Chapter 11. Inequalities

Exercise 11

Solution 1:

In Δ ABC,

AB = AC[Given]

$\therefore \angle ACB = \angle B$[angles opposite to equal sides are equal]

$\angle B = 70^0$[Given]

$\Rightarrow \angle ACB = 70^0$ ....(i)

Now,

$\angle ACB + \angle ACD = 180^0$[BCD is a straight line]

$\Rightarrow 70^0 + \angle ACD = 180^0$

$\Rightarrow \angle ACD = 110^0$ ....(ii)

In Δ ACD,

$\angle CAD + \angle ACD + \angle D = 180^0$

$\Rightarrow \angle CAD + 110^0 + \angle D = 180^0$ [From (ii)]

$\Rightarrow \angle CAD + \angle D = 70^0$

But $\angle D = 40^0$ [Given]

$\Rightarrow \angle CAD + 40^0 = 70^0$

$\Rightarrow \angle CAD = 30^0$ ....(iii)

In Δ ACD,

$\angle ACD = 110^0$[From (ii)]

$\angle CAD = 30^0$[From (iii)]

$\angle D = 40^0$ [Given]

$\therefore \angle D > \angle CAD$

$\Rightarrow AC > CD$

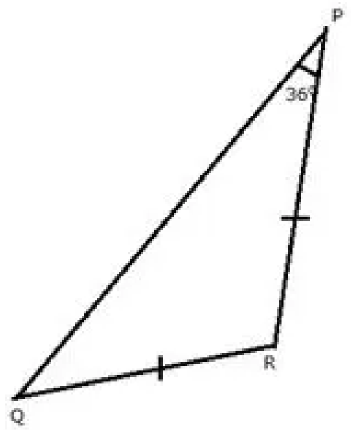
[Greater angle has greater side opposite to it]

Also,

AB = AC[Given]

Therefore, AB > CD.

Solution 2:



In Δ PQR,

QR = PR[Given]

$\therefore \angle P = \angle Q$[angles opposite to equal sides are equal]

$\angle P = 36^0$[Given]

$\Rightarrow \angle Q = 36^0$

In Δ PQR,

$\angle P + \angle Q + \angle R = 180^0$

$\Rightarrow 36^0 + 36^0 + \angle R = 180^0$

$\Rightarrow \angle R + 72^0 = 180^0$

$\Rightarrow \angle R = 108^0$

Now,

$\angle R = 108^0$

$\angle P = 36^0$

$\angle Q = 36^0$

Since $\angle R$ is the greatest, therefore, PQ is the largest side.

Solution 3:

The sum of any two sides of the triangle is always greater than third side of the triangle.

Third side < 13 + 8 = 21 cm.

The difference between any two sides of the triangle is always less than the third side of the triangle.

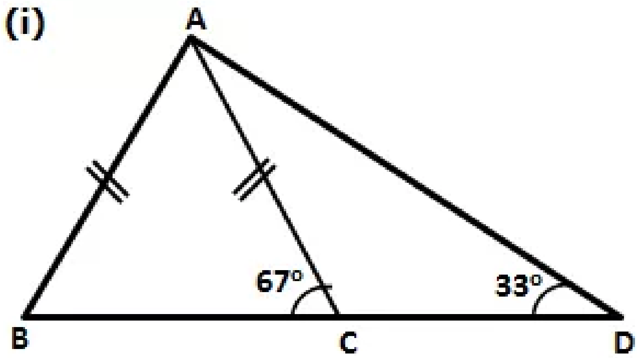
Third side > 13 - 8 = 5 cm.

Therefore, the length of the third side is between 5 cm and 9 cm, respectively.

The value of a = 5 cm and b = 21 cm.

Solution 4:

(i)



In ΔABC,

$AB = AC$

$\Rightarrow \angle ABC = \angle ACB$ (angles opposite to equal sides are equal)

$\Rightarrow \angle ABC = \angle ACB = 67°$

$\Rightarrow \angle BAC = 180° - \angle ABC - \angle ACB$ (angle sum property)

$\Rightarrow \angle BAC = 180° - 67° - 67° = 46°$

Since $\angle BAC < \angle ABC$, we have

$BC < AC$ ....(1)

Now, $\angle ACD = 180° - \angle ACB$ (linear pair)

$\Rightarrow \angle ACD = 180° - 67° = 113°$

Thus, in ΔACD,

$\angle CAD = 180° - \angle ACD - \angle ADC$

$\Rightarrow \angle CAD = 180° - 113° - 33° = 34°$

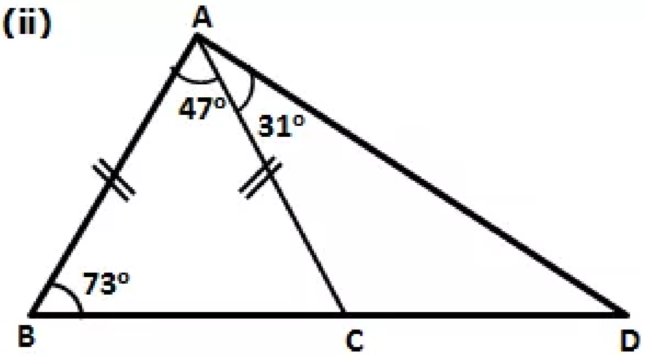
Since $\angle ADC < \angle CAD$, we have

$AC < CD$ ....(2)

From (1) and (2), we have

$BC < AC < CD$

(ii)



In ΔABC,

$\angle BAC < \angle ABC$

$\Rightarrow BC < AC \quad ....(1)$

Now, $\angle ACB = 180° - \angle ABC - \angle BAC$

$\Rightarrow \angle ACB = 180° - 73° - 47°$

$\Rightarrow \angle ACB = 60°$

Now, $\angle ACD = 180° - \angle ACB$

$\Rightarrow \angle ACD = 180° - 60° = 120°$

Now, in ΔACD,

$\angle ADC = 180° - \angle ACD - \angle CAD$

$\Rightarrow \angle ADC = 180° - 120° - 31°$

$\Rightarrow \angle ADC = 29°$

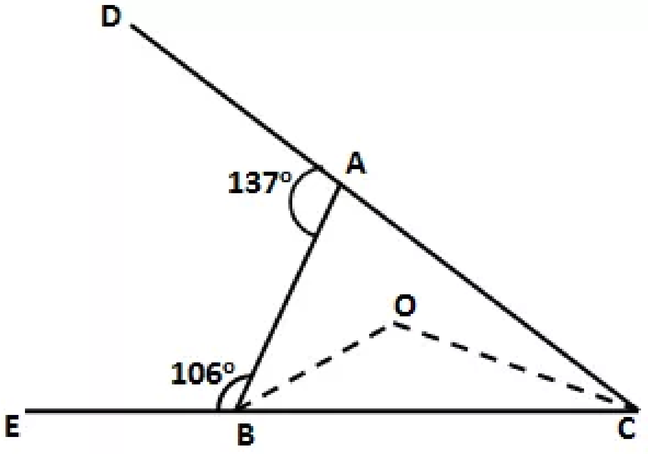
Since $\angle ADC < \angle CAD$, we have

$AC < CD \quad ....(2)$

From (1) and (2), we have

$BC < AC < CD$

Solution 5:



$\angle BAC = 180° - \angle BAD = 180° - 137° = 43°$

$\angle ABC = 180 - \angle ABE = 180° - 106° = 74°$

Thus, in ΔABC,

$\angle ACB = 180° - \angle BAC - \angle ABC$

$\Rightarrow \angle ACB = 180° - 43° - 74° = 63°$

Now, $\angle ABC = \angle OBC + \angle ABO$

$\Rightarrow \angle ABC = 2\angle OBC$ (OB is biosector of $\angle ABC$)

$\Rightarrow 74° = 2\angle OBC$

$\Rightarrow \angle OBC = 37°$

Similarly,

$\angle ACB = \angle OCB + \angle ACO$

$\Rightarrow \angle ACB = 2\angle OCB$ (OC is bisector of $\angle ACB$)

$\Rightarrow 63° = 2\angle OCB$

$\Rightarrow \angle OCB = 31.5°$

Now, in ΔBOC,

$\angle BOC = 180° - \angle OBC - \angle OCB$

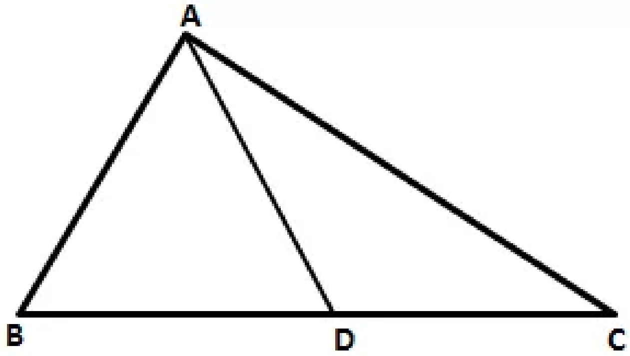
$\Rightarrow \angle BOC = 180° - 37° - 31.5°$

$\Rightarrow \angle BOC = 111.5°$

Since, $\angle BOC > \angle OBC > \angle OCB$, we have

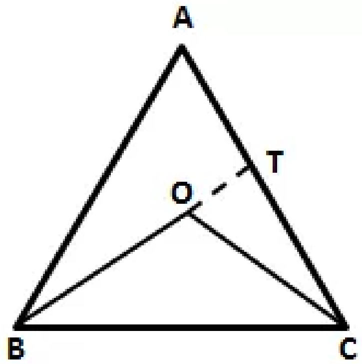
$BC > OC > OB$

Solution 6:



$AD > AC \quad \text{(given)}$
$\Rightarrow \angle C > \angle ADC \quad \dots.(1)$
Now, $\angle ADC > \angle B + \angle BAC \quad$ (Exterior Angle Property)
$\Rightarrow \angle ADC > \angle B \quad \dots.(2)$
From (1) and (2), we have
$\angle C > \angle ADC > \angle B$
$\Rightarrow \angle C > \angle B$
$\Rightarrow AB > AC$

Solution 7:



Construction: Produce BO to meet AC at T.
In ΔABT,
$AB + AT > BT \quad$ (Sum of two sides of a Δ is greater than third side)
$\Rightarrow AB + AT > BO + OT \quad \dots.(1)$
Also, in ΔOCT,
$OT + TC > OC \quad \dots.(2)$
Adding (1) and (2), we have
$AB + AT + OT + TC > BO + OT + OC$
$\Rightarrow AB + AT + TC > BO + OC$
$\Rightarrow AB + AC > OB + OC$
$\Rightarrow OB + OC < AB + AC$

Solution 8:

In Δ BEC,

$\angle B + \angle BEC + \angle BCE = 180^0$

$\angle B = 65^0$ [Given]

$\angle BEC = 90^0$[CE is perpendicular to AB]

$\Rightarrow 65^0 + 90^0 + \angle BCE = 180^0$

$\Rightarrow \angle BCE = 180^0 - 155^0$

$\Rightarrow \angle BCE = 25^0 = \angle DCF$ ....(i)

In Δ CDF,

$\angle DCF + \angle FDC + \angle CFD = 180^0$

$\angle DCF = 25^0$ [From (i)]

$\angle FDC = 90^0$[AD is perpendicular to BC]

$\Rightarrow 25^0 + 90^0 + \angle CFD = 180^0$

$\Rightarrow \angle CFD = 180^0 - 115^0$

$\Rightarrow \angle CFD = 65^0$ ....(ii)

Now, $\angle AFC + \angle CFD = 180^0$[AFD is a straight line]

$\Rightarrow \angle AFC + 65^0 = 180^0$

$\Rightarrow \angle AFC = 115^0$ ....(iii)

In Δ ACE,

$\angle ACE + \angle CEA + \angle BAC = 180^0$

$\angle BAC = 60^0$ [Given]

$\angle CEA = 90^0$[CE is perpendicular to AB]

$\Rightarrow \angle ACE + 90^0 + 60^0 = 180^0$

$\Rightarrow \angle ACE = 180^0 - 150^0$

$\Rightarrow \angle ACE = 30^0$ ....(iv)

In Δ AFC,

$\angle AFC + \angle ACF + \angle FAC = 180^0$

$\angle AFC = 115^0$ [From (iii)]

$\angle ACF = 30^0$[From (iv)]

$\Rightarrow 115^0 + 30^0 + \angle FAC = 180^0$

$\Rightarrow \angle FAC = 180^0 - 145^0$

$\Rightarrow \angle FAC = 35^0$ ....(v)

In Δ AFC,

$\angle FAC = 35^0$[From (v)]

$\angle ACF = 30^0$[From (iv)]

$\therefore \angle FAC > \angle ACF$
$\Rightarrow CF > AF$

In Δ CDF,

$\angle DCF = 25^0$[From (i)]

$\angle CFD = 65^0$[From (ii)]

$\therefore \angle CFD > \angle DCF$
$\Rightarrow DC > DF$

Solution 9:

$\angle ACB = 74^0$ .....(i)[Given]

$\angle ACB + \angle ACD = 180^0$[BCD is a straight line]

$\Rightarrow 74^0 + \angle ACD = 180^0$

$\Rightarrow \angle ACD = 106^0$ ....(ii)

In Δ ACD,

$\angle ACD + \angle ADC + \angle CAD = 180^0$

Given that AC = CD

$\Rightarrow \angle ADC = \angle CAD$

$\Rightarrow 106^0 + \angle CAD + \angle CAD = 180^0$[From (ii)]

$\Rightarrow 2\angle CAD = 74^0$

$\Rightarrow \angle CAD = 37^0 = \angle ADC$...........(iii)

Now,

$\angle BAD = 110^0$[Given]

$\angle BAC + \angle CAD = 110^0$

$\angle BAC + 37^0 = 110^0$

$\angle BAC = 73^0$ ....(iv)

In Δ ABC,

$\angle B + \angle BAC + \angle ACB = 180^0$

$\Rightarrow \angle B + 73^0 + 74^0 = 180^0$[From (i) and (iv)]

$\Rightarrow \angle B + 147^0 = 180^0$

$\Rightarrow \angle B = 33^0$ ....(v)

$\therefore \angle BAC > \angle B$ [From (iv) and (v)]

$\Rightarrow BC > AC$

But,

AC = CD [Given]

$\Rightarrow BC > CD$

Solution 10:

(i) $\angle ADC + \angle ADB = 180^0$[BDC is a straight line]

$\angle ADC = 90^0$[Given]

$90^0 + \angle ADB = 180^0$

$\angle ADB = 90^0$ ....(i)

In ΔADB,

$\angle ADB = 90^0$[From (i)]

$\therefore \angle B + \angle BAD = 90^0$

Therefore, $\angle B$ and $\angle BAD$ are both acute, that is less than 90^0.

$\therefore AB > BD$ ....(ii)[Side opposite 90^0 angle is greater than

side opposite acute angle]

(ii) In ΔADC,

$\angle ADB = 90^0$

$\therefore \angle C + \angle DAC = 90^0$

Therefore, $\angle C$ and $\angle DAC$ are both acute, that is less than 90^0.

$\therefore AC > CD$ ....(iii)[Side opposite 90^0 angle is greater than

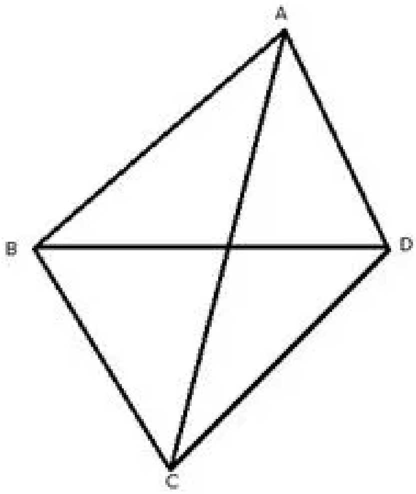
side opposite acute angle]

Adding (ii) and (iii)

$AB + AC > BD + CD$

$\Rightarrow AB + AC > BC$

Solution 11:



Const: Join AC and BD.

(i) In Δ ABC,

AB + BC > AC....(i)[Sum of two sides is greater than the third side]

In Δ ACD,

AC + CD > DA....(ii)[Sum of two sides is greater than the third side]

Adding (i) and (ii)

AB + BC + AC + CD > AC + DA

AB + BC + CD > AC + DA - AC

AB + BC + CD > DA ....(iii)

(ii)In Δ ACD,

CD + DA > AC....(iv)[Sum of two sides is greater than the third side]

Adding (i) and (iv)

AB + BC + CD + DA > AC + AC

AB + BC + CD + DA > 2AC

(iii) In Δ ABD,

AB + DA > BD....(v)[Sum of two sides is greater than the third side]

In Δ BCD,

BC + CD > BD....(vi)[Sum of two sides is greater than the third side]

Adding (v) and (vi)

AB + DA + BC + CD > BD + BD

AB + DA + BC + CD > 2BD

Solution 12:

(i) In ΔABC,

$AB = BC = CA$[ABC is an equilateral triangle]

$\therefore \angle A = \angle B = \angle C$

$$\therefore \angle A = \angle B = \angle C = \frac{180^\circ}{3}$$
$$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$$

In ΔABP,

$\angle A = 60^0$

$\angle ABP < 60^0$

$$\therefore \angle A > \angle ABP$$
$$\Rightarrow BP > PA$$

[Side opposite to greater side is greater]

(ii) In ΔBPC,

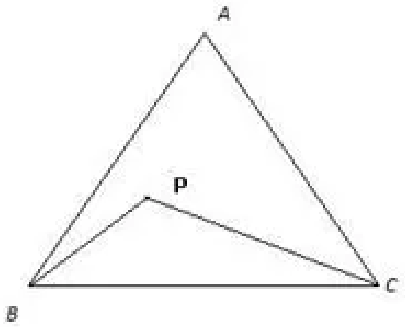
$\angle C = 60^0$

$\angle CBP < 60^0$

$$\therefore \angle C > \angle CBP$$
$$\Rightarrow BP > PC$$

[Side opposite to greater side is greater]

Solution 13:



Let $\angle PBC = x$ and $\angle PCB = y$

then,

$\angle BPC = 180^0 - (x + y)$ ....(i)

Let $\angle ABP = a$ and $\angle ACP = b$

then,

$\angle BAC = 180^0 - (x + a) - (y + b)$

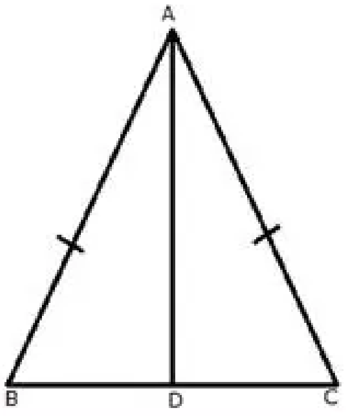
$\Rightarrow \angle BAC = 180^0 - (x + y) - (a + b)$

$\Rightarrow \angle BAC = \angle BPC - (a + b)$

$\Rightarrow \angle BPC = \angle BAC + (a + b)$

$\Rightarrow \angle BPC > \angle BAC$

Solution 14:



We know that exterior angle of a triangle is always greater than each of the interior opposite angles.

$\therefore$ In Δ ABD,

$\angle$ ADC > $\angle$ B ....(i)

In Δ ABC,

AB = AC

$\therefore$ $\angle$ B = $\angle$ C .....(ii)

From (i) and (ii)

$\angle$ ADC > $\angle$ C

(i) In Δ ADC,

$\angle$ ADC > $\angle$ C

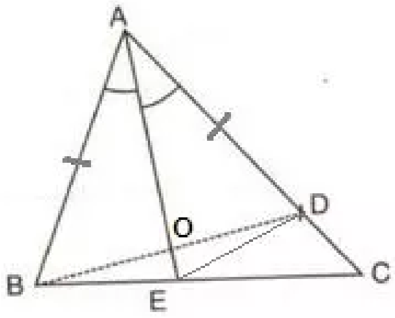
$\therefore$ AC > AD ....(iii) [side opposite to greater angle is greater]

(ii) In Δ ABC,

AB = AC

$\Rightarrow$AB > AD[From (iii)]

Solution 15:



Const: Join ED.

In Δ AOB and Δ AOD,

AB = AD[Given]

AO = AO[Common]

$\angle$ BAO = $\angle$ DAO[AO is bisector of $\angle$ A]

$\therefore$ $\Delta AOB \cong \Delta AOD$ [SAS criterion]

Hence,

BO = OD.........(i)[cpct]

$\angle$ AOB = $\angle$ AOD ....(ii)[cpct]

$\angle$ ABO = $\angle$ ADO $\Rightarrow \angle$ ABD = $\angle$ ADB ....(iii)[cpct]

Now,

$\angle$ AOB = $\angle$ DOE[Vertically opposite angles]

$\angle$ AOD = $\angle$ BOE[Vertically opposite angles]

$\Rightarrow \angle$ BOE = $\angle$ DOE ......(iv)[From (ii)]

(i) In Δ BOE and Δ DOE,

BO = CD[From (i)]

OE = OE[Common]

$\angle$ BOE = $\angle$ DOE[From (iv)]

$\therefore$ $\Delta BOE \cong \Delta DOE$ [SAS criterion]

Hence, BE = DE[cpct]

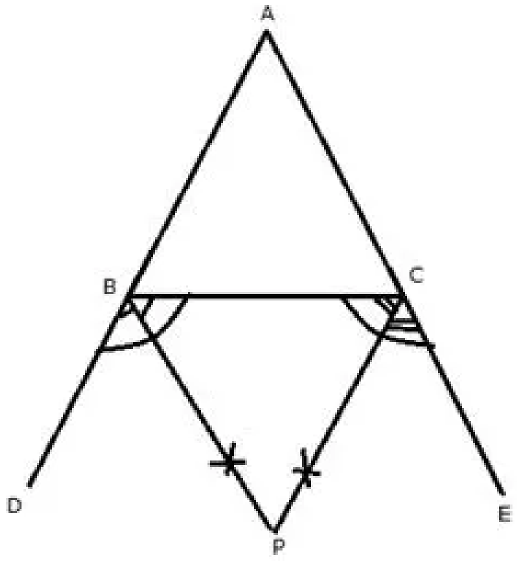
(ii) In Δ BCD,

$\angle$ ADB = $\angle$ C + $\angle$ CBD[Ext. angle = sum of opp. int. angles]

$\Rightarrow \angle$ ADB > $\angle$ C

$\Rightarrow \angle$ ABD > $\angle$ C[From (iii)]

Solution 16:



In Δ ABC,

AB > AC,

$\Rightarrow \angle ABC < \angle ACB$

$\therefore 180^0 - \angle ABC > 180^0 - \angle ACB$

$$\Rightarrow \frac{180^0 - \angle ABC}{2} > \frac{180^0 - \angle ACB}{2}$$

$$\Rightarrow 90^\circ - \frac{1}{2}\angle ABC > 90^\circ - \frac{1}{2}\angle ACB$$

$\Rightarrow \angle CBP > \angle BCP$ [BP is bisector of $\angle CBD$ and CP is bisector of $\angle BCE$]

$\Rightarrow PC > PB$ [side opposite to greater angle is greater]

Solution 17:

Since AB is the largest side and BC is the smallest side of the triangle ABC

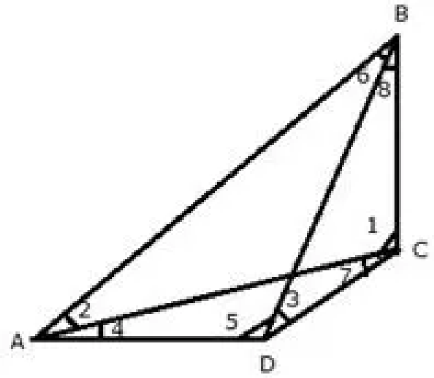
$AB > AC > BC$

$\Rightarrow 180^\circ - z^\circ > 180^\circ - y^\circ > 180^\circ - x^\circ$

$\Rightarrow -z^\circ > -y^\circ > -x^\circ$

$\Rightarrow z^\circ < y^\circ < x^\circ$

Solution 18:



In the quad. ABCD,

Since AB is the longest side and DC is the shortest side.

(i) $\angle 1 > \angle 2$[AB > BC]

$\angle 7 > \angle 4$[AD > DC]

$\therefore \angle 1 + \angle 7 > \angle 2 + \angle 4$

$\Rightarrow \angle C > \angle A$

(ii) $\angle 5 > \angle 6$[AB > AD]

$\angle 3 > \angle 8$[BC > CD]

$\therefore \angle 5 + \angle 3 > \angle 6 + \angle 8$

$\Rightarrow \angle D > \angle B$

Solution 19:

(i) Since AB > AC

$\angle ACB > \angle ABC$

$\Rightarrow 180^\circ - z > 180^\circ - y$
$\Rightarrow -z > -y$
$\Rightarrow z < y$........(i)

Also since AC > BC

$\angle ABC > \angle BAC$

$\Rightarrow 180^\circ - y > 180^\circ - x$
$\Rightarrow -y > -x$
$\Rightarrow y < x$........(ii)

From (i) and (ii)

$z < y < x$

(ii) y > x > z[Given]

Taking y > x

$\Rightarrow (180^\circ - \angle ABC) > (180^\circ - \angle BAC)$
$\Rightarrow -\angle ABC > -\angle BAC$
$\Rightarrow \angle ABC < \angle BAC$
$\Rightarrow AC < BC........(i)$

Again taking x > z

$\Rightarrow (180^\circ - \angle BAC) > (180^\circ - \angle ACB)$
$\Rightarrow -\angle BAC > -\angle ACB$
$\Rightarrow \angle BAC < \angle ACB$
$\Rightarrow BC < AB........(ii)$

From (i) and (ii)

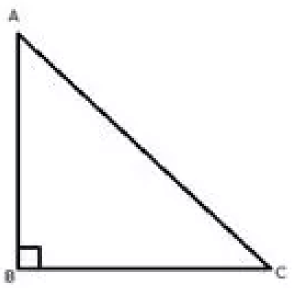
Ac < BC < AB

Writing in descending order

AB > BC > AC

Solution 20:

(i)



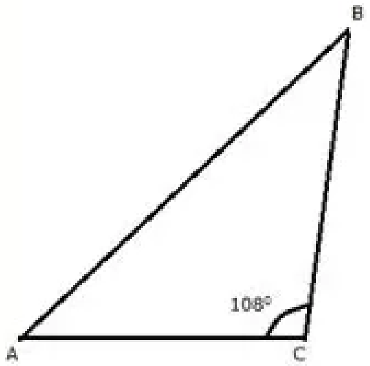
$\because \angle B = 90^\circ$ [Given]
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow \angle A + \angle C + 90^\circ = 180^\circ$
$\Rightarrow \angle A + \angle C = 90^\circ$
$\Rightarrow \angle A < 90^\circ$ and $\angle C < 90^\circ$

Hence, $\angle B > \angle A \Rightarrow AC > BC$

Similarly, $\angle B > \angle C \Rightarrow AC > AB$

Hence, hypotenuse is the greatest side.

(ii)



$\because \angle ACB = 108°$ [Given]

$\angle A + \angle B + \angle ACB = 180°$

$\Rightarrow \angle A + \angle B + 108° = 180°$

$\Rightarrow \angle A + \angle B = 72°$

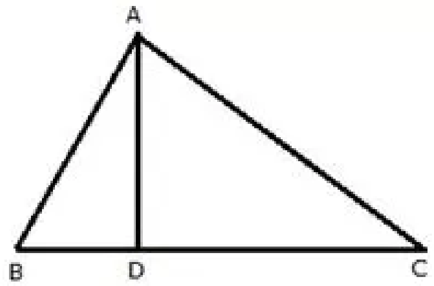
$\Rightarrow \angle A < 72°$ and $\angle B < 72°$

Hence, $\angle ACB > \angle A \Rightarrow AB > BC$

Similarly, $\angle ACB > \angle B \Rightarrow AB > AC$

Therefore, AB is the largest side.

Solution 21:



In Δ ABD,

$AB + BD > AD$ ....(i)

In Δ ACD,

$AC + DC > AD$ ....(ii)

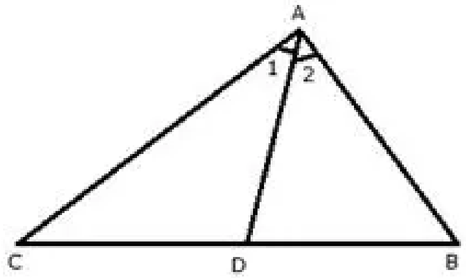
Adding (i) and (ii)

$AB + BD + AC + DC > 2AD$

$AB + BD + DC + AC > 2AD$

$AB + BC + AC > 2AD$

Solution 22:



In $\triangle$ ADC,

$\angle ADB = \angle 1 + \angle C$.............(i)

In $\triangle$ ADB,

$\angle ADC = \angle 2 + \angle B$..................(ii)

But AC > AB[Given]

$\Rightarrow \angle B > \angle C$

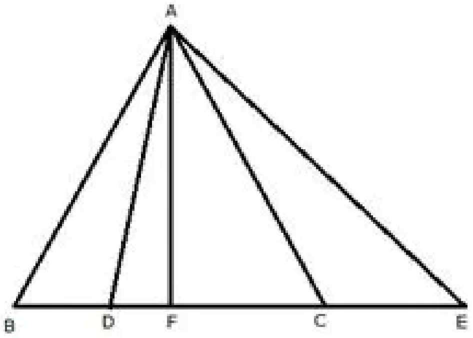
Also given, $\angle 2 = \angle 1$[AD is bisector of $\angle A$]

$\Rightarrow \angle 2 + \angle B > \angle 1 + \angle C$ ....(iii)

From (i), (ii) and (iii)

$\Rightarrow \angle ADC > \angle ADB$

Solution 23:



We know that the bisector of the angle at the vertex of an isosceles triangle bisects the base at right angle.

Using Pythagoras theorem in $\triangle$ AFB,

$AB^2 = AF^2 + BF^2$..............(i)

In $\triangle$ AFD,

$AD^2 = AF^2 + DF^2$..............(ii)

We know ABC is isosceles triangle and AB = AC

$AC^2 = AF^2 + BF^2$ ....(iii)[From (i)]

Subtracting (ii) from (iii)

$AC^2 - AD^2 = AF^2 + BF^2 - AF^2 - DF^2$

$AC^2 - AD^2 = BF^2 - DF^2$

Let 2DF = BF

$AC^2 - AD^2 = (2DF)^2 - DF^2$

$AC^2 - AD^2 = 4DF^2 - DF^2$

$AC^2 = AD^2 + 3DF^2$

$\Rightarrow AC^2 > AD^2$

$\Rightarrow AC > AD$

Similarly, AE > AC and AE > AD.

Solution 24:

The sum of any two sides of the triangle is always greater than the third side of the triangle.

In ΔCEB,

$CE + EB > BC$

$\Rightarrow DE + EB > BC \quad [CE = DE]$

$\Rightarrow DB > BC........(i)$

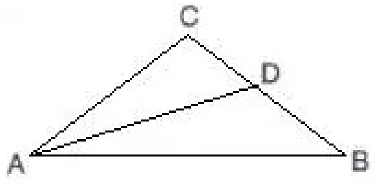
In ΔADB,

$AD + AB > BD$

$\Rightarrow AD + AB > BD > BC \quad [\text{from}(i)]$

$\Rightarrow AD + AB > BC$

Solution 25:



Given that, AB > AC

$\Rightarrow \angle C > \angle B.......(i)$

Also in ΔADC

$\angle ADB = \angle DAC + \angle C \quad [\text{Exterior angle}]$

$\Rightarrow \angle ADB > \angle C$

$\Rightarrow \angle ADB > \angle C > \angle B \quad [\text{From}(i)]$

$\Rightarrow \angle ADB > \angle B$

$\Rightarrow AB > AD$