Polynomials

Selected NCERT Questions

1. Find the zeros of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the co-efficients.

Sol. We have, $p(x) = 6x^2 - 3 - 7x$

$$p(x) = 6x^2 - 7x - 3 \qquad \text{(In general form)}$$
$$= 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$$
$$= (2x - 3)(3x + 1)$$

The zeros of polynomial $p(x)$ is given by

$$p(x) = 0$$
$$\Rightarrow \quad (2x - 3)(3x + 1) = 0$$
$$\Rightarrow \quad x = \frac{3}{2}, -\frac{1}{3}$$

Thus, the zeros of $6x^2 - 7x - 3$ are $\alpha = \frac{3}{2}$ and $\beta = -\frac{1}{3}$

Now, sum of the zeros $= \alpha + \beta = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6}$

and $\quad \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

Therefore, sum of the zeros $= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Again, product of zeros $= \alpha . \beta = \frac{3}{2} \times \left(-\frac{1}{3}\right) = -\frac{1}{2}$

and $\quad \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a} = \frac{-3}{6} = -\frac{1}{2}$

Therefore, product of zeros $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$ Hence verified.

2. Verify that the numbers given alongside the cubic polynomial below are its zeros. Also verify the relationship between the zeros and the coefficients.

$x^3 - 4x^2 + 5x - 2$; 2, 1, 1

Sol. Let $p(x) = x^3 - 4x^2 + 5x - 2$

On comparing with general polynomial $p(x) = ax^3 + bx^2 + cx + d$, we get $a = 1, b = -4, c = 5$ and $d = -2$.

Given zeros 2, 1, 1.

$\therefore \quad p(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

and $\quad p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0.$

Hence, 2, 1 and 1 are the zeros of the given cubic polynomial.

Again, consider $\alpha = 2, \beta = 1, \gamma = 1$

$\therefore \quad \alpha + \beta + \gamma = 2 + 1 + 1 = 4$

and $\quad \alpha + \beta + \gamma = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$$

and $\quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a} = \frac{5}{1} = 5$

$$\alpha\beta\gamma = (2)(1)(1) = 2$$

and $$\alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a} = \frac{-(-2)}{1} = 2$$

3. Find the zeroes of the following quadratic polynomial $4u^2 + 8u$ and verify the relationship between the zeroes and the coefficients.

Sol. $4u^2 + 8u = 4u(u + 2)$

Zeroes are 0 and – 2, so, value of $4u^2 + 8u$ is zero, when

$$4u = 0 \Rightarrow u = 0$$

or $$u + 2 = 0 \Rightarrow u = -2$$

Verification:

$$\alpha = 0, \beta = -2$$

$$\therefore \quad \alpha + \beta = 0 + (-2) = -2$$

$$= -\frac{\text{coefficient of } u}{\text{coefficient of } u^2} = -\frac{8}{4} = -2$$

and $$\alpha\beta = 0(-2) = 0$$

$$= \frac{\text{constant term}}{\text{coefficient of } u^2} = \frac{0}{4} = 0 \quad \text{verified.}$$

4. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Sol. Recall the identity $a^2 - b^2 = (a - b)(a + b)$. Using it, we can write:

$$x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$$

So, the value of $x^2 - 3$ is zero when $x = \sqrt{3}$ or $x = -\sqrt{3}$.

Therefore, the zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$.

Now,

Sum of zeroes = $\sqrt{3} - \sqrt{3} = 0 = \dfrac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

Product of zeroes = $(\sqrt{3})(-\sqrt{3}) = -3 = \dfrac{-3}{1} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

5. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\sqrt{2}, \frac{1}{3}$ **(ii)** $-\frac{1}{4}, \frac{1}{4}$

Sol. (*i*) Let the zeroes of polynomial be α and β.

Then, $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

$\because$ Required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \sqrt{2}x + \frac{1}{3}$$

$$= 3x^2 - 3\sqrt{2}x + 1$$

(*ii*) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = -\frac{1}{4}$ and $\alpha\beta = \frac{1}{4}$

$\because$ Required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

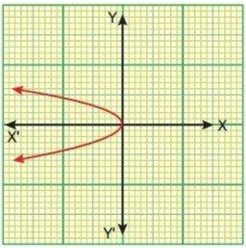
$$= 4x^2 + x + 1 = 0$$

Multiple Choice Questions

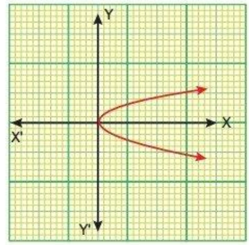
Choose and write the correct option in the following questions.

1. Which of the following graphs could be for the simple polynomial x^2 ?

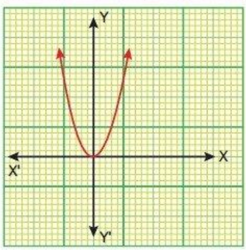
(a)



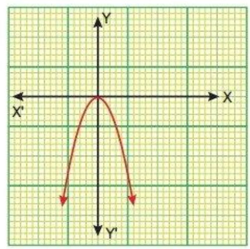
(b)



(c)



(d)



2. The quadratic polynomial, the sum of whose zeros is –5 and their product is 6, is [*CBSE* 2020(30/1/1)]

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$ (c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

3. If one of the zeros of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is – 3, then the value of k is [*NCERT Exemplar*]

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

4. If the zeros of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then [*NCERT Exemplar*]

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. p and q are the zeroes of the polynomial $4y^2 - 4y + 1$. [*CBSE Question Bank*]

What is the value of $\frac{1}{p} + \frac{1}{q} + pq$?

(a) $\frac{-15}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{17}{4}$

6. The zeros of the polynomial $x^2 - 3x - m(m + 3)$ are [*CBSE* 2020(30/2/1)]

(a) $m, m + 3$ (b) $-m, m + 3$ (c) $m, -(m + 3)$ (d) $-m, -(m + 3)$

7. If one of the zeros of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

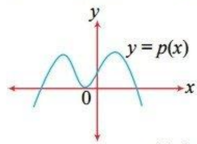
8. The number of polynomials having zeros as – 2 and 5 is [*NCERT Exemplar*]

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) more than 3

9. If one root of the polynomial $p(y) = 5y^2 + 13y + m$ is reciprocal of other, then the value of m is

(*a*) 6 (*b*) 0 (*c*) 5 (*d*) $\frac{1}{5}$

10. The number of zeros for a polynomial $p(x)$ where graph of $y = p(x)$ is given in figure, is [*CBSE* 2020(30/4/1)]



(*a*) 3 (*b*) 4 (*c*) 0 (*d*) 5

11. Given $m + 2$, where m is a positive integer, is a zero of the polynomial $q(x) = x^2 - mx - 6$. Which of these is the value of m? [*Competency Based Question*]

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

12. Consider the expression $x^{(m^2-1)} + 3x^{\frac{m}{2}}$, where m is a constant. For what value of m, will the expression be a cubic polynomial? [*Competency Based Question*]

(*a*) 1 (*b*) 2 (*c*) –1 (*d*) –2

13. Given that one of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeros is [*NCERT Exemplar*]

(*a*) $-\frac{c}{a}$ (*b*) $\frac{c}{a}$ (*c*) 0 (*d*) $-\frac{b}{a}$

14. If one of the zeros of the cubic polynomial $x^3 + ax^2 + bx + c$ is – 1, then the product of the other two zeroes is [*NCERT Exemplar*]

(*a*) $b - a + 1$ (*b*) $b - a - 1$ (*c*) $a - b + 1$ (*d*) $a - b - 1$

15. Which of these is a zero of the polynomial $p(y) = 3y^3 - 16y - 8$?

(*a*) 2 (*b*) 8 (*c*) – 2 (*d*) – 8

16. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is [*CBSE* 2020(30/1/1)]

(*a*) 10 (*b*) – 10 (*c*) – 7 (*d*) – 2

17. The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is [*CBSE* 2020(30/3/1)]

(*a*) 16 (*b*) 9 (*c*) 1 (*d*) 4

18. How many zero(es) does $(x - 2)(x + 3)$ have? [*CBSE Question Bank*]

(*a*) zero (*b*) one (*c*) two (*d*) three

19. Which of these is the polynomial whose zeroes are $\frac{1}{3}$ and $\left(\frac{-3}{4}\right)$? [*CBSE Question Bank*]

(*a*) $12x^2 + 5x - 3$ (*b*) $12x^2 - 5x - 3$ (*c*) $12x^2 + 13x + 3$ (*d*) $12x^2 - 13x - 3$

20. How many zero(es) does the polynomial $293x^2 - 293x$ have? [*CBSE Question Bank*]

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

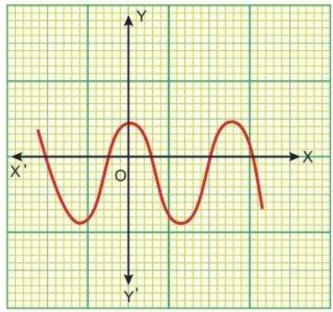
Answers

1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*d*)	**6.** (*b*)	**7.** (*a*)
8. (*d*)	**9.** (*c*)	**10.** (*a*)	**11.** (*d*)	**12.** (*b*)	**13.** (*b*)	**14.** (*a*)
15. (*c*)	**16.** (*b*)	**17.** (*d*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)	

Very Short Answer Questions

Each of the following questions are of 1 mark.

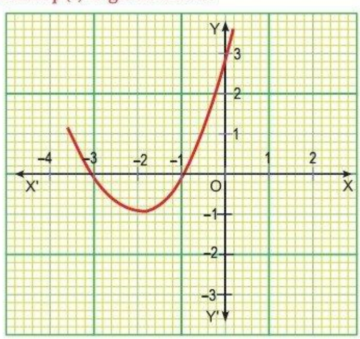
1. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows:



Write the number of zeros of $p(x)$.

Sol. Since the graph of $y = p(x)$ cuts the x-axis at five points, therefore the number of zeros of polynomial $p(x)$ is 5.

2. The graph of the polynomial $p(x)$ is given below:



Write the number of zeros of the polynomial $p(x)$.

Sol. The graph of the polynomial $p(x)$ intersect the x–axis at two points, therefore this polynomial $p(x)$ has two zeros.

3. For what value of k, is –3 a zero of the polynomial $x^2 + 11x + k$?

Sol. Let $p(x) = x^2 + 11x + k$ be the given polynomial, since –3 is the zero of the polynomial $p(x)$.

$\therefore \quad p(-3) = 0 \Rightarrow \quad (-3)^2 + 11(-3) + k = 0$

$\Rightarrow \quad 9 - 33 + k = 0$

$\Rightarrow \quad -24 + k = 0 \Rightarrow \quad k = 24$

4. Form a quadratic polynomial, the sum and product of whose zeros are (– 3) and 2 respectively.

[*CBSE* 2020(30/5/1)]

Sol. Quadratic polynomial is given by

$x^2 - (\text{Sum of zeros}) \cdot x + \text{Product of zeros}$

$= x^2 - (-3)\,x + 2 = x^2 + 3x + 2$

5. **Find the quadratic polynomial whose zeros are –3 and 4.**

Sol. Sum of zeros = –3 + 4 = 1,

Product of zeros = – 3 × 4 = –12

∴ Required polynomial = $x^2 - x - 12$

6. **Can a quadratic polynomial $x^2 + kx + k$ have equal zeros for some odd integer $k > 1$?**

Sol. No, for equal zeros, $k^2 - 4k = 0$ or $k = 0, 4$

$\Rightarrow k$ should be even.

7. **If one zero of the quadratic polynomial $x^2 - 5x - 6$ is 6 then find the other zero.**

Sol. Let α, 6 be the zeros of given polynomial.

Then $\alpha + 6 = 5 \Rightarrow \alpha = -1$.

8. **Check whether $(x + 1)$ is a factor of the polynomial $p(x) = x^3 + 4x + 5$.**

Sol. $p(x) = x^3 + 4x + 5$

Putting $x = -1$, we have

$$p(-1) = (-1)^3 + 4(-1) + 5 = -1 - 4 + 5 = 0$$

$\Rightarrow$ $(x + 1)$ is a factor of polynomial $p(x)$.

Yes, $(x + 1)$ is a factor of the polynomial $p(x) = x^3 + 4x + 5$.

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. **A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to hand over the paper. The following were the answers given by the students:**

$2x + 3,\ 3x^2 + 7x + 2,\ 4x^3 + 3x^2 + 2,\ x^3 + \sqrt{3x} + 7,\ 7x + \sqrt{7},\ 5x^3 - 7x + 2,\ 2x^2 + 3 - \frac{5}{x},$

$5x - \frac{1}{2},\ ax^3 + bx^2 + cx + d,\ x + \frac{1}{x}$

Answer the following questions :

(i) How many of the above ten, are not polynomials ?

(ii) How many of the above ten, are quadratic polynomials? [*CBSE* 2020(30/2/1)]

Sol. (i) $3[(x^3 + \sqrt{3x} + 7,\ 2x^2 + 3 - \frac{5}{x},\ x + \frac{1}{x})]$ are not polynomials.

(ii) $1[(3x^2 + 7x + 2)]$ is a quadratic polynomial.

2. **Find the value of k, if –1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.**

Sol. We have, $p(x) = kx^2 - 4x + k$

Since –1 is a zero of polynomial

∴ $p(-1) = k(-1)^2 - 4(-1) + k = 0$

$\Rightarrow k + 4 + k = 0$

$\Rightarrow 2k + 4 = 0 \Rightarrow 2k = -4 \Rightarrow k = -2$

3. **Find the zeros of the polynomial $p(x) = 4x^2 - 12x + 9$.**

Sol. $p(x) = 4x^2 - 12x + 9 = (2x - 3)^2$

For zeros, $p(x) = 0$

$\Rightarrow (2x - 3)(2x - 3) = 0$

$\Rightarrow x = \frac{3}{2}, \frac{3}{2}$.

4. If α and β are zeros of $p(x) = x^2 + x - 1$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. Here, $\alpha + \beta = -1$, $\alpha\beta = -1$, so $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-1}{-1} = 1$

5. If zeros of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$, then find the value of a.

Sol. Product of (zeros) roots $= \frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha}$

or $2a = 2$

∴ $a = 1$

6. If one of the zeros of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is equal in magnitude but opposite in sign of the other, find the value of k.

Sol. Let one root of the given polynomial be α.

Then the other root = – α

∴ Sum of the roots $= (-\alpha) + \alpha = 0$

$\Rightarrow \quad \frac{-b}{a} = 0 \quad$ or $\quad \frac{8k}{4} = 0 \quad$ or $\quad k = 0$

7. If α and β are zeros of the polynomial $f(x) = ax^2 + bx + c$, then find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

Sol. Given : $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \left[\frac{b^2}{a^2} - \frac{2c}{a}\right] \times \frac{a^2}{c^2} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2} = \frac{b^2 - 2ac}{c^2}$$

8. If the product of two zeros of the polynomial $p(x) = 2x^3 + 6x^2 - 4x + 9$ is 3, then find its third zero.

Sol. Let α, β, γ be the roots of the given polynomial and $\alpha\beta = 3$.

Then $\quad \alpha\beta\gamma = \frac{-d}{a} = -\frac{9}{2}$

$\Rightarrow \quad 3 \times \gamma = \frac{-9}{2} \quad$ or $\quad \gamma = \frac{-3}{2}$

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product. **[*CBSE* 2019(30/1/1)]**

Sol. Sum of zeros $= k + 6$ 1

Product of zeros $= 2(2k - 1)$ 1

Hence $k + 6 = \frac{1}{2} \times 2(2k - 1)$

∴ $k = 7$ 1

[*CBSE Marking Scheme* 2019 (30/1/1)]

2. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Sol. Let α and β be the zeros of the polynomial. Then as per question $\beta = 7\alpha$.

Now sum of zeros $= \alpha + \beta = \alpha + 7\alpha = -\left(\frac{-8}{3}\right)$

$\Rightarrow \quad 8\alpha = \frac{8}{3} \quad$ or $\quad \alpha = \frac{1}{3}$

and $\quad \alpha \times \beta = \alpha \times 7\alpha = \frac{2k+1}{3}$

$\Rightarrow \quad 7\alpha^2 = \frac{2k+1}{3}$

$\Rightarrow \quad 7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3} \quad \left(\because \alpha = \frac{1}{3}\right)$

$\Rightarrow \quad \frac{7}{9} = \frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3} = 2k+1$

$\Rightarrow \quad \frac{7}{3} - 1 = 2k \quad \Rightarrow \quad k = \frac{2}{3}$

3. Quadratic polynomial $2x^2 - 3x + 1$ has zeros as α and β. Now form a quadratic polynomial whose zeros are 3α and 3β. [*Competency Based Question*]

Sol. It is given that α and β are zeros of the polynomial $2x^2 - 3x + 1$.

$\therefore \quad \alpha + \beta = \frac{-(-3)}{2} = \frac{3}{2} \quad$ and $\quad \alpha\beta = \frac{1}{2}$

Now, new quadratic polynomial whose zeros are 3α and 3β is given by

x^2 – (sum of zeros)x + product of zeros

$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$

$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$

$= x^2 - 3 \times \frac{3}{2}x + 9 \times \frac{1}{2}$

$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{1}{2}(2x^2 - 9x + 9)$

4. If $\frac{2}{3}$ and –3 are the zeros of the polynomial $ax^2 + 7x + b$, then find the values of a and b.

[*CBSE* 2019(30/3/3)]

Sol. $ax^2 + 7x + b$

Sum of zeros $= \frac{-b}{a} = \frac{-7}{a}$

But sum of given zeros $= \frac{2}{3} - 3 = -\frac{7}{3}$

Now, $\quad \frac{-7}{a} = \frac{-7}{3}$

$\therefore \quad a = 3$

Product of zeros $= \frac{c}{a} = \frac{b}{a}$

But product of given zeros $= \frac{2}{3} \times (-3) = -2$. Therefore, $-2 = \frac{b}{3}$

$\therefore \quad b = -6$

5. Find the quadratic polynomial whose zeros are reciprocal of the zeros of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$. [*CBSE* 2020(30/1/1)]

Sol. $f(x) = ax^2 + bx + c$

$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$ ½

New sum of zeroes $= \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{b}{c}$ 1

New product of zeroes $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{a}{c}$ 1

$\therefore$ Required quadratic polynomial $= x^2 + \frac{b}{c}x + \frac{a}{c}$ or $cx^2 + bx + a$ ½

[*CBSE Marking Scheme* 2020 (30/1/1)]

6. Find the zeros of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between zeros and the coefficients. [*CBSE* 2019(30/2/1)]

Sol. Given: $p(x) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2) = \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y-2) + 1(3y-2)] = \frac{1}{3}(7y+1)(3y-2)$

Equating $p(x) = 0$

$\Rightarrow \frac{1}{3}(7y+1)(3y-2) = 0 \quad \Rightarrow \quad y = \frac{-1}{7}$ and $y = \frac{2}{3}$

Now, Sum of zeros $= \frac{-b}{a} = \frac{-\left(\frac{-11}{3}\right)}{7} = \frac{11}{21}$ and $\frac{2}{3} - \frac{1}{7} = \frac{14-3}{21} = \frac{11}{21}$

and product of zeros $= \frac{c}{a} = \frac{-\frac{2}{3}}{7} = \frac{-2}{21}$ and $\frac{-1}{7} \times \frac{2}{3} = \frac{-2}{21}$ Hence verified.

7. Find the quadratic polynomial sum and product of whose zeros are –1 and –20 respectively. Also find the zeros of the polynominal so obtained. [*CBSE* 2019(30/4/2)]

Sol. Let α and β be the zeros of the quadratic polynomial.

$\therefore$ Sum of zeros, $\alpha + \beta = -1$

and product of zeros, $\alpha . \beta = -20$

So, quadratic polynomial is given by

$x^2 - (\alpha + \beta).x + \alpha\beta$

$= x^2 - (-1)x - 20$

$= x^2 + x - 20$

Now, for zeros of this polynominal

$x^2 + x - 20 = 0 \quad \Rightarrow \quad x^2 + 5x - 4x - 20 = 0$

$\Rightarrow x(x+5) - 4(x+5) = 0 \quad \Rightarrow \quad (x+5)(x-4) = 0$

$\Rightarrow x = -5, 4$

$\therefore$ Zeros are – 5 and 4.

8. **If α, β, γ be zeros of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.**

[*Competency Based Question*]

Sol. $p(x) = 6x^3 + 3x^2 - 5x + 1$ so $a = 6, b = 3, c = -5, d = 1$

$\because$ α, β and γ are zeros of the polynomial $p(x)$.

$\therefore$ $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-3}{6} = \frac{-1}{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{-5}{6}$

and $\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$

Now $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-5/6}{-1/6} = 5$$

Long Answer Questions

Each of the following questions are of 5 marks.

1. **Find the zeros of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeros are in AP.**

[*Competency Based Question*]

Sol. Let α, β, γ are the zeros of $f(x)$. If α, β, γ are in AP, then,

$\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \alpha + \gamma$ ...(*i*)

$\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-(-12)}{1} = 12 \Rightarrow \alpha + \gamma = 12 - \beta$ ...(*ii*)

From (*i*) and (*ii*)

$2\beta = 12 - \beta$ or $3\beta = 12$ or $\beta = 4$

Putting the value of β in (*i*), we have

$8 = \alpha + \gamma$ ...(*iii*)

$\alpha\beta\gamma = -\frac{d}{a} = \frac{-(-28)}{1} = 28$

$(\alpha\gamma)4 = 28$ or $\alpha\gamma = 7$ or $\gamma = \frac{7}{\alpha}$ ...(*iv*)

Putting the value of $\gamma = \frac{7}{\alpha}$ in (*iii*), we get

$\Rightarrow 8 = \alpha + \frac{7}{\alpha}$ $\Rightarrow 8\alpha = \alpha^2 + 7$

$\Rightarrow \alpha^2 - 8\alpha + 7 = 0$ $\Rightarrow \alpha^2 - 7\alpha - 1\alpha + 7 = 0$

$\Rightarrow \alpha(\alpha - 7) - 1(\alpha - 7) = 0$ $\Rightarrow (\alpha - 1)(\alpha - 7) = 0$

$\Rightarrow \alpha = 1$ or $\alpha = 7$

Putting $\alpha = 1$ in (*iv*), we get

$\gamma = \frac{7}{1}$

or $\gamma = 7$

and $\beta = 4$

$\therefore$ zeros are 1, 4, 7.

Putting $\alpha = 7$ in (*iv*), we get

$\gamma = \frac{7}{7}$

or $\gamma = 1$

and $\beta = 4$

$\therefore$ zeros are 7, 4, 1.

Hence zeros are 1, 4, 7 or 7, 4, 1.

2. Without actually calculating the zeroes, form a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $5x^2 + 2x - 3$. [*CBSE* 2020(30/4/1)]

Sol. Let zeroes of given quadratic polynomial be α and β.

Now, $\alpha + \beta = \frac{-2}{5}$ and $\alpha\beta = \frac{-3}{5}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{-2}{5}}{\frac{-3}{5}} = \frac{2}{3}$$

$$\frac{1}{\alpha\beta} = \frac{-5}{3}$$

Required polynomial is

$$x^2 - \frac{2}{3}x - \frac{5}{3} \quad \text{or} \quad 3x^2 - 2x - 5$$

[*CBSE Marking Scheme* 2020(30/4/1)]

3. Find the zeroes of the quadratic polynomial $pqx^2 + (pr + qs)x + rs$ and verify the relationship between the zeroes and the coefficients. [*Competency Based Question*]

Sol. Given quadratic polynomial is $pqx^2 + (pr + qs)x + rs$.

Here coefficient of $x^2 = pq$, coefficient of $x = pr + qs$ and constant term $= rs$.

Now, $pqx^2 + (pr + qs)x + rs$

$$= (pqx^2 + prx) + (qsx + rs)$$
$$= px(qx + r) + s(qx + r)$$
$$= (px + s)(qx + r)$$

Clearly, $px^2 + (pr + qs)x + rs = 0$

$\Rightarrow \quad (px + s)(qx + r) = 0$

$\therefore \quad x = -\frac{s}{p}$ or $x = -\frac{r}{q}$

Hence zeroes of the given quadratic polynomial are $-\frac{s}{p}$ and $-\frac{r}{q}$.

Second Part:

Sum of zeroes $= -\frac{s}{p} - \frac{r}{q} = -\frac{sq + pr}{pq} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= \left(-\frac{s}{p}\right)\left(-\frac{r}{q}\right) = \frac{sr}{pq} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

4. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial. [*NCERT Exemplar*]

Sol. Let $f(x) = x^3 - 6x^2 + 3x + 10$

Given that $a, (a + b)$ and $(a + 2b)$ are the zeroes of $f(x)$. Then

Sum of the zeroes $= -\frac{(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$

$\Rightarrow \quad a + (a + b) + (a + 2b) = \frac{-(-6)}{1}$

$\Rightarrow \quad 3a + 3b = 6 \quad \Rightarrow \quad a + b = 2 \quad$ ...(*i*)

Sum of product of two zeros at a time $= \frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^3)}$

$\Rightarrow \qquad a + (a + b) + (a + b)(a + 2b) + a(a + 2b) = \frac{3}{1}$

$\Rightarrow \qquad a(a + b) + (a + b)\{(a + b) + b\} + a\{(a + b) + b\} = 3$

$\Rightarrow \qquad 2a + 2(2 + b) + a(2 + b) = 3$ [Using Eq. (*i*)]

$\Rightarrow \qquad 2a + 2(2 + 2 - a) + a(2 + 2 - a) = 3$ [Using Eq. (*i*)]

$\Rightarrow \qquad 2a + 8 - 2a + 4a - a^2 = 3 \qquad \Rightarrow \qquad -a^2 + 8 = 3 - 4a$

$\Rightarrow \qquad a^2 - 4a - 5 = 0$

Using factorisation method

$a^2 - 5a + a - 5 = 0$

$\Rightarrow \qquad a(a - 5) + 1(a - 5) = 0 \qquad \Rightarrow \qquad (a - 5)(a + 1) = 0$

$\Rightarrow \qquad a = -1, 5$

When $a = -1$, then $b = 3$

When $a = 5$ then $b = -3$ [using equation (*i*)]

$\therefore$ Required zeroes of $f(x)$ are

When $a = -1$ and $b = 3$

Then, $a, (a + b), (a + 2b) = -1, (-1 + 3), (-1 + 6)$ or $-1, 2, 5$

When $a = 5$ and $b = -3$ then

$a, (a + b), (a + 2b) = 5, (5 - 3), (5 - 6)$ or $5, 2, -1$

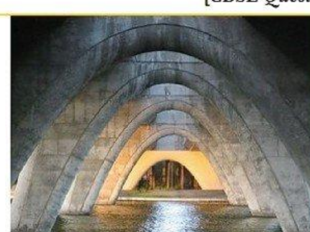
Hence, the required values of a and b are $a = -1$ and $b = 3$ or $a = 5, b = -3$ and the zeroes are $-1, 2$ and 5.

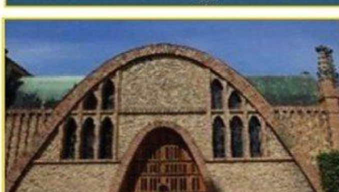
Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (*i*) to (*v*).

The below pictures are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms. **[*CBSE Question Bank*]**





(i) In the standard form of quadratic polynomial, $ax^2 + bx + c$, a, b, and c where

(a) All are real numbers.

(b) All are rational numbers.

(c) 'a' is a non zero real number and b and c are any real numbers.

(d) All are integers.

(ii) If the roots of the quadratic polynomial are equal, where the discriminant $D = b^2 - 4ac$, then

(a) $D > 0$ (b) $D < 0$ (c) $D \geq 0$ (d) $D = 0$

(iii) If α and $\frac{1}{\alpha}$ are the zeros of the quadratic polynomial $2x^2 - x + 8k$ then k is

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{-1}{4}$ (d) 2

(iv) The graph of $x^2 + 1 = 0$ *[Competency Based Question]*

(a) Intersects x-axis at two distinct points.

(b) Touches x-axis at a point.

(c) Neither touches nor intersects x-axis.

(d) Either touches or intersects x-axis.

(v) If the sum of the roots is $-p$ and product of the roots is $-\frac{1}{p}$, then the quadratic polynomial is *[Competency Based Question]*

(a) $k\left(-px^2 + \frac{x}{p} + 1\right)$ (b) $k\left(px^2 + \frac{x}{p} - 1\right)$

(c) $k\left(x^2 + px - \frac{1}{p}\right)$ (d) $k\left(x^2 - px + \frac{1}{p}\right)$

Sol. (i) In the standard form of quadratic polynomial $ax^2 + bx + c$, a is a non zero real number and b and c are any real numbers.

Hence option (c) is correct.

(ii) In case of quadratic polynomial if the roots are equal then the discriminant (D) should be equal to 0.

Hence option (d) is correct.

(iii) Given quadratic polynomial is $2x^2 - x + 8k$.

If α and $\frac{1}{\alpha}$ are zeros then their product $= \alpha \times \frac{1}{\alpha} = \frac{8k}{2}$

$\Rightarrow \quad 1 = 4k \quad \Rightarrow \quad k = \frac{1}{4}$

Hence option (b) is correct.

(iv) Given quadratic polynomial is $x^2 + 1 = 0$.

$\Rightarrow \quad x^2 = -1$

$\Rightarrow$ Zeros can't be find out so its graph neither touches nor intersects x-axis.

Hence option (c) is correct.

(v) Given: Sum of roots $= -p$ and product of roots $= \frac{-1}{p}$

The general form of quadratic polynomial is

$$k(x^2 - (\text{sum of zeros})x + \text{product of zeros})$$

$$\Rightarrow \quad k\left(x^2-(-p)x+\left(\frac{-1}{p}\right)\right) \quad \Rightarrow \quad k\left(x^2+px-\frac{1}{p}\right)$$

Hence option (*c*) is correct.

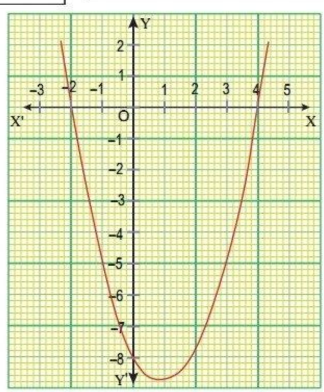
2. Read the following and answer any four questions from (*i*) to (*v*).

An *asana* is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial. **[*CBSE Question Bank*]**

Urdhva Dhanurasana

Trikonasana

Adhomuhasavana



(*i*) The shapes of the poses shown are

(*a*) Spiral (*b*) Ellipse (*c*) Linear (*d*) Parabola

(*ii*) The graph of parabola opens downward, if

(*a*) $a \geq 0$ (*b*) $a = 0$ (*c*) $a < 0$ (*d*) $a > 0$

(*iii*) In the graph, how many zeros are there for the polynomial?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(*iv*) The two zeroes in the above shown graph are

(*a*) 2, 4 (*b*) –2, 4 (*c*) –8, 4 (*d*) 2, –8 (*v*) **The zeros of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are**

(*a*) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (*b*) $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (*c*) $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ (*d*) $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

ol. (*i*) The shape of the poses shown is parabola.
Hence option (*d*) is correct.

(*ii*) The graph of the parabola opens downward if $a < 0$.
Hence option (*c*) is correct.

(*iii*) Since the given graph is intersecting *x*-axis at two places, therefore it should have 2 zeros.
Hence option (*c*) is correct.

(*iv*) Two zeros of the given graph are –2 and 4.
Hence option (*b*) is correct.

(*v*) Given quadratic polynomial is $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$.
By mid term splitting, we can write

$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

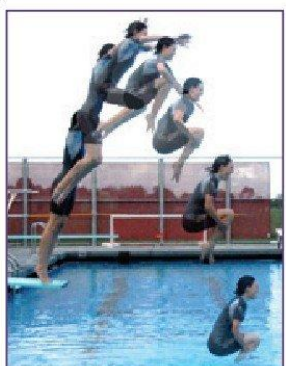
$$\Rightarrow \quad 4x(\sqrt{3}x+2) - \sqrt{3}(\sqrt{3}x+2)$$

$$\Rightarrow \quad (\sqrt{3}x+2)(4x-\sqrt{3}) \quad \Rightarrow \quad x = \frac{-2}{\sqrt{3}}, x = \frac{\sqrt{3}}{4}$$

Hence option (*b*) is correct.

3. The figure given below shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time '*t*' in seconds is given by the polynomial $h(t)$ such that $h(t) = -16t^2 + 8t + k$.



Based on above information answer the following questions.

(*i*) (*a*) **What is the value of *k*?**

(*b*) **At what time will she touch the water in the pool?**

(*ii*) (*a*) **Rita's height (in feet) above the water level is given by another polynomial $p(t)$ with zeroes –1 and 2, then find $p(t)$.**

(*b*) **A polynomial $q(t)$ with sum of zeroes as 1 and the product as –6 is modelling Anu's height in feet above the water at any time *t* (in seconds). Then find $q(t)$.**

Sol. (*i*) (*a*) Initially, at $t = 0$, Annie's height is 48ft.

So, at $t = 0$, h should be equal to 48.

$$h(0) = -16(0)^2 + 8(0) + k = 48$$

So $k = 48$

(*b*) When Annie touches the pool, her height $\equiv 0$ feet

i.e., $-16t^2 + 8t + 48 = 0$ above water level

$\Rightarrow \quad 2t^2 - t - 6 = 0$

$\Rightarrow \quad 2t^2 - 4t + 3t - 6 = 0$

$\Rightarrow \quad 2t(t-2) + 3(t-2) = 0$

$\Rightarrow \quad (2t + 3)(t - 2) = 0$

i.e., $\quad t = 2$ or $t = \dfrac{-3}{2}$

Since time cannot be negative, so $t = 2$ seconds.

(*ii*) (*a*) $\because$ $t = -1$ and $t = 2$ are the two zeroes of the polynomial $p(t)$.

then $\quad p(t) = k\,(t - (-1))(t - 2)$

$= k(t + 1)(t - 2) = k(t^2 - t - 2)$

When $t = 0$ (initially) $h_1 = 48$ ft

$$p(0) = k(0^2 - 0 - 2) = 48$$

i.e., $\quad -2k = 48 \quad \Rightarrow \quad k = -24$

So the polynomial is $-24(t^2 - t - 2) = -24t^2 + 24t + 48$.

(*b*) A polynomial $q(t)$ with sum of zeroes as 1 and the product as -6 is given by $q(t) = k(t^2 -$(sum of zeroes) $t +$ product of zeroes)

$= k(t^2 - 1t + (-6)) = k(t^2 - t - 6)$ ...(*i*)

When $t = 0$ (initially) $q(0) = 48$ ft

$$q(0) = k(0^2 - 1(0) - 6) = 48$$

i.e. $\quad -6k = 48$ or $k = -8$

Putting $k = -8$ in equation (*i*), required polynomial is $-8(t^2 - 1t - 6)$.

$= -8t^2 + 8t + 48$

PROFICIENCY EXERCISE

■ **Objective Type Questions:** **[1 mark each]**

1. Choose and write the correct option in each of the following questions.

(*i*) If 5 is a zero of the quadratic polynomial, $x^2 - kx - 15$ then the value of k is

(*a*) 2 (*b*) -2 (*c*) 4 (*d*) -4

(*ii*) The number of polynomials having zeros 1 and -2 is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) more than 3

(*iii*) A quadratic polynomial, whose zeros are -3 and 4, is

(*a*) $x^2 - x + 12$ (*b*) $x^2 + x + 12$

(*c*) $\dfrac{x^2}{2} - \dfrac{x}{2} - 6$ (*d*) $2x^2 + 2x - 24$

(*iv*) The zeros of the quadratic polynomial $x^2 + kx + k, k \neq 0$

(*a*) cannot be positive (*b*) cannot be negative

(*c*) are always equal (*d*) are always unequal (*v*) If the graph of a polynomial intersects the x-axis at exactly two points, then it

(*a*) cannot be a linear or a cubic polynomial

(*b*) can be a quadratic polynomial only

(*c*) can be a cubic or a quadratic polynomial

(*d*) can be a linear or a quadratic polynomial

■ Very Short Answer Questions: [1 mark each]

2. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

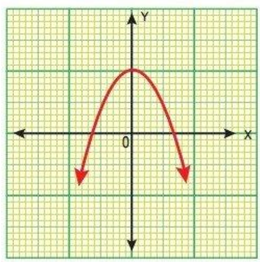
3. If one root of the polynomial $p(y) = 7y^2 + 14y + m$ is reciprocal of other, then find the value of m.

4. Find the other zero of the quadratic polynomial $y^2 + 7y - 60$ if one zero is -12.

5. Find the quadratic polynomial whose zeros are -3 and -5.

6. What number should be added to the polynomial $x^2 + 7x - 35$ so that 3 is the zero of the polynomial?

7. The graph of $y = p(x)$ where $p(x)$ is a polynomial in variable x, is as follows:



Write the number of zeros of $p(x)$.

8. If one zero of the quadratic polynomial $p(x) = x^2 + 4kx - 25$ is negative of the other, find the value of k.

■ Short Answer Questions–I: [2 marks each]

9. If m and n are the zeros of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

[*Competency Based Question*]

10. Find the zeros of the polynomial $5y^2 - 11y + 2$.

11. If one of the zeros of the quadratic polynomial $(k - 2)x^2 - 2x - (k + 5)$ is 4, find the value of k.

12. If α, β are the zeros of the polynomial $x^2 + x - 6$, find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

13. If α, β are the two zeros of the polynomial $f(y) = y^2 - 8y + a$ and $\alpha^2 + \beta^2 = 40$, find the value of a.

14. If the sum of the zeros of the quadratic polynomial $f(x) = kx^2 + 2x + 3k$ is equal to their product, find the value of k.

■ Short Answer Questions–II: [3 marks each]

15. Find the zeros of the following polynomials and verify the relationship between the zeros and the coefficients of the polynomials.

(*i*) $x^2 + \frac{1}{6}x - 2$ (*ii*) $\sqrt{3}x^2 - 11x + 6\sqrt{3}$ (*iii*) $a(x^2 + 1) - x(a^2 + 1)$

16. If α and β are the zeros of the quadratic polynomial $f(t) = t^2 - p(t + 1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$. **[*Competency Based Question*]**

17. If $(x - 2)$ is a factor of $x^3 + ax^2 + bx + 16$ and $b = 4a$, find the values of a and b.

■ Long Answer Questions: [5 marks each]

18. If α and β are zeros of polynomial $f(x) = 2x^2 + 11x + 5$, then find

(i) $\alpha^4 + \beta^4$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

19. Verify that the numbers given alongside the cubic polynomials below are their zeros. Also verify the relationship between the zeros and the coefficients.

(i) $x^3 - 2x^2 - 5x + 6;\ -2, 1, 3$ (ii) $2x^3 + 7x^2 + 2x - 3;\ -3, -1, \frac{1}{2}$

20. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b. **[*Competency Based Question*]**

Answers

1. (i) (a) (ii) (d) (iii) (c) (iv) (a) (v) (c)
2. $a = 1$ 3. $m = 7$ 4. 5 5. $x^2 + 8x + 15$ 6. 5 7. 2 8. $k = 0$
9. $\frac{-145}{12}$ 10. $\frac{1}{5}, 2$ 11. $k = 3$ 12. $\frac{13}{36}$ 13. $a = 12$ 14. $k = \frac{-2}{3}$
15. (i) $\left(\frac{4}{3}, \frac{-3}{2}\right)$ (ii) $\frac{2}{\sqrt{3}}, 3\sqrt{3}$ (iii) $a, \frac{1}{a}$ 17. $a = -2, b = -8$ 18. (i) $\frac{10001}{16}$ (ii) $\frac{-36}{5}$
20. $a = 1, b = \pm\sqrt{2}$

Self-Assessment

Time allowed: 1 hour Max. marks: 40

SECTION A

1. **Choose and write the correct option in the following questions.** $(3 \times 1 = 3)$

(i) A quadratic polynomial with 3 and 2 as the sum and product of its zeros respectively is
(a) $x^2 + 3x - 2$ (b) $x^2 - 3x + 2$
(c) $x^2 - 2x + 3$ (d) $x^2 - 2x - 3$

(ii) The zeros of the quadratic polynomial $x^2 + 99x + 127$ are
(a) both positive (b) both negative
(c) one positive and one negative (d) both equal

(iii) If the product of the zeros of the polynomial $p(x) = ax^3 - 6x^2 + 11x - 6$ is 4, then a equals
(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

2. **Solve the following questions.** $(2 \times 1 = 2)$

(i) If α and β are zeros of a polynomial $x^2 - 4\sqrt{3}x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

(ii) If the square of difference of the zeros of the quadratic polynomial $x^2 + px + 45$ is equal to 144. Find out the value of p. **[*Competency Based Question*]**

SECTION B

- **Solve the following questions.** $(4 \times 2 = 8)$

3. If p and q are the zeros of the polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

4. If α and β are the zeros of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, then evaluate $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

5. If α, β are the zeros of the quadratic polynomial $p(y) = y^2 - 4y + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

6. Find the cubic polynomial with the sum, sum of the products of its zeros taken two at a time, and the products of its zeros as -3, -8 and 2 respectively.

- **Solve the following questions.** $(4 \times 3 = 12)$

7. If $\frac{2}{3}$ and -3 are the zeros of the polynomial $ax^2 + 7x + b$, then find the values of a and b.

8. If $(x + a)$ is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$ then prove that $a = \frac{n - q}{m - p}$.

   [*Competency Based Question*]

9. If α and β are the zeros of the polynomial $2x^2 - 3x + 1$, then find the value of (*i*) $\alpha^2\beta + \alpha\beta^2$ (*ii*) $\alpha^2 + \beta^2$.

10. Find a quadratic polynomial whose zeros are

$$\frac{5 + \sqrt{2}}{5 - \sqrt{2}}, \frac{5 - \sqrt{2}}{5 + \sqrt{2}}.$$

- **Solve the following questions.** $(3 \times 5 = 15)$

11. If the sum of squares of the zeroes of the quadratic polynomial $x^2 - 8x + k$ be 40, find k.

12. If α and β are the zeros of the quadratic polynomial $f(x) = 3x^2 - 7x - 6$, find a polynomial whose zeros are (*i*) α^2 and β^2 (*ii*) $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

13. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$$x^2 - (2a + b)x + 2ab$$

Answers

1. (*i*) (*b*) (*ii*) (*b*) (*iii*) (*d*) **2.** (*i*) $4\sqrt{3} - 3$ (*ii*) ± 18 **3.** $\frac{37}{4}$

4. $\frac{-215}{18}$ **5.** 108 **6.** $x^3 + 3x^2 - 8x - 2$ **7.** $a = 3, b = -6$

9. (*i*) $\frac{3}{4}$ (*ii*) $\frac{5}{4}$ **10.** $23x^2 - 54x + 23$

11. 12 **12.** (*i*) $\frac{1}{9}(9x^2 - 85x + 36)$ (*ii*) $\frac{1}{3}(3x^2 - 35x + 92)$ **13.** $2a, b$