Time Allowed: 120 minutes   **Maximum Marks: 40**

General Instructions:
1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.
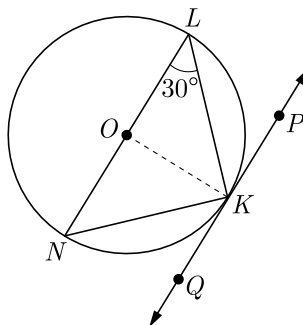
SECTION A

1. Find the positive value of k for which $x^2 - 8x + k = 0$, will have real roots.

<div align="center">OR</div>

If $x^2 + y^2 = 25$, $xy = 12$, then what is the value of x?

2. If seven times the 7th term of an AP is equal to eleven times the 11th term, then what will be its 18th term.

3. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30°$, find $\angle PKL$.

4. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder?

5. The data regarding the height of 50 girls of class X of a school is given below:

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

Change the above distribution to 'more than type' distribution.

6. Convert the following distribution to more than type, cumulative frequency distribution :

Class	50-60	60-70	70-80	80-90	90-100
Frequency	12	18	10	15	5

OR

Find the median for the given frequency distribution:

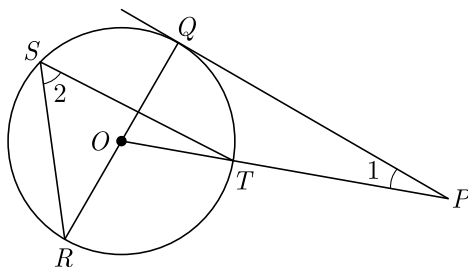| Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

Section B

7. Find the sum of the following series.
$$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$$

8. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are $60°$ and $45°$ respectively. If the height of the tower is 15 m, then find the distance between these points.

9. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

10. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi = \frac{22}{7}$

OR

The radii of two right circular cylinders are in the ratio of $2 : 3$ and their height are in the ratio of $5 : 4$. Calculate the ratio of their curved surface area and radio of their volumes.

Section C

11. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130°$ and S is a point on the circle, find $\angle 1 + \angle 2$.

12. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0-30	30-60	60-90	90-120	120-150	150-180

Frequency	12	21	x	52	y	11

OR

Daily wages of 110 workers, obtained in a survey, are tabulated below :

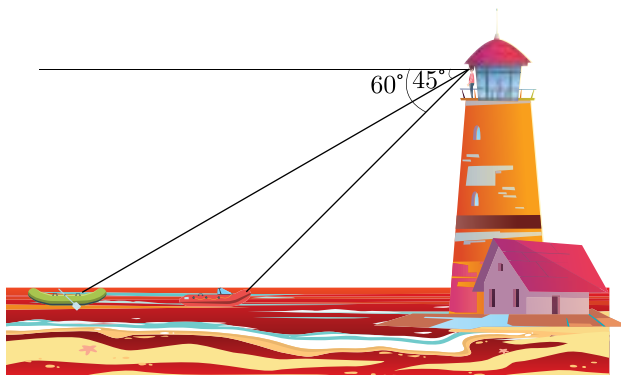| Daily Wages (in ₹)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number of Workers	10	15	20	22	18	12	13

Compute the mean daily wages an modal daily wages of these workers.

13. Nidhi and Ria are very close friends. Nidhi's parents own a Maruti Alto. Ria's parents own a Toyota Liva. Both the families decide to go for a picnic to Somnath temple in Gujrat by their own cars.

Nidhi's car travels x km/h while Ria's car travels 5 km/h more than Nidhi's car. Nidhi's car took 4 hrs more than Ria's car in covering 400 km.
(i)   Which of the following quadratic equation describe the speed of Nidhi's car?
(ii)   How much time did Ria take to travel 400 km?

14. From the observation deck of a seaside building 200 m high, Jignesh sees two fishing boats in the distance. The angle of depression to the nearer boat is 60° while for the boat farther away the angle is 45°.
(i)   How far out to sea is the nearer boat?
(ii)   How far apart are the two boats?

Time Allowed: 120 minutes   **Maximum Marks: 40**

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SECTION A

1. Find the positive value of k for which $x^2 - 8x + k = 0$, will have real roots.

Ans :

We have $x^2 - 8x + k = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = 1, B = -8, C = k$

Since the given equation has real roots,
$B^2 - 4AC > 0$

$(-8)^2 - 4(1)(k) \geq 0$

$64 - 4k \geq 0$

$16 - k \geq 0$

$16 \geq k$

Thus $k \leq 16$

OR

If $x^2 + y^2 = 25,\ xy = 12$, then what is the value of x?

Ans :

We have $x^2 + y^2 = 25$

and $xy = 12$

$x^2 + \left(\dfrac{12}{x}\right)^2 = 25$

$x^4 + 144 - 25x^2 = 0$

$(x^2 - 16)(x^2 - 9) = 0$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9$

$\Rightarrow$ $x = \pm 3$

2. If seven times the 7th term of an AP is equal to eleven times the 11th term, then what will be its 18th term.

Ans :

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

Now $7(a + 6d) = 11(a + 10d)$

$7a + 42d = 11a + 110d$

$11a - 7a = 42d - 110d$

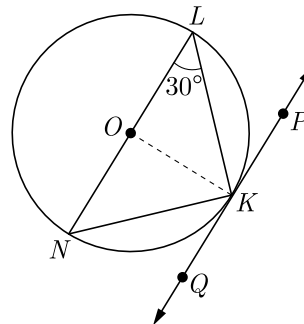, $4a = -68d$

$4a + 68d = 0$

$4(a + 17d) = 0$

$a + 17d = 0$

Hence, $a_{18} = 0$

3. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30°$, find $\angle PKL$.

Ans :

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30°$$
Tangent is perpendicular to the end point of radius,
$$\angle OKP = 90° \qquad \text{(Tangent)}$$
Now $\qquad \angle PKL = \angle OKP - \angle OKL$
$$= 90° - 30° = 60°$$

4. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

Ans :

Volume of cylinder = Volume of sphere,
$$\pi r^2 h = \pi$$
where r and h are radius of base and height of cylinder
$$(0.5)^2 h = 1$$
$$0.25h = 1$$
$$\Rightarrow \qquad h = 4 \text{ cm.}$$

5. The data regarding the height of 50 girls of class X of a school is given below :

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

Change the above distribution to 'more than type' distribution.

Ans :

Heights	No. of Girls
more than 120	50
more than 130	48
more than 140	40
more than 150	28
more than 160	6

6. Convert the following distribution to more than type, cumulative frequency distribution :

Class	50-60	60-70	70-80	80-90	90-100
Frequency	12	18	10	15	5

Ans :

We prepare following cumulative frequency table.

Class	Cumulative Frequency

More than 50	60
More than 60	48
More than 70	30
More than 80	20
More than 90	5

OR

Find the median for the given frequency distribution:

Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

Ans :

Class	Frequency	c.f.
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
	$N = 30$	

We have $\qquad N = 30$; $\frac{N}{2} = 15$
Cumulative frequency just greater than $\frac{N}{2}$ is 19 and the corresponding class is 55-60. Thus median class is 55-60.

Now $\qquad l = 55$, $f = 6$, $F = 13$, $h = 5$

Median, $\qquad M_d = l + \left(\dfrac{\frac{N}{2} - F}{f} \right) \times h$

$$= 55 + \left(\frac{15 - 13}{6} \right) \times 5$$

$$= 55 + \frac{5}{3} = 55 + 1.67$$

$$= 56.67$$

Section B

7. Find the sum of the following series.
$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Ans :

The given series can be written as sum of two series
$(5 + 9 + 13 + ... + 81) +$

$$+(-41)+(-39)+(-37)+(-35)$$
$$...(-5)+(-3)$$
For the series $(5+9+13.....81)$
$$a=5, \ d=4 \text{ and } a_n=81$$
Now
$$a_n=a+(n-1)d$$
$$81=5+(n-1)4$$
$$81=5+(n-1)4$$
$$(n-1)4=76 \Rightarrow n=20$$
$$S_n=\frac{20}{2}(5+81)=860$$
For series $(-41)+(-39)+(-37)+...+(-5)+(-3)$
$$a_n=-3, \ a=-41 \text{ and } d=2$$
$$a_n=-41+(n-1)(2)$$
$$-3=-41+2n-2 \Rightarrow n=20$$
Now
$$S_n=\frac{20}{2}[-41+-3]=-440$$
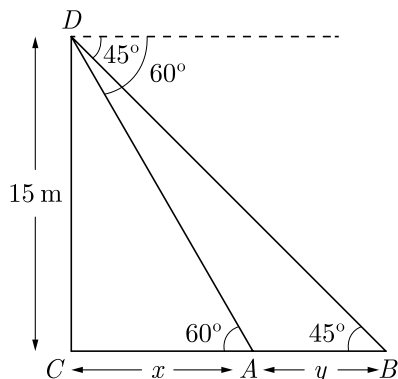
Sum of the series $=860-440=420$

8. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are $60°$ and $45°$ respectively. If the height of the tower is 15 m, then find the distance between these points.

Ans :

Let CD be the tower of height 15 m. Let A and B point on same side of tower As per given in question we have drawn figure below.

In right ΔDCA we have
$$\frac{DC}{CA}=\tan 60°$$
$$\frac{15}{x}=\sqrt{3}$$
$$x=\frac{15}{\sqrt{3}}=5\sqrt{3}$$

In right ΔDCB we have

$$\frac{DC}{CB}=\tan 45°$$
$$\frac{15}{x+y}=1$$
$$x+y=15$$
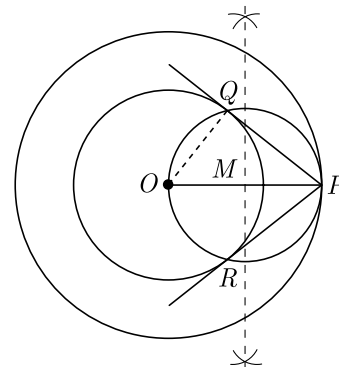$$5\sqrt{3}+y=15$$
$$y=15-5\sqrt{3}$$
$$=5(3-\sqrt{3}) \text{ m}$$
Hence, the distance between points $=5(3-\sqrt{3})$ m

9. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Sol :

Steps of Construction :
1. Draw two circles of radii 5 cm and 3 cm.
2. Take a point P on the outer circle and bisect OP. Let M be the mid-point.
3. Now, draw a circle by taking M as centre and $OM=MP$ as radius. This circle intersects the smaller circle at Q and R.

4. Join PQ and PR. These are the two required tangents. On measuring, we find that $PQ=PR=4$ cm (approx).
   Justification :
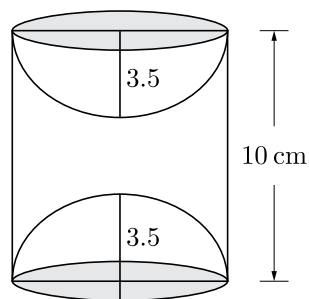   Since $\angle OQP=90°$ by Pythagoras theorem
   $$OP^2=OQ^2+QP^2$$
   $$5^2=3^2+PQ^2$$
   $$PQ=4 \text{ cm}$$

10. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi=\frac{22}{7}$

Ans :

As per question the figure is shown below.

Here radius of toy is equal to the radius of cylinder which is 3.5 cm.

Radius of toy = radius of cylinder = 3.5 cm

Vol. of toy = Vol. of cylinder $- 2 \times$ Vol. of hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3} \pi r^3$$

$$= \pi r^2 \left[h - \frac{4r}{3} \right]$$

$$= \frac{22}{7} \times (3.5)^2 \left[10 - \frac{4 \times 3.5}{3} \right]$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times \left[\frac{30 - 4 \times 3.5}{3} \right]$$

$$= \frac{22}{3} \times 0.5 \times 3.5 \times 16$$

$$= 204.05 \text{ cm}^3.$$

OR

The radii of two right circular cylinders are in the ratio of $2 : 3$ and their height are in the ratio of $5 : 4$. Calculate the ratio of their curved surface area and radio of their volumes.

Ans :

Let the radii of two cylinders be $2r$ and $3r$ and their heights be $5h$ and $4h$ respectively.

Ratio of their curved surface areas,

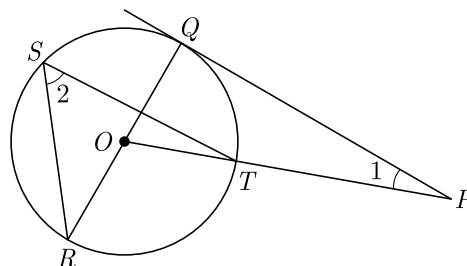$$= \frac{2\pi \times 2r \times 5h}{2\pi \times 3r \times 4h} = \frac{5}{6}$$

Thus their curved surface areas are in the ratio of $5 : 6$.

Ratio of their volumes,

$$= \frac{\pi \times (2r)^2 \times 5h}{2\pi \times 3r \times 4h}$$

$$= \frac{5 \times 4}{4 \times 9} = \frac{5}{9}$$

Hence, their volumes are in the ratio of $5 : 9$ and their $C.SA$ are in the ratio of $5 : 6$.

Section C

11. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130°$ and S is a point on the circle, find $\angle 1 + \angle 2$.

Ans :

Here $\angle OQP = 90°$ because radius is always perpendicular to tangent at point of contact.

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle 2 = \frac{1}{2} \angle TOR = \frac{1}{2} \angle POR$$

$$= \frac{1}{2} \times 130° = 65°$$

Now $\qquad \angle POQ = 180° - 130° = 50°$

$$\angle 1 = 180° - \angle OQP - \angle POQ$$

$$= 180° - 90° - 50° = 40°$$

Now $\qquad \angle 2 + \angle 1 = 65° + 40° = 105°$

12. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

Ans :

We prepare following table to find mean.

Class	x_i (Class marks)	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815

Total		$\sum f = $ $x + y + 96 = 150$	$\sum fx_i = $ $8400 + 75x + 135y$

$$96 + x + y = 150$$
$$x + y = 54 \qquad \text{...(1)}$$
$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$
$$91 = \frac{8400 + 75x + 135y}{150}$$
$$13650 = 8,400 + 75x + 135y$$
$$75x + 135y = 5250$$
$$5x + 9y = 350 \qquad \text{...(2)}$$

Solving equation (1) and (2) we get $x = 34$ and $y = 20$

OR

Daily wages of 110 workers, obtained in a survey, are tabulated below :

Daily Wages (in₹)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number o f Workers	10	15	20	22	18	12	13

Compute the mean daily wages an modal daily wages of these workers.

Ans :

Let $a = 170$ be assumed mean.

CI	x_i	(f)	$x_i - 170$	$\frac{x_i - 170}{20}$	$f_i u_i$
100-120	110	10	-60	-3	-30
120-140	130	15	-40	-2	-30
140-160	150	20	-20	-1	-20
160-180	170	22	0	0	0
180-200	190	18	20	1	18
200-220	210	12	40	2	24
220-240	230	13	60	3	39
		$\sum f_i$ $= 110$			$\sum f_i u_i$ $= 1$

Mean, $\quad \overline{x} = a + \dfrac{\sum f_i u_i}{\sum f_i} \times h$

$$= 170 + \frac{1}{110} \times 20$$

$$= 170 + 0.1818 = 170.1818$$

Hence, mean daily wages of the workers is ₹170.1818 Here the maximum frequency is 22 and the corresponding class is 160-180. So, 160-180 is modal class.

Now $l = 160$, $h = 20$, $f = 22$, $f_1 = 20$ and $f_2 = 18$

Mode $\quad M_o = l + \dfrac{f - f_1}{2f - f_1 - f_2} \times h$

$$= 160 + \frac{22 - 20}{2 \times 22 - 20 - 18} \times 20$$

$$= 160 + \frac{40}{6}$$

$$= 160 + 6.666 = 166.67$$

Hence, modal daily wages of the workers is ₹166.67.

13. Nidhi and Ria are very close friends. Nidhi's parents own a Maruti Alto. Ria's parents own a Toyota Liva. Both the families decide to go for a picnic to Somnath temple in Gujrat by their own cars.

Nidhi's car travels x km/h while Ria's car travels 5 km/h more than Nidhi's car. Nidhi's car took 4 hrs more than Ria's car in covering 400 km.

(i) Which of the following quadratic equation describe the speed of Nidhi's car?

(ii) How much time did Ria take to travel 400 km?

Ans :

(i) As per question,

$$\frac{400}{x} = \frac{400}{x+5} + 4$$
$$400(x+5) = 400x + 4x(x+5)$$
$$2000 = 4x^2 + 20x$$
$$500 = x^2 + 5x$$
$$x^2 + 5x - 500 = 0$$

(ii) Rias car speed $= 20 + 5 = 25$ km/hour
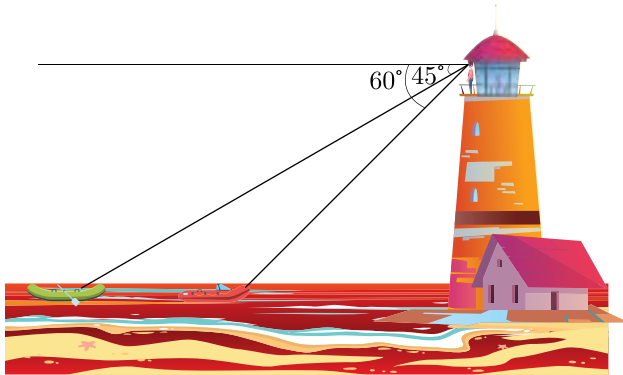
Time taken $= \dfrac{400}{25} = 16$ hour

14. From the observation deck of a seaside building 200 m high, Jignesh sees two fishing boats in the distance. The angle of depression to the nearer boat is 60° while for the boat farther away the angle is
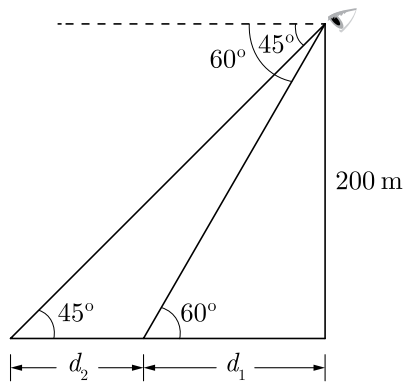
45°.

(i)   How far out to sea is the nearer boat?
(ii)   How far apart are the two boats?

Ans :

Let d_1 be the distance of nearer boat from sea and d_2 be the distance between two boat. We draw a diagram of the situation as shown below.

Now $\tan 60° = \dfrac{150}{d_1}$

         $\sqrt{3} = \dfrac{150}{d_1}$

         $d_1 = \dfrac{150}{\sqrt{3}} = 50\sqrt{3}$

             $= 50 \times 1.732 = 86.6 \text{ m}$

Now $\tan 45° = \dfrac{150}{d_1 + d_2}$

         $1 = \dfrac{150}{d_1 + d_2}$

     $d_1 + d_2 = 150$

Substituting value of d_1 we have

     $86.6 + d_2 = 150$

         $d_2 = 150 - 86.6 = 63.4 \text{ m}$

(i)   Thus distance of nearer boat from seaside is 86.6 m
(ii)   Both boat are 63.4 m apart.