CBSE Board

Class X Mathematics (Standard)

Sample Paper - 3

Term 2 – 2021- 22

Time: 2 hours **Total Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section A

Q1 – Q6 are of 2 mark each.

1. Which term of the AP 121, 117, 113 … is its first negative term?

OR

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

2. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

5. Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	100 – 120	120 – 140	140 –1 60	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

6. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

OR

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Section B

Q7 – Q10 are of 3 mark each.

7. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode.

Number of students per teacher	**Number of states/U.T**
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

8. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

9. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	**Number or leaves f_i**
118 – 126	3
127 – 135	5

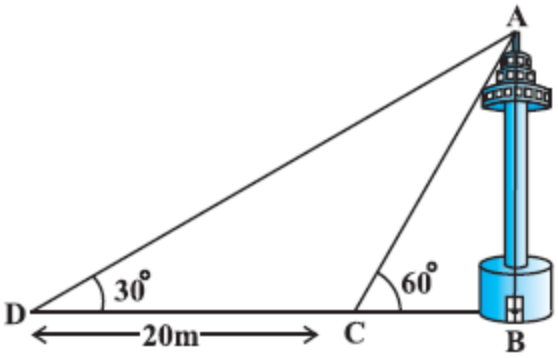
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

OR

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Section C

Q11 – Q14 are of 4 mark each.

11. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure).

12. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

OR

If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then find $\angle POA$

13. There are two temples on each bank of a river. One temple is 70 m high. A man, who is standing on the top of 70 m high temple, observed that the angle of depression of the top and the foot of other temple are 30^o and 60^o respectively. (Take $\sqrt{3} = 1.732$)

i) Draw a labelled figure on the basis of the given information and the approximate width of the river.

ii) Find the approximate height of the other temple

14. Ajay has decided to visit different parts of his state, and to travel from one place to another, he will book a taxi each time. Now the taxi rates are fixed and it costs ₹ 15 for the first km and ₹8 for each additional km. Now using the given data answer the following questions.

i) Form an AP using the given data and find the taxi fare for Ajay, if he travels for 4 km.

ii) If the taxi fare comes out to be ₹63, then find the total kilometers travelled

Solution

Section A

1.

Given AP is 121, 117, 113 …
$a = 121$
$d = 117 - 121 = -4$
$a_n = a + (n - 1)\, d$
$= 121 + (n - 1)\,(-4)$
$= 121 - 4n + 4$
$= 125 - 4n$
We have to find the first negative term of this AP
Therefore, $a_n < 0$
$125 - 4n < 0$
$125 < 4n$
$n > \frac{125}{4}$
$n > 31.25$
Therefore, 32nd term will be the first negative term of this AP

OR

Penalties are in an AP having first term as 200 and common difference as 50.
$a = 200$
$d = 50$
Penalty that has to be paid if he has delayed the work by 30 days = S_{30}
$= \frac{30}{2}\left[2(200) + (30 - 1)50\right]$
$= 15\,[400 + 1450]$
$= 15\,(1850)$
$= 27750$
Therefore, the contractor has to pay Rs 27750 as penalty.

2.

Let the present age of Rehman be x years.
Three years ago, his age was $(x - 3)$ years.
Five years hence, his age will be $(x + 5)$ years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$

$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$

$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$

$\Rightarrow 3(2x+2)=(x-3)(x+5)$

$\Rightarrow 6x+6=x^2+2x-15$

$\Rightarrow x^2-4x-21=0$

$\Rightarrow x^2-7x+3x-21=0$

$\Rightarrow x(x-7)+3(x-7)=0$

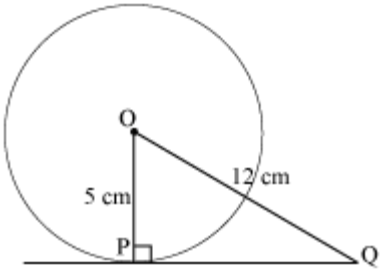
$\Rightarrow (x-7)(x+3)=0$

$\Rightarrow x=7,-3$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

3. Radius is perpendicular to the tangent at the point of contact. So, OP $\perp$ PQ.



Now, applying Pythagoras theorem in ΔOPQ,

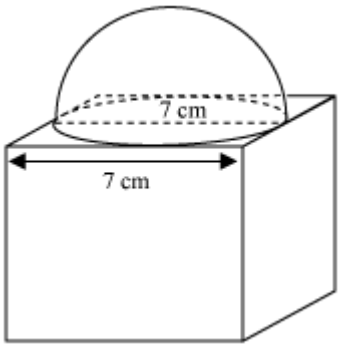
$OP^2 + PQ^2 = OQ^2$

$5^2 + PQ^2 = 12^2$

$PQ^2 = 144 - 25$

$PQ = \sqrt{119}$ cm

4.



From the figure it the greatest diameter possible for such hemisphere is equal to cube's edge i.e. 7cm.

Radius (r) of hemispherical part = $\frac{7}{2}$ = 3.5cm.

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part – area of base of hemispherical part

$= 6\ (\text{edge})^2 + 2\pi r^2 - \pi r^2 = 6\ (\text{edge})^2 + \pi r^2$

$$\text{Total surface area of solid} = 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5 = 332.5\,\text{cm}^2$$

5. Let us find class mark for each interval by using the relation.

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of this data = 20

Now taking 150 as assured mean (a) we may calculate d_i, u_i and f_iu_i as following.

Daily wages (in Rs)	**Number of workers (f_i)**	x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 150}{h}$	f_iu_i
100 –120	12	110	– 40	–2	– 24
120 – 140	14	130	– 20	–1	– 14
140 – 160	8	150	0	0	0
160 –180	6	170	20	1	6
180 – 200	10	190	40	2	20
Total	50				–12

From the table we may observe that

$\sum f_i = 50$

$\sum f_i u_i = -12$

$$\text{Mean}\,\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$= 150 + \left(\frac{-12}{50}\right)20$$

$$= 150 - \frac{24}{5}$$

$$= 150 - 4.8$$

$$= 145.2$$

So mean daily wages of the workers of the factory is Rs.145.20

6. Let the marks in Maths be x.
Then, the marks in English will be $30 - x$.
According to the given question,

$$(x+2)(30-x-3) = 210$$

$$(x+2)(27-x) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow x^2 - 12x - 13x + 156 = 0$$

$$\Rightarrow x(x-12) - 13(x-12) = 0$$

$$\Rightarrow (x-12)(x-13) = 0$$

$$\Rightarrow x = 12, 13$$

If the marks in Maths are 12, then marks in English will be 30 – 12 = 18
If the marks in Maths are 13, then marks in English will be 30 – 13 = 17

OR

Let the shorter side of the rectangle be x m.
Then, larger side of the rectangle = (x + 30) m

Diagonal of the rectangle $= \sqrt{x^2 + (x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$\therefore \sqrt{x^2 + (x+30)^2} = x + 60$

$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x-90) + 30(x-90)$

$\Rightarrow (x-90)(x+30) = 0$

$\Rightarrow x = 90, -30$

However, side cannot be negative. Therefore, the length of the shorter side will be 90 m. Hence, length of the larger side will be (90 + 30) m = 120 m

Section B

7. We may observe from the given data that maximum class frequency is 10 belonging to class interval 30 – 35.

So, modal class = 30 – 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

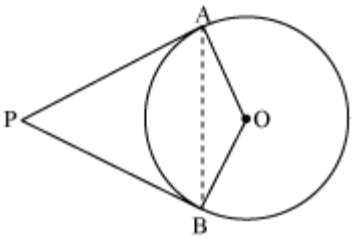
$$= 30 + \left(\frac{10-9}{2(10) - 9 - 3}\right) \times (5)$$

$$= 30 + \left(\frac{1}{20-12}\right) 5$$

$$= 30 + \frac{5}{8} = 30.625$$

Mode $= 30.6$

8.



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to circle which are touching circle at point A and B respectively.
AB is the line segment, joining point of contacts A and B together such that it subtends $\angle$AOB at center O of circle.
As the radius is perpendicular to the tangent at the point of contact, $\angle$OAP = 90°.
Similarly, $\angle$OBP = 90°
In quadrilateral OAPB,
$\angle$OAP + $\angle$APB + $\angle$PBO + $\angle$BOA = 360°
90º + $\angle$APB + 90° + $\angle$BOA = 360°
$\angle$APB + $\angle$BOA = 180°
Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

9. The given data is not having continuous class intervals. Hence continuous class intervals with respective cumulative frequencies can be represented as below –

Length (in mm)	**Number or leaves f_i**	**Cumulative frequency**
117.5 – 126.5	3	3
126.5 – 135.5	5	3 + 5 = 8
135.5 – 144.5	9	8 + 9 = 17
144.5 – 153.5	12	17 + 12 = 29
153.5 – 162.5	5	29 + 5 = 34
162.5 – 171.5	4	34 + 4 = 38
171.5 – 180.5	2	38 + 2 = 40

From the table we may observe that cumulative frequency just greater then $\frac{n}{2}\left(\text{i.e. } \frac{40}{2} = 20\right)$ is 29, belonging to class interval 144.5 – 153.5.

Median class = 144.5 – 153.5
Lower limit (l) of median class = 144.5
Class size (h) = 9
Frequency (f) of median class = 12
Cumulative frequency (cf) of class preceding median class = 17

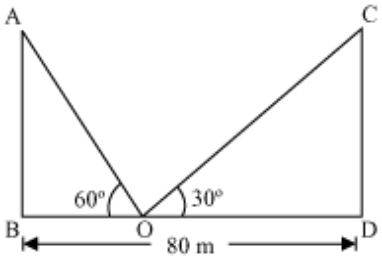
$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 144.5 + \left(\frac{20 - 17}{12} \right) \times 9$$

$$= 144.5 + \frac{9}{4} = 146.75$$

So, median length of leaves is 146.75 mm.

10.



Let AB and CD be the poles and O is the point on the road.
In ΔABO,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}} \qquad \text{... (i)}$$

In ΔCDO,

$\frac{CD}{DO} = \tan 30°$

$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$

$CD\sqrt{3} = 80 - BO$

$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$ [From (i)]

$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$

$CD\left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = 80$ (Since, $AB = CD$)

$CD\left(\frac{3+1}{\sqrt{3}}\right) = 80$

$CD = 20\sqrt{3}$

$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$ m

DO = BD – BO = 80 m – 20 m = 60 m

Thus, the height of the poles is $20\sqrt{3}$ m and the point between the poles is 20 m and 60 m far from these poles.

OR

In ΔABC,

$\frac{AB}{BC} = \tan 60°$

$\frac{AB}{BC} = \sqrt{3}$

$BC = \frac{AB}{\sqrt{3}}$ ... (i)

In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}} \qquad \text{[From (i)]}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

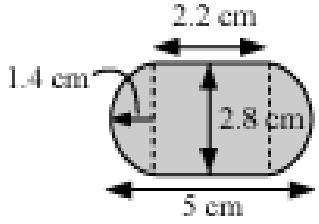
$$AB = 10\sqrt{3}$$

$$\Rightarrow BC = \frac{AB}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$$

Thus, the height of the tower is $10\sqrt{3}$ m and width of canal is 10 m.

Section C

11.



Radius (r) of cylindrical part = radius (r) of hemispherical part = $\frac{2.8}{2} = 1.4\,\text{cm}$

Length of each hemispherical part = radius of hemispherical part = 1.4cm

Length (h) of cylindrical part = 5 – 2 × length of hemispherical part

= 5 – 2 ×1.4 = 2.2cm

Volume of one gulab jamun = volume of cylindrical part + 2 × volume of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

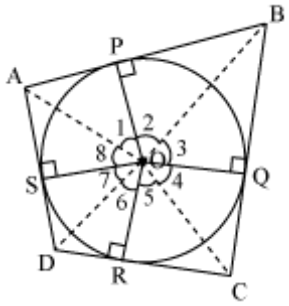
$$= 13.55 + 11.50 = 25.05 \text{ cm}^3$$

Volume of 45 gulab jamuns = $45 \times 25.05 = 1{,}127.25\ cm^3$
Volume of sugar syrup = 30% of volume

$$= \frac{30}{100} \times 1{,}127.279$$

$$\approx 338\ cm^3$$

12.



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S.
Join the vertices of the ABCD to the center of the circle.
Consider $\triangle OAP$ and $\triangle OAS$,

AP = AS	(tangents from same point)
OP = OS	(radius of circle)
OA = OA	(common)
So, $\Delta OAP \cong \Delta OAS$	(SSS congruence rule)

$\therefore \angle POA = \angle SOA$
$\angle 1 = \angle 8$
Similarly,
$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$
$\angle AOB + \angle COD = 180°$
Similarly, $\angle BOC + \angle DOA = 180°$
Hence opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

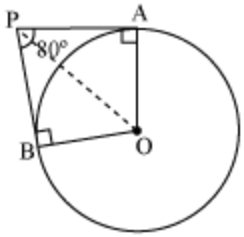
OR

Radius drawn to these tangents will be perpendicular to the tangents.

Therefore, OA $\perp$ PA and OB $\perp$ PB

$\angle OBP = 90^\circ$

$\angle OAP = 90^\circ$



In the quadrilateral AOBP,

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$

$\angle BOA = 100^\circ$

In ΔOPB and ΔOPA,

AP = BP (tangents drawn from an external point are equal in length)

OA = OB (radius of circle)

OP = OP (common side)

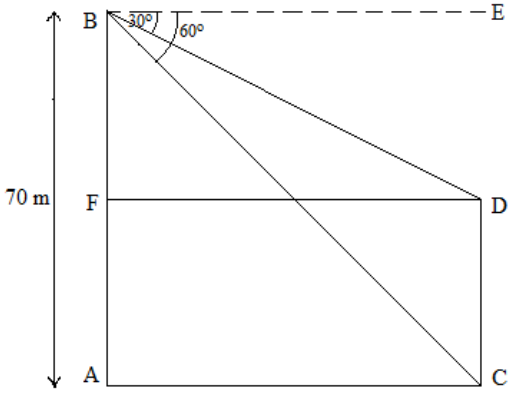
$\therefore \Delta OPB \cong \Delta OPA$

$\therefore \angle POB = \angle POA$

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

13.

i)



AB and CD represents the two temples

We have,

$$\tan 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow AC = \frac{70}{\sqrt{3}} = 40.41$$

So, width of the river is 40.415 m

ii)

$$\tan 30^\circ = \frac{BF}{FD} = \frac{BF}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BF}{70} \times \sqrt{3}$$

$$\Rightarrow BF = \frac{70}{3} = 23.33 \text{ m}$$

CD = FA = AB – BF = 70 – 23.33 = 46.67 m

So, the approximate height of other temple 46.67 m.

14.

i)

AP is 15, 23, 31, 39, 47…

Rate for first km → Rs. 15 = a

Rate for additional km → Rs. 8 = d

Travel fare for 4km = a_4

$$a_n = a + (n-1)d$$

$$a_4 = 15 + (4-1)8 = \text{Rs.}39$$

ii)

Rate for first km → Rs. 15 = a

Rate for additional km → Rs. 8 = d

Travel fare = 63 = a_n

$$a_n = a + (n-1)d$$

$$63 = 15 + (n-1)8$$

$$\therefore n = 7$$