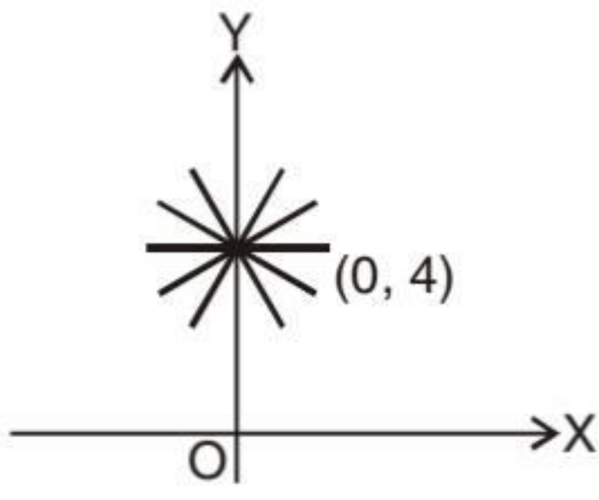
FAMILY OF LINES

We know that the eqn. $y = mx + 4$ represents a straight line having slope m and making an intercept 4 on y-axis.



For different value of m represents different straight lines each making an intercept 4 on y-axis. All these lines (as shown in figure) taken together constitute a family of lines having the common property that each makes an intercept of 4 units on y-axis.

The arbitrary constant m, which is same for one line but is different for different lines, is called the parameter of the family.

Let $\quad S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

and $\quad \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$.

Then, $S = 0$ represents.

(i) a pair of straight lines, if $\Delta = 0$ and $h^2 \geq ab$

(ii) a pair of intersecting lines, if $\Delta = 0$ and $h^2 > ab$

(iii) a pair of parallel lines, if $\Delta = 0$ and $h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$

The equation $ax^2 + 2hxy + by^2 = 0$ represents

(i) two real and different lines if $h^2 > ab$

(iii) two coincident lines if $h^2 = ab$

Consider the equation of pair of lines through the origin, *i.e.*, $S = ax^2 + 2hxy + by^2 = 0$, then

(i) if $y = m_1 x$ and $y = m_2 x$ are two lines represented by $S = 0$, then

$m_1 + m_2 = -2h/b$ and $m_1 m_2 = a/b$

(ii) The angle θ between the lines represented by $S = 0$, is given by $\theta = \tan^{-1}\left|\dfrac{2\sqrt{(h^2 - ab)}}{a+b}\right|$

(*iii*) The lines given by S = 0 are parallel if $h^2 = ab$.

(*iv*) The line given by S = 0 are perpendicular if $a + b = 0$, *i.e.*, if coeff. of x^2 + coeff. of $y^2 = 0$

(*v*) The equation of the pair of bisectors of the angles between the lines S = 0 is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}.$$