CBSE Class 10 Mathematics Basic

Sample Paper - 03 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Write whether the rational number $\frac{7}{75}$ will have terminating decimal expansion or a nonterminating decimal.

OR

Write the denominator of the rational number $\frac{129}{2^2 \times 5^7}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

2. Solve: $2x^2 + x - 6 = 0$
3. Find whether the following pair of equations has no solution, unique solution or

infinitely many solutions.

$5x - 8y + 1 = 0;$

$3x - \frac{24}{5}y + \frac{3}{5} = 0$

4. If a circle can be inscribed in a parallelogram how will the parallelogram change?
5. The fee charged from a student every month by a school for the whole session, when the monthly fee is Rs 400. Do the lists of numbers involved form an AP? Give reasons for your answer.

OR

Find the indicated terms of the sequence whose nth terms are: $A_n = n(n - 1)(n - 2)$; a_5 and a_8

6. In an A.P., if $d = -2$, $n = 5$ and $a_n = 0$, then find the value of a.
7. Find the value of k for which the given value is a solution of the given equation: $x^2 + 3ax + k = 0, x = -a$

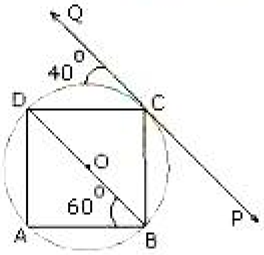
OR

For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

8. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them?
9. How many tangents can a circle have?

OR

In the given figure, ABCD is a cyclic quadrilateral and PQ is a tangent to the circle at C. If BD is a diameter, $\angle OCQ = 40^\circ$ and $\angle ABD = 60^\circ$, find $\angle BCP$



10. The areas of two circles are in the ratio 9: 4, then what is the ratio of their circumferences?

11. If S_n denotes the sum of first n terms of an AP, prove that $S_{12}=3(S_8 - S_4)$.

12. If 5 tan θ = 4, write the value of $\frac{(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)}$.

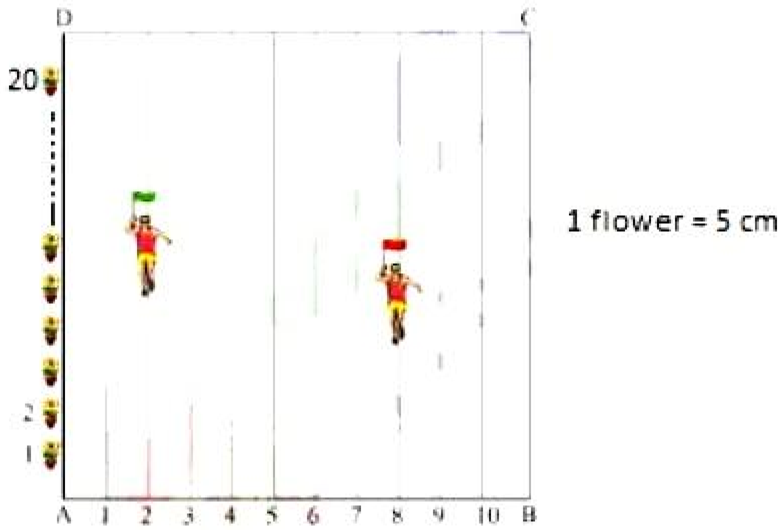
13. Prove that: $\cot^4 A - 1 = \operatorname{cosec}^4 A - 2\operatorname{cosec}^2 A$

14. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

15. Find the sum of all odd numbers between 0 and 50.

16. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing
    i. an ace
    ii. a '4' of spades
    iii. a '9' of a black suit
    iv. a red king.

17. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs the distance AD on the 2nd line and posts a green flag. Preet runs the distance AD on the eighth line and posts a red flag. (take the position of feet for calculation)



    i. In the distance, Niharika posted the green flag:
        a. 5
        b. 15
        c. 25
        d. 20
    ii. The coordinates of the green flag are:

a. (2, 15)

b. (25, 2)

c. (2, 5)

d. (2, 25)

iii. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

a. 20.5 m on the 5th line

b. 22.5 m on the 5th line

c. 25.5 m on the 5th line

d. 24.5 m on the 5th line

iv. What is the distance between both the flags?

a. $\sqrt{61}\,m$

b. $\sqrt{63}\,m$

c. $\sqrt{60}\,m$

d. $\sqrt{62}\,m$

v. The coordinates of the Red flag are:

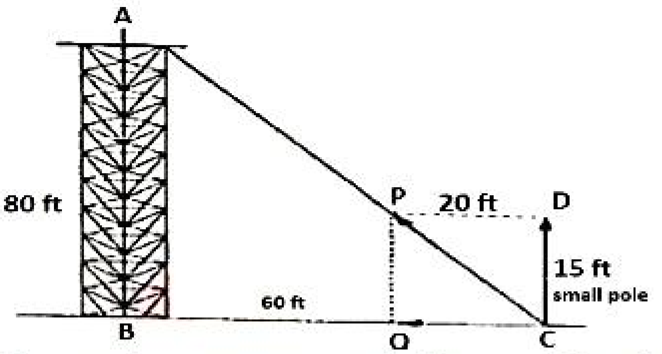
a. (8, 4)

b. (4, 8)

c. (8, 20)

d. (2, 25)

18.



There exist a tower near the house of Shankar. The top of the tower AB is tied with steel wire and on the ground, it is tied with string support.

One day Shankar tried to measure the longest of the wire AC using Pythagoras theorem.

i. In the figure, the length of wire AC is: (take BC = 60 ft)

a. 75 ft

b. 100 ft

c. 120 ft

d. 90 ft

ii. What is the area of $\triangle ABC$?

a. 2400 ft^2

b. 4800 ft^2

c. 6000 ft^2

d. 3000 ft^2

iii. What is the length of the wire PC?

a. 20 ft

b. 30 ft

c. 25 ft

d. 40 ft

iv. What is the length of the hypotenuse in $\triangle ABC$?

a. 100 ft

b. 80 ft

c. 60 ft

d. 120 ft

v. What is the area of a $\triangle POC$?

a. 100 ft^2

b. 150 ft^2

c. 200 ft^2

d. 250 ft^2

19. 

Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute was recorded and summarised as follows:

Number of heartbeats per minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86

Number of women	2	4	3	8	7	4	2

i. Find the mean heartbeats per minute for these women.

a. 75.9

b. 78.9

c. 77.9

d. 59.9

ii. Find the modal class of the given data.

a. 74-77

b. 77-80

c. 65-68

d. 68-71

iii. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its:

a. mean

b. median

c. mode

d. all the three above

iv. The sum of the upper limit and lower limit of the median class is:

a. 141

b. 161

c. 151

d. 162

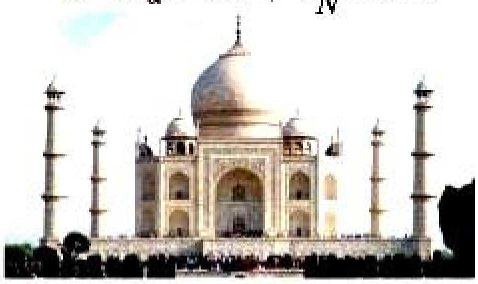
v. Formula for median is:

a. $M_d = L + \frac{\frac{N}{2} - cf}{f}$

b. $M_d = L + \frac{\frac{N}{2} - cf}{f} \times h$

c. $M_d = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$

d. $M_d = A + \frac{f_i u_i}{N} \times h$

20. 

A mathematics teacher took her grade X students to the Taj Mahal. It was an educational trip. She was interested in history also. On reaching there she told them about the history and facts about the seventh wonder. She also told them that the structure of the monument is a combination of several solid figures. There are 4 pillars that are cylindrical in shape. A big dome in the center and 2 more small domes on both sides of the big dome on its side. The domes are hemispherical. The pillars also have domes on them.

i. How much cloth material will be required to cover a big dome of a diameter of 7m?

   a. 77 m^2

   b. 78 m^2

   c. 79 m^2

   d. 80 m^2

ii. Write the formula to calculate the volume of the pillar.

   a. $\pi r^2 h + \pi r^3$

   b. $\pi r^2 h + \frac{2}{3}\pi r^2 l$

   c. $\pi r l + \frac{2}{3}\pi r^3$

   d. $\pi r^2 h + \frac{2}{3}\pi r^3$

iii. How much is the volume of the hemisphere if the radius of the base is 3 m?

   a. 65.57 m^3

   b. 75.77 m^3

   c. 56.57 m^3

   d. 85.57 m^3

iv. Find the curved surface area of 4 pillars if the height of pillars is 7.5 m and the radius of the base is 2.5 m.

   a. 768.56 m^2

   b. 658.56 m^2

   c. 766.56 m^2

   d. 628.57 m^2

v. What is the ratio of the sum of volumes of two-cylinder of radius 1 cm and height 2 cm each to the volume of a sphere of radius 3 cm?

   a. 2:3

b. 3:2

c. 1:1

d. 1:2

Part-B

21. Prove that $2\sqrt{3}-1$ is an irrational number.

22. Find the coordinates of a point A, where AB is a diameter of the circle with centre (3, -1) and the point B is (2, 6).

OR

Show that the points (a, a), (-a, -a) and ($-\sqrt{3}$a, $\sqrt{3}$a) are the vertices of an equilateral triangle.

23. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and -2.

24. Draw two tangents to a circle of radius 3.5 cm from a point P at a distance of 6.2 cm from its centre.

25. If cos 2θ = sin 4θ, where 2θ and 4θ are acute angles, find the value of θ.

OR

Prove the trigonometric identity: $(cosec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

26. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

27. Show that $(2+\sqrt{3})$ is an irrational number.

28. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.

OR

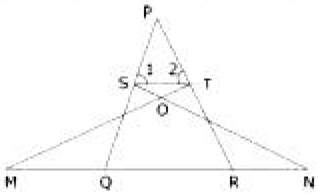
Solve for x : $2(\frac{x-1}{x+3}) - 7(\frac{x+3}{x-1}) = 5$; given that $x \neq -3, x \neq 1$.

29. If α and β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

30. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

OR

In the given Fig, if $\angle 1 = \angle 2$ and, $\triangle NSQ \cong \triangle MTR$ Then prove that $\triangle PTS \sim \triangle PRQ$



31. 5 cards the ten, jack, queen, king and ace of diamonds are well shuffled with their faces downward. One card is then picked up at random.
    a. What is the probability that the drawn card is the queen?
    b. If the queen is drawn and put aside and a second card is drawn, find the probability that the second card is (i) an ace, (ii) a queen.
32. A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall.
33. Find the mean, median and mode of the following data:

Classes:	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Frequency:	2	3	5	6	5	3	1

34. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.
35. Form the pair of linear equations in the problem, and find its solution (if it exists) by the elimination method:
    A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Mona paid Rs.27 for a book kept for seven days, while Tanvy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.
36. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

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Solution

Part-A

1. $Number = \frac{7}{75}$

$= \frac{7}{3\times25} = \frac{7}{3\times5^2\times2^0}$

Since the denominator of this rational number is not of form $2^m \times 5^n$.

Hence this number has non-terminating decimal expansion.

OR

The given number is $\frac{129}{2^2\times5^7}$.

It's seen that, $2^2 \times 5^7$ is of the form $2^m \times 5^n$, where m = 2 and n = 7.

So, the given number has terminating decimal expansion.

$\therefore \frac{129}{2^2\times5^7} = \frac{129\times2^5}{2^2\times5^7\times2^5} = \frac{129\times32}{(2\times5)^7} = \frac{4182}{10^7} = \frac{4128}{10000000} = 0.0004182$

2. $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ or $x = \frac{3}{2}$

3. $a_1 = 5, b_1 = -8, c_1 = 1$ and $a_2 = 3, b_2 = \frac{-24}{5}, c_2 = \frac{3}{5}$

$\frac{a_1}{a_2} = \frac{5}{3}$ ...(i)

$\frac{b_1}{b_2} = \frac{-8}{-24/5} = \frac{5}{3}$ ...(ii)

and $\frac{c_1}{c_2} = \frac{1}{3/5} = \frac{5}{3}$ ...(iii)

Form (i), (ii) and (iii)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ The pair of equations has infinitely many solutions.

4. It changes into a rectangle or a square.

5. The fee charged from a student every month by a school for the whole session is

400, 400, 400, 400, ...

which from an AP, with common difference, d = 400 – 400 = 0

OR

$A_n = n(n - 1)(n - 2)$

Put n = 5

$A_5 = 5(5 - 1)(5 - 2)$

$= 5(4)(3) = 60$

Put n = 8

$A_8 = 8(8 - 1)(8 - 2)$

$= 8(7)(6) = 336$

6. Given:

   $d = -2, n = 5$ and $a_n = 0$

   We know that , $a_n = a + (n - 1)\ d$

   $0 = a + (n - 1)\ d$

   $[\ a_n = 0]$

   $0 = a + (\ 5 - 1\ )\ (-2)$

   $0 = a + 4 \times -2$

   $0 = a - 8$

   $0 + 8 = a$

   $a = 8$

   Hence, the value of a is 8.

7. For the given equation x = -a

   $x^2 + 3ax + k = 0$

   $\Rightarrow (-a)^2 + 3 \times a \times (-a) + k=0$

   $\Rightarrow a^2 - 3a^2 + k = 0$

   $\Rightarrow -2a^2 + k = 0$

   Hence, $k = 2a^2$

OR

Compare $x^2 + 4x + k = 0$ with $ax^2 + bx + c = 0$

a = 1 , b = 4, c = k

Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow$ Discriminant $D \geq 0$

$\Rightarrow b^2 - 4ac = (4)^2 - 4(1)(k) = 16 - 4k \geq 0$

$\Rightarrow k \leq 4$

8. We know , tangent and radius of a circle are perpendicular to each other at point of contact

$\therefore$ Angle between the radii = $360^o - 90^o - 90^o - 30^o = 150^o$

9. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

OR

$\because$ BD is a diameter

$\therefore \angle BCD = 90^o$ [Angle in the semi-circle]

$\therefore \angle BCP = 180^o - 90^o - 40^o = 50^o$

10. Let A_1 = Area of the first circle

A_2 = Area of the second circle

given,

$A_1 : A_2 = 9 : 4$

Let,

r_1 = radius of the first circle

r_2 = radius of the second circle

Now,

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

$\Rightarrow \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2}$

$\Rightarrow \frac{9}{4} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \left(\frac{3}{2}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$

$\therefore r_1 : r_2 = 3 : 2$

11. Let a be the first term and d be the common difference of the given AP. Then,

$S_n = \frac{n}{2} \cdot [2a+(n-l)d]$,

$\therefore 3(S_8 - S_4) = 3[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)]$

$= 3[4(2a + 7d) - 2(2a + 3d)] = 6(2a + 11d)$

$= \frac{12}{2} \cdot (2a + 11d) = S_{12}$.

Hence, $S_{12} = 3(S_8 - S_4)$.

12. Given, $5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

Now

$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} = \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{\frac{5-4}{5}}{\frac{5+4}{5}} = \frac{1}{9}$.

13. To prove: $\cot^4 A - 1 = \operatorname{cosec}^4 A - 2\operatorname{cosec}^2 A$.

LHS = $\cot^4 A - 1$

$= (\operatorname{cosec}^2 A - 1)^2 - 1$ [$\because \cot^2 A = \operatorname{cosec}^2 A - 1$, therefore $\cot^4 A = (\operatorname{cosec}^2 A - 1)^2$]

$= \operatorname{cosec}^4 A - 2\operatorname{cosec}^2 A + 1 - 1$

$= \operatorname{cosec}^4 A - 2\operatorname{cosec}^2 A$ = RHS

Hence proved.

14. Let radius of the sphere = r

Ratio = $\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$

$\therefore$ Total surface area of hemisphere : Square of radius = $3\pi : 1$

15. We know that odd numbers between 0 and 50 are 1,3,5,7,9,....,49

Let, sum = 1 + 3 + 5 + 7 + ...+49.

As this forms an A.P, whose first term (a) = 1 and last term (T_n) = 49.

common difference (d) = 3 - 1 = 2

Let, total no. of odd numbers lie between 0 and 50 = n

We know,

$T_n = a + (n-1)d$

or, $49 = 1 + (n-1)2$

$\Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 49 - 1 = 2n - 2$

$\Rightarrow 48 + 2 = 2n$

$\Rightarrow 50 = 2n$

$\Rightarrow n = \frac{50}{2}$

$\Rightarrow n = 25.$

Therefore

sum = $\frac{n}{2}(a + T_n)$

sum = $\frac{25}{2}(1 + 49)$

sum = $25 \times \frac{50}{2} = 25 \times 25$

sum = 625.

16. Total number of all possible outcomes = 52
    i. Total number of aces = 4

       Therefore, P(getting an ace) = $\frac{4}{52} = \frac{1}{13}$
    ii. Number of 4 of spades = 1

       Therefore, P(getting a 4 of spade) = $\frac{1}{52}$
    iii. Number of 9 of a black suit = 2

       Therefore, P (getting a 9 of a black suit) = $\frac{2}{52} = \frac{1}{26}$
    iv. Number of red kings = 2

       Therefore, P (getting a red king) = $\frac{2}{52} = \frac{1}{26}$
17. i. (c) It can be observed that Niharika posted the green flag at 5th position flower of the distance AD i.e., 5×5 = 25m from the starting point of 2nd line.
    ii. (d) The coordinates of the Green flag are (2, 25).
    iii. (b) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A(x, y)

       Now by midpoint formula,

       $(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

       $x = \frac{2+8}{2} = 5$

       $y = \frac{25+20}{2} = 22.5$

       Hence, A(x, y) = (5, 22.5)

       Therefore, Rashmi should post her blue flag at 22.5 m on the 5th line.
    iv. (a) According to the distance formula,

       Distance between these flags by using the distance formula, D

       $= [(8 - 2)^2 + (25 - 20)^2]^{1/2} = (36 + 25)^{1/2} = \sqrt{61}\,m$
    v. (c) Preet posted a red flag at the distance of 4th flower position of AD i.e., 4×5 = 20m from the starting point of the 8th line. Therefore, the coordinates of this point R are (8, 20).

18. i. (b) 100 ft

ii. (a) 2400 ft^2

iii. (c) 25 ft

iv. (a) 100 ft

v. (b) 150 ft^2

19. i. (a) 75.9

ii. (a) 74-77

iii. (b) Median

iv. (c) 151

v. (b) $M_d = L + \frac{\frac{N}{2} - cf}{f} \times h$

20. i. (a) 77 m^2

ii. (d) $\pi r^2 h + \frac{2}{3}\pi r^3$

iii. (c) 56.57 m^3

iv. (d) 628.57 m^2

v. (c) 1:1

Part-B

21. Let us assume that $2\sqrt{3} - 1$ is a rational. number

Then, there exist positive co-primes a and b such that

$2\sqrt{3} - 1 = \frac{a}{b}$

$2\sqrt{3} = \frac{a}{b} + 1$

$2\sqrt{3} = \frac{a+b}{b}$

$\sqrt{3} = \frac{a+b}{2b}$

Here $\frac{a+b}{b}$ is a rational number, so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number

Hence $2\sqrt{3} - 1$ is irrational

22. Centre of circle is O.(3, - 1)

This O point acts a mid point of line segment AB.

So, let the coordinates of A be (x_1, y_1) then we have

$3 = \frac{x_1+2}{2}$ (Mid point formula)

$6 = x_1 + 2$

$\Rightarrow x_1 = 4$

Also,

$-1 = \frac{y_1+6}{2}$

$\Rightarrow y_1 = -8$

So, coordinates of A are (4, -8)

OR

Let A(a, a), B(- a, - a) and C $(-\sqrt{3}\,a, \sqrt{3}\,a)$

Here, AB = $\sqrt{(a+a)^2+(a+a)^2} = 2\sqrt{2}a$

BC = $\sqrt{(-a+\sqrt{3}a)^2+(-a-\sqrt{3}a)^2} = 2\sqrt{2}a$

AC = $\sqrt{(a+\sqrt{3}a)^2+(a-\sqrt{3}a)^2} = 2\sqrt{2}a$

Since, AB = BC = AC, therefore ABC is an equilateral triangle.

23. Given polynomial is $x^4 + x^3 - 34x^2 - 4x + 120$

Since, the two zeroes of the polynomial given is 2 and -2

So, factors are $(x + 2)(x - 2) = x^2 + 2x - 2x - 4 = x^2 - 4$

	$x^2 + x - 30$
$x^2 - 4$	$x^4 + x^3 - 34x^2 - 4x + 120$
	$x^4 \quad - 4x^2$
	$x^3 - 30x^2 - 4x + 120$
	$x^3 \quad - 4x$
	$-30x^2 + 120$
	$-30x^2 + 120$
	0

dividend = divisor × quotient + remainder

dividend= $x^4 + x^3 - 34x^2 - 4x + 120$

divisor = $x^2 - 4$

quotient = $x^2 + x - 30$

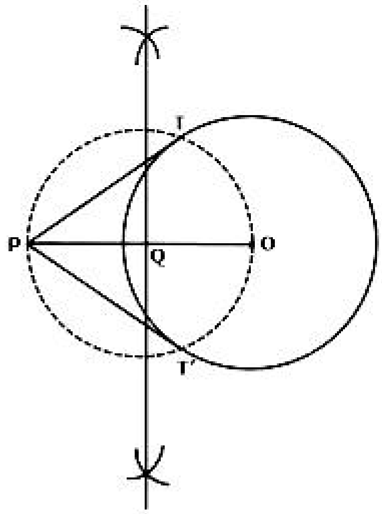
remainder = 0

So, $x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + x - 30)$

$= (x - 2)(x + 2)(x^2 + 6x - 5x - 30)$

$= (x - 2)(x + 2)(x + 6)(x - 5)$

Therefore, the zeroes of the polynomial = x = 2, -2, -6, 5

24. 

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of radius 3.5 cm.
2. Mark a point P at a distance of 6.2 cm from the centre O and join OP.
3. Draw a right bisector of OP, intersecting OP at Q.
4. Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T'.
5. Join PT and PT' to get the required tangents.

25. According to the question,

$\cos 2\theta = \sin 4\theta$

$\Rightarrow \sin(90^\circ - 2\theta) = \sin 4\theta \; [\because \cos\theta = \sin(90^\circ - \theta)]$

$\Rightarrow 90^\circ - 2\theta = 4\theta$

$\Rightarrow -2\theta - 4\theta = -90^\circ$

$\Rightarrow -6\theta = -90^\circ \Rightarrow \theta = \frac{-90^\circ}{-6} = 15^\circ$

OR

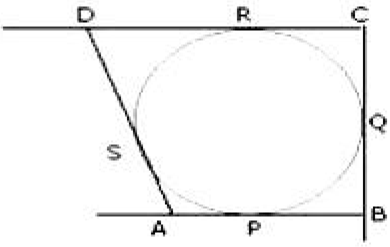
We have, LHS = $(\text{cosec}\,\theta - \cot\theta)^2$

$\Rightarrow \text{LHS} = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$\Rightarrow \text{LHS} = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$\Rightarrow \text{LHS} = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} [\because \sin^2\theta = 1-\cos^2\theta]$

$\Rightarrow \text{LHS} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$

26. 

$\because$ AP, AS are tangents from a point A (Outside the circle) to the circle.

$\therefore$ AP = AS

Similarly BP = BQ

CQ = CR

DR = DS

Now AB + CD = AP + PB + CR + RD

= AS +BQ + CQ +DS

= (AS +DS) + (BQ + CQ)

= AD + BC

AB + CD = AD + BC

27. Suppose that $(2+\sqrt{3})$ is rational number.

$\Rightarrow$ there must exist co-prime numbers between a and b $(b \neq 0)$ such that

$(2+\sqrt{3}) = \frac{a}{b}$

$\Rightarrow \sqrt{3} = \frac{a}{b} - 2 \Rightarrow \sqrt{3} = \frac{a-2b}{b}$

Since a and b are integers, so $\frac{a-2b}{b}$ is rational number

$\Rightarrow \sqrt{3}$ is also rational number .

But, this contradicts the fact that $\sqrt{3}$ is irrational.

$\therefore$ our assumption is incorrect.

$\therefore (2+\sqrt{3})$ is irrational.

28. Let the faster pipe takes x minutes to fill the cistern and the slower pipe will take (x + 3) minutes.

According to question,

$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

$\Rightarrow \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \frac{x+3+x}{x^2+3x} = \frac{13}{40}$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x - 5) + 24(x - 5) = 0$

$\Rightarrow x = 5$ or $x = \frac{-24}{13}$ (Neglected)

Faster pipe will take 5 minutes and slower pipe will take 8 minutes to fill the cistern

OR

The given equation is :

$2(\frac{x-1}{x+3}) - 7(\frac{x+3}{x-1}) = 5$

By putting $\frac{x-1}{x+3} = y$ the given equation becomes

$2y - 7 \times \frac{1}{y} = 5$

$\Rightarrow \frac{2y^2-7}{y} = 5$

$\Rightarrow 2y^2 - 7 = 5y$

$\Rightarrow 2y^2 - 5y - 7 = 0$

Factorize above equation we get,

$2y^2 + 2y - 7y - 7 = 0$

$\Rightarrow 2y(y + 1) - 7(y + 1) = 0$

$\Rightarrow (2y - 7)(y + 1) = 0$

$2y - 7 = 0, y + 1 = 0$

$y = 7/2, y = -1$

Therefore, either $y = \frac{7}{2}$ or $y = -1$

Now When $y = \frac{7}{2}$, then $\frac{x-1}{x+3} = \frac{7}{2}$

$\Rightarrow 2x - 2 = 7x + 21 \Rightarrow x = \frac{-23}{5}$

and when $y = -1$, then $\frac{x-1}{x+3} = -1$

$\Rightarrow x - 1 = -x - 3 \Rightarrow x = -1$

29. It is given that α and β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$

Sum of Zeroes = $\alpha + \beta = -\frac{5}{2}$ and, product of zeroes $\alpha\beta = \frac{k}{2}$

We have,

$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$\Rightarrow \alpha^2 + \beta^2 + \alpha\beta + \alpha\beta - \alpha\beta = \frac{21}{4}$

$\Rightarrow (\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$

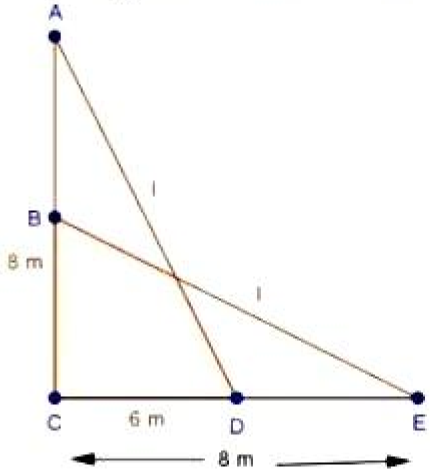
$\Rightarrow (\alpha+\beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow -\frac{k}{2} = -1$

$\Rightarrow k = 2$

30. Let, length of ladder be AD = BE = l m



In ΔACD, by pythagoras theorem

$AD^2 = AC^2 + CD^2$

$\Rightarrow l^2 = 8^2 + 6^2$ ....(i)

In ΔBCE, by pythagoras theorem

$BE^2 = BC^2 + CE^2$

$\Rightarrow l^2 = BC^2 + 8^2$ ....(ii)

Compare (i) and (ii)

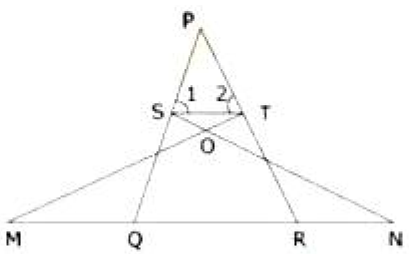
$BC^2 + 8^2 = 8^2 + 6^2$

$\Rightarrow BC^2 = 6^2$

Taking square root on both sides, we get,

$\Rightarrow$ BC = 6 m

OR



Since $\triangle NSQ \cong \triangle MTR$

$\therefore \angle SQN = \angle TRM$

$\Rightarrow \angle Q = \angle R$ in $\triangle PQR$

$= 90^\circ - \frac{1}{2}\angle P$

Again $\angle 1 = \angle 2$ [given in $\triangle PST$]

$\therefore \angle 1 = \angle 2 = \frac{1}{2}(180^\circ - \angle P)$

$= 90^\circ - \frac{1}{2}\angle P$

Thus in $\triangle PTS$ and $\triangle PRQ$

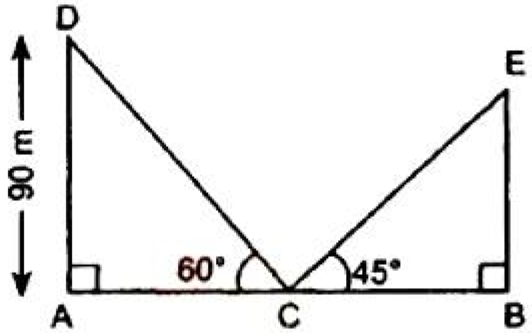
$\therefore \angle 1 = \angle Q$ [Each $= 90^\circ - \frac{1}{2}\angle P$]

$\angle 2 = \angle R$, $\angle P = \angle P$ [Common]

$\triangle PTS \sim \triangle PRQ$

31. Total number of card = 5.
    a. Number of queens = 1.
       Therefore, P(getting a queen) = $\frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}} = \frac{1}{5}$
    b. When the queen has put aside, number of remaining cards = 4.
       i. The number of aces = 1.
          Therefore, P(getting an ace) = $\frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}} = \frac{1}{4}$
       ii. Number of queens = 0.
          Therefore, P(getting a queen now) = $\frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}} = \frac{0}{4} = 0$

32. Let AB is path



In rt. $\triangle DAC$, $\frac{DC}{AD} = cosec\, 60^\circ$

$\Rightarrow \frac{DC}{90} = \frac{2}{\sqrt{3}}$

$DC = \frac{2}{\sqrt{3}} \times 90m = \frac{180}{\sqrt{3}}m$

Now, $DC = CE$

$\therefore CE = \frac{180}{\sqrt{3}}m$

In rt. $\triangle EBC$,

$\frac{BE}{CE} = \sin 45^{\circ}$
$\Rightarrow \text{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}} \text{m}$
$\Rightarrow BE = 73.47\text{m}$

33.

Class interval	Mid value (x)	Frequency (f)	fx	Cumulative frequency
0 – 50	25	2	50	2
50 – 100	75	3	225	5
100 – 150	125	5	625	10
150 – 200	175	6	1050	16
200 – 250	225	5	1127	21
250 – 300	275	3	825	24
300 – 350	325	1	325	25
		N = 25	$\Sigma fx = 4225$	

$\text{Mean} = \frac{\Sigma fx}{N} = \frac{4225}{25} = 169$

We have,

N = 25

Then, $\frac{N}{2} = \frac{25}{2} = 12.5$

The cumulative frequency just greater than $\frac{N}{2}$ is 16, then the median class is 150 - 200 such that

l = 150, h = 200 – 150 = 50, f = 6, F = 10

$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$

$= 150 + \frac{12.5 - 10}{6} \times 50$

$= 150 + \frac{125}{6}$

= 150 + 20.83

= 170.83

Here the maximum frequency is 6, then the corresponding class 150 - 200 is the modal class

l = 150, h = 200 – 150 = 50, f = 6, f_1 = 5, f_2 = 5

$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

$= 150 + \frac{6-5}{2\times6-5-5} \times 50$

$= 150 + \frac{50}{2}$

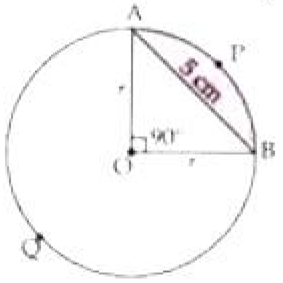
= 150 + 25

= 175

34. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that θ = 90°

Area of $\triangle$OAB = $\frac{1}{2}$ Base $\times$ Altitude = $\frac{1}{2}$ r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2 \theta}{360^\circ}$ – Area of $\triangle$AOB

$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2}r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ ...(i)

Area of major segment AQB = Area of circle – Area of minor segment

$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ ...(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

35. Suppose the fixed charge be Rs. x and the extra charge per day be Rs y.

According to the question, Mona paid Rs 27 for a book kept for 7 days,

$\Rightarrow$ x + 4y = 27........(i)

Tanvy paid Rs.21 for a book kept for 5 days,

$\Rightarrow$ x + 2y = 21.........(ii)

Subtracting (ii) from (I),

$\Rightarrow$ 2y = 6

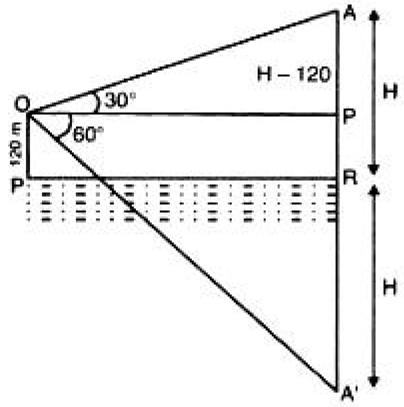
$\Rightarrow$ y = 3

Substituting y = 3 in (ii), we get x = 15

The fixed charge is Rs. 15 and the charge per day is Rs 3.

36. In $\triangle AOP, \tan 30^\circ = \frac{H-120}{OP}$

$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$



$\Rightarrow OP = (H-120)\sqrt{3}$ ...(i)

In $\triangle$ OPA'

$\frac{PA'}{OP} = \frac{H+120}{(H-120)\sqrt{3}} = tan60^o = \sqrt{3}$

H + 120 = 3H - 360

H = 240 meter

Hence height of cloud is 240 meter