frac{11}{4} \int \frac{dx}{x + 1} - \frac{5}{2} \int \frac{dx}{(x + 1)^2} - \frac{11}{4} \int \frac{dx}{x + 3}$

$I = \frac{11}{4} \log|x + 1| + \frac{5}{2(x + 1)} - \frac{11}{4} \log|x + 3| + c$

Indefinite Integrals Ex 19.30 Q28

Let $\frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\Rightarrow \quad 2x+1 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$$
$$= (A+B)x^2 + (-6A - B + C)x + (9A - 6B + 2C)$$

Equating similar terms, we get,

$A + B = 0 \quad \Rightarrow \quad A = -B$
$-6A - B + C = 2 \quad \Rightarrow \quad 5B + C = 2$
$9A - 6B + 2C = 1 \Rightarrow \quad -15B + 2C = 1$

Solving, we get, $B = \frac{3}{25}$, $C = \frac{7}{5}$, $A = -\frac{3}{25}$

Thus,

$$I = -\frac{3}{25}\int \frac{dx}{x+2} + \frac{3}{25}\int \frac{dx}{x-3} + \frac{7}{5}\int \frac{dx}{(x-3)^2}$$

$$I = -\frac{3}{25}\log|x+2| + \frac{3}{25}\log|x-3| - \frac{7}{5(x-3)} + c$$

Indefinite Integrals Ex 19.30 Q29

Let $\frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$

$$\Rightarrow \quad x^2+1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$$
$$= (A+C)x^2 + (A+B-4C)x + (-6A + 3B + 4C)$$

Equating similar terms, we get,

$A + C = 1$, $A + B - 4C = 0$, $-6A + 3B + 4C = 1$

Solving, we get, $A = \frac{3}{5}$, $B = 1$, $C = \frac{2}{5}$

Thus,

$$I = \frac{3}{5}\int \frac{dx}{x-2} + \int \frac{dx}{(x-2)^2} + \frac{2}{5}\int \frac{dx}{x+3}$$

$$I = \frac{3}{5}\log|x-2| - \frac{1}{(x-2)} + \frac{2}{5}\log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q30

Let $\frac{x}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$

$x=A(x-1)(x+2)+B(x+2)+C(x-1)^2$

Substituting x = 1, we obtain

$B=\frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A=\frac{2}{9}$ and $C=\frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)}=\frac{2}{9(x-1)}+\frac{1}{3(x-1)^2}-\frac{2}{9(x+2)}$$

$$\Rightarrow \int\frac{x}{(x-1)^2(x+2)}dx=\frac{2}{9}\int\frac{1}{(x-1)}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{(x+2)}dx$$

$$=\frac{2}{9}\log|x-1|+\frac{1}{3}\left(\frac{-1}{x-1}\right)-\frac{2}{9}\log|x+2|+\mathrm{C}$$

$$=\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+\mathrm{C}$$

Indefinite Integrals Ex 19.30 Q31

Let $\frac{x^2}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$

$$\Rightarrow \quad x^2=A(x+1)^2+B(x-1)(x+1)+C(x-1)$$
$$=(A+B)x^2+(2A+C)x+(A-B-C)$$

Equating similar terms,

$A+B=1,\ 2A+C=0,\ A-B-C=0$

Solving, we get, $A=\frac{1}{4},\ B=\frac{3}{4},\ C=-\frac{1}{2}$

Thus,

$$I=\frac{1}{4}\int\frac{dx}{x-1}+\frac{3}{4}\int\frac{dx}{x+1}-\frac{1}{2}\int\frac{dx}{(x+1)^2}$$
$$=\frac{1}{4}\log|x-1|+\frac{3}{4}\log|x+1|+\frac{1}{2(x+1)}+c$$

$$I=\frac{1}{4}\log|x-1|+\frac{3}{4}\log|x+1|+\frac{1}{2(x+1)}+c$$

Indefinite Integrals Ex 19.30 Q32

Let $\frac{x^2+x-1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$

$$\Rightarrow \quad x^2+x-1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$
$$= (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

Equating similar terms

$A+C=1,\ 3A+B+2C=1,\ 2A+2B+C=-1$

Solving, we get, $A=0,\ B=-1,\ C=1$

Thus,

$$I = 0.\int \frac{dx}{x+1} + (-1)\int \frac{dx}{(x+1)^2} + 1.\int \frac{dx}{(x+2)}$$
$$= +\frac{1}{x+1} + \log|x+2| + c$$

$$I = \frac{1}{x+1} + \log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q33

Let $\frac{2x^2+7x-3}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$

$$\Rightarrow \quad 2x^2+7x-3 = Ax(2x+1) + B(2x+1) + Cx^2$$

Equating similar terms, we get,

$2A+C=2,\ A+2B=7,\ B=-3$

Solving, we get, $A=13,\ C=-24$

Thus,

$$I = \int \frac{13dx}{x} - \int \frac{3dx}{x^2} - 24\int \frac{dx}{2x+1}$$

$$I = 13\log|x| + \frac{3}{x} - 12\log|2x+1| + c$$

Indefinite Integrals Ex 19.30 Q34

Let $I = \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

$= \int \frac{5x^2 + 20x + 6}{x(x+1)^2} dx$

Now,

Let $\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\Rightarrow \quad 5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$

Equating similar terms, we get,

$A + B = 5,\ 2A + B + C = 20,\ A = 6$

Solving, we get, $B = -1,\ C = 9$

Thus,

$I = \int \frac{6dx}{x} - 1\int \frac{dx}{x+1} + 9\int \frac{dx}{(x+1)^2}$

$\therefore \quad I = 6\log|x| - \log|x+1| - \frac{9}{x+1} + c$

Indefinite Integrals Ex 19.30 Q35

Let $\frac{18}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx + C}{x^2+4}$

$\Rightarrow \quad 18 = A(x^2+4) + (Bx + C)(x+2)$

$18 = (A+B)x^2 + (2B + C)x + (4A + 2C)$

Equating similar terms, we get,

$A + B = 0,\ 2B + C = 0,\ 4A + 2C = 18$

Solving, we get, $A = \frac{9}{4},\ B = -\frac{9}{4},\ C = \frac{9}{2}$

Thus,

$I = \frac{9}{4}\int \frac{dx}{x+2} + \left(-\frac{9}{4}\right)\int \frac{x}{x^2+4} dx + \frac{9}{2}\int \frac{dx}{x^2+4}$

$I = \frac{9}{4}\log|x+2| - \frac{9}{8}\log|x^2+4| + \frac{9}{4}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{9}\right]$

Indefinite Integrals Ex 19.30 Q36

Let $\frac{5}{\left(x^2+1\right)\left(x+2\right)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow \quad 5 = \left(Ax+B\right)\left(x+2\right) + C\left(x^2+1\right)$

Equating similar terms, we get,

$A + C = 0,\ 2A + B = 0,\ 2B + C = 5$

Solving, we get, $A = -1,\ B = 2,\ C = 1$

Thus,

$$I = \int \frac{-x+2}{x^2+1}dx + \int \frac{dx}{x+2}$$
$$= \int \frac{-xdx}{x^2+1} + 2\int \frac{dx}{x^2+1} + \int \frac{dx}{x+2}$$

$$I = -\frac{1}{2}\log\left|x^2+1\right| + 2\tan^{-1}x + \log\left|x+2\right| + c$$

Indefinite Integrals Ex 19.30 Q37

Let $\frac{x}{\left(x+1\right)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow \quad x = A\left(x^2+1\right) + \left(Bx+C\right)\left(x+1\right)$

Equating similar terms, we get,

$A + B = 0,\ B + C = 1,\ A + C = 0$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$$I = -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$I = -\frac{1}{2}\log\left|x+1\right| + \frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q38

Let $I=\int \frac{dx}{1+x+x^2+x^3}$

$\Rightarrow \quad I=\int \frac{dx}{\left(x^2+1\right)\left(x+1\right)}$

Now,

Let $\frac{1}{\left(x^2+1\right)\left(x+1\right)}=\frac{Ax+B}{x^2+1}+\frac{C}{x+1}$

$\Rightarrow \quad 1=\left(Ax+B\right)\left(x+1\right)+C\left(x^2+1\right)$

Equating similar terms, we get,

$A+C=0,\ A+B=0,\ B+C=1$

Solving, we get, $A=-\frac{1}{2},\ B=\frac{1}{2},\ C=\frac{1}{2}$

Thus,

$I=-\frac{1}{2}\int \frac{xdx}{x^2+1}+\frac{1}{2}\int \frac{dx}{x^2+1}+\frac{1}{2}\int \frac{dx}{x+1}$

$I=-\frac{1}{4}\log\left|x^2+1\right|+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log\left|x+1\right|+c$

Indefinite Integrals Ex 19.30 Q39

Let $\frac{1}{\left(x+1\right)^2\left(x^2+1\right)}=\frac{A}{x+1}+\frac{B}{\left(x+1\right)^2}+\frac{Cx+D}{x^2+1}$

$$\Rightarrow \quad 1=A\left(x+1\right)\left(x^2+1\right)+B\left(x^2+1\right)+\left(Cx+D\right)\left(x+1\right)^2$$
$$=\left(A+C\right)x^3+\left(A+B+2C+D\right)x^2+\left(A+C+2D\right)x+\left(A+B+D\right)$$

Equating similar terms, we get,

$A+C=0,\ A+B+2C+D=0,\ A+C+2D=0,\ A+B+D=1$

Solving, we get, $A=\frac{1}{2},\ B=\frac{1}{2},\ C=-\frac{1}{2},\ D=0$

Thus,

$I=\frac{1}{2}\int \frac{dx}{x+1}+\frac{1}{2}\int \frac{dx}{\left(x+1\right)^2}-\frac{1}{2}\int \frac{xdx}{x^2+1}$

$I=\frac{1}{2}\log\left|x+1\right|-\frac{1}{2\left(x+1\right)}-\frac{1}{4}\log\left|x^2+1\right|+c$

Indefinite Integrals Ex 19.30 Q40

Let $I = \int \frac{2x}{x^3 - 1} dx = \int \frac{2x}{(x-1)(x^2+x+1)} dx$

Now,

Let $\frac{2x}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$

$$\Rightarrow \quad 2x = A(x^2+x+1) + (Bx+C)(x-1)$$

$$= (A+B)x^2 + (A-B+C)x + (A-C)$$

Equating similar terms,

$A+B = 0,\ A-B+C = 2,\ A-C = 0,$

Solving, we get, $A = \frac{2}{3},\ B = -\frac{2}{3},\ C = \frac{2}{3}$

Thus,

$$I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\int \frac{(x-1)dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}.\frac{1}{2}\int \frac{(2x-2)dx}{x^2+x+1}$$

$$\Rightarrow \quad I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1} dx + \int \frac{dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1} dx + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{2}{3}\log|x-1| - \frac{1}{3}\log\left|x^2+x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

Hence,

$$I = \frac{2}{3}\log|x-1| - \frac{1}{3}\log\left|x^2+x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.30 Q41

Let $\frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{x^2+4}$

$$\Rightarrow \quad 1 = \left(Ax+B\right)\left(x^2+4\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(A+C\right)x^3 + \left(B+D\right)x^2 + \left(4A+C\right)x + 4B + D$$

Equating similar terms, we get,

$A+C=0,\ B+D=0,\ 4A+C=0,\ 4B+D=1$

Solving, we get, $A=0,\ B=\frac{1}{3},\ C=0,\ D=-\frac{1}{3}$

Thus,

$$I = \int \frac{\frac{1}{3}dx}{\left(x^2+1\right)} - \int \frac{\frac{1}{3}dx}{\left(x^2+4\right)}$$

$$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$\therefore \quad I = \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.30 Q42

Let $\frac{x^2}{\left(x^2+1\right)\left(3x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{\left(3x^2+4\right)}$

$$\Rightarrow \quad x^2 = \left(Ax+B\right)\left(3x^2+4\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(3A+C\right)x^3 + \left(3B+D\right)x^2 + \left(4A+C\right)x + 4B + D$$

Equating similar terms, we get,

$3A+C=0,\ 3B+D=1,\ 4A+C=0,\ 4B+D=0$

Solving, we get, $A=0,\ B=-1,\ C=0,\ D=4$

Thus,

$$I = \int \frac{-dx}{\left(x^2+1\right)} + \int \frac{4dx}{\left(3x^2+4\right)}$$

$$= -\tan^{-1}x + \frac{4}{3}\int \frac{dx}{x^2+\left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= -\tan^{-1}x + \frac{4}{3}.\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$$

$$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q43

To evaluate the integral follow the steps:

$$\int \frac{3x+5}{x^3-x^2-x+1}dx$$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

For $x=1 \quad B=4$

For $x=-1 \; C=\frac{1}{2}$

For $x=0 \quad A=-\frac{1}{2},$

Therefore

$$\int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int \frac{dx}{x-1} + 4\int \frac{dx}{(x-1)^2} + \frac{1}{2}\int \frac{dx}{x+1}$$

$$= -\frac{1}{2}\ln|(x-1)| - \frac{4}{(x-1)} + \frac{1}{2}\ln|(x+1)| + c$$

$$= \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + c$$

Indefinite Integrals Ex 19.30 Q44

Let $I = \int \frac{x^3-1}{x^3+x}dx$

$$= \int 1 - \frac{(x+1)}{x^3+x}dx$$

$$= \int dx - \int \frac{x+1}{x^3+x}dx$$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow \quad x+1 = A(x^2+1) + (Bx+C)(x)$$

$$= (A+B)x^2 + (B+C)x + A$$

Equating similar terms, we get,

$A+B=0,\ C=1,\ A=1$

Solving, we get, $A=1,\ B=-1,\ C=1$

Thus,

$$I = -\int \frac{dx}{x} - \int \frac{-x+1}{x^2+1}dx + \int dx$$

$$= -\log|x| + \int \frac{xdx}{x^2+1} - \int \frac{dx}{x^2+1} + \int dx$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + c$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q45

To evaluate the integral follow the steps:

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx$$

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$

$x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

For $x=-1$ $B=1$

For $x=-2$ $C=3$

For $x=0$ $A=-2$,

Therefore

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = -2\int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2} + 3\int \frac{dx}{x+2}$$

$$= -2\ln|x+1| - \frac{1}{x+1} + 3\ln|x+2| + c$$

Indefinite Integrals Ex 19.30 Q46

Let $\frac{1}{x(x^4+1)} = \frac{A}{x} + \frac{Bx^3+Cx^2+Dx+E}{x^4+1}$

$$\Rightarrow \quad 1 = A(x^4+1) + (Bx^3+Cx^2+Dx+E)x$$
$$= (A+B)x^4 + Cx^3 + Dx^2 + Ex + A$$

Equating similar terms, we get,

$A+B=0$, $C=0$, $D=0$, $E=0$, $A=1$

$\therefore \quad B=-1$

Thus,

$$I = \int \frac{dx}{x} + \int -\frac{x^3dx}{x^4+1}$$

$$= \log|x| - \frac{1}{4}\log|x^4+1| + c$$

$$I = \frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right| + c$$

Indefinite Integrals Ex 19.30 Q47

Consider the integral

$$I=\int \frac{1}{x\left(x^3+8\right)}dx$$

Rewriting the above integrand, we have,

$$I=\int \frac{x^2}{x^3\left(x^3+8\right)}dx$$

$$=\frac{1}{3}\int \frac{3x^2}{x^3\left(x^3+8\right)}dx$$

Now substituting $x^3 = t$, we have,

$3x^2dx = dt$

$$\Rightarrow I = \frac{1}{3}\int \frac{dt}{t(t+8)}$$

Let us separata the integrand by partial fractions.

Thus,

$$\frac{1}{t(t+8)} = \frac{A}{t} + \frac{B}{t+8}$$

$$\Rightarrow \frac{1}{t(t+8)} = \frac{A(t+8)+Bt}{t(t+8)}$$

$$\Rightarrow 1 = A(t+8)+Bt$$

$$\Rightarrow 1 = At + 8A + Bt$$

Comparing the coefficients, we have,

A+B=0 and 8A=1

$$\Rightarrow A = \frac{1}{8} \text{ and } B = -\frac{1}{8}$$

Therefore,

$$I = \frac{1}{3}\int \frac{dt}{t(t+8)}$$

$$= \frac{1}{3}\int \left[\frac{\frac{1}{8}}{t} - \frac{\frac{1}{8}}{t+8}\right]dt$$

$$= \frac{1}{3}\times\frac{1}{8}\int \frac{dt}{t}dt - \frac{1}{3}\times\frac{1}{8}\int \frac{dt}{t+8}$$

$$= \frac{1}{24}\log t - \frac{1}{24}\times\log(t+8)+C$$

$$= \frac{1}{24}\log x^3 - \frac{1}{24}\times\log\left(x^3+8\right)+C$$

$$= \frac{3}{24}\log x - \frac{1}{24}\times\log\left(x^3+8\right)+C$$

$$= \frac{1}{8}\log x - \frac{1}{24}\times\log\left(x^3+8\right)+C$$

Indefinite Integrals Ex 19.30 Q48

Let $\frac{3}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$\Rightarrow \quad 3 = A(1+x^2) + (Bx+C)(1-x)$

$= (A-B)x^2 + (B-C)x + (A+C)$

Equating similar terms, we get,

$A - B = 0,\ B - C = 0,\ A + C = 3$

Solving we get,

$A = C = \frac{3}{2}$ and $B = \frac{3}{2}$

Thus,

$I = \frac{3}{2}\int \frac{dx}{1-x} + \frac{3}{2}\int \frac{xdx}{1+x^2} + \frac{3}{2}\int \frac{dx}{1+x^2}$

$= -\frac{3}{2}\log|1-x| + \frac{3}{2}\log|1+x^2| + \frac{3}{2}\tan^{-1}x + c$

$I = \frac{3}{4}\left[\log\left|\frac{1+x^2}{(1-x)^2}\right| + 2\tan^{-1}x + c\right]$

Indefinite Integrals Ex 19.30 Q49

Let

$\sin x = t$

$\Rightarrow \quad \cos x = dt$

$\therefore \quad \int \frac{\cos x}{(1-\sin x)^3(2+\sin x)} = \int \frac{1}{(1-t)^3(2+t)}dt$

Let $f(t) = \frac{1}{(1-t)^3(2+t)}$

Then, suppose

$\frac{1}{(1-t)^3(2+t)} = \frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{c}{(1-t)^3} + \frac{D}{(2+t)}$

$\Rightarrow \quad 1 = A(1-t)^2(2+t) + B(1-t)(2+t) + C(2+t) + D(1-t)^3$

Put $t = 1$

$1 = 3C$

$\Rightarrow \quad C = \frac{1}{3}$

Put $t = -2$

$1 = 27D$

$\Rightarrow \quad D = \frac{1}{27}$

Similarly, we can find that $A = \frac{-1}{27}$ and $B = \frac{+1}{9}$

$\therefore \quad \int \frac{1}{(1-t)^3(2+t)}dt = \frac{-1}{27}\int \frac{1}{1-t}dt + \frac{1}{9}\int \frac{dt}{(1-t)^2} + \frac{1}{3}\int \frac{dt}{(1-t)^3} + \frac{1}{27}\int \frac{dt}{2+t}$

$= \frac{-1}{27}\log|1-t| + \frac{1}{9(1-t)} + \frac{1}{6(1-t)^2} + \frac{1}{27}\log|2+t| + c$

Putting $t = \sin x$, we get

$\int \frac{\cos x}{(1-\sin x)^3(2+\sin x)}dx$

$= \frac{-1}{27}\log|1-\sin x| + \frac{1}{9(1-\sin x)} + \frac{1}{6(1-\sin x)^2} + \frac{1}{27}\log|2+\sin x| + C$

Indefinite Integrals Ex 19.30 Q50

Consider the integral

$$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

Now let us separate the fraction $\frac{2x^2+1}{x^2\left(x^2+4\right)}$ through partial fractions.

Substitute $x^2=t$, then

$$\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2t+1}{t\left(t+4\right)}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)}=\frac{A}{t}+\frac{B}{t+4}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)}=\frac{A\left(t+4\right)+Bt}{t\left(t+4\right)}$$

$$\Rightarrow 2t+1=A\left(t+4\right)+Bt$$

$$\Rightarrow 2t+1=At+4A+Bt$$

Comparing the coefficients, we have,

$A+B=2$ and $4A=1$

$\Rightarrow A=\frac{1}{4}$ and $B=\frac{7}{4}$

$$\Rightarrow \frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$$

Thus, we have,

$$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

$$=\frac{1}{4}\int \frac{dx}{x^2}+\frac{7}{4}\int \frac{dx}{\left(x^2+4\right)}$$

$$=-\frac{1}{4x}+\frac{7}{4}\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$$

$$=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$$

Indefinite Integrals Ex 19.30 Q51

To evaluate the integral follow the steps:

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$$

Let $1-\sin x=t$ and

$$-\cos x dx=dt$$

Therefore

$$-\int \frac{dt}{t(1+t)}=-\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$$

$$=\ln\left|(t+1)\right|-\ln|t|+c$$

$$=\ln\left|\frac{t+1}{t}\right|+c$$

$$=\ln\left|\frac{2-\sin x}{1-\sin x}\right|+c$$

Indefinite Integrals Ex 19.30 Q52

Let $\frac{2x+1}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$

$\Rightarrow \quad 2x+1 = A(x-3) + B(x-2)$
$= (A+B)x + (-3A-2B)$

Equating similar terms, we get,

$A + B = 2$, and $-3A - 2B = 1$

Thus,

$I = -5\int \frac{dx}{x-2} + 7\int \frac{dx}{x-3}$

$= -5\log|x-2| + 7\log|x-3| + c$

$I = \log\left|\frac{(x-3)^7}{(x-2)^5}\right| + c$

Indefinite Integrals Ex 19.30 Q53

Let $x^2 = y$

Then $\frac{1}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$

$\Rightarrow \quad 1 = A(y+2) + B(y+1)$
$= (A+B)y + (2A+B)$

Equating similar terms, we get,

$A + B = 0$, and $2A + B = 1$

Solving, we get,

Thus,

$I = \int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+2}$

$I = \tan^{-1} x - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$

Indefinite Integrals Ex 19.30 Q54

To evaluate the integral follow the steps:

$$\int \frac{1}{x(x^4-1)}dx$$

Let $\frac{1}{x(x^4-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{x^2+1}$

$$1 = A(x+1)(x-1)(x^2+1) + Bx(x-1)(x^2+1) + Cx(x+1)(x^2+1) + Dx(x+1)(x-1)$$

For $x=0 \quad A=-1,$

For $x=1 \quad C=\frac{1}{4}$

For $x=-1$ $B=\frac{1}{4}$

For $x=2$ $D=\frac{1}{4}$

Therefore

$$\int \frac{1}{x(x^4-1)}dx = -\int \frac{1}{x}dx + \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int \frac{dx}{x^2+1}$$

$$= -\ln|x| + \frac{1}{4}\ln|(x+1)| + \frac{1}{4}\ln|(x-1)| + \frac{1}{4}\ln|(x^2+1)| + c$$

$$= \frac{1}{4}\ln\left|\frac{x^4-1}{x^4}\right| + c$$

Indefinite Integrals Ex 19.30 Q55

To evaluate the integral follow the steps:

$$\int \frac{1}{(x^4-1)}dx$$

Let $\frac{1}{(x^4-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x^2+1}$

$$1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + C(x+1)(x-1)$$

For $x=1 \quad B=\frac{1}{4}$

For $x=-1$ $A=-\frac{1}{4}$

For $x=0 \quad C=-\frac{1}{2},$

Therefore

$$\int \frac{1}{(x^4-1)}dx = -\frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= -\frac{1}{4}\ln|(x+1)| + \frac{1}{4}\ln|(x-1)| - \frac{1}{2}\tan^{-1}x + c$$

$$= \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q57

Let $I = \int \frac{dx}{\cos x\left(5 - 4\sin x\right)}$

$= \int \frac{\cos x dx}{\cos^2 x\left(5 - 4\sin x\right)}$

$= \int \frac{\cos x dx}{\left(1 - \sin^2 x\right)\left(5 - 4\sin x\right)}$

Let $\sin x = t \quad \Rightarrow \quad \cos x dx = dt$

$\therefore \quad I = \int \frac{dt}{\left(1 - t^2\right)\left(5 - 4t\right)}$

Now,

Let $\frac{1}{\left(1 - t^2\right)\left(5 - 4t\right)} = \frac{A}{1 - t} + \frac{B}{1 + t} + \frac{C}{5 - 4t}$

$\Rightarrow \quad 1 = A\left(1 + t\right)\left(5 - 4t\right) + B\left(1 - t\right)\left(5 - 4t\right) + C\left(1 - t^2\right)$

Put $t = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $t = -1$

$\Rightarrow \quad 1 = 18B \quad \Rightarrow \quad B = \frac{1}{18}$

Put $t = \frac{5}{4}$

$\Rightarrow \quad 1 = -\frac{9C}{16} \Rightarrow \quad C = -\frac{16}{9}$

Thus,

$I = \frac{1}{2}\int \frac{dt}{1 - t} + \frac{1}{18}\int \frac{dt}{1 + t} - \frac{16}{9}\int \frac{dt}{5 - 4t}$

$= -\frac{1}{2}\log\left|1 - t\right| + \frac{1}{18}\log\left|1 + t\right| + \frac{4}{9}\log\left|5 - 4t\right| + c$

Hence,

$I = -\frac{1}{2}\log\left|1 - \sin x\right| + \frac{1}{18}\log\left|1 + \sin x\right| + \frac{4}{9}\log\left|5 - 4\sin x\right| + c$

Indefinite Integrals Ex 19.30 Q58

Let $I = \int \frac{1}{\sin x\left(3+2\cos x\right)} dx$

$= \int \frac{\sin x dx}{\sin^2 x\left(3+2\cos x\right)}$

$= \int \frac{\sin x dx}{\left(1-\cos^2 x\right)\left(3+2\cos x\right)}$

Let $\cos x = t$

$\Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \frac{dt}{\left(t^2-1\right)\left(3+2t\right)}$

Now,

Let $\frac{1}{\left(t^2-1\right)\left(3+2t\right)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{3+2t}$

$\Rightarrow \quad 1 = A\left(t+1\right)\left(3+2t\right) + B\left(t-1\right)\left(3+2t\right) + C\left(t^2-1\right)$

Put $t = 1$

$\Rightarrow \quad 1 = 10A \Rightarrow \quad A = \frac{1}{10}$

Put $t = -1$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Put $t = -\frac{3}{2}$

$\Rightarrow \quad 1 = \frac{5}{4}.C \Rightarrow \quad C = \frac{4}{5}$

Thus,

$I = \frac{1}{10}\int \frac{dt}{t-1} - \frac{1}{2}\int \frac{dt}{t+1} + \frac{5}{4}\int \frac{dt}{3+2t}$

$= \frac{1}{10}\log\left|t-1\right| - \frac{1}{2}\log\left|t+1\right| + \frac{2}{5}\log\left|3+2t\right| + c$

Hence,

$I = \frac{1}{10}\log\left|\cos x - 1\right| - \frac{1}{2}\log\left|\cos x + 1\right| + \frac{2}{5}\log\left|3+2\cos x\right| + c$

Indefinite Integrals Ex 19.30 Q59

Let $I = \int \frac{1}{\sin x + \sin 2x} dx$

$= \int \frac{dx}{\sin x + 2\sin x \cos x}$

$= \int \frac{\sin x dx}{\left(1 - \cos^2 x\right) + 2\left(1 - \cos^2 x\right)\cos x}$

Let $\cos x = t \quad \Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \frac{dt}{\left(t^2 - 1\right) + 2\left(t^2 - 1\right)t}$

$= \int \frac{dt}{\left(t^2 - 1\right)\left(1 + 2t\right)}$

Let $\int \frac{1}{\left(t^2 - 1\right)\left(1 + 2t\right)} = \frac{A}{t - 1} + \frac{B}{t + 1} + \frac{C}{1 + 2t}$

$\Rightarrow \quad 1 = A\left(t + 1\right)\left(1 + 2t\right) + B\left(t - 1\right)\left(1 + 2t\right) + C\left(t^2 - 1\right)$

Put $t = 1$

$\Rightarrow \quad 1 = 6A \Rightarrow \quad A = \frac{1}{6}$

Put $t = -1$

$\Rightarrow \quad 1 = 2B \Rightarrow \quad B = \frac{1}{2}$

Put $t = -\frac{1}{2}$

$\Rightarrow \quad 1 = -\frac{3}{4}C \Rightarrow \quad C = -\frac{4}{3}$

Thus,

$I = \frac{1}{6}\int \frac{dt}{t - 1} + \frac{1}{2}\int \frac{dt}{t + 1} - \frac{4}{3}\int \frac{dt}{1 + 2t}$

$= \frac{1}{6}\log\left|t - 1\right| + \frac{1}{2}\log\left|t + 1\right| - \frac{2}{3}\log\left|1 + 2t\right| + c$

Hence,

$I = \frac{1}{6}\log\left|\cos x - 1\right| + \frac{1}{2}\log\left|\cos x + 1\right| - \frac{2}{3}\log\left|1 + 2\cos x\right| + c$

Indefinite Integrals Ex 19.30 Q60

Let $I = \int \frac{x+1}{x\left(1+xe^x\right)}dx$

$= \int \frac{\left(x+1\right)\left(1+xe^x - xe^x\right)}{x\left(1+xe^x\right)}dx$

$= \int \frac{\left(x+1\right)\left(1+xe^x\right)}{x\left(1+xe^x\right)}dx - \int \frac{\left(x+1\right)\left(xe^x\right)}{x\left(1+xe^x\right)}dx$

$= \int \frac{\left(x+1\right)}{x}dx - \int \frac{e^x\left(x+1\right)}{1+xe^x}dx$

$= \int \frac{\left(x+1\right)e^x}{xe^x}dx - \int \frac{e^x\left(x+1\right)}{1+xe^x}dx$

$= \log\left|xe^x\right| - \log\left|1+xe^x\right| + c$

$\therefore \quad I = \log\left|\frac{xe^x}{1+xe^x}\right| + c$

Indefinite Integrals Ex 19.30 Q61

$f\left(x\right) = \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$

Now,

$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$

$= \frac{x^4+3x^2+2}{x^4+7x^2+12}$

$= \frac{\left(x^4+7x^2+12\right)-4x^2-10}{x^4+7x^2+12}$

$= 1 - \frac{4x^2+10}{x^4+7x^2+12}$

Now,

$\frac{4x^2+10}{x^4+7x^2+12} = \frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)}$

Let $\frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+4}$

$\Rightarrow \quad 4x^2+10 = \left(Ax+B\right)\left(x^2+4\right) + \left(Cx+D\right)\left(x^2+3\right)$

Let $x = 0$, we get

$10 = 4B + 3D \quad$ ---(i)

If $x = 1$, we get

$14 = 5\left(A+B\right) + 4\left(C+D\right) = 5A + 5B + 4C + 4D \quad$ ---(ii)

if $x = -1$, we get

$14 = 5\left(-A+B\right) + 4\left(-C+D\right) = -5A + 5B - 4C + 4D \quad$ ---(iii)

Applying (ii) and (iii), we get

$28 = 10B + 8D$

$\Rightarrow \quad 14 = 5B + 4D \quad$ ---(iv)

From (i), we get

$10 = 4B + 3D \quad$ ---(i)

Multiplying equation (iv) by 3 and (i) by 4 and subtracting, we get

$42 - 40 = 15B - 16B$

$\Rightarrow \quad 2 = -B$

or $\quad B = -2 \quad$ ---(v)

Putting value of B in (i), we get

$10 = 4\left(-2\right) + 3D$

$\frac{10+8}{3} = D$

$\Rightarrow \quad D = 6 \quad$ ---(vi)

Comparing coefficients of x^3 in

$$4x^2+10=(Ax+B)\left(x^2+4\right)+(Cx+4)\left(x^2+3\right)$$, we get,

$$0=A+C \qquad \text{---(vii)}$$

Comparing coefficients of x, we get

$$0=4A+3C \qquad \text{---(viii)}$$

$\Rightarrow \quad A=C=0$

$$\therefore \quad f(x)=1-\frac{(-2)}{x^2+3}-\frac{6}{x^2+4}$$

$$=1+\frac{2}{x^2+3}-\frac{6}{x^2+4}$$

$$\therefore \quad \int f(x)\,dx=\int 1+\frac{2}{x^2+3}-\frac{6}{x^2+4}dx$$

$$=x+\frac{2}{\sqrt{3}}\tan^{-1}\times\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+c$$

Indefinite Integrals Ex 19.30 Q62

Let $x^2=y$

$$\therefore \quad \frac{4x^4+3}{\left(x^2+2\right)\left(x^2+3\right)\left(x^2+4\right)}=\frac{4y^2+3}{(y+2)(y+3)(y+4)}$$

Now,

Let $\frac{4y^2+3}{(y+2)(y+3)(y+4)}=\frac{A}{y+2}+\frac{B}{y+3}+\frac{C}{y+4}$

$$\Rightarrow \quad 4y^2+3=A(y+3)(y+4)+B=(y+2)(y+4)+C(y+2)(y+3)$$

$$=(A+B+C)y^2+(7A+6A+5C)y+12A+8B+6C$$

Equating similar terms,

$A+B+C=4,\ 7A+6A+5C=0,\ 12A+8B+6C=3$

Solving, we get

$A=\frac{19}{2},\ B=-39,\ C=\frac{67}{2}$

Thus,

$$I=\frac{19}{2}\int\frac{dx}{x^2+2}+(-39)\int\frac{dx}{x^2+3}+\frac{67}{2}\int\frac{dx}{x^2+4}$$

$$\Rightarrow \quad I=\frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right)+c$$

Hence,

$$I=\frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right)+c$$

Indefinite Integrals Ex 19.30 Q63

$$\frac{x^4}{(x-1)(x^2+1)} = \frac{x^4}{x^3 - x^2 + x - 1}$$

$$= \frac{x(x^3 - x^2 + x - 1) + 1(x^3 - x^2 + x - 1) + 1}{(x^3 - x^2 + x - 1)}$$

$$= x + 1 + \frac{1}{(x-1)(x^2+1)}$$

Now, suppose

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx + C}{x^2+1}$$

$\Rightarrow \quad 1 = A(x^2+1) + (Bx + C)(x-1)$

Put $x = 1$

$1 = 2A$

$\Rightarrow \quad A = \frac{1}{2}$

Put $x = 0$

$1 = A - C$

$\Rightarrow \quad C = A - 1 = -\frac{1}{2}$

Put $x = -1$

$1 = 2A + 2B - 2C = 2(A - C) + 2B$

$\Rightarrow \quad 1 = 2 + 2B$

$\Rightarrow \quad 2B = -1$

$\Rightarrow \quad B = -\frac{1}{2}$

$\therefore \quad \int \frac{x^4}{(x-1)(x^2+1)} dx = \int x dx + \int 1 dx + \frac{1}{2}\int \frac{1}{x-1} dx - \frac{1}{2}\int \frac{x+1}{x^2+1} dx$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + C$$

Ex 19.31

Indefinite Integrals Ex 19.31 Q1

$$I = \int \frac{x^2+1}{x^4+x^2+1}dx$$

$$= \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $x-\frac{1}{x}=t \Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2+3}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+c$$

$$\therefore \quad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+c$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot\theta}\,d\theta$$

Let $\cot\theta = x^2$

$\Rightarrow \quad -\operatorname{cosec}^2\theta\, d\theta = 2x\,dx$

$\Rightarrow \quad d\theta = \dfrac{-2x}{\operatorname{cosec}^2\theta}dx$

$= \dfrac{-2x}{1+\cot^2\theta}dx$

$= \dfrac{-2x}{1+x^4}dx$

$\therefore \quad I = -\int \dfrac{2x^2}{1+x^4}dx$

$= -\int \dfrac{2}{\dfrac{1}{x^2}+x^2}dx$

Dividing numerator and denominator by x^2

$$= -\int \frac{1+\frac{1}{x^2}+1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= -\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2} - \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}$$

Let $x - \dfrac{1}{x} = t \Rightarrow \quad \left(1+\dfrac{1}{x^2}\right)dx = dt$

and $\quad x + \dfrac{1}{x} = z \Rightarrow \quad \left(1-\dfrac{1}{x^2}\right)dx = dz$

$\Rightarrow \quad I = -\int \dfrac{dt}{t^2+2} - \int \dfrac{dz}{z^2-2}$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot\theta-1}{\sqrt{2\cot\theta}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{\cot\theta+1-\sqrt{2\cot\theta}}{\cot\theta+1-\sqrt{2\cot\theta}}\right| + c$$

Indefinite Integrals Ex 19.31 Q3

Let $I = \int \dfrac{x^2+9}{x^4+81}dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}}dx$$

$$= \int \frac{1+\frac{9}{x^2}}{\left(x-\frac{9}{x}\right)^2+18}dx$$

Let $\left(x-\dfrac{9}{x}\right) = t \Rightarrow \quad \left(1+\dfrac{9}{x^2}\right)dx = dt$

$\therefore \quad I = \int \dfrac{dt}{t^2+18}$

$\Rightarrow \quad I = \dfrac{1}{3\sqrt{2}}\tan^{-1}\left(\dfrac{t}{3\sqrt{2}}\right) + c$

Thus,

$$I = \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right) + c$$

Indefinite Integrals Ex 19.31 Q4

Let $I = \int \frac{1}{x^4+x^2+1}dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}-1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\left\{\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3} - \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-1}dx\right\}$$

Let $\left(x-\frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

and $\quad \left(x+\frac{1}{x}\right) = z$

$\Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dz$

$\therefore \quad I = \frac{1}{2}\int \frac{dt}{t^2+3} - \frac{1}{2}\int \frac{dz}{z^2-1}$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}}tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4}log\left|\frac{z-1}{z+1}\right| + c$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{1}{4}log\left|\frac{x^2+1-x}{x^2+1+x}\right| + c$

Indefinite Integrals Ex 19.31 Q5

Let $I = \int \frac{x^2-3x+1}{x^4+x^2+1}dx$

Dividing numerator and denominator by x^2

$\therefore \quad I = \int \frac{1-\frac{3}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3} - \int \frac{3x}{x^4+x^2+1}dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt \qquad$ [For Ist part]

Let $\quad x^2 = z$

$\Rightarrow \quad 2xdx = dz \qquad$ [For IInd part]

$\therefore \quad I = \int \frac{dt}{t^2+3} - \frac{3}{2}\int \frac{dz}{z^2+z+1}$

$\Rightarrow \quad = \int \frac{dt}{t^2+3} - \frac{3}{2}\int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$

$\Rightarrow \quad = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{3}{2}\times\frac{2}{\sqrt{3}}tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$

$\Rightarrow \quad I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \sqrt{3}\ tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right) + c$

Hence,

$$I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \sqrt{3}\ tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q6

Let $I = \int \frac{x^2+1}{x^4-x^2+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2-1+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+1}$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\Rightarrow \quad I = \int \frac{dt}{t^2+1}$$

$$= tan^{-1} t + c$$

$$\therefore \quad I = tan^{-1}\left(\frac{x^2-1}{x}\right)+c$$

Indefinite Integrals Ex 19.31 Q7

Let $I = \int \frac{x^2-1}{x^4+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1-\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2}dx$$

Let $\left(x+\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2-2}$$

$$= \frac{1}{2\sqrt{2}} log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+c$$

So,

$$I = \frac{1}{2\sqrt{2}} log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|+c$$

Indefinite Integrals Ex 19.31 Q8

Let $I = \int \frac{x^2+1}{x^4+7x^2+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+7+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+9}dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2+9}$$

$$= \frac{1}{3}tan^{-1}\left|\frac{t}{3}\right| + c$$

Hence,

$$I = \frac{1}{3}tan^{-1}\left(\frac{x^2-1}{3x}\right) + c$$

Indefinite Integrals Ex 19.31 Q9

Let $\quad I = \int \frac{(x-1)^2}{x^4+x^2+1}dx$

$$= \int \frac{x^2-2x+1}{x^4+x^2+1}dx$$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{1-\frac{2}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx - \int \frac{2x}{x^4+x^2+1}dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt \qquad \text{[For Ist part]}$$

Let $\quad x^2 = z$

$$\Rightarrow \quad 2xdx = dz \qquad \text{[For IInd part]}$$

$$\therefore \quad I = \int \frac{dt}{t^2+3} - \int \frac{dz}{z^2+z+1}$$

$$= \int \frac{dt}{t^3+3} - \int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q10

Let $I = \int \frac{1}{x^4 + 3x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right) - \left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \frac{1}{2} \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx$$

Let $\left(x - \frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt$$

and $x + \frac{1}{x} = z$

$$\Rightarrow \quad \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore \quad I = \frac{1}{2} \int \frac{dt}{t^2 + 5} - \frac{1}{2} \int \frac{dz}{z^2 + 1}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) - \frac{1}{2} \tan^{-1}(z) + c$$

Hence,

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{5}x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q11

Consider the integral

$$I = \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

Divide both the numerator and the denominator by $\cos^4 x$, we have,

$$I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{\sec^2 x \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{\left(\tan^2 x + 1\right) \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

Substituting $\tan x = t$; $\sec^2 x dx = dt$

Thus,

$$I = \int \frac{\left(1 + t^2\right) dt}{t^4 + t^2 + 1}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} + 1\right)}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} - 2 + 2 + 1\right)}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

Substituting $z = t - \frac{1}{t}$; $dz = \left(1 + \frac{1}{t^2}\right) dt$

$$I = \int \frac{dz}{z^2 + 3}$$

$$\Rightarrow I = \int \frac{dz}{z^2 + \left(\sqrt{3}\right)^2}$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{z}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{3}}\right) + C$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x - \cot x}{\sqrt{3}}\right) + C$$

Ex 19.32

Indefinite Integrals Ex 19.32 Q1

Let $\quad I = \int \frac{1}{(x-1)\sqrt{x+2}}\,dx$

Let $x + 2 = t^2$

$\therefore \quad I = \int \frac{2tdt}{\left(t^2 - 3\right)t}$

$= 2\int \frac{dt}{t^2 - 3}$

$= \frac{2}{2\sqrt{3}} \log\left|\frac{t - \sqrt{3}}{t + \sqrt{3}}\right| + c$

Thus,

$$I = \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.32 Q2

Let $\quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $\quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $2x + 3 = t^2$

$\Rightarrow \quad 2dx = 2tdt$

$\therefore \quad I = \int \frac{t\,dt}{\left(\frac{t^2 - 3}{2} - 1\right)t}$

$= 2\int \frac{dt}{t^2 - 5}$

$= \frac{2}{2\sqrt{5}} \log\left|\frac{t - \sqrt{5}}{t + \sqrt{5}}\right| + c$

Thus,

$$I = \frac{1}{\sqrt{5}} \log\left|\frac{\sqrt{2x+3} - \sqrt{5}}{\sqrt{2x+3} + \sqrt{5}}\right| + c$$

Indefinite Integrals Ex 19.32 Q3

Let $I = \int \frac{1}{(x-1)\sqrt{x+2}}dx$

$I = \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}}dx$

$I = \int \frac{dx}{\sqrt{x+2}} + 2\int \frac{dx}{(x-1)\sqrt{x+2}}$ ---(A)

Now,

$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$

and,

$\int \frac{dx}{(x-1)\sqrt{x+2}}$

Let $x + 2 = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$

$= \frac{2\times 1}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_2$

$= \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| +_2$

Thus, form (A),

$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_2$

Hence,

$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$

Indefinite Integrals Ex 19.32 Q4

Let $I = \int \frac{x^2}{(x-1)\sqrt{x+2}}dx$

$= \int \frac{(x^2-1+1)}{(x-1)\sqrt{x+2}}dx$

$= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$

$= \int \frac{(x+1)}{\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$

$= \int \frac{(x+2)-1}{\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$

$I = \int \sqrt{x+2}dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}}$ ---(A)

Now,

$\int \sqrt{x+2}dx = \frac{2}{3}(x+2)^{\frac{3}{2}} + c_1$

and,

$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$

$\int \frac{dx}{(x-1)\sqrt{x+2}}$

Let $x + 2 = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$

$= \frac{2}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_3$

$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_3$

Thus, form (A)

$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$ [when $c = c_1 + c_2 + c_3$]

Indefinite Integrals Ex 19.32 Q5

Let $I = \int \frac{x}{(x-3)\sqrt{x+1}}dx$

$= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}}dx$

$I = \int \frac{dx}{\sqrt{x+1}} + 3\int \frac{dx}{(x-3)\sqrt{x+1}}$ ---(A)

Now,

$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$

and,

$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$

$\int \frac{dx}{(x-3)\sqrt{x+1}}$

Let $x + 1 = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = 2\int \frac{tdt}{(t^2-4)t}$

$= 2\left|\frac{dt}{t^2-4}\right|$

$= \frac{2}{2\times 2} log\left|\frac{t-2}{t+2}\right| + c_2$

$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2} log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c_2$

Thus, form (A)

$I = 2\sqrt{x+1} + \frac{3}{2} log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c$ [when $c = c_1 + c_2$]

Indefinite Integrals Ex 19.32 Q6

Let $I = \int \frac{x}{\left(x^2+1\right)\sqrt{x}}dx$

Let $x = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad 2\int \frac{tdt}{\left(t^2+1\right)t}$

$= 2\left|\frac{dt}{t^4+1}\right|$

Dividing numerator and denominator by t^2

$$I = 2\int \frac{\frac{t}{t^2}}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt - \int \frac{\left(1-\frac{1}{t^2}\right)}{\left(1+\frac{1}{t}\right)^2-2}dt$$

Let $t-\frac{1}{t} = z$

$\Rightarrow \quad \left(1+\frac{1}{t^2}\right)dt = dz \qquad$ [For Ist part]

and,

$t+\frac{1}{t} = y$

$\Rightarrow \quad \left(1-\frac{1}{t^2}\right)dt = dy \qquad$ [For IInd part]

$$\therefore \quad I = \int \frac{dz}{z^2+2} - \int \frac{dy}{y^2-2}$$

$$= \frac{1}{\sqrt{2}}tan^{-1}\left(\frac{z}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) - \frac{1}{2\sqrt{2}}log\left|\frac{x+1-\sqrt{2x}}{x+1+\sqrt{2x}}\right| + c$$

Indefinite Integrals Ex 19.32 Q7

Let $I=\int \frac{x}{\left(x^2+2X+2\right)\sqrt{x+1}}dx$

Let $x+1=t^2$

$\Rightarrow \quad dx=2tdt$

$$=2\int \frac{\left(t^2-1\right)tdt}{\left(t^4+1\right)t}$$

$$=2\int \frac{\left(t^2-1\right)dt}{\left(t^4+1\right)}$$

$$=2\int \frac{\left(1-\frac{1}{t^2}\right)dt}{t^2+\frac{1}{t^2}}$$

$$=2\int \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-2}$$

Let $t+\frac{1}{t}=y$

$\Rightarrow \quad \left(1-\frac{1}{t^2}\right)dt=dy$

$\therefore \quad I=2\int \frac{dy}{y^2-2}$

$$=\frac{2}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right|+c$$

Thus,

$$I=\frac{1}{\sqrt{2}}\log\left|\frac{t^2+1-\sqrt{2}t}{t^2+1+\sqrt{2}t}\right|+c$$

Hence,

$$I=\frac{1}{\sqrt{2}}\log\left|\frac{x+2-\sqrt{2(x+1)}}{x+2+\sqrt{2(x+1)}}\right|+c$$

Indefinite Integrals Ex 19.32 Q8

Let $I=\int \frac{1}{(x-1)\sqrt{x^2+1}}dx$

Let $x-1=\frac{1}{t}$

$\Rightarrow \quad dx=-\frac{1}{t^2}dt$

$\therefore \quad I=-\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}+1\right)^2+1}}$

$$=-\int \frac{dt}{\sqrt{2t^2+2t+1}}$$

$$=-\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2+t+\frac{1}{2}}}$$

$$=-\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{1}{4}}}$$

$\therefore \quad I=-\frac{1}{\sqrt{2}}\log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{1}{4}}\right|+c \quad \left[\text{When } t=\frac{1}{x-1}\right]$

Indefinite Integrals Ex 19.32 Q9

Let $I = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$

Let $x+1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t^2}+\frac{1}{t}-1\right)}}$

$= -\int \frac{dt}{\sqrt{1+t-t^2}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(\frac{1}{4}-t+t^2\right)}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{1}{2}\right)^2}}$

$= -\sin^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right)+c$

$\therefore \quad I = -\sin^{-1}\left(\frac{2t-1}{\sqrt{5}}\right)+c \qquad \left[\text{When } t = \frac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q10

Let $I = \int \frac{1}{(x^2-1)\sqrt{x^2+1}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\left(\frac{1}{t^2}-1\right)\sqrt{\left(\frac{1}{t^2}+1\right)}}$

$= -\int \frac{tdt}{(1-t^2)\sqrt{1+t^2}}$

Let $1+t^2 = u^2$

$\Rightarrow \quad 2tdt = 2udu$

$I = \int \frac{u\,du}{(u^2-2)u}$

$= \int \frac{du}{u^2-2}$

$\therefore \quad I = \frac{1}{2\sqrt{2}}\log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right|+c$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{1+t^2}-\sqrt{2}}{\sqrt{1+t^2}+\sqrt{2}}\right|+c$

Hence,

$I = -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+\sqrt{x^2+1}}{\sqrt{2}x-\sqrt{x^2+1}}\right|+c$

Indefinite Integrals Ex 19.32 Q11

$$I=\int \frac{x}{\left(x^2+4\right)\sqrt{x^2+1}}dx$$

Let $x^2+1=u^2$

$\Rightarrow \quad 2x dx = 2u\, du$

$$\therefore \quad I=\int \frac{u}{\left(u^2+3\right)u}du$$

$$=\int \frac{1}{u^2+3}du$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)+C$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x^2+1}{3}}\right)+C$$

Indefinite Integrals Ex 19.32 Q12

Let $\quad I=\int \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}}dx$

Let $x=\frac{1}{t}$

$\Rightarrow \quad dx=-\frac{1}{t^2}dt$

$$\therefore \quad I=\int \frac{-\frac{1}{t^2}dt}{\left(\frac{1}{t^2}+1\right)\sqrt{\left(1-\frac{1}{t^2}\right)}}$$

$$=-\int \frac{tdt}{\left(t^2+1\right)\sqrt{t^2-1}}$$

Let $t^2-1=u^2$

$\Rightarrow \quad 2tdt=2udu$

$$I=-\int \frac{udu}{\left(u^2+2\right)u}$$

$$=-\int \frac{du}{u^2+2}$$

$$=-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$$

$$=-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right)+c$$

Thus,

$$I=-\frac{1}{\sqrt{2}}\tan^{-1}\left|\sqrt{\frac{1-x^2}{2x^2}}\right|+c$$

Indefinite Integrals Ex 19.32 Q13

Let $I = \int \frac{1}{\left(2x^2+3\right)\sqrt{x^2-4}}\,dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$$\therefore \quad I = \int \frac{-\frac{1}{t^2}dt}{\left(\frac{2}{t^2}+3\right)\sqrt{\left(\frac{1}{t^2}-4\right)}}$$

$$= -\int \frac{t\,dt}{\left(2+3t^2\right)\sqrt{1-4t^2}}$$

Let $1-4t^2 = u^2$

$\Rightarrow \quad -8t\,dt = 2u\,du$

$$\therefore \quad I = \frac{1}{4}\int \frac{u\,du}{\frac{\left(11-3u^2\right)}{4}u}$$

$$= \frac{1}{3}\int \frac{du}{\frac{11}{3}-u^2}$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{u-\sqrt{\frac{11}{3}}}{u+\sqrt{\frac{11}{3}}}\right|+c$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{1-4t^2}-\sqrt{\frac{11}{3}}}{\sqrt{1-4t^2}+\sqrt{\frac{11}{3}}}+c\right|$$

Hence,

$$I = \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{11}x+\sqrt{3x^2-12}}{\sqrt{11}x-\sqrt{3x^2-12}}\right|+c$$

Indefinite Integrals Ex 19.32 Q14

$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+9}}\,dx$

Let $x^2+9 = u^2$

$\Rightarrow \quad 2x\,dx = 2u\,du$

$$\therefore \quad I = \int \frac{u}{\left(u^2-5\right)u}\,du$$

$$= \int \frac{du}{u^2-5}$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{u-\sqrt{5}}{u+\sqrt{5}}\right)+C$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{\sqrt{x^2+9}-\sqrt{5}}{\sqrt{x^2+9}+\sqrt{5}}\right)+C$$