Exponents and Powers

- **Laws of rational exponents of real numbers:**

Let *a* and *b* be two real numbers and *m* and *n* be two rational numbers then

  - $a^p \cdot a^q = a^{p+q}$
  - $(a^p)^q = a^{pq}$
  - $\frac{a^p}{a^q} = a^{p-q}$
  - $a^p b^p = (ab)^p$
  - $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
  - $a^{-p} = \frac{1}{a^p}$

Example:

$\sqrt[3]{(512)^{-2}}$

$= \left[(512)^{-2}\right]^{\frac{1}{3}}$

$= (512)^{\frac{-2}{3}} \quad [(a^m)^n = a^{mn}]$

$= (8^3)^{\frac{-2}{3}}$

$= (8)^{3 \times \frac{-2}{3}} \quad [(a^m)^n = a^{mn}]$

$= (8)^{-2}$

$= \frac{1}{8^2} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{64}$

- If a natural number m can be expressed as n^2, where n is also a natural number, then m is a square number. The square numbers are also called **perfect squares.**

  Examples: 1, 4, 9, 16, 25, 36 … are perfect squares.

Prime factorization method of finding the square roots of numbers

The square root of 67600 can be found by prime factorization method as follows:
The number 67600 can be prime factorized as:
$\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times \underline{13 \times 13}$
The numbers, 2, 2, 5, 13, occur in pairs. Therefore,
$\sqrt{67600} = 2 \times 2 \times 5 \times 13 = 260$

Example: Find the smallest number by which 252 can be multiplied to make it a perfect square.
Solution: We have, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$
The number 7 does not occur in pair. Therefore, if we multiply 252 by 7, then it will become a perfect square.
Therefore, $252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$
That is, 1764 is a perfect square and $\sqrt{1764} = 42$

Finding square root of perfect squares by division method

The steps of finding the square root of 1369 by division method are as follows:
Step1: Firstly, place bars over every pair of digits starting from the digit at ones place. We obtain $\overline{13}\,\overline{69}$.
Step2: Find the largest number whose square is less than or equal to the number under the extreme left bar.
Take this number as the divisor and the number under the extreme left bar as the dividend. Divide and obtain the remainder.

	3
3	$\overline{13}\,\overline{69}$
	–9
9	4

Step3: Bring down the number under the next bar to the right of the remainder.
Therefore, the new dividend is 469.
Double the divisor and enter it with the blank on its right.

	3.
3	13.69
	–9
9	4.69

Step 4: Guess the largest possible digit to fill the blank, which becomes the new digit in the quotient, such that when
the new digit is multiplied to the new quotient, the product is less than or equal to the dividend.
In this case, $97 \times 7 = 469$
Therefore, the quotient is 7.
Also, the remainder becomes 0 and no bar is left.
Therefore, $\sqrt{1369} = 3.7$

- Perfect squares exhibit some special properties.
  - The square of even numbers are even and square of odd numbers are odd.
  - The unit place of a perfect square can never be 2, 3, 7 and 8.
  - By observing the last digit of a number, we can find the last digit of the square of the number.
    - If a number has 1 or 9 at its units place, then its square ends in 1.
    - If a number ends with 4 or 6, then its square end with 6.
    - If a number ends with 2 or 8, then its square ends with 4.
    - If a number ends with 5, then its square ends with 5.
    - If a number ends with 0, then its square also ends with 0.
    - If a number ends with 3 or 7, then its square ends with 9.
  - If a square number ends with 0, then the number of zeroes at the end is even.
  - If a number ends with n number of zeroes, then its square ends with $2n$ zeroes.

- A number is said to be a **perfect cube** if each of its prime factors appears in group of three.
- Prime factorization method can be used to check whether a number is a perfect cube or not.

For example, 5832 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

Here, each of the prime factors occurs in groups of three. Hence, 5832 is a perfect cube.

- Cube root is the inverse operation of finding a cube. The symbol $\sqrt[3]{\ }$ denotes cube-root.

Example:

$\sqrt[3]{64} = 4$ since 4 × 4 × 4 = 64

- The cube root of a perfect cube can be found by prime factorization method.

Example:

Cube root of 287496 is 66.

Prime factorization of 287496

2	287496
2	143748
2	71874
3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

Thus, the number 287496 can be expressed as a product of its prime factors as

$$287496 = \underline{2\times 2\times 2}\times \underline{3\times 3\times 3}\times \underline{11\times 11\times 11} = 2^3\times 3^3\times 11^3 = \left(2\times 3\times 11\right)^3$$

$$\therefore \sqrt[3]{287496} = 2\times 3\times 11 = 66$$