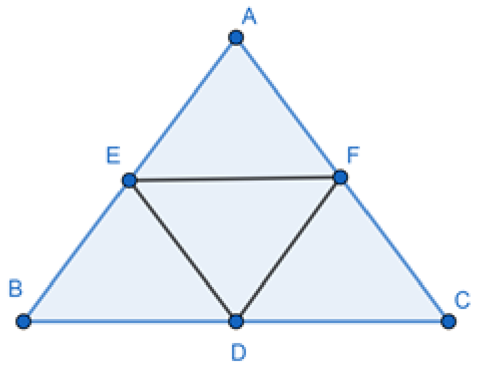
9. Transversal and Mid-Point Theorem

Let us Work Out 9

1. Question

In the triangle ABC, D is the midpoint of the side BC; From the point D, the parallel straight lines of CA and BA intersect the sides BA and CA at the points E and F respectively. Let us prove that, $EF = \frac{1}{2}BC$.

Answer



In Δ ABC, since D is the mid-point of BC and as FD || AB,

Then by the theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side.

⇒ F is the mid-point of AC

By using the above theorem, we can also prove that E is the midpoint of AB as DE || AC.

Now as E and F have been proved to be the midpoint of AB and AC respectively, by applying the theorem:-

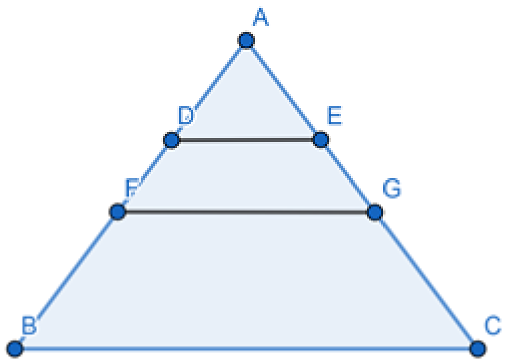
The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

$$\Rightarrow EF = \frac{1}{2} BC$$

2. Question

D and E lie on AB and AC respectively of the triangle ABC such that, $AD = \frac{1}{4}$ AB and $AE = \frac{1}{4}AC$. Let us prove that, DE || BC and $DE = \frac{1}{4}BC$.

Answer



In ΔABC, Let F and G be the midpoint of AB and AC respectively.

So, in ΔAFG, $= \frac{1}{2}$ AF and AE $= \frac{1}{2}$ AG ,

⇒ D and E are the midpoint of AF and AG respectively.

By applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ $DE = \frac{1}{2}$ FG and DE || FG ……… (1)

Also, by using above theorem in ΔABC, we get,

⇒ $FG = \frac{1}{2}$ BC and FG || BC ……… (2)

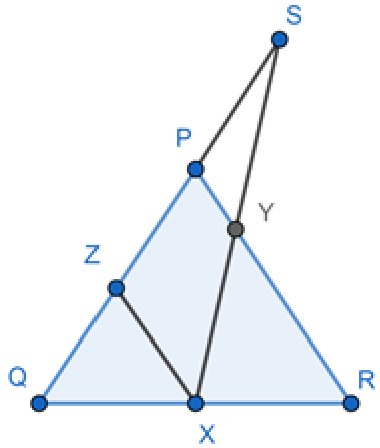
From equations (1) and (2), we get

⇒ $DE = \frac{1}{4}$ BC and DE || BC

3. Question

In the triangle PQR, the midpoints of the sides QR and QP are X and Z respectively. The side QP is extended upto the points S so that PS = ZP. SX intersects the side PR at the point Y. Let us prove that, $PY = \frac{1}{4}PR$.

Answer



In ΔPQR, as X and Z are the midpoint of QR and PQ respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

$\Rightarrow$ XZ $= \frac{1}{2}$ PR ……… (1)

Also, in ΔSZX since ZX || PY and P is the midpoint of SZ,

By applying theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side.

$\Rightarrow$ PY $= \frac{1}{2}$ ZX ……… (2)

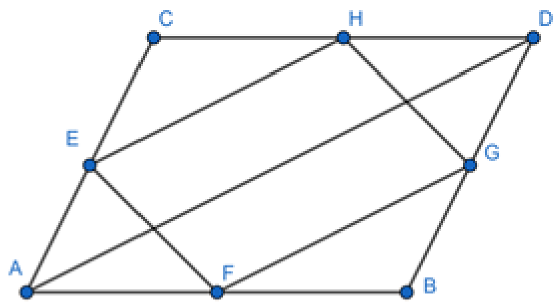
From equations (1) and (2), we get

$$\Rightarrow \text{PY} = \frac{1}{4}\ \text{PR}$$

4. Question

Let us prove that, the quadrilateral formed by joining midpoints of consecutive sides of a parallelogram is a parallelogram.

Answer



In ΔACD, as E and H are the midpoints of AC and CD respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ $EH = \frac{1}{2}$ AD and EH || AD ……… (1)

Also, by using above theorem in ΔABD, as F and G are midpoints of AB and BD respectively,

⇒ $FG = \frac{1}{2}$ AD and FG || AD ……… (2)

From equations (1) and (2) we see that, EH = FG and EH || AD.

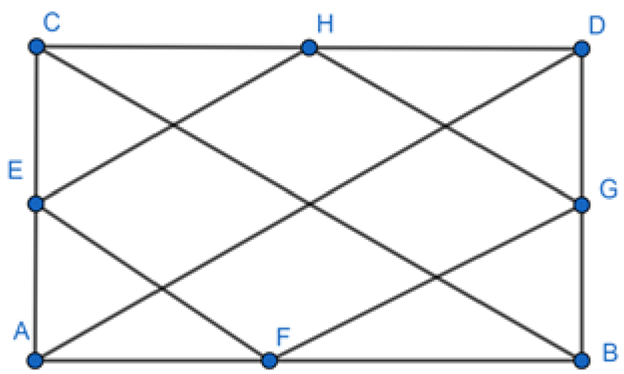
As both the above conditions are sufficient for a parallelogram,

⇒ EFGH is a parallelogram

5. Question

Let us prove that, the quadrilateral formed by joining midpoints of consecutive sides of a rectangular figure is not a square figure but a rhombus.

Answer



In ΔACD, as E and H are the midpoints of AC and CD respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ $EH = \frac{1}{2}$ AD and EH || AD ……… (1)

Also, by using above theorem in ΔABD, as F and G are midpoints of AB and BD respectively,

⇒ $FG = \frac{1}{2}$ AD and FG || AD ……… (2)

From equations (1) and (2) we see that, EH = FG and EH || AD.

Similarly for ΔBCD, we have G and H as midpoints of BD and CD respectively, by applying above theorem, we get,

⇒ $GH = \frac{1}{2}$ Bc and GH || BC .... (3)

And also in ΔABC,

⇒ $EF = \frac{1}{2}$ BC and EF || BC .... (4)

From equations (3) and (4) we see that, GH = EF and GH || EF.

As ABCD is a rectangle, the length of diagonals are equal

⇒ AD = BC

⇒ HG = GF = FE = EH

But, we also see that since ABCD is a rectangle not square BD is not equal to CD,

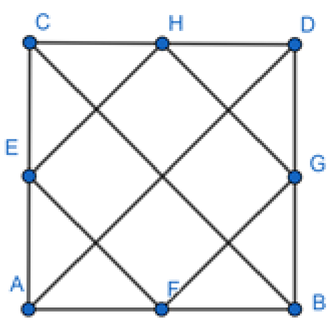
∴ ∠ DCB is not equal to ∠ DBC.

Hence, the angle between the sides in EFGH cannot be 90° and as a result EFGH is not a square figure but a rhombus.

6. Question

Let us prove that, the quadrilateral formed by joining midpoints of consecutive sides of a square is a square.

Answer



In ΔACD, as E and H are the midpoints of AC and CD respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ $EH = \frac{1}{2}$ AD and EH || AD .... (1)

Also, by using above theorem in ΔABD, as F and G are midpoints of AB and BD respectively,

$\Rightarrow FG = \frac{1}{2}AD$ and FG || AD ……… (2)

From equations (1) and (2) we see that, EH = FG and EH || AD.

Similarly for ΔBCD, we have G and H as midpoints of BD and CD respectively, by applying above theorem, we get,

$\Rightarrow GH = \frac{1}{2}Bc$ and GH || BC ……… (3)

And also in ΔABC,

$\Rightarrow EF = \frac{1}{2}BC$ and EF || BC ……… (4)

From equations (3) and (4) we see that, GH = EF and GH || EF.

As ABCD is a square, the length of diagonals are equal

⇒ AD = BC

⇒ HG = GF = FE = EH

⇒ AE = AF, this means ∠AEF = ∠AFE = ∠ABC = ∠ACB = 45°

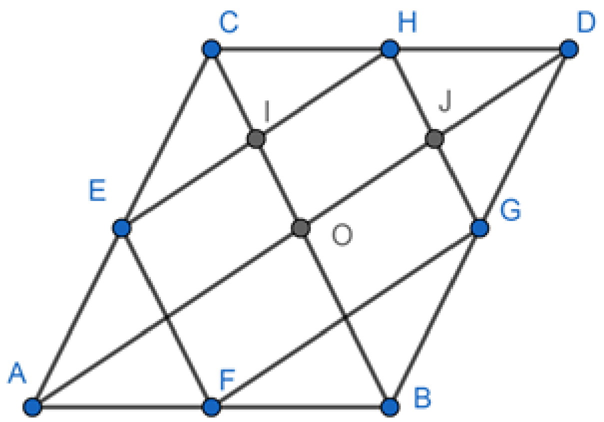
∴ We can easily prove that angle between the sides of the figure FGHE is equal to 90°

Hence, Quadrilateral FGHE is a square.

7. Question

Let us prove that, the quadrilateral formed by joining midpoints of a rhombus is a rectangle.

Answer



In ΔACD, as E and H are the midpoints of AC and CD respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ $EH = \frac{1}{2}$ AD and EH || AD ……… (1)

Also, by using above theorem in ΔABD, as F and G are midpoints of AB and BD respectively,

⇒ $FG = \frac{1}{2}$ AD and FG || AD ……… (2)

From equations (1) and (2) we see that, EH = FG and EH || AD.

Similarly for ΔBCD, we have G and H as midpoints of BD and CD respectively, by applying above theorem, we get,

⇒ $GH = \frac{1}{2}$ Bc and GH || BC ……… (3)

And also in ΔABC,

⇒ $EF = \frac{1}{2}$ BC and EF || BC ……… (4)

From equations (3) and (4) we see that, GH = EF and GH || EF.

So, EHGF is a parallelogram and as IH lie on EH and HJ lie on HG.

⇒ IHJO is also a parallelogram.

As we know that, the angle subtended by the diagonals in a triangle is equal to 90°, and also in a parallelogram, the opposite angles are equal.

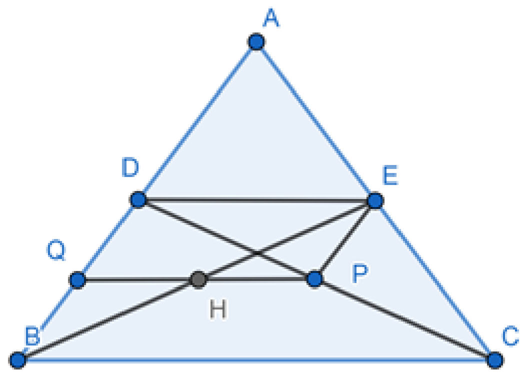
⇒ ∠EHG = ∠IOJ = 90°

Hence, quadrilateral HGFE is a rectangle

8. Question

In the triangle ABC, the midpoints of AB and AC are D and E respectively; the midpoints of CD and BD are P and Q respectively. Let us prove that, BE and PQ bisect each other.

Answer



In ΔBDC, as Q and P are the midpoints of BD and CD respectively.

So by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

$\Rightarrow PQ = \frac{1}{2}$ BC and PQ || BC .... (1)

Similarly, applying above theorem on ΔABC where D and E are midpoints of AB and AC respectively

$\Rightarrow DE = \frac{1}{2}$ BC and DE || BC .... (2)

From equations (1) and (2),

⇒ PQ = DE and PQ||DE, ... (3)

⇒ PQDE is a parallelogram.

Also in ΔBDE, Q is the midpoint to BD and QH||DE as PQDE is a parallelogram,

$\Rightarrow QH = \frac{1}{2} DE$ .... (4)

From equations (3) and (4), it proves that:

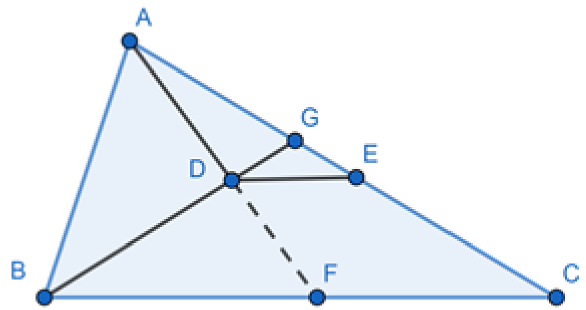
$$QH = \frac{1}{2} PQ$$

⇒ BE and PQ bisect each other

9. Question

In the triangle ABC, AD is perpendicular on the bisector of ∠ABC. From the point D, a straight line DE parallel to the side BC is drawn which intersects the side AC at the point E. Let us prove that AE = EC.

Answer



We extend the perpendicular till F such that it intersects the line BC at point F.

As BG is the angle bisector of ∠ABC,

⇒ ∠ABD = ∠FBD = θ

Since AD is perpendicular to BG, so ∠BAD = 90 – θ

In ΔABF, as sum of all sides of triangle is equal to 180°

⇒ ∠ABF + ∠BAF + ∠BFA = 180

⇒ 2θ + (90 – θ) + ∠BFA = 180

⇒ ∠BFA = 90 – θ

We know that sides opposite to equal angles in a triangle are equal and as ∠BFA = ∠BAD.

⇒ BA = BF

In ΔBAD and ΔBFD,

∠ABD = ∠FBD (given)

∠ADB = ∠FDB = 90° (given)

BA = BF (proved above)

⇒ ΔBAD ≅ ΔBFD by RHS congruency.

⇒ AD = DF, that is, D is the midpoint of AF.

Now, In ΔACF, as D is midpoint of AF and DE || FC, then by applying theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side, we get,

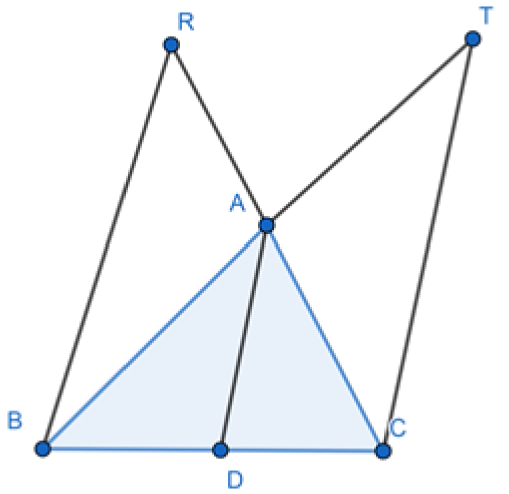
⇒ E is the midpoint of AC

∴ AE = EC, hence proved.

10. Question

In the triangle ABC, AD is median. From the points B and C, two straight lines BR and CT, parallel to AD are drawn, which meet extended BA and CA at the points T and R respectively. Let us prove that $\frac{1}{AD} = \frac{1}{RB} + \frac{1}{TC}$.

Answer



Since it is given that AD is the median to side BC,

⇒ D is midpoint to BC

Also, it is given that AD||CT||BR, so by applying theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side.

$$\Rightarrow AD = \frac{1}{2} CT = \frac{1}{2} RB$$

⇒ TC = RB ……… (1)

As $AD = \frac{1}{2}$ TC, it can also be written as, $\frac{1}{AD} = \frac{2}{TC}$

$$\Rightarrow \frac{1}{AD} = \frac{1}{TC} + \frac{1}{TC}$$

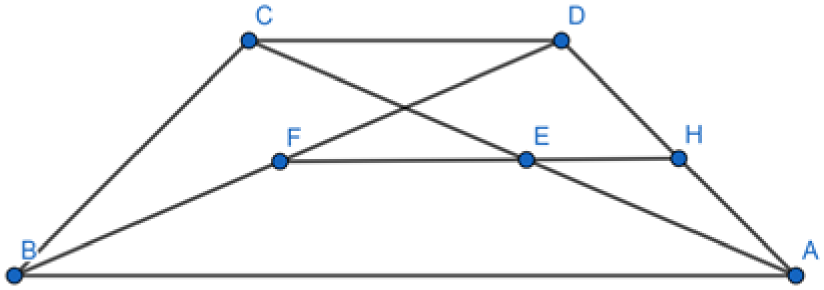
Using equation (1) in above equation,

$$\Rightarrow \frac{1}{AD} = \frac{1}{TC} + \frac{1}{RB}$$

11. Question

In the trapezium ABCD, AB || DC and AB > DC; the midpoints of two diagonals AC and BD are E and F respectively. Let us prove that, $EF = \frac{1}{2}(AB - DC)$.

Answer



As E and F are the midpoints of sides AC and BD respectively, therefore by the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get,

⇒ EF || CD

Now, we have extended EF to H, so in ΔACD, since E is midpoint of AC and EH||CD, so by applying theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side.

$$\Rightarrow \mathrm{EH} = \frac{1}{2}\ \mathrm{CD}$$

Similarly, applying above theorem in ΔADB as F is midpoint of BD and FH||AB,

$$\Rightarrow \mathrm{FH} = \frac{1}{2}\mathrm{AB}$$

As FE = FH – EH,

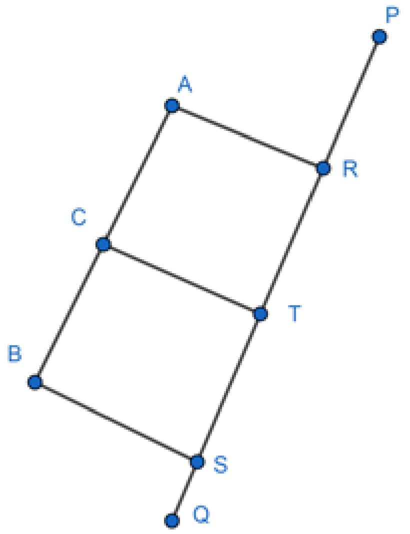
So by substituting above values, we get,

$$\Rightarrow \mathrm{EF} = \frac{1}{2}\ (\mathrm{AB} - \mathrm{CD})$$

12. Question

C is the midpoint of the line segment AB and PQ is any straight line. The minimum distances of the line PQ from the points A, B and C are AR, BS and CT respectively; let us prove that, AR + BS = 2CT.

Answer



As AR, BS, CT are the minimum distances of the line PQ from the points A, B and C, this can be only achieved when AB||PQ and AR, CT and BS are perpendicular to it.

This makes ARTC and CTSB as a parallelogram.

⇒ CT = AR from ARTC parallelogram,

And CT = BS from CTSB parallelogram.

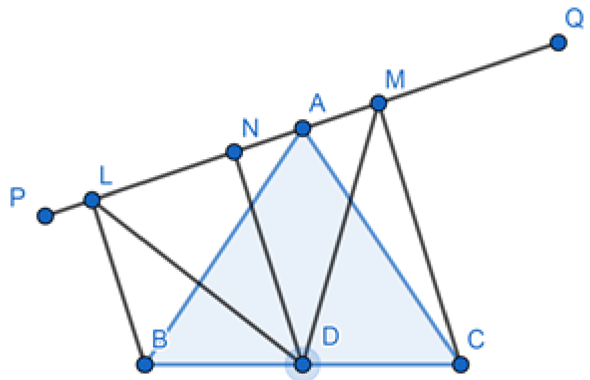
On adding the above two equations, we get

⇒ AR + BS = 2CT

13. Question

In a triangle ABC, D is the midpoint of the side BC; through the point A, PQ is any straight line. The perpendiculars from the points B, C and D on PQ are BL, CM and DN respectively; let us prove that, DL = DM.

Answer



Since BL||DN||CM and D is midpoint of BC, by using the theorem:-

If three or more parallel straight lines make equal intercepts from a traversal, then they will make equal intercepts from another traversal.

⇒ LN = MN

In ΔDLN and ΔDMN,

DN = DN (common)

∠DNL = ∠DNM = 90° (given)

LN = MN (proved above)

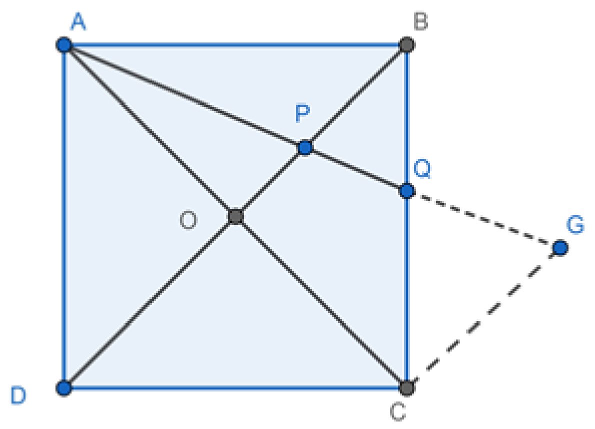
∴ ΔDLN ≅ ΔDMN by SAS congruency.

As a result of it, DL = DM

14. Question

ABCD is a squared figure. The two diagonals AC and BD intersect each other at the point O. The bisector of ∠BAC intersects BO at the point P and BC at the point Q. Let us prove that, $OP = \frac{1}{2}CQ$.

Answer



We have extended PQ till G such that OP||CG,

As O is intersection of Diagonals of a square, therefore O is mid point of AC.

Using the theorem,

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

⇒ $OP = \frac{1}{2}CG$ ……… (1)

As AP is angle bisector of ∠BAO,

⇒ ∠BAP = ∠OAP = θ

⇒ in ΔABQ, ∠AQB = 90 – θ

∠GQC = ∠AQB = 90 – θ (Opposite angles)

Similarly in ΔACG, as ∠ACG = 90° ,

$\Rightarrow \angle AGC = 90 - \theta$

As in ΔGQC, $\angle AGC = \angle GQC = 90 - \theta$ and as sides opposite to equal angles in a triangle are equal

$\Rightarrow CQ = CG$ ……… (2)

From equations (1) and (2), $OP = \frac{1}{2}\ CQ$

15 A. Question

In the triangle PQR, ∠PQR = 90° and PR = 10 cm. If S is the midpoint of PR, then the length of QS is

A. 4 cm

B. 5 cm

C. 6 cm

D. 3 cm

Answer

From the given question, we can see that,

$$QS = \frac{1}{2}\ PR$$

Since, S is the midpoint of PR,

$\Rightarrow QS = SR = 5\ cm$

15 B. Question

In the trapezium ABCD, AB || DC and AB = 7 cm and DC = 5 cm. The midpoints of AD and BC are E and F respectively, the length of EF is

A. 5 cm

B. 7 cm

C. 6 cm

D. 12 cm

Answer

Since AB||DC and E and F are midpoints of AD and BC,

By using the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

$\Rightarrow$ AB||CD||EF

Also,

$$EF = \frac{1}{2}(AB + CD)$$

$$\Rightarrow EF = \frac{1}{2}(5 + 7) = 6 \text{ cm}$$

15 C. Question

In the triangle ABC, E is the midpoint of the median AD; the extended BE intersects AC at the point F. If AC=10.5 cm, then the length of AF is

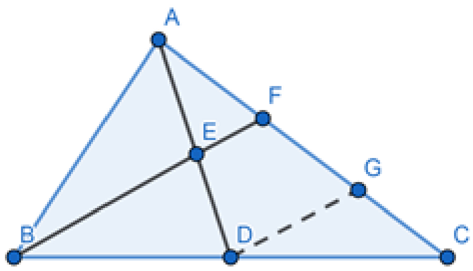
A. 3 cm

B. 5 cm

C. 2.5 cm

D. 3.5 cm

Answer



In Δ ADG, AE = ED and EF||DG, therefore F is mid point of AG.

Similarly, In Δ BFC, BD = DC and BF||DG, therefore G is mid point of FC.

$$\Rightarrow AF = \frac{1}{3}AC$$

$\Rightarrow$ AF = 3.5 cm

15 D. Question

In the triangle ABC, the midpoints of BC, CA and AB are D, E and F respectively; BE and DF intersect at the point X and CF and DE intersect at the point Y, the length of XY is equal to

A. $\frac{1}{2}BC$

B. $\frac{1}{4}$BC

C. $\frac{1}{3}$ BC

D. $\frac{1}{8}$ BC

Answer

By using the theorem,

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

⇒ X and Y are them id points of DF and DE respectively.

In ΔABC, by using above theorem,

$\Rightarrow \text{EF} = \frac{1}{2}\text{BC}$

Similarly applying it in ΔDFE,

$\Rightarrow \text{XY} = \frac{1}{2}\text{EF}$

From above two equations, we get $\text{XY} = \frac{1}{4}\text{BC}$

15 E. Question

In the parallelogram ABCD, E is the midpoint of the side BC; DE and extended AB meet at the point F. The length of AF is equal to

A. $\frac{3}{2}$ AB

B. 2AB

C. 3AB

D. $\frac{5}{4}$ AB

Answer

AF = AB + BF,

We can see that ΔBEF ≅ ΔCED by AAA congruency

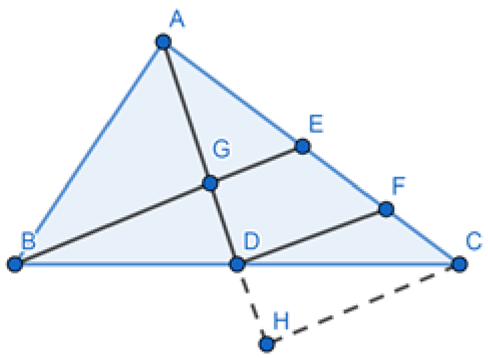
⇒ BF = CD and CD = AB as ABCD is a parallelogram.

⇒ AF = 2AB

16 A. Question

In the triangle ABC, AD and BE are two medians and DF parallel to BE, meets AC at the point F. If the length of the side AC is 8 cm., then let us write the length of the side CF.

Answer



We extend the median AD till H such that GE||CH

In Δ BEC, BD = DC and BF||DG, therefore D is midpoint of BC,

By applying theorem:-

Through the mid-point of any side, if a line segment is drawn parallel to second side, then it will bisect the third side and the line segment intercepted by the two sides of the triangle is equal to half of the second side.

$\Rightarrow \mathrm{EF} = \mathrm{FC}$

$\Rightarrow \mathrm{FC} = \frac{1}{2}\ \mathrm{EC}$

Also, as E is midpoint of AC,

$\Rightarrow \mathrm{EC} = \frac{1}{2}\mathrm{AC}$

From above two equations, we get

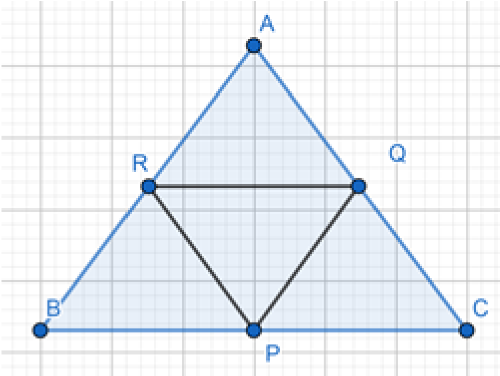
$\Rightarrow \mathrm{FC} = \frac{1}{4}\mathrm{AC} = \frac{1}{4} \times 8 = 2\ \mathrm{cm}$

∴ length of FC is 2 cm

16 B. Question

In the triangle ABC, the midpoints of BC, CA and AB are P, Q and R respectively; if AC = 21 cm, BC = 29 cm. and AB = 30 cm, then let us write the perimeter of the quadrilateral ARPQ.

Answer



If R, Q and P are the midpoints of AB, AC and BC, and by applying the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

$\Rightarrow$ $PQ = \frac{1}{2} AB = AR$ and PQ||AR

Also, $RP = \frac{1}{2} AC = AQ$ and PR || AQ

Since the above conditions are sufficient for a parallelogram, therefore ARPQ is a parallelogram.

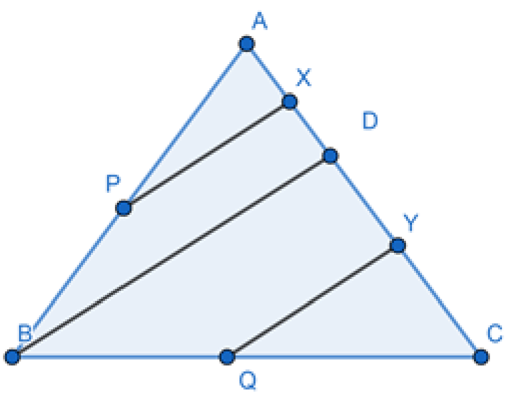
Perimeter of parallelogram ARPQ = (2×AR) + (2×AQ)

$\Rightarrow$ Perimeter = AB + AC = 30 + 21 = 51 cm

16 C. Question

In the triangle ABC, D is any point on the side AC. The midpoints of AB, BC, AD and DC are P, Q, X, Y respectively. If PX = 5 cm, then let us write the length of the side QY.

Answer



In ΔABD, as X is midpoint of AD and P is midpoint of AB, we apply the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

$\Rightarrow \text{PX} = \frac{1}{2}\ \text{BD}$ and PX parallel to BD

⇒ BD = 2 × PX = 10 cm

Similarly, In ΔBDC, as Y is midpoint of DC and Q is midpoint of BC, we apply the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

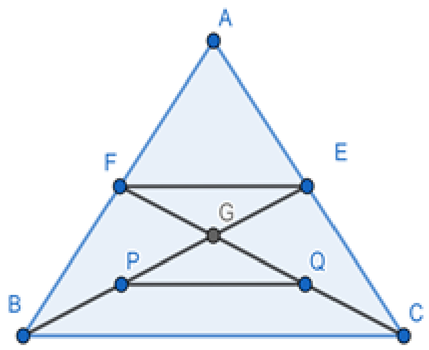
$\Rightarrow \text{QY} = \frac{1}{2}\ \text{BD} = \frac{1}{2} \times 10$ and QY is parallel to BD

⇒ QY = 5 cm

16 D. Question

In the triangle ACB, the medians BE and CF intersects at the point G. The midpoints of BG and CG are P and Q respectively. If PQ = 3 cm, then let us write the length of BC.

Answer



In ΔABC, as F is midpoint of AB and E is midpoint of AC, we apply the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

$\Rightarrow \text{EF} = \frac{1}{2}\ \text{BC}$ and EF||BC

Similarly, In ΔOBC, as P is midpoint of BG and Q is midpoint of GC, we apply the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

$\Rightarrow \text{PQ} = \frac{1}{2}\ \text{BC}$ and PQ||BC

⇒ BC = 2× PQ

⇒ BC = 2 × 3

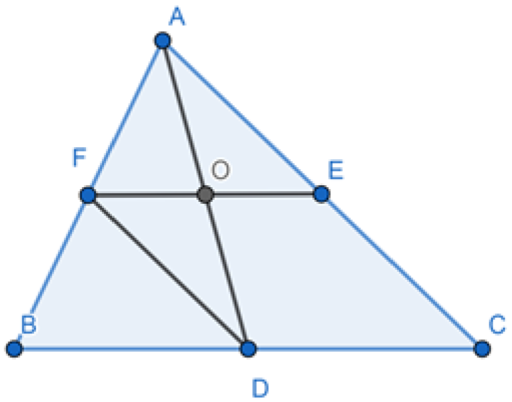
⇒ BC = 6 cm

∴ The length of BC is 6 cm.

16 E. Question

In the triangle ABC, the midpoint of BC, CA and AB are D, E and F respectively; FE intersects AD at the point O. If AD = 6 cm, let us write the length of AO.

Answer



In ΔABC, as F is midpoint of AB and D is midpoint of BC, we apply the theorem:-

The line segment joining the midpoints of two side of a triangle is parallel to the third side and equal to half of it, we get

⇒ $FD = \frac{1}{2}$ AC and FD|| AC

Also, it is given that E is the midpoint of AC

$\Rightarrow AE = \frac{1}{2}AC$

From above two equations, we get, AE = FD

In ΔFOD and ΔEOA,

∠FOD = ∠EOA (Opposite angles)

∠FDO = ∠EAO (Alternate Interior angles as FD || AE)

Also, ∠OFD = ∠OEA (Alternate Interior angles as FD || AE)

⇒ ΔFOD ≅ ΔEOA

∴ AO = OD

$$\Rightarrow AO = \frac{1}{2} AD$$

$$\Rightarrow AO = \frac{1}{2} \times 6 = 3 \text{ cm}$$

∴ The length of AO is 3 cm.