CONSTRUCTION OF TRIANGLES

10.1 Revise the chapters containing concept of triangle, its properties and congruence of triangles before studying this chapter.

We classify the triangles on the basis of sides and angles like as equilateral, isosceles and scalene triangles on the basis of sides and acute angled triangle, right angled triangle, obtuse angled triangle on the basis of angles etc.

We'll learn to construct different types of triangle in this chapter.

It is not necessary to know the measures of all the six elements (3 sides and 3 angles) to make a required triangle; the same has been studied in the chapter related with congruence of triangles. We can construct required triangle if we are given measures from any one of the groups given below. Here required is a unique triangle constructed on the basis of given measures.

1. Three sides.
2. Two sides and the angle between them.
3. Two angles and the side between them.
4. The hypotenuse and a leg in the case of a right-angled-triangle.

10.2 Construction of a triangle when the lengths of its three sides are given

Example 1 Construct a triangle ABC, given that AB = 5 cm. BC = 6 cm. and AC = 7 cm.

Solution **Step-1**

First, we draw a rough sketch with given measure.

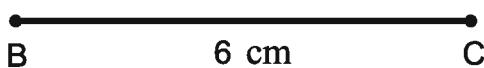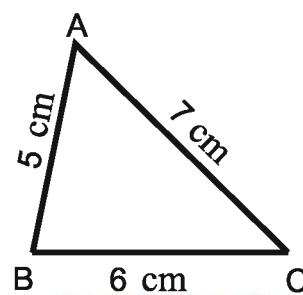**Step-2**

Draw a line segment BC of length 6 cm.

Rough sketch
Fig. (i)

B ————— 6 cm ————— C
Fig. (ii)

Step-3

From B, point A is at a distance of 5 cm. So, with B as centre, draw an arc of radius 5 cm.
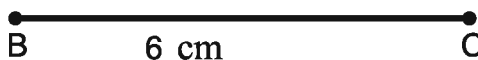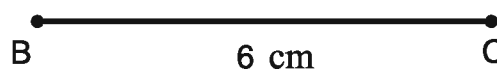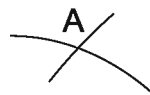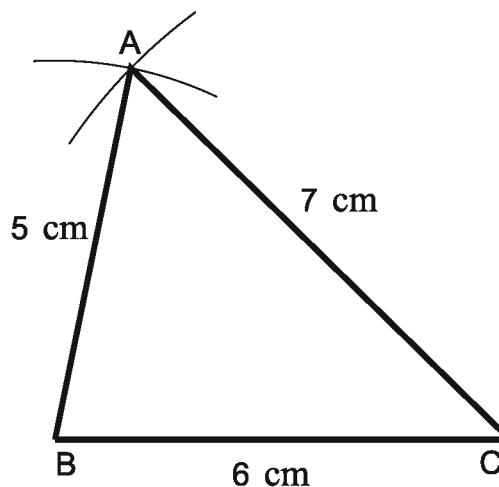
B ————— 6 cm ————— C
Fig. (iii)

Step- 4

AC = 7cm, So, with C as centre, draw an arc of radius 7 cm. in such a way that this arc intersects the arc drawn from point B.

Figure (iv)

Step- 5

A has to be on both these arcs drawn. So, it is the point of intersection of arcs. Mark the point of intersection of arcs as A. Join AB and AC. Triangle ABC is now ready.

Figure (v)

> ### Do and Learn ◆
> 1. Construct △XYZ in which XY = 4.5 cm. YZ = 5 cm. and ZX = 6 cm.
> 2. Construct an equilateral triangle of side 5.5 cm.
> 3. Draw △PQR with PQ = 4 cm. QR = 3.5 cm. and PR = 4 cm.
>   What type of triangle is this?

10.3 Construction of a triangle when the lengths of two sides and the measure of the angle between them are known
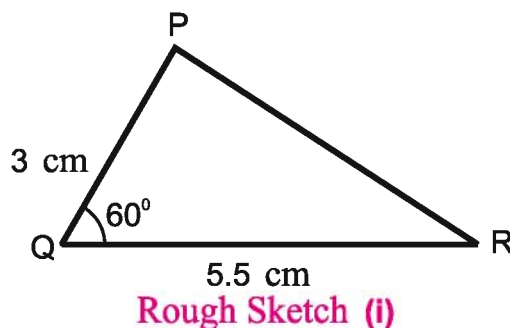
Here, we are given two sides and the angle between them. We first draw a rough sketch. The other steps follow according to example 2.
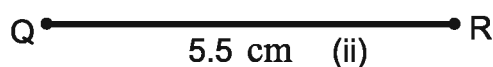
Example 2   Construct a triangle ΔPQR, given that PQ = 3 cm. , QR = 5.5 cm. and ΔPQR = 60°.
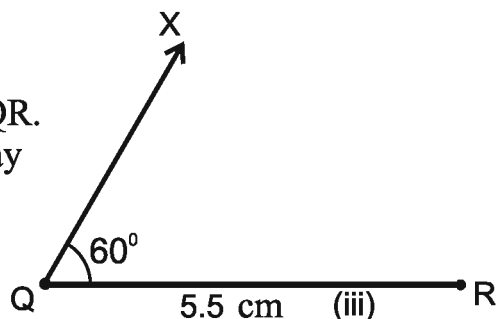
Solution

Step-1 First, we draw a rough sketch with given measures. (This helps us to determine the procedure in construction) figure (i)
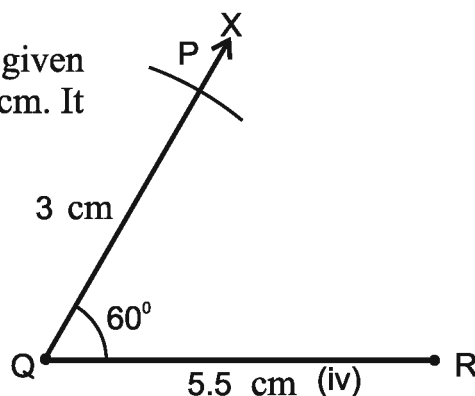
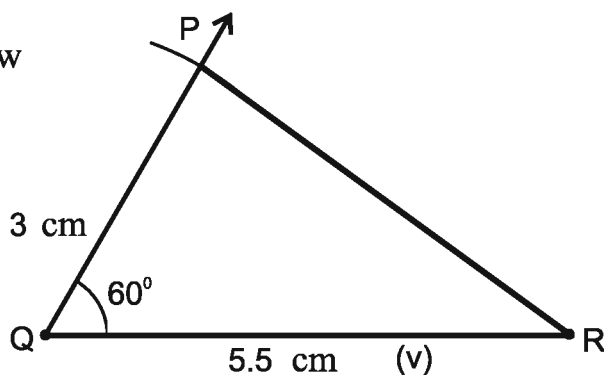**Step-2** Draw a line segment QR of length 5.5 cm.figure (ii)

Step-3 At Q, draw QX making 60° with QR. (The point P must be somewhere on this ray of the angle.) figure (iii)

Step-4 To fix P, the distance QP has been given With Q as centre, draw an arc of radius 3 cm. It cuts QX at the point P.(figure (iv)).

Step-5 Join PR. Triangle PQR is now obtained.(figure (v))

Think, and Discuss

Teacher- In a triangle ABC if the measures are AB = 3 cm, AC = 5 cm and, $\angle C = 30^0$ then. Can we draw this triangle?

Krishna, Vikram and Sarla try to construct it.

Krishna - We may draw AC = 5 cm. and draw $\angle C = 30^0$.

Vikram – CA is one arm of $\angle C$. Point B should be lying on the other arm of C.

Teacher – Observe that point B cannot be located uniquely. Thus, we can conclude that a unique triangle can be constructed only if the lengths of its two sides and the measure of the included angle between them is given.

Do and Learn ◆

(I) Construct ΔDEF such that DE = 5 cm., DF = 3 cm. and $\angle EDF = 90°$.

(ii) Construct an isosceles triangle in which the lengths of each of its equal sides is 6.5 cm. and the angle between them is 110°.

(iii) Construct ΔABC with BC = 7.5 cm., AC = 5 cm. and $\angle C= 60°$.

10.4 Construction of a triangle when the measures of two of its angles and the length of the side included between them is given

As before, draw a rough sketch. Now, draw the given line segment. Make angles on the two ends. See the example 3.

Example 3 Construct ΔXYZ if it is given that XY = 6 cm, $\angle ZXY = 30°$ and $\angle XYZ = 100°$.

Solution

Step-1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed) (figure (I))
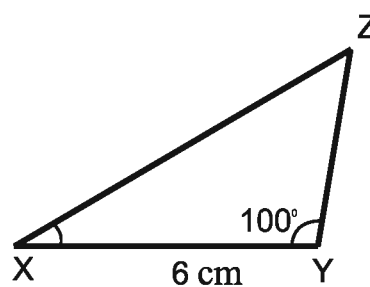
Fig. (i)

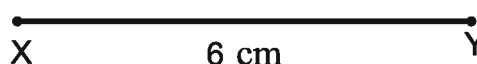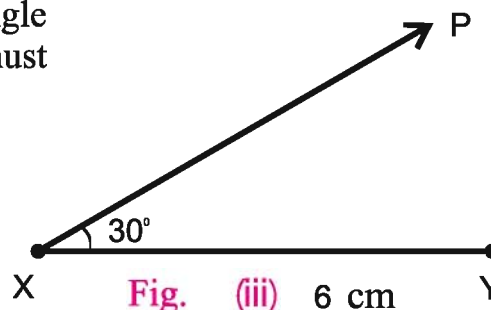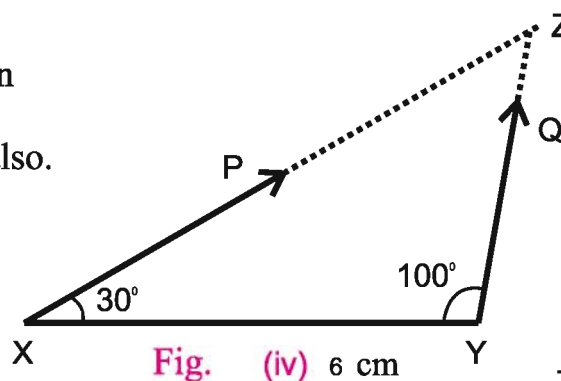**Step- 2** Draw a line segment XY of length 6 cm.(figure (ii))

Fig. (ii)

Step - 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.(figure (iii))

Fig. (iii)   6 cm

Step-4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also. (figure (iv))

Fig. (iv) 6 cm

Step- 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.(figure (v))

Fig. (v)   6 cm
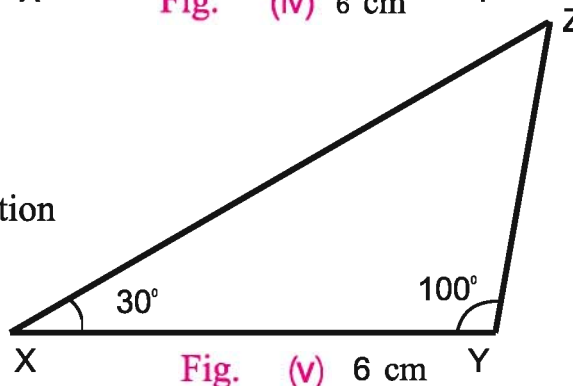
Do and Learn ◆

1. Construct $\triangle ABC$, given $\angle A = 60^0$, $\angle B = 30^0$ and AB = 5.8 cm

2. Construct $\triangle PQR$ if PQ =5 cm, $\angle PQR = 105^0$ and $\angle QRP = 40^0$

    (Hint: Recall angle- sum property of a triangle)

3. Examine whether you can construct $\triangle DEF$ such that

    EF = 7.2 cm, $\angle E = 110^0$ and $\angle F = 80^0$, Justify your answer.

10.5 Construction of a right angled triangle - when the length of one leg and its hypotenuse are given. Here it is easy to make the sketch. Now, draw a line segment as per the given side. Make a right angle on one of its end. Use compasses to mark length of side and hypotenuse of the triangle. Complete the triangle. Consider the following example:

Example 4 Construct $\triangle LMN$, right-angled at M, given that LN = 5 cm and MN = 3 cm.

Solution **Step-1** Draw a rough sketch and mark the measure. Remember to mark the right angle. (figure (I))
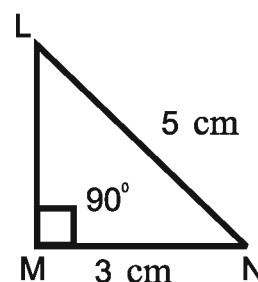
Fig. (i)

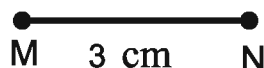**Step -2** Draw line segment MN of length 3 cm. (figure (ii))

Fig. (ii)

Step -3 At M, draw MX $\perp$ MN. For this draw a 90^0 angle at M.(L should be somewhere on this perpendicular) (figure (iii))
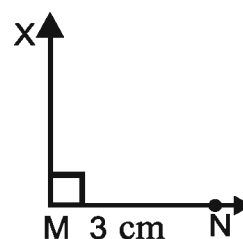
Fig. (iii)

Step -4 With N as centre, draw an arc of radius 5 cm. (L must be on this arc, since it is at a distance of 5 cm. from N)(figure (iv))
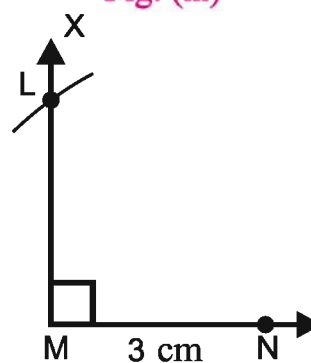
Fig. (iv)

Step- 5 L has to be on the perpendicular line MX as well as on the arc drawn with centre N. Therefore, L is the intersecting point of these two. Join LN (figure (v))
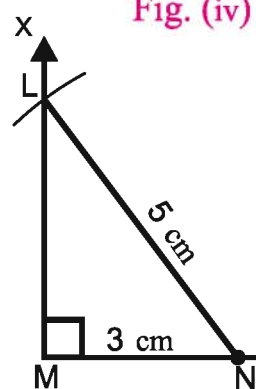
Fig. (v)

Do and Learn ◆

1. Construct the right angled triangle PQR, where $\angle Q = 90^0$, QR = 8 cm and PR = 10 cm
2. Construct a right- angled triangle whose hypotenuse is 6 cm. long and one of the legs is 4 cm. long.

⤜∞⊂ Exercise 10 ⊃∞⤛

1. Construct ΔPQR, when PQ = 4 cm, QR = 3 cm and RP = 5.5 cm
2. Construct Δ XYZ, when XZ = 6 cm, XY = 4.5 cm and $\angle$X = 50°.
3. Construct Δ ABC , when AB = 5 cm, $\angle$A = 45° and $\angle$B = 60°.
4. Conbstruct Δ DEF, when hypotenuse DE = 5 cm, base DF = 3 cm and $\angle$D = 90°.
5. Construct an equilateral triangle of side 4 cm.
6. Construct Δ PQR, where PQ = 5 cm, $\angle$P = 75° and $\angle$R = 55°

⤜✕ We Learnt ✕⤛

1. In this chapter, we studied the methods of constructions of triangles using ruler and compasses.
2. We studied the method of drawing a triangle, using indirectly the concept of congruence of triangles.
3. In this chapter we studied constructions of triangle using following sets of measures.
    - (i)   SSS: Given the three side lengths of a triangle.
    - (ii)   SAS : Given the lengths of any two sides and the measure of the angle between these sides.
    - (iii)   ASA : Given the measures of two angles and the length of side included between them.
    - (iv)   RHS : Given the length of hypotenuse of a right-angled triangle and the length of one of its legs.