CBSE Class 11 Mathematics
Sample Papers 06 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} Find $(A \cap B) \cap (B \cup C)$

OR

Is the set of letters in the English alphabet finite or infinite?

2. Find the distance between (-3, 7, 2) and (2, 4, -1) pairs of points.
3. Express as the product of sines and cosines: cos 12x + cos 8x.

OR

Find the value of $\sin\frac{3\pi}{4}$.

4. Find the sum $\left(i^n + i^{n+1} + i^{n+2} + i^{n+3}\right)$, where n $\in$ N.
5. In how many ways can 6 pictures be hung from 4 picture nails on a wall?

OR

If there are six periods in each working day of a school, then in how many ways we can arrange 5 subjects such that each subject is allowed at least one period?

6. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, prove that x, y, z are in A.P.
7. Find the domain of the real valued function of real variable: f(x) = $\frac{2x+1}{x^2-9}$.

OR

Let f: R $\rightarrow$ R : f(x) = x^2 + 1. Find f^{-1} {10}

8. Find the equation of the circle whose centre is (2, -3) and radius is 8.
9. Suppose we throw a die once. Find the probability of getting a number greater than 4.

OR

A die is thrown. Find the probability of getting a multiple of 3.

10. Consider the experiment of rolling a die. Let A be the event getting a prime number, B be the event getting an odd number. Write the sets representing the event A or B
11. What is the locus of a point (x, y, z) for which y = 0, z = 0?
12. How many different words can be formed with the letters of the word CAPTAIN? In how many of these C and T are never together?
13. If tan α = x + 1, tan β = x - 1, show that 2 cot $(\alpha - \beta)$ = x^2.
14. Prove that: $\frac{\sin A+\sin 3A}{\cos A+\cos 3A} = \tan 2A$

15. Prove that $\frac{\cos x+\cos y}{\cos y-\cos x}=\cot\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$.

16. Find the range of the function given by f (x) = 1 + 3 cos2x.

(Hint: $-1\leq\cos 2x\leq 1\Rightarrow -3\leq 3\cos 2x\leq 3\Rightarrow -2\leq 1+3\cos 2x\leq 4$)

Section - II

17. **Read the Case study given below and attempt any 4 sub parts:**

Schools do not take IQ tests of the majority of students anymore. IQ tests are mostly used for children who struggle in school in order to determine if they are eligible to receive special services. Do you think this is a mistake? If money were not an issue, what do you think of the idea of giving an IQ test to every child in upper elementary school? Is this a good idea or a bad idea? What would you see as the pros and cons of doing so? Write an opinion piece answering these questions, concluding with your recommendation either to give all children IQ tests or not.

IQ of a person is given by the formula IQ = $\frac{MA}{CA}\times 100$ where MA is mental age and CA is the chronological age. If 80$\leq$ IQ $\leq$140 for a group of 12 years old children, find:

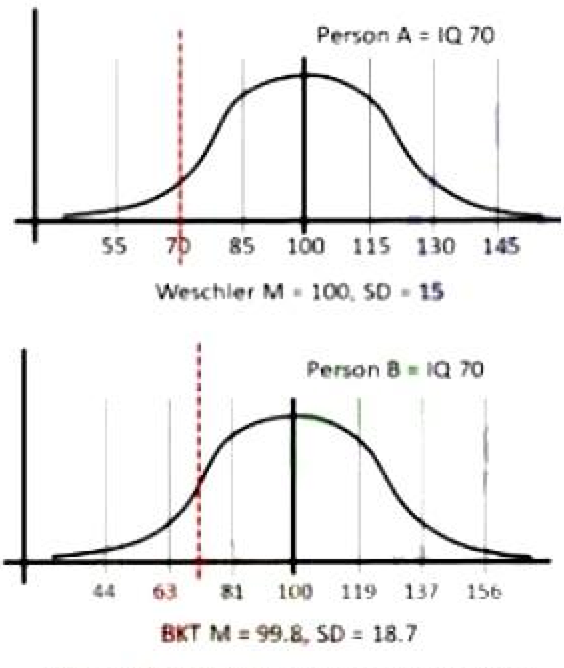
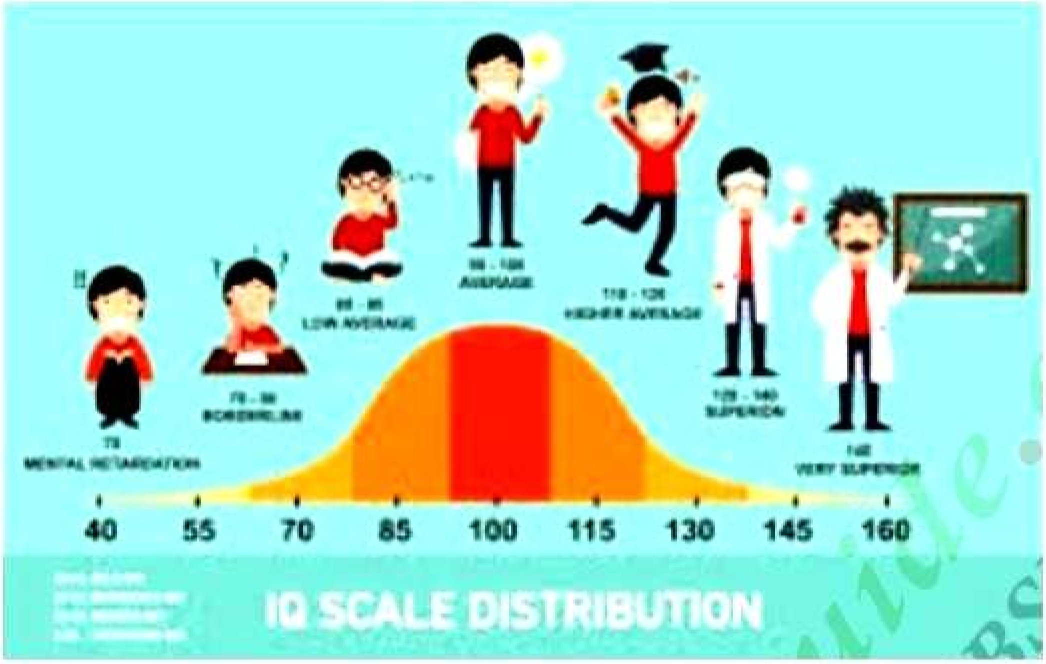


Figure 1: IQ distribution of two different SDs



i. the range of their mental age.

a. 9.6 to 14.8

b. 9.6 and 16.8

c. 9.6 to 16.8

d. 14.8 to 16.8 2.

ii. Jake is younger than Sophie. Sophie is 14 years old.

Write an inequality that compares Jake's age in years, j to Sophie's age.

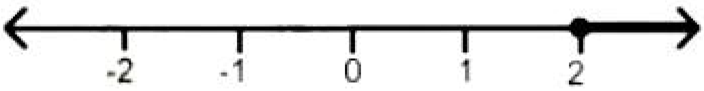
a. $j < 14$

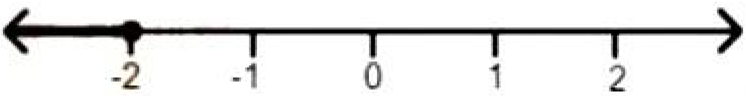
b. $j > 14$

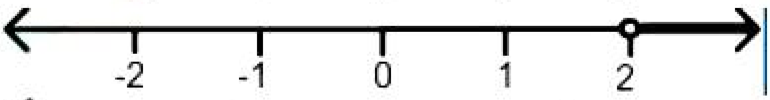
c. $j \leq 14$

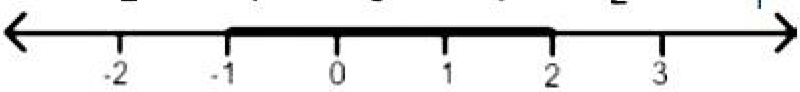
d. $j \geq 14$

iii. The graphical representation of $x > 2$ on the number line is

a. 

b. 

c. 

d. 

iv. {14,15,16} is the solution set of

a. $x > 14$ and $x > 17$, $x \in Z$

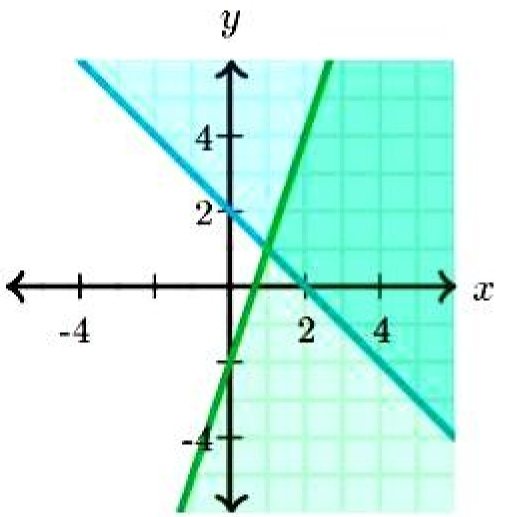
b. $x \geqslant 14$ and $x \leqslant 17$, $x \in Z$

c. x > 14 and x < 17, x∈Z

d. x ⩾ 14 and x < 17, x∈Z

v. y>−x+2 and y≤3x−2 Which graph represents the system of inequalities

a.



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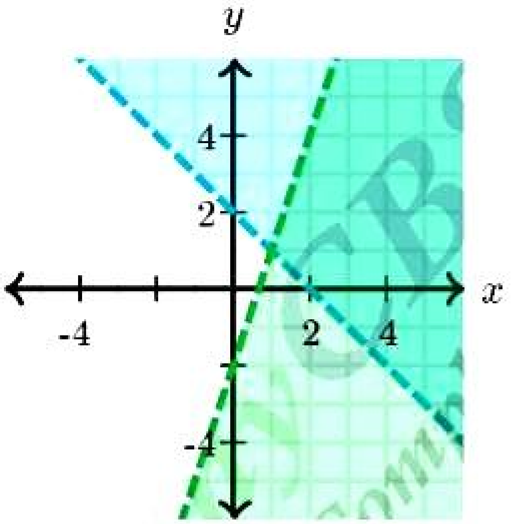

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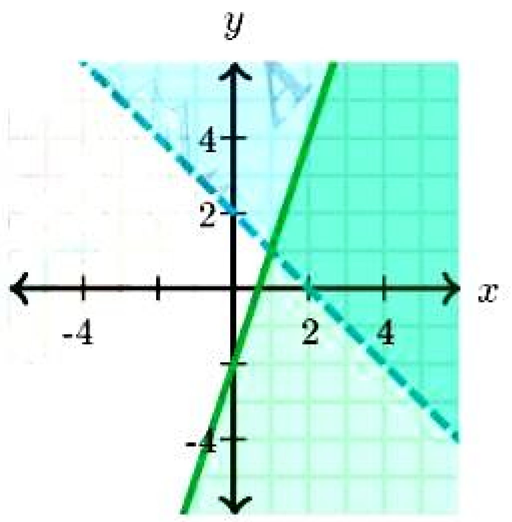
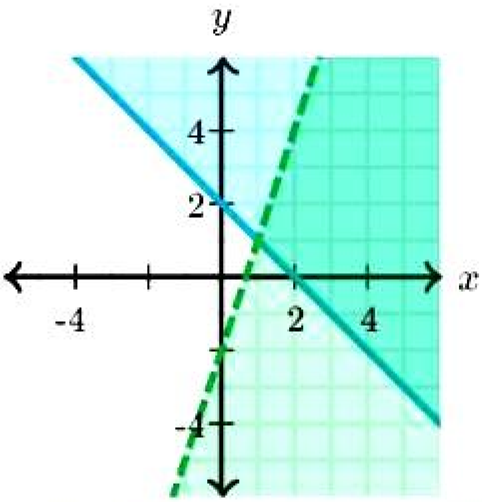
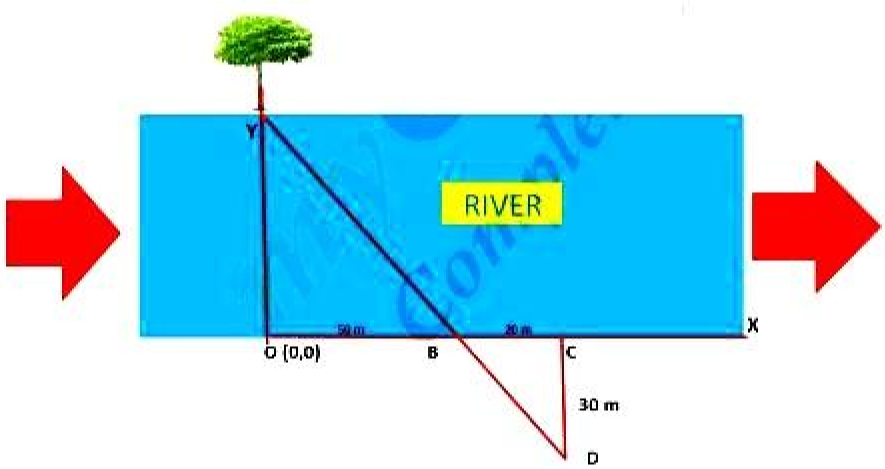


d. 

18. **Read the Case study given below and attempt any 4 sub parts:**

A surveyor was measuring the width of a river. For this, he selected a tree at Y on the other side of the river. He is standing at Point O(0,0). From O he walks 50 m in the right direction, at point B he fixes a stick. From B in the right side at distance 20 m at C fixes a stick. Now from C, he walks perpendicular to line OC. Further, he fixes a stick D so the stick D, B and the tree are in the same straight line approximately. He finds that CD = 30m.

We assume that OC is the x-axis and OY is the y-axis.



Now answer the following questions:

i. What are the coordinates of point D?

a. (50, 30)

b. (70, -30)

c. (50, 20)

d. (70, 30)

ii. What are the coordinates of point C?

   a. (0, 500)
   b. (0, -70)
   c. (0, 70)
   d. (70, 30)

iii. What is the width of the river?

   a. 75 m
   b. 50 m
   c. 200 m
   d. 300 m

iv. What are the coordinates of point Y?

   a. (0, 100)
   b. (0 ,-70)
   c. (75, 0)
   d. (0, 75)

v. What is the equation of straight line BD?

   a. 3x + 2y = 150
   b. 3x + 2y = 100
   c. 5x + 2y = 150
   d. 5x - 3y = 150

Part - B Section - III

19. Let A and B be two sets. Then, prove that A = B ⇔ $A \subseteq B$ and $B \subseteq A$.

20. Suppose a set A = {January, February, August} and set B = {28,15, 30}. Write a relation R given by $R = \{(a, b) \in A \times B,$ where a is month and b has number of days}. Also, find R^{-1}.

    Which ordered pair in R represent an independence day?

    OR

    If the ordered pairs (x, -1) and (5, y) belong to the set {(a, b) : b = 2a - 3}, find the values of x and y.

21. If tan x + sin x = m and tan x - sin x = n, show that $m^2 - n^2 = 4\sqrt{mn}$.

22. If z = 2 + i, prove that $z^3 + 3z^2 - 9z + 8 = (1 + 14i)$

23. Prove that $(1-i)^n\left(1-\frac{1}{i}\right)^n = 2^n$ for all values of n ∈ N.

OR

For any two complex numbers z_1 and z_2, prove that: $|z_1 - z_2| \geq |z_1| - |z_2|$.

24. Find the domain and the range of the real functions: $f(x) = \frac{1}{x}$

25. If f is a real valued function defined by f(x) = $x^2 + 4x + 3$, then find f'(1) and f'(3)

26. Differentiate the functions with respect to x: cos (x + a).

27. Let A = {a, b, c, d}, B = {a, b, c} and C = {b, d}. Find all sets X such that: X ⊂ A and X ▯ B.

28. Solve: $x^2 - x + 1 = 0$.

OR

Solve the equation $4x^2 - 12x + 25 = 0$ quadratic equations by factorization method only.

Section - IV

29. Find the derivative of the function $f(x) = \frac{4x+5\sin x}{3x+7\cos x}$

30. If the letters of the word ASSASSINATION are arranged at random. Find the probability that: Two I's and two N's come together.

31. A man arranges to pay-off a debt of Rs. 3600 by 40 annual installments, which are in AP. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid. Find the 8^{th} installment.

OR

Find all the sequence which are simultaneously AP and GP.

32. Find the centre, the lengths of the axes, eccentricity, foci of the ellipse: $4x^2 + y^2 - 8x + 2y + 1 = 0$.

33. Write the number of ways in which 6 men and 5 women can dine at a round table if no two women sit together.

34. List all the element of the sets: H = {x : x ∈ Z, |x| ≤ 2}

OR

The Given 'Set' is null or not? Set of odd natural numbers divisible by 2.

35. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of x and y.

Section - V

36. The lengths of three unequal edges of a rectangular solid block are in GP. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. Find the length of the longest edge.

OR

The interior angles of a polygon are in AP. The smallest angle is 52° and the common difference is 8°. Find the number of sides of the polygon.

37. Where x is a positive integer. Determine the mean and standard deviation of the marks.

OR

Find the mean deviation about the mean for the following data:

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4

38. Solve the following system of linear inequalities.

2(2x + 3) - 10 < 6(x - 2)

and $\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3}$

OR

Solve: the inequality 3 (2 – x) $\geq$ 2 (1 – x) for real x.

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Solution

Part - A Section - I

1. $A \cap B$ = {7, 9, 11}
   $B \cup C$ = {7, 9, 11, 13, 15}
   $(A \cap B) \cap (B \cup C)$ = {7, 9, 11}

OR

The set of letters in the English alphabet is a finite set because there are 26 letters in the English alphabet.

2. Let A(-3, 7, 2) and B(2, 4, -1) be two points. Then
   $AB = \sqrt{[2-(-3)]^2 + (4-7)^2 + (-1-2)^2}$ [using distance formula]
   $= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} = \sqrt{25+9+9} = \sqrt{43}$ units
3. Let y = cos 12x + cos 8x, then
   y = 2 cos $\left(\frac{12x+8x}{2}\right)$ cos $\left(\frac{12x-8x}{2}\right)$ {∵ cos A + cos B = 2 cos $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$}
   = 2 cos 10x cos 2x

OR

$$Sin\frac{3\pi}{4} = Sin(\pi - \frac{\pi}{4}) = Sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

4. We have $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

   $= i^n + i^n.i + i^n.i^2 + i^n.i^3$

   $= i^n (1 + i + i^2 + i^3)$ [$i^2 = -1$ & $i^3 = -i$]

   $= i^n (1 + i - 1 - i)$

   $= i^n (0) = 0$
5. There are 6 pictures to be placed in 4 places.
   Therefore, a permutation of 6 different objects in 4 places is
   $P(6,4) = \frac{6!}{(6-4)!}$

$= \frac{6!}{2!} = \frac{720}{2} = 360.$

This can be done by 360 ways

OR

Six periods can be arranged for 5 subjects in ${}^6P_5$ ways

$= \frac{6!}{1!} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

One period is left, which can be arranged for any of the 5 subjects. One left period can be arranged in 5 ways.

$\therefore$ Required number of arrangements $= 720 \times 5 = 3600$

6. We are given that,

$a^{1/x} = b^{1/y} = c^{1/z} = \lambda$(say)

$\Rightarrow a = \lambda^x, b = \lambda^y$ and $c = \lambda^z$

Now, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow (\lambda^y)^2 = \lambda^x \times \lambda^z$

$\Rightarrow \lambda^{2y} = \lambda^{x+z}$

$\Rightarrow 2y = x + z$

$\Rightarrow$ x, y, z are in A.P.

7. Clearly, f(x) is defined for all real values of x, except for the case when $x^2 - 9 = 0$.

$x^2 - 9 = 0$

$= x^2 - 32 = 0$

$= (x + 3)(x - 3) = 0$

$= x + 3 = 0$ or $x - 3 = 0$

$= x = \pm 3$

When $x = \pm 3$, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {–3, 3}

OR

Here we have, $f(x) = x^2 + 1$

To find inverse of f(x), Let y = f(x)

Then, $y = x^2 + 1$

$y - 1 = x^2$

$x = \sqrt{y-1}$

$f^{-1}(x) = \sqrt{x-1}$

Substituting x = 10

$f^{-1}(10) = \sqrt{10-1} = \sqrt{9} = 3$

8. The required equation of the the circle is

   $(x - 2)^2 + (y - (-3))^2 = 8^2$

   $(x - 2)^2 + (y + 3)^2 = 82$

   or, $x^2 + y^2 - 4x + 6y - 51 = 0$.

9. Suppose A be the event of getting a number greater than 4.

   Number of possible outcomes = Total number of outcomes on throwing a die = 6

   Number of favourable outcomes = $\{5, 6\} = 2$

   ∴ Probability of getting a number greater than 4 is,

   $P(A) = \frac{2}{6} = \frac{1}{3}$

OR

As 3, 6 are multiples up to 6, so the desired outcomes are 3, 6, and total outcomes are 1, 2, 3, 4, 5, 6

Therefore, total no.of outcomes are 6, and total no.of desired outcomes are 2

Probability of getting multiple of 3 = $\frac{2}{6} = \frac{1}{3}$

Conclusion: Probability of getting multiple of 3 when die is thrown is $\frac{1}{3}$

10. Here sample space s= {1, 2, 3, 4, 5, 6}, A = {2, 3, 5} and B = {1, 3, 5}

    'A or B' = A ∪ B = {1, 2, 3, 5}

11. On the x-axis both y and z coordinates are zero.

    Hence, the locus of a point for which y = 0, z = 0 is x-axis.

12. To find: number of words such that C and T are never together

    Number of words where C and T are never the together = Total numbers of words - Number of words where C and T are together

    Total number of words = $\frac{7!}{2!}$ = 2520

    Let C and T be denoted by a single letter Z

    New word is APAINZ

This can be permuted in $\frac{6!}{2!}$ = 360 ways

Z can be permuted among itself in 2 ways

number of words where C and T are together = 360 $\times$ 2 = 720

number of words where C and T are never together = 2520 - 720 = 1800

There are 1800 words where C and T are never together

13. We have given that tan α = x + 1 and tan β = x - 1

To prove $2\cot(\alpha-\beta)$ = = x^2

LHS $= 2\cot(\alpha-\beta)$

$= \frac{2(1+\tan\alpha\tan\beta)}{[\tan\alpha-\tan\beta]}$

$= \frac{2+2(x+1)(x-1)}{(x+1-x+1)}$

$= \frac{2+2x^2-2}{2}$

$= \frac{2x^2}{2}$

= x^2

= RHS

Hence proved.

14. LHS $= \frac{\sin 3A+\sin A}{\cos 3A+\cos A}$

$= \frac{2\sin\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)}{2\cos\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)}$ [$\because$ sin x + sin y = 2 sin $\frac{x+y}{2}$ cos $\frac{x-y}{2}$ and cos x + cos y = 2 cos $\frac{x+y}{2}$ cos $\frac{x-y}{2}$]

$= \frac{\sin 2A\cos A}{\cos 2A\cos A}$ = tan 2A = RHS

15. To prove: $\frac{\cos x+\cos y}{\cos y-\cos x} = \cot\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$.

Now L.H.S = $\frac{\cos x-\cos y}{\cos y-\cos x}$

$= \frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{y-x}{2}}$

$= \frac{\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{\sin\frac{x+y}{2}\sin\frac{y-x}{2}}$

$= \cot\frac{x+y}{2}\cot\frac{x-y}{2}$

16. Here we have f (x) = 1 + 3 cos2x.

We know that, $-1 \le \cos 2x \le 1$

$\Rightarrow \quad -3 \le 3\cos 2x \le 3$

$\Rightarrow \quad -2 \le 1+3\cos 2x \le 4$

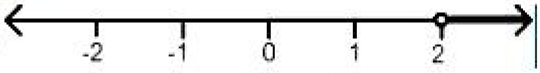
$\Rightarrow -2 \leq f(x) \leq 4$

$\therefore$ Range of f = [-2, 4]

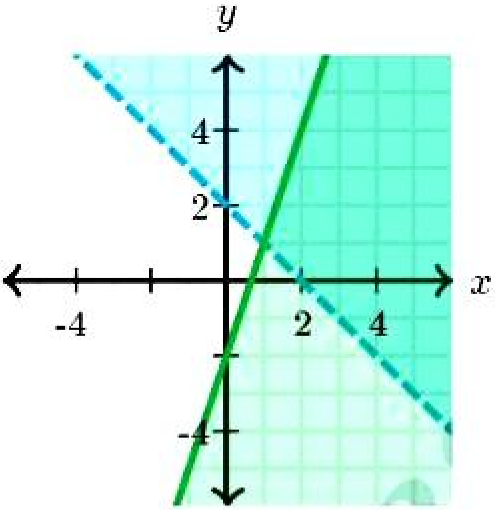
Section - II

17. i. (b) 9.6 and 16.8

ii. (a) j <14

iii. (c) 

iv. (d) x ⩾ 14 and x < 17, x∈Z

v. (c) 

18. i. (b) (70, -30)

ii. (c) (0, 70)

iii. (a) 75 m

iv. (d) (0, 75)

v. (a) 3x + 2y = 150

Part - B Section - III

19. **Given:** $A = B$

To Prove: $A \subseteq B$ and $B \subseteq A$

Proof: As we know that every element of A is in B and every element of B is in A in equal sets

$\therefore A \subseteq B$ and $B \subseteq A$

$\therefore A = B \Rightarrow A \subseteq B$ and $B \subseteq A$

Now, Suppose $A \subseteq B$ and $B \subseteq A$

By the definition of a subset, if $A \subseteq B$ then it follows that every element of A is in B and if $B \subseteq A$ then it follows that every element of B is in A.

$\therefore A = B$

$\therefore A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$

Hence Proved.

20. Given sets are:

A ={January, February, August}

B ={28,15,30}

∴ R={(January,28), (January,15), (January, 30), (February,28), (February,15), (February, 30), (August, 28), (August, 15), (August, 30)}

R^{-1}={(28, January), (15, January), (30, January), (28, February), (15, February), (30, February), (28, August), (15, August), (30, August)}

The ordered pair which represent Independence day is (August, 15).

OR

We have,

$(x, -1) \in \{(a, b) : b = 2a - 3\}$

and, $(5, y) \in \{(a, b) : b = 2a - 3\}$

$\Rightarrow -1 = 2 \times x - 3$ and $y = 2 \times 5 - 3$

$\Rightarrow -1 = 2x - 3$ and $y = 10 - 3$

$\Rightarrow 3 - 1 = 2x$ and $y = 7$

$\Rightarrow 2 = 2x$ and $y = 7$

$\Rightarrow x = 1$ and $y = 7$

21. We have, tan x + sin x = m and tan x - sin x = n.

$\therefore m^2 - n^2 = (\tan x + \sin x)^2 - (\tan x - \sin x)^2 = 4 \tan x \sin x$ ... (i)

and, $4\sqrt{mn} = 4\sqrt{(\tan x + \sin x)(\tan x - \sin x)} = 4\sqrt{\tan^2 x - \sin^2 x}$

$= 4\sqrt{\frac{\sin^2 x}{\cos^2 x} - \sin^2 x} = 4\sqrt{\frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}} = 4\sqrt{\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}}$

$= 4\sqrt{\frac{\sin^4 x}{\cos^2 x}} = 4\frac{\sin^2 x}{\cos x}$ = 4 tan x sin x ... (ii)

From (i) and (ii), we obtain that $m^2 - n^2 = 4\sqrt{mn}$.

22. We have,

$z = 2 + i \Rightarrow z - 2 = i$

$\Rightarrow (z - 2)^2 = i^2$

$\Rightarrow z^2 - 4z + 4 = -1$

$\Rightarrow z^2 - 4z + 5 = 0$ ...(i)

$\therefore z^3 + 3z^2 - 9z + 8$

$= z(z^2 - 4z + 5) + 7(z^2 - 4z + 5) + 14z - 27$

$= (z \times 0) + (7 \times 0) + 14z - 27 = (14z - 27)$ [using (i)]

$= 14(2 + i) - 27 = (1 + 14i)$

Hence, $z^3 + 3z^2 - 9z + 8 = (1 + 14i)$.

23. L.H.S. = $(1-i)^n\left(1-\frac{1}{i}\right)^n$

$= (1-i)^n\left(1-i^{-4^* 1+3}\right)^n$ [$i^{4n} = 1$]

$= (1-i)^n\left(1-i^3\right)^n$ [$i^3 = -i$]

$= (1 - i)^n(1 + i)^n$

Applying $a^n b^n = (ab)^n$

$= ((1 - i)(1 + i))^n$

$= \left(1-i^2\right)^n$

$= 2^n$

L.H.S. = R.H.S.

Hence Proved.

OR

We have,

$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$\therefore -1 \le \cos\ (\theta_1 - \theta_2) \le 1$

$\Rightarrow \cos(\theta_1 - \theta_2) \le 1$

$\Rightarrow -\cos(\theta_1 - \theta_2) \ge -1$

$\Rightarrow -2|z_1||z_2|\cos(\theta_1 - \theta_2) \ge -2|z_1||z_2|$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos\ (\theta_1 - \theta_2) \ge |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow |z_1 + z_2|^2 \ge (|z_1| - |z_2|)^2$

$\Rightarrow |z_1 - z_2| \ge |z_1| - |z_2|$

Hence proved.

24. Here we are given that, $f(x) = \frac{1}{x}$

Need to find: where the function is defined.

Let, $f(x) = \frac{1}{x} = y$ ......(i)

To find the domain of the function f(x) we need to equate the denominator of the function to 0

Therefore,

x = 0

It means that the denominator is zero when x = 0

So, the domain of the function is the set of all the real numbers except 0

The domain of the function, $D_{\{f(x)\}} = (-\infty, 0) \cup (0, \infty)$

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes, $\frac{1}{y} = x$

$\Rightarrow y = \frac{1}{x} = f(x_1)$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

x = 0

It means that the denominator is zero when x = 0

So, the range of the function is the set of all the real numbers except 0.

The range of the function, $R_{f(x)} = (-\infty, 0) \cup (0, \infty)$

25. We are given that, $f(x) = x^2 + 4x + 3$,

$\therefore f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$\Rightarrow f'(1) = \lim_{h\to 0} \frac{\left((1+h)^2+4(1+h)+3\right]-(1^2+4\times 1+3)}{h}$

$\Rightarrow f'(1) = \lim_{h\to 0} \frac{(h^2+6h+8)-8}{h} = \lim_{h\to 0} \frac{h^2+6h}{h} = \lim_{h\to 0} h + 6 = 6$

and, $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$

$\Rightarrow f'(3) = \lim_{h\to 0} \frac{\left\{(3+h)^2+4(3+h)+3\right\}-\{3^2+4\times 3+3\}}{h}$

$\Rightarrow f'(3) = \lim_{h\to 0} \frac{(h^2+10h+24)-24}{h} = \lim_{h\to 0} \frac{h^2+10h}{h} = \lim_{h\to 0} h + 10 = 10$

26. we have,

$\frac{d}{dx}(\cos(x + a))$

$= \frac{d}{dx}(\cos x.\cos a - \sin x.\sin)$ [$\because$(cos (x + a) = cos x cos a - sin x sin a]

$= \cos a \frac{d}{dx}(\cos x) - \sin a \frac{d}{dx}(\sin x)$

$= \cos a(-\sin x) - \sin a(\cos x)$

= -(sin x cos a + cos x sin a)

= -sin (x + a).

$\therefore \frac{d}{dx}$ (cos (x + a)) = -sin (x + a)

27. Here, we have $X \subset A$ and $X \not\subset B$

$\Rightarrow$ X is a subset of A but X is not a subset of B

$\Rightarrow X \in P(A)$ but $X \notin P(B)$,we get

$\Rightarrow$ X = {d}, {a, b, d}, {b, c, d}, {a, c, d}, {a, d}, {b, d}, {c, d}, {a, b, c, d}.

28. $x^2 - x + 1 = 0$

$\Rightarrow x^2 - x + \frac{1}{2} + \frac{3}{4} = 0$

$\Rightarrow x^2 - 2(x)(\frac{1}{2}\} + (\frac{1}{2})^2 - \frac{3}{4}i^2 = 0$

$\Rightarrow (x - \frac{1}{2})^2 - \left(\frac{\sqrt{3}i}{2}\right)^2 = 0$

$\Rightarrow \left(x - \frac{1}{2} + \frac{\sqrt{3}}{2}\right)\left(x - \frac{1}{2} - \frac{\sqrt{3}}{2}\right) = 0$

$\Rightarrow \left(x - \frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 0$ or $\left(x - \frac{1}{2} - \frac{\sqrt{3}}{2}\right) = 0$

$\Rightarrow x = \frac{1}{2} - \frac{i\sqrt{3}}{2}$ or $x = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

Hence, the roots of the equation are $\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

OR

$4x^2 - 12x + 25 = 0$

$\Rightarrow 4x^2 - 12x + 9 + 16 = 0$

$\Rightarrow (2x)^2 + 3^2 - 2 \times 2x \times 3 - (4i)^2 = 0$

$\Rightarrow (2x - 3)^2 - (4i)^2 = 0$ $[(a-b)^2 = a^2 - 2ab + b^2]$

$\Rightarrow (2x - 3 + 4i)(2x - 3 - 4i) = 0$ $[a^2 - b^2 = (a + b)(a - b)]$

$\Rightarrow (2x - 3 + 4i) = 0$ or, $(2x - 3 - 4i) = 0$

$\Rightarrow 2x = 3 - 4i$ or, $2x = 3 + 4i$

$\Rightarrow x = \frac{3}{2} - 2i$ or, $x = \frac{3}{2} + 2i$

Hence, the roots of the equation are $\frac{3}{2} - 2i$ and $\frac{3}{2} = 2i$

Section - IV

29. Here $f(x) = \frac{4x+5\sin x}{3x+7\cos x}$

$$\therefore f'(x) = \frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x) - (4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$

$= \frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$
$= \frac{12x+15x\cos x+28\cos x+35\cos^2 x-12x+28x\sin x-15\sin x+35\ \sin^2 x}{(3x+7\cos x)^2}$
$= \frac{15x\cos x+28\cos x+28x\sin x-15\sin x+35(\cos^2 x+\sin^2 x)}{(3x+7\cos x)^2}$
$= \frac{15x\cos x+28\cos x+28x\sin x-15\sin x+35}{(3x+7\cos x)^2}$

30. As we have to get the probability that two I's and two N's come together,

I	I	N	N	A	S	S	A	S	S	A	T	O

So, now numbers of letters is 1(two I's and two N's are counted as one letters) + 9 = 10

$\therefore\ n(E) = \frac{4!}{2!2!} \times \frac{10!}{3!4!}$

$\therefore$ Required Probability $= \frac{\frac{4!10!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}}$

$= \frac{10!4!}{3!4!2!2!} \times \frac{3!4!2!2!}{13!}$
$= \frac{10!\times 4!}{13\times 12\times 11\times 10!}$
$= \frac{4\times 3\times 2\times 1}{13\times 12\times 11}$
$= \frac{2}{143}$

31. Here, total debt, S = Rs. 3600

and total installments, n = 40

Let a and d be the first installment and increment in installment.

By using sum of n terms of an A.P., we get

$\therefore 3600 = \frac{40}{2}$ [2a + (40 - 1) d]

$\Rightarrow$ 180 = 2a + 39 d ... (i)

Now, after 30 installments, one-third of the debt is unpaid.

i.e., $\frac{3600}{3}$ = Rs. 1200 is unpaid. Thus paid money = 3600 - 1200 = Rs. 2400

So, again by using sum of n terms, we get,

$S_{30} = 2400 = \frac{30}{2}$ [2a + (30 - 1) d]

$\Rightarrow$ 160 = 2a + 29 d ... (ii)

On solving Eqs. (i) and (ii), we get

a = 51, d = 2

We know that, n^{th} term, $T_n = a + (n - 1) d$

$\therefore 8^{th}$ installment, $T_8 = a + (8 - 1) d = 51 + 7 \times 2$

= 51 + 14 = Rs. 65

Hence, the 8^{th} installment is Rs. 65.

OR

Let T_1, T_2, T_3, ... be a sequence which is AP as well as GP.

Let $T_n = a + (n - 1) d, \forall n \in N$

So, the sequence is a, a + d, a + 2d,...

This is also a GP.

$\therefore \frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}, \forall n \in N$

$\Rightarrow \frac{a+nd}{a+(n-1)d} = \frac{a+(n+1)d}{a+nd}$

$\Rightarrow (a + nd)^2 = [(a + nd) + d] [(a + nd) - d]$

$\Rightarrow (a + nd)^2 = (a + nd)^2 - d^2$

$\Rightarrow d^2 = 0$

$\Rightarrow d = 0$

So, the sequence is a, a + 0, a + 2 (0),... i.e., a, a, a...

32. $4x^2 + y^2 - 8x + 2y + 1 = 0$

$\Rightarrow 4(x^2 - 2x) + (y^2 + 2y) = -1$

$\Rightarrow 4(x^2 - 2x + 1) + (y^2 + 2y + 1) = -1 + 4 + 1$

$\Rightarrow 4(x - 1)^2 + (y + 1)^2 = 4$

$\Rightarrow \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$

Centre = $(x_1, y_1) = (1, -1)$

Here, x_1 = 1a and y_1 = -1

Also, a = 1 and b = 2

Major axis = 2b

$= 2 \times 2$

Major axis = 4

Minor axis = 2a

$= 2 \times 1$

Minor axis = 2

eccentricity, $e = \sqrt{1 - \frac{a^2}{b^2}}$

$\Rightarrow e = \sqrt{1 - \frac{1}{4}}$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

Foci = $(x_1, y_1 \pm be)$

= $(1, -1 \pm \sqrt{3})$

33. 6 men can be arranged in (6-1)! = 5! ways to dine at a round table.

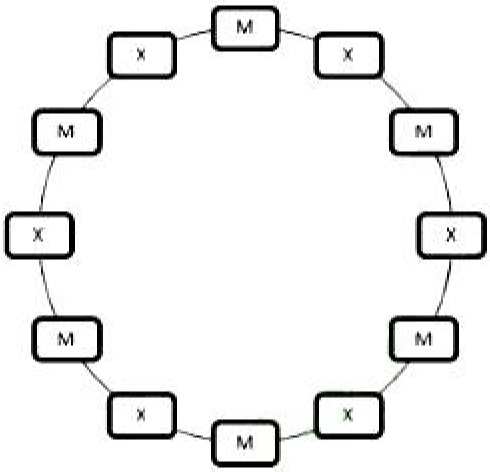
Now, if we place 5 women in 6 empty seats between them so that no two women will be together, and this can be done in ${}^6P_5$ ways i.e. in

$\frac{6!}{(6-5)!}$ = 6! ways.

As, the operations are dependent, so, the number of ways in which 6 men and 5 women can dine at a round table if no two women sit together.

= $5! \times 6!$

The discussion can be shown pictorially as:



[X = 6 empty seats between the 6 men(M)].

34. Here, $x \in Z$ and $|x| \leq 2$

Z is a set of integers ,the theref

Integers are ...-3, -2 , -1, 0, 1, 2, 3, ...

Now, if we take x = -3 then we have to check that it satisfies the given condition $|x| \leq 2$

|-3| = 3 > 2

Therefore, $-3 \notin H$

If x = -2 then |-2| = 2 [satisfying $|x| \leq 2$]

$\Rightarrow -2 \in H$

If x = -1 then |-1| = 1 [satisfying $|x| \leq 2$]$\Rightarrow -1 \in H$

If x = 0 then |0| = 0 [satisfying $|x| \leq 2$]

$\Rightarrow 0 \in H$

If x = 1 then |1| = 1 [satisfying $|x| \leq 2$]

$\Rightarrow 1 \in H$

If x = 2 then |2| = 2 [satisfying $|x| \leq 2$]

$\Rightarrow 2 \in H$

If x = 3 then |3| = 3 > 2 [satisfying $|x| \leq 2$]

$\Rightarrow 3 \notin H$

Therefore, H = {-2, -1, 0, 1, 2}

OR

Set of odd natural numbers divisible by 2 is an empty set because odd natural numbers are not divisible by 2.

35. Here $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

$\therefore \frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ and $y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ and $y=\frac{3}{3}$

$\Rightarrow$ x = 2 and y = 1

Section - V

36. Let the edges of rectangular block in GP be a, ar and ar^2, respectively.... (i)

Now, Volume =216 cm^3

$\Rightarrow a(ar)(ar^2) = 216$ [$\because$ volume of cuboid = $l \times b \times h$]

$\Rightarrow (ar)^3 = (6)^3$

$\Rightarrow$ ar = 6 cm [taking cube root] ...(ii)

and total surface area = 252 cm

$\Rightarrow 2[a(ar) + ar(ar^2) + a(ar^2)] = 252$ [$\because$ sufrace area of cuboid = 2 (lb + bh + hl)]

From Eq. (ii), we get

2 (6a +36r +36) = 252

$\Rightarrow$ 12 (a + 6r + 6) = 252

$\Rightarrow$ a + 6r = 15 [divide both sides by 12] ...(iii)

$\Rightarrow a + 6 \times \left(\frac{6}{a}\right) = 15$ [from Eq. (ii)]

$\Rightarrow a^2 - 15a + 36 = 0 \Rightarrow (a - 12)(a - 3) = 0$

$\Rightarrow a = 3, 12$

From Eq. (iii), we get

When $a = 3$, then $3 + 6r = 15 \Rightarrow r = 2$

and when $a = 12$, then $12 + 6r = 15 \Rightarrow r = \frac{1}{2}$

On putting above values in Eq. (i),

edges are $3, 3 \times 2, 3 \times (2)^2$ or $12, 12 \times \left(\frac{1}{2}\right), 12 \times \left(\frac{1}{2}\right)^2$

i.e., 3, 6, 12 or 12, 6, 3.

Hence, the length of the longest edge is 12 cm.

OR

Interior angles of a polygon are in A.P

Smallest angle = $a = 52°$

Common difference = $d = 8°$

Let the number of sides of a polygon = n

Angles are in the following order

$52°, 52° + d, 52° + 2d, \ldots, 52° + (n-1) \times d$

Sum of n terms in A.P = $s = \frac{n}{2}\{2a + (n - 1) d\}$

Sum of angles of the given polygon is $\frac{n}{2}(2 \times 52°) + (n - 1) \times 8°$

Hint:

Sum of interior angles of a polygon of n sides is $(n - 2) \times 180°$

Therefore,

$(n - 2) \times 180° = \frac{n}{2}\ 104° + (n - 1) \times 8°$

$180n - 360 = 52n + n(n - 1) \times 4$

$4n^2 + 48n = 180n - 360$

$4n^2 - 132n + 360 = 0$

$n^2 - 33n + 90 = 0$

$(n - 3)(n - 30) = 0$

$n = 3$ & $n = 30$

$\therefore$ It can be a triangle or a 30 sided polygon.

The number of sides of the polygon is 3 or 30

37. It is given that there are 60 students in the class.

$\therefore (x - 2) + x + x^2 + (x + 1)^2 + 2x + (x + 1) = 60$

$\Rightarrow 2x^2 + 7x - 60 = 0$

$\Rightarrow (2x + 15)(x - 4) = 0$

$\Rightarrow x - 4 = 0$ [$\because x > 0 \therefore 2x + 15 \neq 0$]

$\Rightarrow x = 4$

Thus, we obtain the following frequency distribution:

Marks:	0	1	2	3	4	5
Frequency:	x-2	x	x^2	$(x + 1)^2$	2x	x + 1

Computation of mean and standard deviation

Marks (x_i)	**Frequency (f_i)**	**f_ix_i**	**$f_ix_i^2$**
0	2	0	0
1	4	4	4
2	16	32	64
3	25	75	225
4	8	32	128
5	5	25	125
	$N = \Sigma f_i = 60$	**$\Sigma f_ix_i = 168$**	**$\Sigma f_ix_i^2 = 546$**

Here, $N = 60$, $\Sigma f_ix_i = 168$, $\Sigma f_ix_i^2 = 546$

$\therefore$ Mean $= \frac{1}{N} \Sigma f_ix_i = \frac{168}{60}$ 2.8

and, Variance $= \left(\frac{1}{N}\Sigma f_i x_i^2\right) - \left(\frac{1}{N}\Sigma f_i x_i\right)^2 = \frac{546}{60} - \left(\frac{168}{60}\right)^2 = 9.1 - 7.84 = 1.26$

Hence, S.D. $= \sqrt{\text{Variance}} = \sqrt{1.26} = 1.122$

OR

We have

$N = \sum_{i=1}^{6} f_i = (6 + 8 + 1\,5 + 3 + 8 + 4) = 44$

$\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{N} = \frac{(6\times3)+(8\times5)+(15\times7)+(3\times9)+(8\times11)+(4\times13)}{44}$

$= \frac{(18+40+105+27+88+52)}{44} = \frac{330}{44} = \frac{15}{2}$ = 7.5

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4
cf	6	14	29	32	40	44

Here we have, N = 44, which is even.

Therefore, median $= \frac{1}{2} \cdot \{ \frac{N}{2}$ th observation + $\left(\frac{N}{2}+1\right)$ th observation}

$= \frac{1}{2}$ (22nd observation + 23rd observation)

$= \frac{1}{2}$ (7 + 7) = 7

Thus, M = 7.

Now, we have:

$\lvert x_i - M \rvert$	4	2	0	2	4	6
f_i	6	8	15	3	8	4
$f_i \lvert x_i - M \rvert$	24	16	0	6	32	24

$\therefore \sum_{i=1}^{6} f_i = 44$ and $\sum_{i=1}^{6} f_i \lvert x_i - M \rvert = 102$

$\therefore \text{MD}(\bar{x}) = \frac{\sum_{i=1}^{6} f_i \lvert x_i - M \rvert}{N} = \frac{102}{44}$ = 2.32

38. The given system of linear inequalities is

2(2x + 3) -10 < 6(x - 2) ....(i)

and $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$ ...(ii)

From inequality (i), we get

2(2x + 3) - 10 < 6(x - 2)

⇒ 4x + 6 - 10 < 6x - 12

⇒ 4x - 4 < 6x - 12

⇒ 4x - 4 + 4 < 6x - 12 + 4 [adding 4 on both sides]

⇒ 4x < 6x - 8

⇒ 4x - 6x < 6x - 8 - 6x [subtracting 6x from both sides]

⇒ -2x < -8

$\Rightarrow$ 2x > 8 [dividing both sides by - 1 and then inequality sign will change]

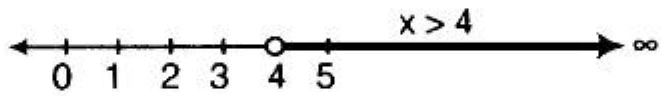
$\Rightarrow \quad \frac{2x}{2} > \frac{8}{2}$ [dividing both sides by 2]

$\therefore \quad x > 4$ ...(iii)

Thus, any value of x greater than 4 satisfies the inequality.

$\therefore$ Solution set is $x \in (4, \infty)$

The representation of solution of inequality (i) is



From inequality (ii), we get

$\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3} \Rightarrow \frac{2x-3+24}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \quad \frac{2x+21}{4} \geq \frac{6+4x}{3} \Rightarrow 3(2x+21) \geq 4(6+4x)$

$\Rightarrow \quad 6x + 63 \geq 24 + 16x$

$\Rightarrow -10x \geq -39 \Rightarrow 10x \leq 39$

$\Rightarrow \quad \frac{10x}{10} \leq \frac{39}{10}$

$\Rightarrow \quad x \leq 3.9$ ...(iv)

Thus, any value of x less than or equal to 3.9 satisfies the inequality.

$\therefore$ Solution set is $x \in (-\infty, 3.9]$.

Its representation on number line is

From Eqs. (iii) and (iv), it is clear, that there is no common value of x, which satisfies both inequalities (iii) and (iv).

Hence, the given system of inequalities has no solution.

OR

Here $3(2-x) \geqslant 2(1-x)$

$\Rightarrow 6 - 3x \geqslant 2 - 2x$

$\Rightarrow -3x + 2x \geqslant 2 - 6$

$\Rightarrow -x \leqslant -4$

Dividing both sides by -1, we have

$\frac{-x}{-1} < \frac{-4}{-1} \Rightarrow x \leqslant 4$

Thus the solution set is $(-\infty, 4]$.