Previous Years Paper

02nd June 2023 (Shift 3)

Q1. Corner points of a feasible bounded region are (0, 10), (4, 2), (3, 7) and (10, 6). Maximum value 50 of objective function $z = ax +$ by occurs at two points (0, 10) and (10, 6). The value of a and b are:
(a) a = 5, b = 2
(b) a = 4, b = 5
(c) a = 2, b = 5
(d) a = 5, b = 4

Q2. Degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = y^2 + e^x$ is:
(a) 1
(b) 2
(c) 3
(d) 4

Q3. General solution of the differential equation $\frac{2ydx-3xdy}{y} = 0$ is (c is an arbitrary constant)
(a) y = cx
(b) $y^3 = x^2$
(c) $y^3 = cx^2$
(d) y = cx^2

Q4. Which of the following is the probability of x successes in a binomial distribution with number of trails n and probability of success as θ $(0 < \theta < 1)$ in each trail?
(a) ${}^np_x\theta^x(1-\theta)^{n-x}, x = 0, 1, 2 ----, n$
(b) ${}^nc_x\theta^x(1-\theta)^{n-x}, x = 0, 1, 2 ----, n$
(c) ${}^nc_x\theta\,(1-\theta), x = 0, 1, 2 ----, n$
(d) ${}^nc_x\theta^x\,(1-\theta)^x, x = 0, 1, 2 ----, n$

Q5. $\int_2^3 |2x-1|\,dx =$
(a) 4
(b) 5
(c) 6
(d) 8

Q6. If $A = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$ and A + A' = 2I, then x is equal to
(a) 0
(b) –1
(c) 0.5
(d) 1

Q7. Let X be a random variable whose probability distribution is given by the table

X	1	3	5	7
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Then variance of x is
(a) $\sqrt{\frac{57}{3}}$
(b) $\frac{19}{3}$
(c) 4
(d) $\frac{67}{3}$

Q8. Differentiation of $\log_5(\log x^2)$ w.r.t.x is
(a) $\frac{2}{x^2(\log 5)(\log x^2)}$
(b) $\frac{2}{x(\log 5)(\log x)}$
(c) $\frac{1}{x(\log 5)(\log x^2)}$
(d) $\frac{1}{x(\log 5)(\log x)}$

Q9. If B is a non-singular 4 × 4 matrix and A is its adjoint such that $|A| = 125$, then $|B|$ is
(a) 5
(b) 25
(c) 125
(d) 625

Q10. Value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} =$
(a) 0
(b) 3 (a – b – c)
(c) 2a – 2b – 2c
(d) –(a + b + c)

Q11. Match List I with List II

	LIST I		LIST II
A.	Slope of the tangent to curve $x^3 - 2x$ at $x = 2$	I.	–81
B.	Slope of line passing through the points (0, 2) and (5, –6)	II.	10
C.	Point at which the tangent to the curve $y = \sqrt{4x-3}$ has its slope $\frac{2}{3}$	III.	$-\frac{8}{5}$
D.	Slope of normal to the curve $y = \frac{x-2}{x-1}$ at x = 10	IV.	(3, 3)

Choose the correct answer from the options given below:
(a) A-II, B-III, C-IV, D-I
(b) A-III, B-II, C-I, D-IV
(c) A-III, B-I, C-IV, D-II
(d) A-I, B-IV, C-II, D-III

Q12. The corner points of the feasible region determined by system of linear constraints are (60, 0), (120, 0), (40, 20) and (60, 30). Let z = ax + by, a, b>0 be the objective function. Find condition on a and b so that the maximum of z occurs at (120, 0) and (60, 30).
(a) $b = \frac{a}{3}$
(b) 2b = a
(c) 2a = b
(d) $a = \frac{b}{3}$

Q13. The sum of values of a and b such that the function $f(x)$ defined by

$f(x) = \begin{cases} 3, & x \leq 1 \\ ax+b, 1 & 1 < x < 5 \\ 10, & x \geq 5 \end{cases}$ is a continuous function is

(a) $\frac{11}{9}$
(b) $\frac{9}{4}$
(c) 3
(d) $\frac{1}{3}$

Q14. The tangent to the circle centre at (0, 0) and with radius = 1 at point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ on it is given by
(a) $x - y = 0$
(b) $x + y = \sqrt{2}$
(c) $2\sqrt{2}x - 3\sqrt{2}\,y = -1$
(d) $3\sqrt{2}\,x + \sqrt{2}\,y = 4$

Q15. $\int \frac{dx}{x^a} =$
(a) $\frac{x^{1-a}}{1-a}$+C for all $a \in R$
(b) $2\sqrt{x+1} + C$ for $a = \frac{1}{2}$
(c) $\frac{3}{2}x^{\frac{3}{2}} + C$ for $a = \frac{-1}{2}$
(d) $\frac{3}{4}x^{\frac{4}{3}} + C$ for $a = \frac{-1}{3}$

Q16. $\sin\left(2\tan^{-1}\frac{5}{12}\right)$ is equal to
(a) $\frac{120}{169}$
(b) $-\frac{120}{169}$
(c) $\frac{169}{120}$
(d) $-\frac{169}{120}$

Q17. Position vector of four points A, B, C, D are $-\hat{\imath} + \hat{\jmath} + \hat{k}, 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}, 4\hat{\imath} - \lambda\hat{\jmath} - \hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively. The value of λ for which the points A, B, C, D are coplanar is
(a) 7
(b) –7
(c) $\frac{1}{7}$
(d) $\frac{-1}{7}$

Q18. If A is a square matrix of order 3 such that $|2(adj\ A)| = 288$, then the value of $|A|$ is
(a) 144
(b) 36
(c) ± 12
(d) ± 6

Q19. If m and M are respectively minimum and maximum values of $f(x) = \big|2 - |x|\big|, -3 \leq x \leq 3$, then
(a) m = 0 and M = 2
(b) m = 1 and M = 2
(c) m = 0 and M = 4
(d) m = 0 and M = 1

Q20. If A (adj A) = $\begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$, then the value of |A| is
(a) –125
(b) –5
(c) 5
(d) 125

Q21. The function $f(x) = \frac{1}{12}(3x^4 + 4x^3 - 12x^2)$ decreases in
(a) $(-\infty, -2) \cup (0, 1)$
(b) $(-\infty, 2]$
(c) $[-2, 0]$
(d) $(-\infty, -2] \cap [0, 1]$

Q22. If the distance of the point (4, 6, 8) from the plane $\vec{r}.\left(6\hat{\imath} - 12\hat{\jmath} + 4\hat{k}\right) = a$ is 1, then a is:
(a) 2 or 30
(b) 1 or 15
(c) –2 or –30
(d) – or –15

Q23. Let R be an equivalence relation on the set A = {1, 2, 3, 4, 5} given by R = {(x, y):2 divides (x – y)}. Then equivalence class of 3 is:
(a) {1, 5}
(b) {1, 3, 5}
(c) {3, 5}
(d) {2, 4}

Q24. If 3x + y = 8 is a tangent to the curve $y^2 = \alpha + \beta x^3$ at (2, 2), then value of $\alpha - \beta$ is
(a) 12
(b) –1
(c) 11
(d) 13

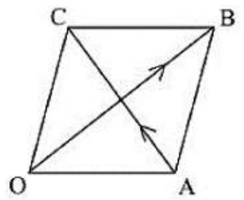
Q25. The value of the integral $\int_0^{\frac{\pi}{4}} \log_e(1 + \tan x)dx$ is:
(a) $\frac{\pi}{12}\log_e 4$
(b) $\frac{\pi}{12}\log_e 2$
(c) $\frac{\pi}{16}\log_e 4$
(d) $\frac{\pi}{16}\log_e 2$

Q26. The feasible region corresponding to an LPP represented by the constraints $x \geq 7, y \geq 4, x + 2y \geq 8$ is
(a) bounded and feasible
(b) Unbounded and feasible
(c) bounded and not feasible
(d) Concave polygon, unbounded and feasible

Q27. If $f(x) = \begin{cases} \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x} & x \neq \frac{\pi}{4} \\ k & x = \frac{\pi}{4} \end{cases}$ is continuous at x =$\frac{\pi}{4}$, then the value of k will be equal to
(a) 1
(b) 2
(c) $\frac{1}{2}$
(d) –1

Q28. If $\vec{a}$and $\vec{b}$ are two vectors such that $|\vec{a}| = 7$ and $|\vec{b}| = 4$, then the value of scalar product of vectors $2\vec{a} - 3\vec{b}$ and $2\vec{a} + 3\vec{b}$ is
(a) 52
(b) 25
(c) 340
(d) 430

Q29. OA BC is a parallelogram. If $\overrightarrow{OB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$, then $\overrightarrow{OA}$ is equal to



(a) $\vec{a} - \vec{b}$
(b) $\frac{\vec{a}-\vec{b}}{2}$
(c) $\vec{a} + \vec{b}$
(d) $\frac{\vec{a}+\vec{b}}{2}$

Q30. If A is a square matrix such that $A^2 = 2I$, then the value of $(A - I)^2 + (A + I)^2 - 7A$ is
(a) 2I
(b) −7A + 6I
(c) −8I-7A
(d) 8I+7A

Q31. Area of the region bounded by $|x| + |y| \leq 2$ is:
(a) 16
(b) 4
(c) 12
(d) 8

Q32. If A is a square matrix of order 3 and |A| = −3, then value of $|2AA^T|$ is
(a) −36
(b) −72
(c) 72
(d) 36

Q33. If $\tan^{-1}(-3x) + \tan^{-1}(-2x) = \frac{\pi}{4}$, then the values of x are
(a) $-1, \frac{-1}{6}$
(b) $-1, \frac{1}{16}$
(c) $1, \frac{-1}{6}$
(d) $1, \frac{1}{6}$

Q34. Let R be the relation on N (set of Natural numbers) defined by R = {(a, b) : a, b ∈ N and b is divisible by a}. Then the relation R is
(a) Reflexive, symmetric but not Transitive.
(b) Reflexive, Transitive but not symmetric.
(c) Not Reflexive, not transitive, not symmetric.
(d) Equivalence relation.

Q35. If A is a matrix and |A|≠ 0, then solution of the equation XA = B is:
(a) $X = A^{-1}B$
(b) $X = \frac{1}{|A|}B$
(c) $X = BA^{-1}$
(d) $X = AB^{-1}$

Q36. If $f: R \to R$ is a function given by $f(x) = [x]$ (greatest integer function), then which of the following is/are correct.
(a) f is one-one
(b) f is not onto
(c) Range of f is I (set of the integers)
(d) f (2. 5) = 2
(e). f is bijective
Choose the correct answer from the options given below:
(a) C, E only
(b) B, C, D only
(c) A, B only
(d) C, D only

Q37. The point which does not lie in the solution region of $2x - y \leq 4$ is
(a) (1, -1)
(b) (5, 1)
(c) (0, 2)
(d) (1, 1)

Q38. If the equation of a line PQ is $\frac{x+1}{2} = \frac{2-y}{5} = \frac{z+6}{7}$, then the direction cosines of a line parallel to PQ are
(a) $\frac{2}{\sqrt{78}}, \frac{5}{\sqrt{78}}, \frac{7}{\sqrt{78}},$
(b) $\frac{-5}{\sqrt{78}}, \frac{-2}{\sqrt{78}}, \frac{7}{\sqrt{78}},$
(c) $\frac{5}{\sqrt{78}}, \frac{-2}{\sqrt{78}}, \frac{7}{\sqrt{78}},$
(d) $\frac{2}{\sqrt{78}}, \frac{-5}{\sqrt{78}}, \frac{7}{\sqrt{78}},$

Q39. Find equation of a line through the point (-2, 1, 3) and parallel to the line $\frac{x-2}{4} = \frac{y+3}{-3}, z = -2$
(a) $\frac{x+2}{4} = \frac{y-1}{-3}, z = 3$
(b) $\frac{x+2}{4} = \frac{y-1}{-3}, z = -3$
(c) $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z-3}{1}$
(d) $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+3}{1}$

Q40. For a Binomial distribution B (n, p), $\frac{E(x)}{V(x)}$is equal to: (symbols have their usual meaning)
(a) $\frac{1}{p^2}$
(b) 1−p
(c) $\frac{1}{1+p}$
(d) $\frac{1}{1-p}$

Q41. If A = $\begin{bmatrix} 0 & -3 & 2 \\ a & b & 8 \\ -2 & c & 0 \end{bmatrix}$ is skew-symmetric matrix, then
(a) b = 0 and a + c =5
(b) a = 0 and b + c = 5
(c) b = 0 and a + c = -5
(d) c = 0 and a + b = -5

Q42. The number of arbitrary constants in a particular solution of a differential equation of 4th order is
(a) 3
(b) 4
(c) 5
(d) 0

Q43. The area enclosed by the curve $\frac{x^2}{16}+\frac{y^2}{25}=1$ is:
(a) 2π
(b) 20π
(c) 40π
(d) 5π

Q44. The integrating factor of the differential equation $x\frac{dy}{dx}-2y=x^3$ is
(a) x^2
(b) $\frac{1}{x^2}$
(c) $-x^2$
(d) $\frac{-1}{x^2}$

Q45. The area enclosed by the parabola $x^2=6y$ and the line $x-6y+2=0$ is:
(a) $\frac{1}{3}$
(b) $\frac{3}{8}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$

Q46. If $y=\frac{e^{-x}+e^{x}}{e^{-x}-e^{x}}$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{4}{(e^{-x}-e^{x})^2}$
(b) $\frac{2}{(e^{-x}-e^{x})^2}$
(c) $\frac{-4}{(e^{-x}-e^{x})^2}$
(d) $\frac{-2}{(e^{-x}-e^{x})^2}$

Q47. The surface area of an open box with a square base in 36 units. It's maximum volume (in cubic units) is:
(a) $9\sqrt{3}$
(b) $12\sqrt{3}$
(c) 15
(d) 18

Q48. The integral $\int\frac{e^{-x}}{9+4e^{-2x}}dx$ is equal to
(a) $\frac{1}{6}\tan^{-1}\left(\frac{2e^{-x}}{3}\right)+C$
(b) $\frac{1}{6}\tan^{-1}\left(\frac{2e^{-x}}{3}\right)+C$
(c) $\frac{1}{12}\tan^{-1}\left(\frac{2e^{-x}}{3}\right)+C$
(d) $-\frac{1}{12}\tan^{-1}\left(\frac{2e^{-x}}{3}\right)+C$

Q49. Kashvi, Kalyani and Sara of class 12 are given a problem in Accounts whose respective probabilities of solving are $\frac{2}{5},\frac{1}{4}$ and $\frac{1}{6}$. They were asked to solve it independently.
The probability that the problem is solved is:
(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{5}{8}$
(d) $\frac{7}{8}$

Q50. Kashvi, Kalyani and Sara of class 12 are given a problem in Accounts whose respective probabilities of solving are $\frac{2}{5},\frac{1}{4}$and $\frac{1}{6}$. They were asked to solve it independently.
The probability that either only Kalyani or Kashvi or Sara solves it is:
(a) $\frac{9}{20}$
(b) $\frac{7}{20}$
(c) $\frac{5}{20}$
(d) $\frac{3}{20}$

SOLUTIONS

S1. Ans. (c)

Sol. Given points are (0, 10), (4, 2), (3, 7) and (10, 6).
Also given maximum value of z is 50 which occurs at (0, 10) & (10, 6), then we have

$a(0) + b(10) = a(10) + b(6) = 50$

$10b = 10a + 6b = 50$

If $10b = 50$

$\Rightarrow b = 5$

$10a + 6b = 50 \Rightarrow 10a + 6 \times 5 = 50$

$10a = 20$

$a = 2$

S2. Ans. (b)

Sol. Given differential equation can be written as

$-3\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = \frac{d^2y}{dx^2} - y^2 - e^x$

On squaring both sides, we get

Degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = y^2 + e^x$ is 2.

S3. Ans. (c)

Sol. Given differential equation is

$\frac{2ydx - 3xdy}{y} = 0$

$2ydx = 3xdy$

$3\frac{dy}{y} = 2\frac{dx}{x}$

Integrating

$3\log y = 2\log x + \log c$

$\log y^3 = \log x^2 + \log c$

$\log y^3 = \log cx^2$

$y^3 = cx^2$

S4. Ans. (b)

Sol. ${}^nc_x\theta^x(1-\theta)^{n-x}, x = 0, 1, 2 - - - -, n$

S5. Ans. (a)

Sol. Given integral is $\int_2^3 |2x-1|\,dx$

Put $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$\Rightarrow \int_2^3 |2x-1|\,dx = \int_2^3 (2x-1)dx = (x^2 - x)_2^3$

$= [(3)^2 - 3] - [(2)^2 - 2]$

$= 6 - 2 = 4$

S6. Ans. (d)

Sol. Given

$A = \begin{bmatrix} x & -y \\ y & x \end{bmatrix} \Rightarrow A' = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

$A + A' = 2I$

$\Rightarrow \begin{bmatrix} x & -y \\ y & x \end{bmatrix} + \begin{bmatrix} x & y \\ -y & x \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow 2x = 2 \Rightarrow x = 1$

S7. Ans. (b)

Sol. Given

$P(1) = \frac{1}{3}, P(3) = \frac{1}{6}, P(5) = \frac{1}{6}, P(7) = \frac{1}{3}$

$E(x) = 1 \times \frac{1}{3} + 3 \times \frac{1}{6} + 5 \times \frac{1}{6} + 7 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{2} + \frac{5}{6} + \frac{7}{3}$

$= 4$

$E(x^2) = 1^2 \times \frac{1}{3} + 3^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 7^2 \times \frac{1}{3} = \frac{1}{3} + \frac{9}{6} + \frac{25}{6} + \frac{49}{3}$

$= \frac{134}{6}$

Variance $= E(x^2) - [E(x)]^2 = \frac{134}{6} - (4)^2$

$= \frac{134}{6} - 16 = \frac{134 - 96}{6} = \frac{38}{6} = \frac{19}{3}$

S8. Ans. (d)

Sol. We have

$\log_5(\log x^2) = \frac{\log(\log x^2)}{\log 5}$

d. w. r. to x,

$\frac{d}{dx}\{\log_5(\log x^2)\} = \frac{1}{\log 5} \times \frac{d}{dx}[\log(\log x^2)]$

$= \frac{1}{\log 5} \times \frac{1}{\log x^2} \times \frac{1}{x^2} \times 2x$

$= \frac{2}{xlog\, 5 \log x^2} = \frac{2}{x \log 5 \times 2\log x} = \frac{1}{x \log 5\, logx}$

S9. Ans. (a)

Sol. Given

$A = adj(B)$ & $|A| = 125$

$|A| = |adj(B)|$

We have

$|adj(B)| = |B|^{4-1} = |B|^3$

$|A| = |B|^3$

$|B| = |A|^{\frac{1}{3}} = (125)^{\frac{1}{3}} = 5$

S10. Ans. (a)

Sol. Given

$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Since all elements of first row are zero.

S11. Ans. (a)

Sol. (A) Given curve

$y = x^3 - 2x$ at $x = 2$

Slope of tangent $\left(\frac{dy}{dx}\right) = 3x^2 - 2$

At $x = 2$

$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 2 = 12 - 2 = 10$

(B) Slope of line passing through the points (0, 2) and (5, –6)$= \frac{-6-2}{5-0} = -\frac{8}{5}$

(C) Given curve

$y = \sqrt{4x-3}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4x-3}} \times 4 = \frac{2}{\sqrt{4x-3}}$

ATQ.

$\frac{2}{\sqrt{4x-3}} = \frac{2}{3}$

$\sqrt{4x-3} = 3$

Squaring both sides

$4x - 3 = 9 \Rightarrow 4x = 12 \Rightarrow x = 3$

$y = \sqrt{4(3)-3} = \sqrt{9} = 3$

Points are (3, 3)

(D) Given curve

$y = \frac{x-2}{x-1}$

$\frac{dy}{dx} = \frac{(x-1)\times 1 - (x-2)\times 1}{(x-1)^2} = \frac{1}{(x-1)^2}$

Slope of tangent at $x = 10$

$= \frac{1}{(10-1)^2} = \frac{1}{81}$

Slope of normal $= -81$

S12. Ans. (c)

Sol. The corner points of the feasible region determined by system of linear constraints are (60, 0), (120, 0), (40, 20) and (60, 30).

Also given

$z = ax + by$

ATQ.

$a(120) + b(0) = a(60) + b(30)$

$120a = 60a + 30b$

$60a = 30b$

$2a = b$

S13. Ans. (c)

Sol. We have

$$f(x) = \begin{cases} 3, & x \le 1 \\ ax+b, 1 & 1 < x < 5 \\ 10, & x \ge 5 \end{cases}$$

Continuity at $x = 1$

L.H.L. = 3

R.H.L.

$\lim_{x\to 1^-} ax + b = \lim_{h\to 0} a(1-h) + b = a + b$

$f(1) = 3$

$a + b = 3$

S14. Ans. (b)

Sol. Equation of circle is

$x^2 + y^2 = 1$

d. w. r. to x

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

Slope of tangent at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Slope $= -1$

Equation of tangent is

$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$

$y - \frac{1}{\sqrt{2}} = -x + \frac{1}{\sqrt{2}}$

$y + x = \sqrt{2}$

S15. Ans. (d)

Sol. We have

$\int \frac{dx}{x^a} = \int x^{-a} dx = \frac{x^{-a+1}}{-a+1} + C$

$= \frac{x^{1-a}}{1-a} + C = \frac{3}{4}x^{\frac{4}{3}} + C \; for \; a = -\frac{1}{3}$

S16. Ans. (a)

Sol. We have

$\sin\left(2\tan^{-1}\frac{5}{12}\right) = \sin\left\{sin^{-1}\left(\frac{2\times\frac{5}{12}}{1+\left(\frac{5}{12}\right)^2}\right)\right\} = \frac{\frac{10}{12}}{\frac{144+25}{144}}$

$= \frac{10}{12} \times \frac{144}{169} = \frac{120}{169}$

S17. Ans. (b)

Sol. Position vector of four points A, B, C, D are $-\hat{\imath} + \hat{\jmath} + \hat{k}, 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}, 4\hat{\imath} - \lambda\hat{\jmath} - \hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively.

We have

$\overrightarrow{AB} = (3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) - (-\hat{\imath} + \hat{\jmath} + \hat{k}) = 4\hat{\imath} - 3\hat{\jmath} + \hat{k}$

$\overrightarrow{AC} = (4\hat{\imath} - \lambda\hat{\jmath} - \hat{k}) - (-\hat{\imath} + \hat{\jmath} + \hat{k}) = 5\hat{\imath} + (-1 - \lambda)\hat{\jmath} - 2\hat{k}$

$\overrightarrow{AD} = (\hat{\imath} + \hat{\jmath} + \hat{k}) - (-\hat{\imath} + \hat{\jmath} + \hat{k}) = 2\hat{\imath} + 0.\hat{\jmath} + 0.\hat{k}$

Vectors are coplanar if

$[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$

$$\begin{vmatrix} 4 & -3 & 1 \\ 5 & -1-\lambda & -2 \\ 2 & 0 & 0 \end{vmatrix} = 0$$

$4(0) + 3(0+4) + 1(0 + 2 + 2\lambda) = 0$

$12 + 2 + 2\lambda = 0$

$14 + 2\lambda = 0$

$\lambda = -7$

S18. Ans. (d)

Sol. If A is a square matrix of order 3 such that $|2(adj\,A)| = 288.$

$2^3|adj(A)| = 288$

$2^3|A|^2 = 288$

$|A|^2 = \frac{288}{8} = 36$

$|A| = \pm 6$

S19. Ans. (a)

Sol. Given

$f(x) = |2 - |x||, -3 \le x \le 3$

$|x| \le 3$

$f(x) = |2 - 0| = 2$ (maxima)

$f(x) = |2 - 2| = 0$ (minima)

Sol. Given

$$A\,(\text{adj } A) = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

We have

$A.(adjA) = |A|I$

$$|A|I = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$\Rightarrow |A| = -5$

S21. Ans. (a)

Sol. Given

$f(x) = \dfrac{1}{12}(3x^4 + 4x^3 - 12x^2)$

$f'(x) = \dfrac{1}{12}(12x^3 + 12x^2 - 24x) = x^3 + x^2 - 2$

$f'(x) = 0 \Rightarrow x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x-1)(x+2) = 0$

$x = 0, 1, -2$

Intervals are $(-\infty, -2), (-2, 0), (0, 1)$ & $(1, \infty)$

$f'(x) < 0$ in $(-\infty, -2) \cup (0, 1)$

S22. Ans. (c)

Sol. If the distance of the point (4, 6, 8) from the plane $\vec{r}.(6\hat{\imath} - 12\hat{\jmath} + 4\hat{k}) = a$ is 1

$\left|\dfrac{6(4) - 12(6) + 4(8) - a}{\sqrt{(6)^2 + (-12)^2 + (4)^2}}\right| = 1$

$\left|\dfrac{24 - 72 + 32 - a}{\sqrt{36 + 144 + 16}}\right| = 1$

$\left|\dfrac{-16 - a}{\sqrt{196}}\right| = 1 \Rightarrow \left|\dfrac{16 + a}{14}\right| = 1$

$\Rightarrow \dfrac{16 + a}{14} = \pm 1$

$16 + a = 14 \Rightarrow a = -2$

$16 + a = -14 \Rightarrow a = -30$

S23. Ans. (b)

Sol. A = {1, 2, 3, 4, 5} given by R = {(x, y):2 divides (x – y)}.

Reflexive: We have

$(x, x) \in R$

Since 2 divides $x - x$ i.e. 0

So, R is reflexive.

Symmetric: Let $(x, y) \in R$, then 2 divides $x - y$.

If 2 divides $x - y$, then 2 also divides $-(x - y)$ i.e. $y - x$

$\Rightarrow (y, x) \in R$

So, R is symmetric.

Transitive: Let $(x, y), (y, z) \in R$, then 2 divides $x - y$ and $y - z$.

Since 2 divides $x - y$ and $y - z$.

Then, 2 divides their sum $(x - y + y - z)$ i.e. $x - z$

$\Rightarrow (x, z) \in R$

Equivalence class of 3 is $\{1, 3, 5\}$

S24. Ans. (d)

Sol. Given curve is

$y^2 = \alpha + \beta x^3$

$2y\dfrac{dy}{dx} = 0 + 3\beta x^2$

$\dfrac{dy}{dx} = \dfrac{3\beta x^2}{2y}$

Tangent at (2, 2) is

$\left.\dfrac{dy}{dx}\right|_{(2,2)} = \dfrac{3\beta(2)^2}{2 \times 2} = 3\beta$

Now equation of tangent is

$y - 2 = 3\beta(x - 2)$

$y - 3\beta x = 2 - 6\beta$ .... (i)

Also given tangent to the curve is

$3x + y = 8$ .... (ii)

Equating (i) & (ii)

$-3\beta = 3 \Rightarrow \beta = -1$

$y^2 = \alpha + \beta x^3 \Rightarrow (2)^2 = \alpha + (-1)(2)^3 \Rightarrow 4 = \alpha - 8 \Rightarrow \alpha = 12$

$\alpha - \beta = 12 - (-1) = 13$

S25. Ans. (c)

Sol. We have

$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$ .... (i)

$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$

$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right] dx = \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan x}\right] dx$ .... (ii)

Adding (i) & (ii), we get

$I + I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx + \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1 + \tan x}\right] dx$

$2I = \int_0^{\frac{\pi}{4}} \log 2\, dx$

$2I = \dfrac{\pi}{4} \log 2$

$I = \dfrac{\pi}{8} \log 2$

$I = \dfrac{\pi}{16} \times 2 \log 2 = \dfrac{\pi}{16} \log 4$

S26. Ans. (b)

Sol. Given

$x \geq 7, y \geq 4, x + 2y \geq 8$ is

$x = 7$

$y = 4$

$x + 2y = 8$

Unbounded and feasible

S27. Ans. (c)

Sol. Given

$$f(x) = \begin{cases} \dfrac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x} & x \neq \dfrac{\pi}{4} \\ k & x = \dfrac{\pi}{4} \end{cases}$$

Continuity At $x = \frac{\pi}{4}$

L.H.L.

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x} \quad \{\text{Form } \tfrac{0}{0}\}$$

By L' Hospital rule

$$\lim_{x\to\frac{\pi}{4}} \frac{\sec^2\left(\frac{\pi}{4}-x\right)\times -1}{-cosec^2 2x \times 2}$$

$$= \frac{\sec^2(0)}{2cosec^2\left(\frac{\pi}{2}\right)} = \frac{1}{2}$$

$$f\left(\frac{\pi}{4}\right) = k = \frac{1}{2}$$

S28. Ans. (a)

Sol. $|\vec{a}| = 7$ and $|\vec{b}| = 4$, then

$(2\vec{a} - 3\vec{b}).(2\vec{a} + 3\vec{b}) = 4|\vec{a}|^2 - 9|\vec{b}|^2$

$= 4 \times (7)^2 - 9 \times (4)^2 = 196 - 144 = 52$

S29. Ans. (b)

Sol. Given

$\overrightarrow{OB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{AB} \Rightarrow \overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{AO}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} \Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AC} - \overrightarrow{AO} = \overrightarrow{OB} - \overrightarrow{OA}$

$\vec{b} + \overrightarrow{OA} = \vec{a} - \overrightarrow{OA}$

$2\overrightarrow{OA} = \vec{a} - \vec{b}$

$\overrightarrow{OA} = \frac{\vec{a}-\vec{b}}{2}$

S30. Ans. (b)

Sol. Given

$A^2 = 2I$

Now

$(A-I)^2 + (A+I)^2 - 7A = A^2 + I^2 - 2AI + A^2 + I^2 + 2AI - 7A$

$= 2A^2 + 2I - 7A$

$= 2(2I) + 2I - 7A = 6I - 7A$

S31. Ans. (d)

Sol. Given

$|x| + |y| \le 2$

Here, given function is |x| + |y| = 2

The equations of the straight lines in each quadrant.

For 1st quadrant $x + y = 2; x > 0$ and y > 0

For 2nd quadrant $-x + y = 2; x < 0$ and $y > 0$

For 3rd quadrant $-x - y = 2; x < 0$ and $y < 0$

For 4th quadrant $x - y = 2; x > 0$ and $y < 0$

∴ The co-ordinates we get, (2,0), (0,2), (-2,0), (0,-2)

So, by plotting them, we have a Square with length of each side as $2\sqrt{2}$ unit.

∴ Area enclosed by the function $= 2\sqrt{2} \times 2\sqrt{2} = 8$ sq. unit

S32. Ans. (c)

Sol. Given

A is a square matrix of order 3 and |A| = –3, then

$|2AA^T| = 2^3|A||A^T| = 8 \times (-3) \times (-3) = 72$

{Since $|kA| = k^n|A|$ & $|A^T| = |A|$ }

S33. Ans. (c)

Sol. We have

$$\tan^{-1}(-3x) + \tan^{-1}(-2x) = \frac{\pi}{4}$$

$$-\tan^{-1}3x - \tan^{-1}2x = \frac{\pi}{4}$$

$$-(\tan^{-1}3x + \tan^{-1}2x) = \frac{\pi}{4}$$

$$-\tan^{-1}\left(\frac{3x+2x}{1-3x\times 2x}\right) = \frac{\pi}{4}$$

$$-\tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = \tan\left(-\frac{\pi}{4}\right)$$

$$\frac{5x}{1-6x^2} = -1$$

$$5x = 6x^2 - 1 \Rightarrow 6x^2 - 5x - 1 = 0$$

$$x = \frac{5\pm\sqrt{25+24}}{12} = \frac{5\pm 7}{12}$$

$$x = 1, -\frac{1}{6}$$

S34. Ans. (b)

Sol. Given

R = {(a, b) : a, b ∈ N and b is divisible by a}

Reflexive: We have

$(x, x) \in R$ since x is divisible by x.

Symmetric: Let $(x, y) \in R \Rightarrow y$ is divisible by x.

$\Rightarrow x$ is not divisible by y.

$\Rightarrow (y, x) \notin R$

So, R is not symmetric.

Transitive: Let $(x, y) \in R$, $(y, z) \in R$, then y divides x & z divides y.

Since z divides y and y divides x.

$\Rightarrow z$ divides x.

$\Rightarrow (x, z) \in R$

So, R is transitive.

S35. Ans. (c)

Sol. If A is a matrix and |A|≠ 0, then solution of the equation XA = B is $X = BA^{-1}$

S36. Ans. (b)

Sol. Given

$f(x) = [x]$

f is neither one – one nor onto

Range of f is I (set of the integers)

f (2. 5) = 2

S37. Ans. (b)

Sol. Given

$2x - y \le 4$

Since (5, 1) does not satisfy the inequality. So, (5, 1) not lie in the solution of $2x - y \le 4$.

S38. Ans. (d)

Sol. Given equation of line is

$$\frac{x+1}{2} = \frac{y-2}{-5} = \frac{z+6}{7}$$

Direction ratios are 2, 5, 7

So, direction cosines are

$$\frac{2}{\sqrt{(2)^2+(-5)^2+(7)^2}}, \frac{-5}{\sqrt{(2)^2+(-5)^2+(7)^2}}, \frac{7}{\sqrt{(2)^2+(-5)^2+(7)^2}}$$

i.e. $\frac{2}{\sqrt{78}}, \frac{-5}{\sqrt{78}}, \frac{7}{\sqrt{78}}$

S39. Ans. (a)

Sol. Given equation of line is

$\frac{x-2}{4} = \frac{y+3}{-3}, z = -2$ i.e. $\frac{x-2}{4} = \frac{y+3}{-3} = \frac{z+2}{1}$

Equation of line through the point $(-2, 1, 3)$ and parallel to the line $\frac{x-2}{4} = \frac{y+3}{-3} = \frac{z+2}{1}$ is

$$\frac{x+2}{4} = \frac{y-1}{-3} = \frac{z-3}{1}$$

Or $\frac{x+2}{4} = \frac{y-1}{-3}, z = 3$

S40. Ans. (d)

Sol. For binomial distribution we have

$\text{Var(X)} = E(X)(1-p)$

$1 - p = \frac{\text{Var(X)}}{E(X)}$

$\frac{E(X)}{Var(X)} = \frac{1}{p-1}$

S41. Ans. (c)

Sol. Given $A = \begin{bmatrix} 0 & -3 & 2 \\ a & b & 8 \\ -2 & c & 0 \end{bmatrix}$ is skew-symmetric matrix, then

$\Rightarrow a = 3, b = 0, c = -8$

$a + c = 3 - 8 = -5, b = 0$

S42. Ans. (d)

Sol. The number of arbitrary constants in a particular solution of a differential equation of 4th order is 0.

S43. Ans. (b)

Sol. Given curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$\frac{x^2}{16} = 1 - \frac{y^2}{25}$$

$$x = \frac{4}{5}\sqrt{25 - y^2}$$

Area of bounded region $= 4\int_0^5 x dy =$

$4\int_0^5 \frac{4}{5}\sqrt{(5)^2 - y^2}\, dy$

$= \frac{16}{5}\left\{\frac{y}{2}\sqrt{25 - y^2} + \frac{25}{2}\sin^{-1}\left(\frac{y}{5}\right)\right\}_0^5 = \frac{16}{5} \times \frac{25}{2} \times \frac{\pi}{2} = 20\pi$

S44. Ans. (b)

Sol. Given differential equation is

$x\frac{dy}{dx} - 2y = x^3$

$$\frac{dy}{dx} - \frac{2}{x}y = x^2$$

Here $P = -\frac{2}{x}$

Integrating factor $= e^{\int P dx} = e^{\int -\frac{2}{x} dx} =$

$e^{-2\log x} = x^{-2} = \frac{1}{x^2}$

S45. Ans. (d)

Sol. Given

parabola $x^2 = 6y$ and the line $x - 6y + 2 = 0$

$x^2 = 6y$ & $y = \frac{x+2}{6}$

Solving above equations, we get

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

Area of bounded region $= \int_{-1}^{2}\left(\frac{x+2}{6} - \frac{x^2}{6}\right)dx$

$= \frac{1}{6}\int_{-1}^{2}(x + 2 - x^2)dx$

$= \frac{1}{6}\left\{\frac{x^2}{2} + 2x - \frac{x^3}{3}\right\}_{-1}^{2} = \frac{1}{6}\left\{2 + 4 - \frac{8}{3}\right\} - \frac{1}{6}\left\{\frac{1}{2} - 2 + \frac{1}{3}\right\}$

$= \frac{3}{4}$

S46. Ans. (a)

Sol. Given

$y = \frac{e^{-x} + e^x}{e^{-x} - e^x} = \frac{e^{-x}(1+e^{2x})}{e^{-x}(1-e^{2x})} = \frac{1+e^{2x}}{1-e^{2x}}$

$$\frac{dy}{dx} = \frac{(1-e^{2x}) \times 2e^{2x} - (1+e^{2x}) \times -2e^{2x}}{(1-e^{2x})^2}$$

$$= \frac{2e^{2x} - 2e^{4x} + 2e^{2x} + 2e^{4x}}{(1-e^{2x})^2} = \frac{4e^{2x}}{(1-e^{2x})^2}$$

$$= \frac{4e^{2x}}{e^{2x}(e^{-x} - e^x)^2} = \frac{4}{(e^{-x} - e^x)^2}$$

S47. Ans. (b)

Sol. Let length & height of square box be x and y respectively.

Area of metal used $= x^2 + 4xy$

ATQ.

$x^2 + 4xy = 36$

$y = \frac{36 - x^2}{4x}$

Volume $= x^2 y$

$V = x^2\frac{36-x^2}{4x} = \frac{36x - x^3}{4}$

$\frac{dV}{dx} = \frac{36 - 3x^2}{4}$

$\frac{dV}{dx} = 0 \Rightarrow \frac{36 - 3x^2}{4} = 0 \Rightarrow 3x^2 = 36 \Rightarrow x^2 = 12 \Rightarrow x = 2\sqrt{3}$

$\frac{d^2V}{dx^2} = \frac{-6x}{4} = \frac{-6(2\sqrt{3})}{4} < 0$ (Maxima)

Maximum volume $= \frac{36(2\sqrt{3}) - (2\sqrt{3})^3}{4} =$

$\frac{72\sqrt{3} - 24\sqrt{3}}{4} = \frac{48\sqrt{3}}{4} = 12\sqrt{3}$

S48. Ans. (b)

Sol. Given integral is

$$\int \frac{e^{-x}}{9 + 4e^{-2x}}dx = \frac{1}{4}\int \frac{e^{-x}}{(e^{-x})^2 + \left(\frac{3}{2}\right)^2}dx$$

Put $e^{-x} = t \Rightarrow -e^{-x}dx = dt \Rightarrow e^{-x}dx = -dt$

$$= -\frac{1}{4}\int \frac{dt}{t^2 + \left(\frac{3}{2}\right)^2}$$

$$= -\frac{1}{4} \times \frac{2}{3} \times \tan^{-1}\left(\frac{2t}{3}\right) + C$$

$$= -\frac{1}{6}\tan^{-1}\left(\frac{2e^{-x}}{3}\right) + C$$

S49. Ans. (c)

Sol. Let

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{6}$

$P(A') = 1 - \frac{2}{5} = \frac{3}{5}$, $P(B') = 1 - \frac{1}{4} = \frac{3}{4}$, $P(C') = 1 - \frac{1}{6} = \frac{5}{6}$

P(probability that problem not solved)=

$P(A') \times P(B') \times P(C') = \frac{3}{5} \times \frac{3}{4} \times \frac{5}{6} = \frac{3}{8}$

P(probability that problem is solved)= $1 - \frac{3}{8} = \frac{5}{8}$

S50. Ans. (a)

Sol. We have

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{6}$

$P(A') = 1 - \frac{2}{5} = \frac{3}{5}$, $P(B') = 1 - \frac{1}{4} = \frac{3}{4}$, $P(C') = 1 - \frac{1}{6} = \frac{5}{6}$

Required probability $= P(A)P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C)$

$= \frac{2}{5} \times \frac{3}{4} \times \frac{5}{6} + \frac{3}{5} \times \frac{1}{4} \times \frac{5}{6} + \frac{3}{5} \times \frac{3}{4} \times \frac{1}{6}$

$= \frac{1}{4} + \frac{1}{8} + \frac{3}{40} = \frac{10+5+3}{40} = \frac{18}{40} = \frac{9}{20}$