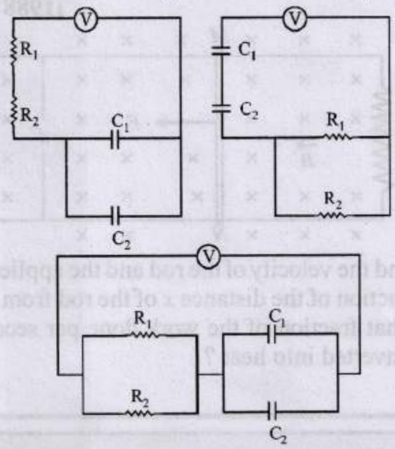
Chapter
Alternating Current

Topic-1: AC Circuit, LCR Circuit, Quality and Power Factor

1 MCQs with One Correct Answer

1. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased **[2010]**
   (a) the bulb glows dimmer
   (b) the bulb glows brighter
   (c) total impedance of the circuit is unchanged
   (d) total impedance of the circuit increases

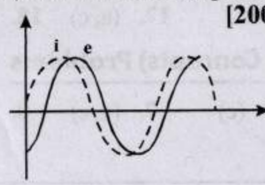
2. Find the time constant (in μs) for the given RC circuits in the given order respectively **[2006 - 3M, –1]**



$R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 4\mu F$, $C_2 = 2\mu F$

(a) $18, 4, \frac{8}{9}$ (b) $18, \frac{8}{9}, 4$ (c) $4, 18, \frac{8}{9}$ (d) $4, \frac{8}{9}, 18$

3. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements **[2003S]**
   (a) $R = 1k\Omega, C = 10\,\mu F$
   (b) $R = 1k\Omega, C = 1\,\mu F$
   (c) $R = 1k\Omega, L = 10\,H$
   (d) $R = 1k\Omega, L = 1\,H$



2 Integer Value Answer

4. Consider an LC circuit, with inductance $L = 0.1\ H$ and capacitance $C = 10^{-3} F$, kept on a plane. The area of the circuit is $1 m^2$. It is placed in a constant magnetic field of strength B_0 which is perpendicular to the plane of the circuit. At time $t = 0$, the magnetic field strength starts increasing linearly as $B = B_0 + \beta t$ with $\beta = 0.04\ T s^{-1}$. The maximum magnitude of the current in the circuit is ____ mA. **[Adv. 2022]**

5. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is **[Adv. 2016]**

3 Numeric / New Stem Based Questions

Stem for Qs. 6-7

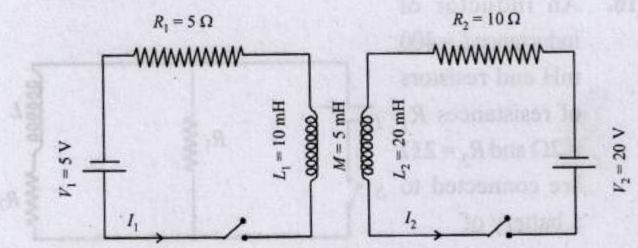
In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take *rms* values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is φ.

Assume, $\pi\sqrt{3} \approx 5$.

6. The value of C is ____. **[Adv. 2021]**

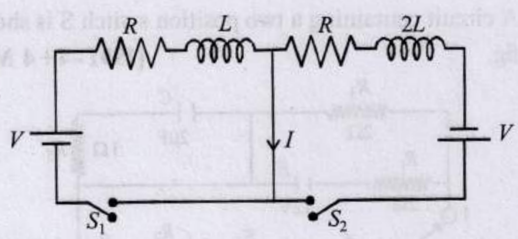
7. The value of φ is ____. **[Adv. 2021]**

8. The inductors of two LR circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF in the inductors by the time the currents reach their steady state values is ________ mJ. **[Adv. 2020]**



6 MCQs with One or More than One Correct Answer

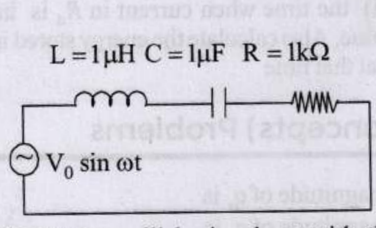
9. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true? **[Adv. 2018]**



(a) $I_{max} = \dfrac{V}{2R}$ (b) $I_{max} = \dfrac{V}{4R}$

(c) $\tau = \dfrac{L}{R}\ln 2$ (d) $\tau = \dfrac{2L}{R}\ln 2$

10. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **[Adv. 2017]**



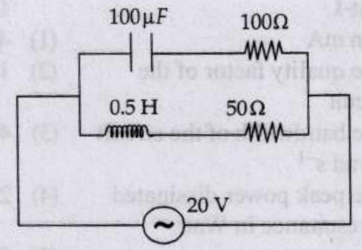
(a) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}

(b) The frequency at which the current will be in phase with the voltage is independent of R

(c) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(d) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

11. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are) **[2012]**



(a) The current through the circuit, I is 0.3 A.

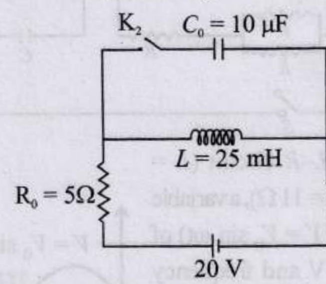
(b) The current through the circuit, I is $0.3\sqrt{2}A$

(c) The voltage across 100 Ω resistor = $10\sqrt{2}V$

(d) The voltage across 50 Ω resistor = 10 V

7 Match the Following

12. The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1. After a long time, the current attains a steady state value I_2. Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option. **[Adv. 2024]**

List-I	List-II
(P) The value of I_1 in Ampere is	(1) 0
(Q) The value of I_2 in Ampere is	(2) 2
(R) The value of ω_0 in kilo-radians/s is	(3) 4
(S) The value of V_0 in Volt is	(4) 20
	(5) 200

(a) P → 1; Q → 3; R → 2; S → 5
(b) P → 1; Q → 2; R → 3; S → 5
(c) P → 1; Q → 3; R → 2; S → 4
(d) P → 2; Q → 5; R → 3; S → 4

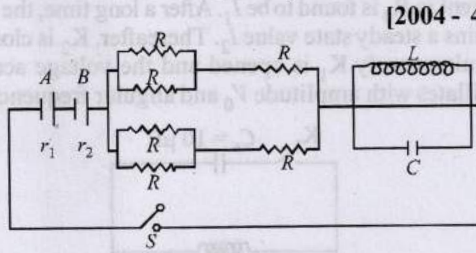
13. A series LCR circuit is connected to a 45 sin(ωt) Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad s^{-1}, the current amplitude in the circuit is $0.05\, I_0$. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option. **[Adv. 2023]**

	List-I		List-II
(P)	I_0 in mA	(1)	44.4
(Q)	The quality factor of the circuit	(2)	18
(R)	The bandwidth of the circuit in rad s^{-1}	(3)	400
(S)	The peak power dissipated at resonance in Watt	(4)	2250
		(5)	500

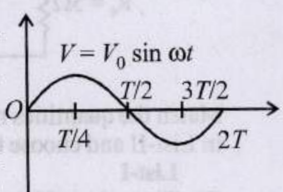
(a) $P \to 2, Q \to 3, R \to 5, S \to 1$
(b) $P \to 3, Q \to 1, R \to 4, S \to 2$
(c) $P \to 4, Q \to 5, R \to 3, S \to 1$
(d) $P \to 4, Q \to 2, R \to 1, S \to 5$

10 Subjective Problems

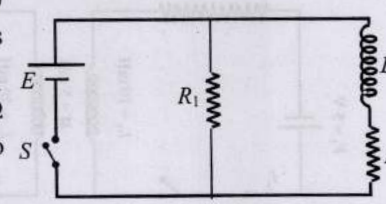
14. In the figure both cells A and B are of equal emf. Find R for which potential difference across battery A will be zero, long time after the switch is closed. Internal resistance of batteries A and B are r_1 and r_2 respectively ($r_1 > r_2$). **[2004 - 4 Marks]**



15. In a series L–R circuit (L = 35 mH and R = 11 Ω), a variable emf source ($V = V_0 \sin \omega t$) of V_{rms} = 220 V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph (π=22/7). **[2004 - 4 Marks]**

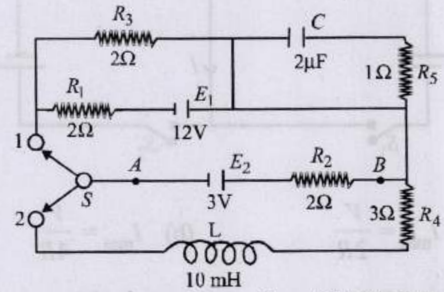


16. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2Ω and R_2 = 2Ω are connected to a battery of emf E = 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time t = 0. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? **[2001-5 Marks]**



17. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? **[1996 - 3 Marks]**

18. A circuit containing a two position switch S is shown in fig. **[1991 - 4 + 4 Marks]**



(a) The switch S is in position '1'. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

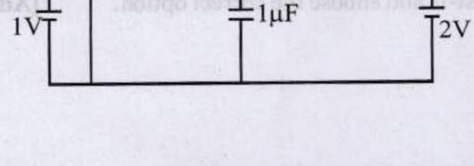
(b) If now the switch S is put in position 2 at t = 0 find
(i) steady current in R_4 and
(ii) the time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time

Topic-2: Miscellaneous (Mixed Concepts) Problems

3 Numeric / New Stem Based Questions

Stem for Qs. No. 1-2

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



1. The magnitude of q_1 is _____ . **[Adv. 2021]**

2. The magnitude of q_2 is _____ . **[Adv. 2021]**

6 MCQs with One or More than One Correct Answer

3. A series R– C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_c across the capacitor are compared in the two cases. Which of the following is/are true? **[2011]**

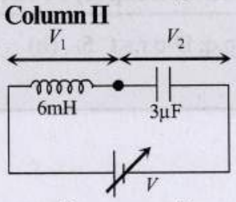
(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$
(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

7 Match the Following

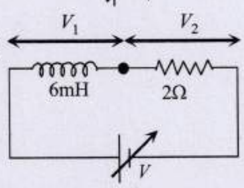
4. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2, (indicated in circuits) are related as shown in **Column I.** Match the two **[2010]**

Column I | **Column II**

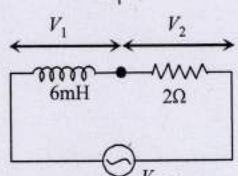
(A) $I \neq 0, V_1$ is proportional to I (p)



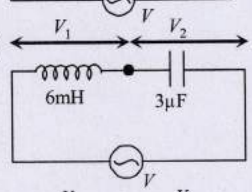
(B) $I \neq 0, V_2 > V_1$ (q)



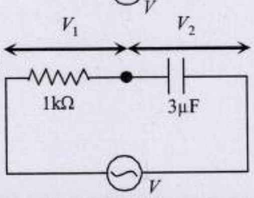
(C) $V_1 = 0, V_2 = V$ (r)



(D) $I \neq 0, V_2$ is proportional to I (s)



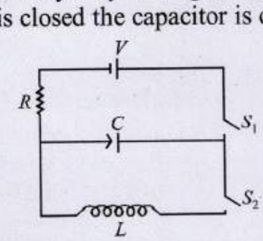
(t)



8 Comprehension/Passage Based Questions

Passage 1

In the given circuit the capacitor (C) may be charged through resistance R by a battery V by closing switch S_1. Also when S_1 is opened and S_2 is closed the capacitor is connected in series with inductor (L).



5. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is τ. Which of the following is correct **[2006 – 5M, –2]**

(a) after time interval τ, charge on the capacitor is $\frac{CV}{2}$

(b) after time interval 2τ, charge on the capacitor of $CV(1-e^{-2})$

(c) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(d) after time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$

6. When the capacitor gets charged completely, S_1 is opened and S_2 is closed. Then, **[2006 – 5M, –2]**

(a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy

(b) at any time $t > 0$, current in the circuit is in the same direction

(c) at $t > 0$, there is no exchange of energy between the inductor and capacitor

(d) at any time $t > 0$, instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$

7. Given that the total charge stored in the LC circuit is Q_0, for $t \geq 0$, the charge on the capacitor is **[2006 – 5M, –2]**

(a) $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$ (b) $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

(c) $Q = -LC\frac{d^2Q}{dt^2}$ (d) $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Passage 2

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with power factor unity. All the currents and voltages mentioned are rms values. **[Adv. 2013]**

8. If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation| (in %) during transmission is

(a) 20 (b) 30 (c) 40 (d) 50

9. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1 (c) 100 : 1 (d) 50 : 1

Answer Key

Topic-1 : AC Circuit, LCR Circuit, Quality and Power Factor

1. (b)	**2.** (b)	**3.** (a)	**4.** (4)	**5.** (8)	**6.** (100.00)							
7. (60.00)	**8.** (55.00)	**9.** (b, d)	**10.** (b, c)	**11.** (a, c)	**12.** (a)	**13.** (b)						

Topic-2 : Miscellaneous (Mixed Concepts) Problems

1. (1.33)	**2.** (0.67)	**3.** (a,c)	**4.** A-r,s,t; B-q,r,s,t; C-p,q; D-q,r,s,t	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (a)
9. (b)							

Hints & Solutions

Topic-1: AC Circuit, LCR Circuit, Quality and Power Factor

1. **(b)** $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$

As ω increases, I_{rms} through the bulb increases. Hence the bulb glows brighter.

2. **(b)** Time constant of $R - C$ circuit, $\tau = R_{eq} C_{eq}$

(i) R_1 & R_2 in series and C_1 & C_2 in parallel.

$\tau_1 = (2 + 1)(2 + 4) = 18\ \mu s$.

(ii) R_1 & R_2 in parallel and C_1 & C_2 in series.

$$\tau_2 = \left(\frac{2\times1}{2+1}\right)\left(\frac{2\times4}{2+4}\right) = \frac{8}{9}\mu s$$

(iii) R_1 & R_2 in parallel and C_1 & C_2 in parallel.

$$\tau_3 = \left(\frac{2\times1}{2+1}\right)\times(4+2) = 4\mu s$$

3. **(a)** From $e = E_0 \sin(100t)$ $\therefore$ $\omega = 100$

From given graph, current leads emf,

Hence this is an $R - C$ circuit.

$$\tan\phi = \frac{X_C}{R}$$

$$\text{Here } \phi = 45° = \frac{\pi}{4} \quad \therefore \quad X_C = R$$

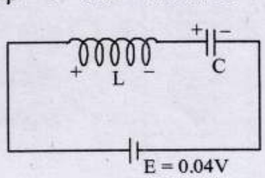
$$\frac{1}{\omega C} = R \Rightarrow RC\omega = 1$$

or $RC = \frac{1}{100}s^{-1}$ $\therefore R = 1k\,\Omega, C = 10\mu F$

4. **(4)** Induced Emf, $E = \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}(BA) = \frac{Ad}{dt}\left[(B_0 + \beta t)\right]$

$= A\beta = 1 \times 0.04 = 0.04$ volt

So, circuit can be drawn as



By KVL, $E = L\frac{di}{dt} + \frac{q}{c}$

$$\Rightarrow L\frac{di}{dt} = E - \frac{q}{c}$$

$$\Rightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}(q - CE)$$

Comparing it with equation of SHM, we get

$q = CE + A\sin(\omega t + \phi)$, where $\omega = \frac{1}{\sqrt{LC}}$

So, $i = A\omega\cos(\omega t + \phi)$

at $t = 0$, $q = 0$ and $i = 0$

So, $O = CE + A\sin\phi \Rightarrow A\sin\phi = -CE$ ....(i)

$O = \cos\phi \Rightarrow \phi = \frac{\pi}{2}$ ....(ii)

from (i) and (ii), we get

$A = -CE$

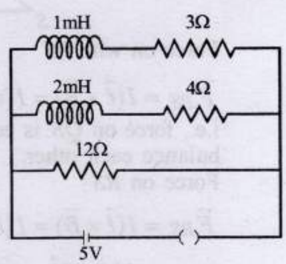
So, $i = -CE\,\omega\cos(\omega t + \pi/2)$

$= CE\,\omega\sin\omega t$

Therefore, $i_{max} = CE\,\omega = 10^{-3} \times 0.04 \times \frac{1}{\sqrt{0.1\times10^{-3}}} = 4mA$

5. **(8)** At $t = 0$ $I_{min} = \frac{5}{12}$

At $t = \infty$



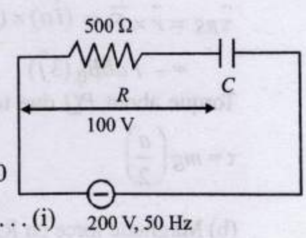
$$I_{max} = \frac{5}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3}$$

$$\left[\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}\right]$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{10}{3} \times \frac{12}{5} = 8$$

For Questions No. 6 and 7

6. **(100.00)** **7.** **(60.00)**



From $V_{RMS} = \sqrt{V_C^2 + V_R^2}$

$\Rightarrow V_C^2 + 100^2 = 200^2$

or, $V_C^2 + 10000 = 40000$

$\therefore V_C = 100\sqrt{3}V$ . . . (i)

$$\tan\phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100}$$

$\therefore \phi = 60°$ . . . (ii)

Power consumed, $P = I_{rms} V_{rms} \cos\phi = \frac{1}{2}\frac{V_{rms}^2}{z}$

$$\Rightarrow 500 = \frac{200}{z}\frac{1}{2}$$

$\therefore\ z = 40\,\Omega$ ...(iii)

$$\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{40}$$

$\therefore\ R = 20$

And $X_C = \sqrt{z^2 - R^2} = \sqrt{40^2 - 20^2} = 20\sqrt{3}\,\Omega$

$$X_C = \frac{1}{C\omega} \Rightarrow 20\sqrt{3} = \frac{1}{C2\pi f}$$

$$\therefore\ C = \frac{1}{2\pi f(20\sqrt{3})} = \frac{1}{20\pi\sqrt{3}\times 100}$$

$= 10^{-4}\,F = \mathbf{100}\,\mu\text{F}$

8. **(55.00)** Given: Mutual inductance, M = 5mH
$L_1 = 10$ mH, $V_1 = 5$V, $L_2 = 20$ mH & $V_2 = 20$V

$$I_1 = \frac{V_1}{R_1} = \frac{5}{5} = 1\text{A};\ I_2 = \frac{V_2}{R_2} = \frac{20}{10} = 2\text{A}$$

After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF = increase in magnetic energy

$$\therefore W = \Delta U = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2$$

$$= \frac{1}{2}\times(10\times10^{-3})1^2 + \frac{1}{2}\times(20\times10^{-3})\times 2^2$$

$+ (5\times10^{-3})\times 1\times 2$

$= (5 + 40 + 10)\times 10^{-3}$J

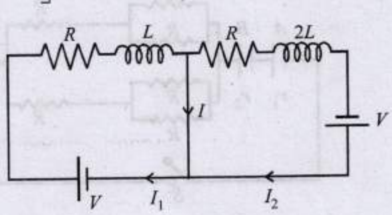
$\therefore W = 55$ mJ

9. **(b, d)** Here $I + I_2 = I_1 \quad \therefore\ I = I_1 - I_2$

$$\therefore\ I = \frac{V}{R}\left[1 - e^{\frac{-Rt}{2L}}\right] - \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$$

$$\Rightarrow I = \frac{V}{R}\left[e^{\frac{-Rt}{L}} - e^{\frac{-Rt}{2L}}\right] \quad ...(i)$$

For I_{max}, $\frac{dI}{dt} = 0$



$$\therefore\ \frac{V}{R}\left[\frac{-R}{L}e^{\frac{-Rt}{L}} - \left(\frac{-R}{2L}\right)e^{\frac{-Rt}{2L}}\right] = 0$$

$$\therefore\ e^{\frac{-Rt}{2L}} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2 \qquad \therefore\ t = \frac{2L}{R}\ln 2$$

This is the time when I is maximum
Putting this value of time in eq.(i)

$$\text{Further } I_{max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ell n 2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ell n 2\right)}\right]$$

$$\Rightarrow I_{max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right] = \frac{V}{4R}$$

10. **(b, c)** The frequency at which the current is in phase with the voltage is resonance frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{(10^{-6}\times10^{-6})^{1/2}} = 10^6\,\text{rad s}^{-1}$$

This frequency is independent of 'R'

At $\omega \approx 0$, the current $i = \dfrac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

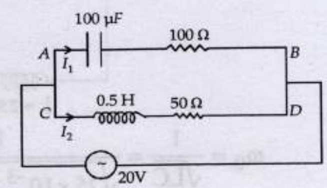
i.e, current through the circuit nearly becomes zero.
If $\omega >> \omega$, $X_L > X_C$ so circuit behaves like an inductor.

11. **(a, c)** Impedance across *AB*, *RC* part of the circuit

$$Z_1 = \sqrt{X_c^2 + R_1^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R_1^2}$$

$= \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}$



$$\therefore\quad I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}}$$

[leads emf by ϕ_1]

where $\cos\phi_1 = \dfrac{R}{Z_1} = \dfrac{100}{100\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

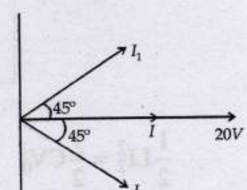
Impedance across *CD*, *LR* part of the circuit.

$$Z_2 = \sqrt{X_L^2 + R_2^2} = \sqrt{(\omega L)^2 + R_2^2}$$

$= \sqrt{(0.5\times100)^2 + (50)^2} = 50\sqrt{2}\,\Omega$

$$\therefore\quad I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}}$$

[leads emf by ϕ_2]

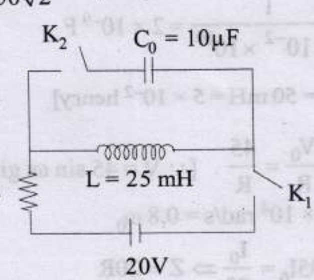


where $\cos\phi_2 = \dfrac{R}{Z_2} = \dfrac{50}{50\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45°$

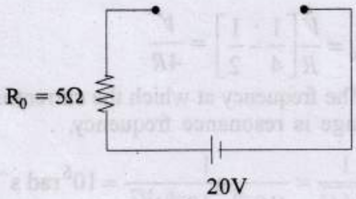
$\therefore$ Current *I* from the circuit

$$I = \frac{20}{100\sqrt{2}} + \frac{20}{50\sqrt{2}} = I_1 + I_2 \simeq 0.3\text{ A}$$

12. **(a)** $R_0 = 5\Omega$

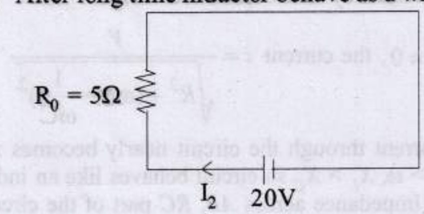


(P) When key K_1 is closed, current in R_0 in I_1
At t = 0; circuit is as follows



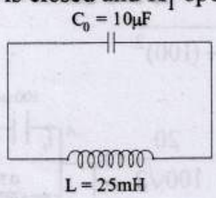
$I_1 = 0$

(Q) After long time inductor behave as a wire so I_2.



$I_2 = \frac{V}{R} = \frac{20}{5} = 4A$

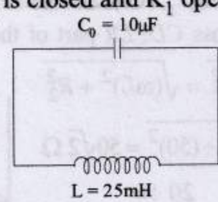
(R) When K_2 is closed and K_1 open



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25\times10^{-3}\times10\times10^{-6}}} = \frac{1}{5\times10^{-4}}$$

$\therefore \quad \omega_0 = 2\times10^3$ rad/s $\therefore \omega_0 = 2$ kilo-radian /s

(S) when K_2 is closed and K_1 open



$$\frac{1}{2}LI_2^2 = \frac{1}{2}CV_0^2$$

$25\times10^{-3}\times(4)^2 = 10\times10^{-6}\times V_0^{\,2}$

$V_0^2 = 2500\times16$ or, $V_0 = 50\times4 = 200V$

13. (b) As, $V = V_0 \sin \omega t$ and resonant angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = 10^5 \text{ rad/s} = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2}$$

$$= \frac{1}{5\times10^{-2}\times10^{10}} = 2\times10^{-9}\text{ F}$$

[$\because$ L = 50 mH = 5×10^{-2} henry]

$I_0 = \frac{V_0}{R} = \frac{45}{R}$ [$\because$ V = 45 sin ωt given]

$\omega = 8\times10^4$ rad/s $= 0.8\,\omega_0$

$I = 0.05I_0 = \frac{I_0}{20} \Rightarrow Z = 20R$

$X_L = L\omega = 8\times10^4\times5\times10^{-2}\,\Omega = 4\,k\Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{8\times10^4\times2\times10^{-9}} = \frac{1}{16}\times10^5\Omega = \frac{25}{4}k\Omega$$

$$Z^2 = R^2 + (X_C - X_L)^2 \text{ or, } 400R^2 = R^2 + \left(\frac{9}{4}k\Omega\right)^2$$

$$\therefore R = \frac{\frac{9}{4}k\Omega}{\sqrt{399}} = \frac{9}{80}k\Omega = \frac{900}{8}\Omega$$

$$I_0 = \frac{V_0}{R} = \frac{45\times8}{900} = \frac{8}{20}A \approx 0.4A = 400mA$$

Quality factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{8}{900}\sqrt{\frac{5\times10^{-2}}{2\times10^{-9}}} = \frac{8}{900}\sqrt{25\times10^6}$$

$$= \frac{8}{900}\times5000 = 44.4$$

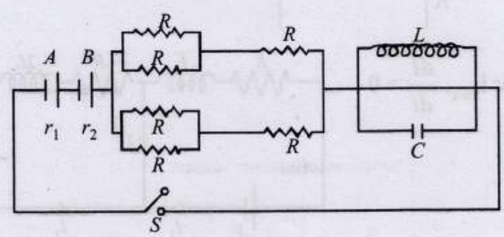
$$Q = \frac{\omega_0}{\Delta_\omega} \therefore \text{Bandwidth, } \Delta\omega = \frac{\omega_0}{Q} = \frac{10^5}{44.4} = 2250.0$$

Peak Power dissipated, $P_{max} = I_0^2R = \frac{45^2}{R^2}\times R = \frac{45^2}{R}$

$$= \frac{45^2}{900}\times8 = 18.4W \approx 18W$$

14. After a long time capacitor will be fully charged, hence no current will flow through capacitor and all the current will flow from inductor. Thus resistance across capacitor becomes infinite and across an inductor becomes zero.

$$\therefore \quad R_{eq} = \left(\frac{R}{2}+R\right)\times\frac{1}{2} + r_1 + r_2 = \frac{3R}{4} + r_1 + r_2$$



$$I = \frac{\varepsilon+\varepsilon}{R_{eq}} \Rightarrow I = \frac{2\varepsilon}{R_{eq}} = \frac{2\varepsilon}{(3R/4)+r_1+r_2}$$

Resistance 'R' for which potential difference across battery *A* is zero

$$\varepsilon - Ir_1 = 0 \quad \Rightarrow \quad \varepsilon = \frac{2\varepsilon}{(3R/4)+r_1+r_2}r_1$$

$$\Rightarrow \quad r_1 = r_2 + 3R/4 \quad \text{or} \quad R = \frac{4}{3}(r_1 - r_2)$$

15. Given, $V_{rms} = 220\text{ V}, \nu = 50\text{ Hz}, L = 35\text{ mH}, R = 11\Omega$

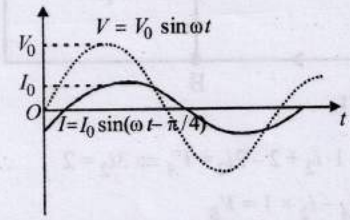
Impedance $Z = \sqrt{(\omega L)^2 + R^2} = 11\sqrt{2}\,\Omega$

Also, current amplitude, $I_0 = \dfrac{V_0}{Z}$

$V_0 = V_{rms}\sqrt{2}$ $\quad\therefore\ I_0 = \dfrac{V_{rms}\sqrt{2}}{Z} = 20A$

$\cos\phi = \dfrac{R}{Z} = \dfrac{11}{11\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $\quad\therefore\ \phi = \dfrac{\pi}{4}$ phase

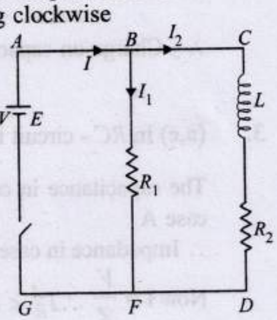
In *L-R* circuit, voltage leads the current. $I_{instantaneous} = 20 \sin\left(\omega t - \pi/4\right)$, the current time graph as shown below.



16. Given: $R_1 = R_2 = 2\Omega, E = 12V$ and $L = 400\text{mH}$

The circuit given is an *R-L* circuit in which the current grows as soon as current is switched on.

Applying Kirchhoff's law in the loop ABCDFGA we get, starting from G moving clockwise



$$E - L\frac{dI_2}{dt} - I_2R_2 = 0$$

or $I_2 = \dfrac{E}{R_2}\left[1 - e^{-\frac{R}{L}t}\right]$

Also we know that the emf (V) produced across the inductor

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}[LI_2] = -L\frac{dI_2}{dt}$$

$$= -L\frac{d}{dt}\left[\frac{E}{R_2}\left(1 - e^{\frac{-R_2}{L}t}\right)\right]$$

$V = -E\,e^{\frac{R_2}{L}t}$. Here the negative sign shows the opposition to the growth of current.

$\therefore\quad V = 12e^{\frac{2}{400\times10^{-3}}t} = 12\,e^{-5t}\text{ V}$

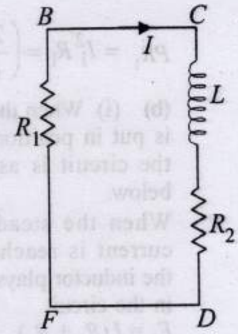
When the switch is open, the current flows through the circuit BCDF only in the direction as shown in figure.

Applying Kirchhoff 's law

$$I(R_1 + R_2) - \left(-\frac{L\,dI}{dt}\right) = 0$$

$$\therefore\quad \frac{dI}{I} = -\left(\frac{R_1 + R_2}{L}\right)dt$$

$\therefore$ On integrating,



$$\int_{I_0}^{I}\frac{dI}{I} = -\frac{(R_1 + R_2)}{L}\int_0^t dt$$

$\therefore\quad I = I_0\,e^{-\frac{(R_1+R_2)t}{L}}$

Here, $\dfrac{R_1 + R_2}{L} = \dfrac{2+2}{400\times10^{-3}} = 10$

and $I_0 = \dfrac{E}{R_1 + R_2} = \dfrac{12}{4} = 3\text{ A}$

$\therefore\quad I = 3e^{-10t}A$, clockwise.

17. As we know, energy $U = \dfrac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\dfrac{1}{4}$th of its maximum value when current is reached half of its maximum value.

In *L-R* circuit, equation of current growth

$i = i_0\,(1 - e^{-t/\tau L})$

Here, i_0 = Maximum value of current

τ_L = Time constant = $L/R = \dfrac{10H}{2\Omega} = 5s$

$\therefore\ i = i_0/2 = i_0\,(1 - e^{-t/5})$

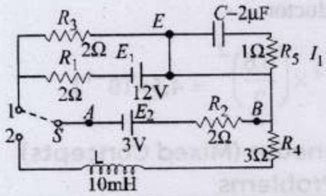
or $\dfrac{1}{2} = 1 - e^{-t/5} \Rightarrow e^{-t/5} = \dfrac{1}{2}$

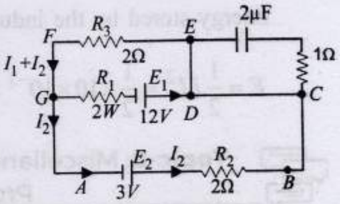
or $-t/5 = \ln\left(\dfrac{1}{2}\right) \Rightarrow t/5 = \ln(2) = 0.693$

$\therefore\ t = (5)(0.693)\ or\ t = 3.465s$

18. **(a)** (i) When the switch S is in position '1'

In steady state, current through the capacitor is zero.





Using Kirchhoff's law in loop ABCDGA

$$+3 - I_2\times2 - 12 + I_1\times2 = 0$$

$\Rightarrow\quad 2I_1 - 2I_2 = 9$ ...(i)

Applying Kirchhoff 's law in loop DEFGD

$$-2I_1 + 12 - (I_1 + I_2)2 = 0$$

$\Rightarrow\ 2I_1 + I_2 = 6$ ...(ii)

From eq. (i) and (ii) $I_1 = \dfrac{21}{6}A.$

$\therefore\quad$ From (ii) $I_2 = -1\,A.$

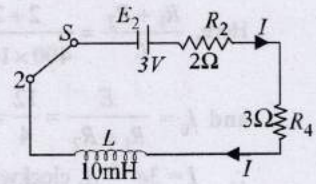
Potential difference between A and B

$$V_A + 3 - (-1)\times2 = V_B \quad\Rightarrow\quad V_A - V_B = -5V$$

Rate of production of heat in R_1

$PR_1 = I_1^2 R_1 = \left(\frac{21}{6}\right)^2 \times 2 = 24.5W$

(b) (i) When the switch is put in position 2 then the circuit is as shown below.



When the steady state current is reached then the inductor plays no role in the circuit

$E_2 = I(R_2 + R_4)$

$\Rightarrow \quad I = \frac{E_2}{R_2 + R_4} = \frac{3}{5} = 0.6\,\text{A}.$

(ii) The growth of current in L–R circuit

$I = I_0\left[1 - e^{-\frac{R}{L}t}\right]$

When $I = \frac{I_0}{2}$ half the steady value, then $\frac{I_0}{2} = I_0\left[1 - e^{-\frac{R}{L}t}\right]$

$\Rightarrow \quad \frac{1}{2} = 1 - e^{\frac{-R}{L}t} \quad \Rightarrow \quad e^{-\frac{R}{L}t} = \frac{1}{2}$

Taking log on both sides

$\log_e e^{-\frac{R}{L}t} = \log_e \frac{1}{2}$

$\Rightarrow \quad \frac{R}{L}t = 0.693 \Rightarrow t = 0.693\frac{L}{R} = \frac{0.6930 \times 10 \times 10^{-3}}{(2+3)}$

when $R = R_2 + R_4$

$\therefore t = 1.386 \times 10^{-3}\,\text{s}$

Energy stored by the inductor

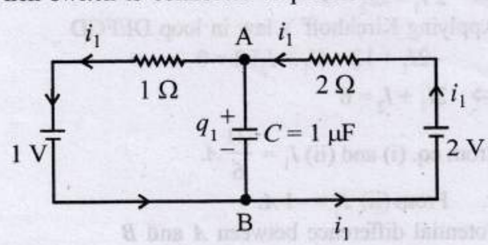
$E = \frac{1}{2}LI^2 = \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{0.6}{2}\right)^2 = 4.5 \times 10^{-4}\,\text{J}$

Topic-2: Miscellaneous (Mixed Concepts) Problems

For Questions No. 1 and 2

1. (1.33) **2. (0.67)**

Sol. When switch is connected to position P



From KVL,

$V_A - 1 \cdot i_1 - 1 + 2 - 2i_1 = V_A \Rightarrow 3i_1 = 1 \quad \therefore i_1 = \frac{1}{3}\text{A}$

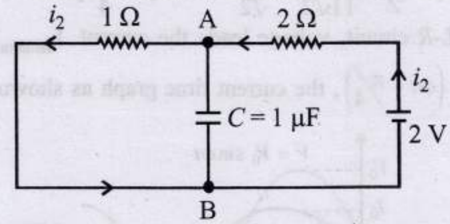
Again $V_A - 1 \cdot i_1 - 1 = V_B$ or, $V_A - V_B = 1 + i_1 = \frac{4}{3}\text{V}$

Potential drop across capacitor $\Delta V = \frac{4}{3}\text{V}$

$\therefore$ Charge on capacitor, $q_1 = C\Delta V = 1 \times \frac{4}{3}\mu\text{C}$

$q_1 = \mathbf{1.33\ \mu C}$

When switch is connected to position Q



From KVL,

$V_A - 1 \cdot i_2 + 2 - 2i_2 = V_A \Rightarrow 3i_2 = 2 \quad \therefore i_2 = \frac{2}{3}\text{A}$

Again, $V_A - i_2 \times 1 = V_B$

$V_A - V_B = i_2 \times 1 = \frac{2}{3}\text{V}$

Potential difference across capacitor $\Delta V = \frac{2}{3}\text{V}$

$\therefore$ Charge on capacitor, $q_2 = C\Delta V = \mathbf{1 \times \frac{2}{3} = 0.67\mu C}$

3. **(a,c)** In RC - circuit impedance, $Z = \sqrt{R^2 + \left(\frac{1}{WC}\right)^2}$

The capacitance in case B is four times the capacitance in case A

$\therefore$ Impedance in case B is less then that of case A ($Z_B < Z_A$)

Now $I = \frac{V}{Z} \quad \therefore I_R^A < I_R^B$.

and $V_R^A < V_R^B$. $\Rightarrow V_C^A > V_C^B$

[$\because$ If V is the applied potential difference access series R-C circuit then $V = \sqrt{V_R^2 + V_C^2}$]

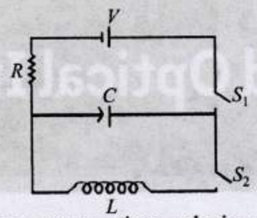
4. **A-r,s,t; B-q,r,s,t; C-p,q; D-q,r,s,t**

For DC circuit, in steady state, the current I through the capacitor (c) is zero. In case of L-C circuit, the potential difference (v) across the inductor (L) is zero and that across the capacitor = applied potential difference. In case of L-R circuit, = (V) across inductor (L) = across (R) = applied voltage.

For AC circuit in steady state, I_{rms} current flows through the capacitor (c), inductor (R) and (L) and resistor (R). The potential difference across resistor, inductor and capacitor I. And for changing current, the potential difference across (V) inductor (L), capacitor (c) or resistor (R) $\propto$ Current (I).

5. **(b)** When charging is complete, the potential difference between the capacitor plates will be V and the charge stored in this case will be maximum.

$\therefore Q_0 = CV.$



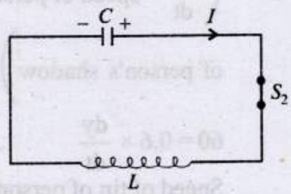
When $t = 2\tau$, $Q = CV\left[1 = e^{\frac{-2\tau}{\tau}}\right]$

$= CV[1 - e^{-2}]$

6. **(d)** Instantaneous charge on plates at any time t during discharging

$$Q_{inst} = Q_0 \cos \omega t$$

$\therefore$ Instantaneous current,



$$I_{instan} = \frac{dQ}{dt} = Q_0\, \omega \sin \omega t$$

$\therefore \quad I_{max} = Q_0\omega$

Here $Q_0 = CV$ and $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore \quad I_{max} = CV \times \frac{1}{\sqrt{LC}} = V\sqrt{\frac{C}{L}}$$

7. **(c)** Applying Kirchhoff 's law

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \Rightarrow \frac{Q}{C} = L\frac{dI}{dt}$$

$$\Rightarrow \quad Q = LC\,\frac{d}{dt}\left(-\frac{dQ}{dt}\right) = -LC\frac{d^2Q}{dt^2}$$

8. **(a)** Step up transformer $\frac{N_s}{N_p} = \frac{V_s}{V_p} \Rightarrow \frac{10}{1} = \frac{V_s}{4000}$

$\therefore V_s = 40{,}000\ V$

Step down transformer $\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40{,}000}{200} = \frac{200}{1}$

9. **(b)** Power $P = V \times I$

$$\Rightarrow \quad I = \frac{P}{V} = \frac{600 \times 1000}{4000} = 150\ A$$

Total resistance $= 0.4 \times 20 = 8\ \Omega$

$\therefore$ Power dissipated as heat $= I^2R = (150)^2 \times 8$
$= 180{,}000W = 180\ kW$

$$\therefore \%\ loss = \frac{180}{600} \times 100 = 30\%$$