Applied Mathematics

NOTES

Olympiad

MATHEMATICS Comprehensive

Set

Set is a collection of well-defined objects which are distinct from each other. The objects in the set are called its elements. Sets are usually denoted by capital Letters A, B, C, and elements are usually denoted by small letters a, b, c,

For example/ the set of all even natural numbers less than 10 can be represented by

 $N = \{2, 4, 6, 8\}.$

Methods for Describing a Set

(i) **Roster Method:** In this method, a set is described by listing elements, separated by commas, within braces.

e.g. $A = \{a, e, i, o, u\}$

Note: This method is also called listing method or tabular form method.

(ii) Set Builder Method: In this method, we write down a rule which gives us all the elements of the set by that rule.

e.g. $A = \{x : x \text{ is a vowel of English alphabet}\}$

Finite Set

A set containing finite number of elements or no element, is called a finite set, eg. The set of all persons in India is a finite set.

Infinite Set

A set containing infinite number of elements is called an infinite set.

Cardinality of a Finite Set

The number of elements in a given finite set is called cardinal number of finite set, denote by n (a), where A is the given set.

e.g. P = $\{5, 15, 25, 35, 45\} \Rightarrow n(P) = 5$

Universal Set (U)

A set consisting of all possible elements which occurs under consideration is called a universal set.

e.g. Let the set U defines the set of all natural numbers then set of all odd natural numbers is another subset of U and the set of all even natural numbers is another subset of U.

Equal Sets

Two sets A and B are called equal, if every element of A is a member of B and every element of B is a member of A, Thus we write A = B.

e.g. $A = \{2, 4, 6, 8, 10\}$ and {all the even natural numbers less than or equal to 10} i.e., A and B are equal sets.

> Example:

Find cardinal number of a set A of the composite numbers between 10 and 25.

(a) 4	(b) 6	
(c) 8	(d) 9	
(e) None of these		
Answer (d)		
Explanation: Here, $A = \{12, 14, 15, 16, 18, 20, 21, 22, 24\}$		
\Rightarrow n(A) = 9 \Rightarrow Cardinal number of set A = 9		

Average

Average is a calculated central value of a set of given number. The average is the sum of the numbers divided by the count.

So, Average = $\frac{\text{sum of the number}}{\text{count}}$

For example. Average of first ten natural numbers

is
$$\frac{1+2+3+4+5+6+7+8+9+10}{10} = \frac{55}{10} = 5.5$$

Percentage

The word 'percent', means 'per hundred' or 'out of hundred'. Symbol % is used to express percentage.

• A fraction can be converted into percentage by multiplying the fraction by 100. If fraction = $\frac{x}{v}$, then its percentage

$$=\left(\frac{x}{y}\times 100\right)\%$$

Percentage can be converted into fraction by dividing the percentage by 100. if percentage is x%, then its fraction

will be $\frac{x}{100}$.

- A decimal can be converted into percentage by multiplying 100 to It, e.g. 0, 05 = 5%,
- A percentage can be converted into decimal by dividing 100 to it. e.g. 10% "= 0.10

Some Basic Terms

- (i) Cost Price (CP): It is the price paid by a customer or shopkeeper to purchase an article.
- (ii) **Selling Price (SP):** It is the price at which a shopkeeper sells an article.
- (iii) Gain (Profit) = S.P C.P
- (iv) Loss = C.P S.P.
- (v) Profit/loss% = $\left(\frac{\text{Amount of profit / loss}}{\text{C.P}} \times 100\right)$ %

Some Useful Results

- If a quantity is increased by x% then x% $\left(=\frac{\text{increase in quantity}}{\text{original quantity}} \times 100\right)$ % and new quantity $\left(\frac{100 + x}{100}\right) \times \text{ ix}$ Original quantity.
- If a quantity is decreased by x% then percentage decrease $\left(=\frac{\text{decrease in quantity}}{\text{original quantity}} \times 100\right)\%$ and new quantity

 $=\left(\frac{100-x}{100}\right)\times$ original quantity.

> Example:

By how much is 30% of 450 more than 70% of 140?

- (a) 37 (b) 72 (c) 33 (d) 45
- (e) None of these

Answer (d)

Explanation: 30% of 450 $=\frac{30}{100} \times 450 = 135$

70% of 140 $\frac{70}{100}$ × 140 = 98 ∴ 135 - 98 = 37 ⇒ 30% of 450 is 37 more than 70% of 140.

> Example:

The cost price of 10 pens is equal to selling price of 8 pens. Find the profit/loss earned on selling a pen at Rs 25.

(a) Rs 10	(b) Rs 3
(c) Rs 5	(d) Rs 8

(e) None of these

Answer (c)

Explanation: Let C.P. of 1 pen = Rs x and S.P of 1 pen = Rs y

- $\therefore 10x = 8y \Rightarrow y = \text{Rs}\frac{10x}{8} = \text{Rs}\frac{5x}{4} \Rightarrow y > x \Rightarrow \text{profit}$
- \therefore Profit on selling a pen = Rs $\left(\frac{5x}{4} x\right)$ = Rs $\frac{x}{4}$

So, profit $\% = \frac{x/4}{x} = 25\%$

Thus, C.P. of the pen, whose S.P. is Rs 25, is $=\frac{100}{(100+25)} \times 25 = \frac{2500}{125} = \text{Rs } 20$

 \therefore Profit = S.P – C.P Rs (25 – 20) = Rs 5