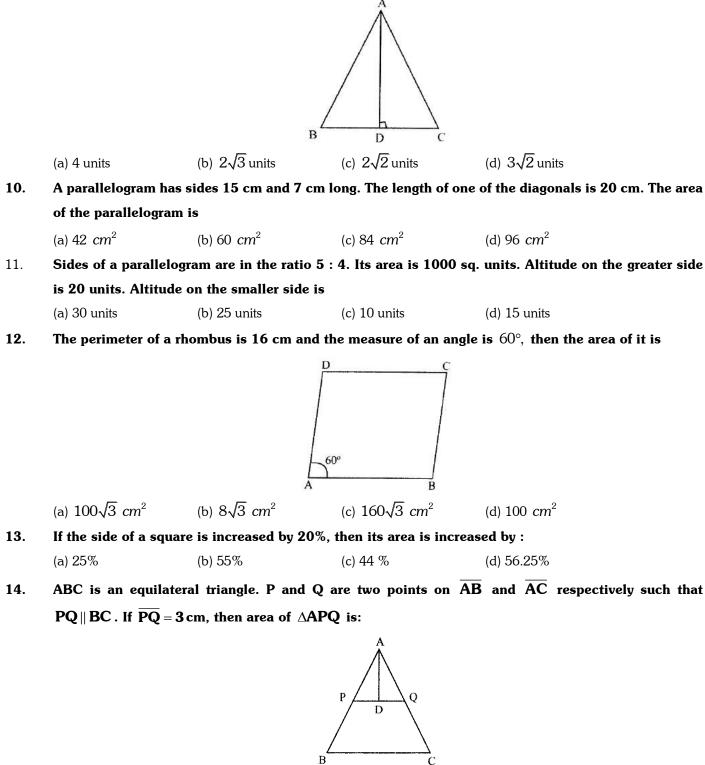
Areas of Parallelograms and Triangles

			OLYMPIAD Excellence Book	MATHEMATICS		
		Ç	UESTIONS			
1.	The difference of the areas of two squares drawn on two line segments of different lengths is 51 sq.					
	cm. Find the lengt	h of the greater line	segment if one is longer tl	han the other by 3 cm.		
	(a) 7 cm	(b) 9 cm	(c) 11 cm	(d) 16 cm		
2.	A kite in the shape of a square with a diagonal 32 cm attached to an equilateral triangle of the base					
	8 cm. approximately how much paper has been used to make it? (use $\sqrt{3}$ = 1.732)					
	(a) 539.712 cm ²	(b) 538.721 <i>cm</i> ²	(c) 540.712 cm^2	(d) 539.217 cm ²		
3.	A field is in the shape of a trapezium whose parallel sides are 25m and 10m. The non parallel sides					
	are 14m and 13m. Then the area of the field is					
	(a) 190 <i>m</i> ²	(b) 180 <i>m</i> ²	(c) 196 <i>m</i> ²	(d) 195 <i>m</i> ²		
4.	The area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m, is					
	(a) 380 m ²	(b) 370 <i>m</i> ²	(c) 374 <i>m</i> ²	(d) $384 m^2$		
5.	The length of a room floor exceeds its breadth by 20 m. The area of the floor remains unaltered when					
	the length is decreased by 10 m but the breadth is increased by 5 m. The area of the floor (in square					
	metres) is					
	(a) 280	(b) 325	(c) 300	(d) 420		
6.	The length and breadth of a rectangle are increased by 20 % and 10% respectively. The increase in					
		ulting rectangle will				
_	(a) 60%	(b) 50%	(c) 40%	(d) 32%		
7.	In the given figure an isosceles triangle, the measure of each of equal sides is 10 cm and the angle					
	between them is 45°, the area of the triangle is Λ					
			$B \xrightarrow{c} a \xrightarrow{A} 10 \text{ cm}$			
	(a) 25 cm^2	(b) $\frac{25}{2}\sqrt{2}cm^2$	(c) $25\sqrt{2} \ cm^2$	(d) $25\sqrt{3} \ cm^2$		

- 8. ABC is an equilateral triangle of side 4 cm. with A, B, C as vertex and radius 2 cm three arcs are drawn. The area of the region within the triangle bounded by the three area is
 - (a) $\left(3\sqrt{3} \frac{\pi}{2}\right) cm^2$ (b) $\left(\sqrt{3} - \frac{3\pi}{2}\right) cm^2$ (c) $4\left(\sqrt{3} - \frac{\pi}{2}\right) cm^2$ (d) $\left(\frac{\pi}{2} - \sqrt{3}\right) cm^2$

9. The area of an isosceles triangle is 4 square unit. If the length of the third side is 4 unit, the length of each equal side is





- 15. ABCD is a parallelogram. BC is produced to Q such that BC = CQ. Then
 - (a) area (ΔBCP) = area (ΔDPQ)
 - (b) area (ΔBCP) > area (ΔDPQ)
 - (c) area (ΔBCP) < area (ΔDPQ)
 - (d) area (ΔBCP) + area (ΔDPQ) = area (ΔBCD)
- 16. In $\triangle PQR$, the line drawn from the vertex P intersects QR at a point S. If QR = 4.5 cm and SR = 1.5 cm then the ratios of the area of triangle PQS and triangle PSR is (a) 4:1 (b) 3:1 (c) 3:2 (d) 2:1
- 17. ABCD is a parallelogram X and Y are the mid points of sides BC and CD respectively. If the area of \triangle ABC is 16 cm², then the area of \triangle AXY is
 - (a) $12 \ cm^2$ (b) $8 \ cm^2$ (c) $9 \ cm^2$ (d) $10 \ cm^2$
- 18. PQR is a right angles triangle Q being the right angle. Mid-points of QR and PR are respectively Q' and P' Area of $\Delta P'Q'R'$ is

(a)
$$\frac{1}{2} \times area \text{ of } \Delta PQR$$

(b) $\frac{2}{3} \times area \text{ of } \Delta PQR$
(c) $\frac{1}{4} \times area \text{ of } \Delta PQR$
(d) $\frac{1}{8} \times area \text{ of } \Delta PQR$

- 19. From any point inside an equilateral triangle, the lengths of perpendiculars 015 the sides are 'a' cm
 'b' cm and 'c' cms. Its area (in cm²) is
 - (a) $\frac{\sqrt{2}}{3}(a+b+c)$ (b) $\frac{\sqrt{3}}{3}(a+b+c)^{2}$ (c) $\frac{\sqrt{3}}{3}(a+b+c)$ (d) $\frac{\sqrt{2}}{3}(a+b+c)^{2}$
- 20. ABCD is a square. Draw a triangle QBC on side BC considering BC as base and draw a triangle PAC on AC as its base such that $\triangle QBC \sim PAC$.

Then,
$$\frac{Area of \Delta QBC}{Area of \Delta PAC}$$
 is equal to
(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

21. A rectangular park 60 metre long and 40 metre wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 metre² then the width of the road is

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(a) 3 metre (b) 5 metre (c) 6 metre (d) 2 metre
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Direction (Question 22 to 23): Each of the questions Mow consists of a questions followed by statements. You have to study the questions and the statements and decide which of the statement (s) is/are necessary to answer the question?

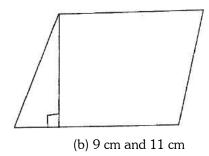
- 22. What is the area of rectangular field?
 - I. The perimeter of the field is 110 metres.
 - II. The length is 5 metres more than the width.
 - III. The ratio between length and width is 6:5 respectively,
 - (a) I and II only

(c) I, and either II or III only

(d) None of these

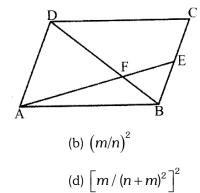
(b) Any two of the three

- 23. A path runs around a rectangular lawn. What is the width of the path?
 - I. The length and breadth of the lawn are in the ratio of 2:1 respectively,,
 - II. The width of the path is twenty times the length of the lawn.
 - III. The cost of gravelling the path @ Rs. 50 per nr is Rs. 4416.
 - (a) All I, II and III (b) III, and either I or II
 - (c) I and III only (d) II and III only
- 24. The area of a rectangle lies between 40 cm² and 45 cm². If one of the sides is 5cm, then its diagonal lies between

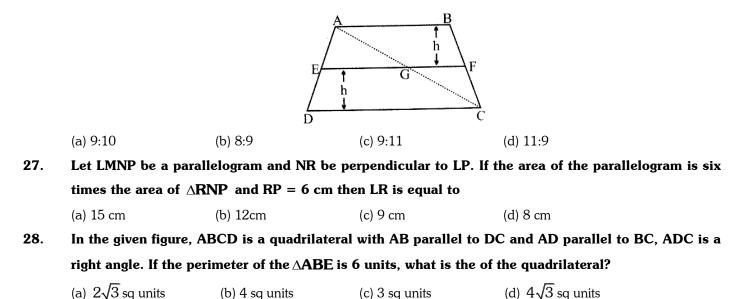


(a) 8 cm and 10 cm(c) 10 cm and 12 cm

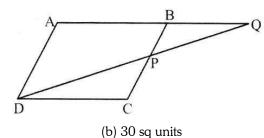
- (d) 11 cm and 13 cm
- 25. ABCD is a parallelogram. E is a point BC such that BE : EC = m : n. If AE and BD intersect in F, then what is the ratio of the area of \triangle PEB to the area of \triangle AFD ?



- (a) *m/n*
- (c) $(n/m)^2$ (d) [m/(n+m)]
- 26. ABCD is a trapezium with parallel sides AB = 2cm and DC = 3cm. E and F are the mid- points of the non-parallel sides. The ratio of area of ABFE to area of EFCD is



29. In the figure given below, ABCD is a parallelogram. P is a point in BC such that PR : PC = 1 : 2, DP produced meets AB produced at Q, If the area of the ΔBPQ is 20 sQ units, what is the area of the ΔDCP ?



(a) 20 sq units(c) 40 sq units

(d) none of the above

30. From a point within an equilateral triangle, perpendicular are drawn to its sides. The lengths of these perpendicular are 6 m, 7m and 8m. Find the area of the triangle

(a) 160 sq. m (b) $147\sqrt{3}$ sq. m (c) $210\sqrt{3}$ sq.m. (d) $27\sqrt{3}$ sq.nz

ANSWER KEY & HINTS

1. (b): Let the length of the smaller line segment =x cm. The length of larger line segment = (x+3) cm. According to the question,

$$(x+3)^{2} - x^{2} = 51$$

$$\Rightarrow x^{2} + 6x - 9 - x^{2} = 51$$

$$\Rightarrow 6x = 51 - 9 = 42$$

$$\Rightarrow x = \frac{42}{6} = 7$$

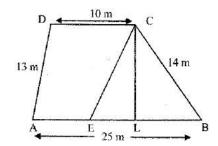
The required length

$$= x + 2 = 7 + 2 = 9 \,\mathrm{cm}$$

2. (a): Area of paper = Area of square + Area of equilateral triangle

$$= \frac{1}{2} (diagonal)^{2} + \frac{\sqrt{3}}{4} \times (side)^{2}$$
$$= \frac{1}{2} \times 32 \times 32 + \frac{\sqrt{3}}{4} \times 8 \times 8$$
$$512 + 16 \times 1.732$$
$$512 + 27.712 = 539.712 \text{ cm}^{2}$$





From C, draw $CE \parallel DA$. Clearly, ADCE is a parallelogram having $AD \parallel CE$ and $DC \parallel AE$ such that AD = 13 m and DC = 10 m.

$$\therefore AE = DC = 10 \text{ m} \text{ and } CE = AD = 13 \text{ m}$$

$$\Rightarrow BE = AB - AE = (25 - 10)m = 15 \text{ m}$$

Thus in $\triangle BCE$, we have

BC = 14 m, CE = 13 m and BE = 15 m

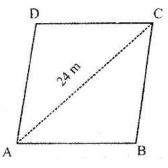
Let's be the semi - perimeter of $\Delta\!BC\!E$. Then,

2s = BC + CE + BE = 14 + 13 + 15 = 42

 $\Rightarrow s = 21$ Area of $\Delta BCE = \sqrt{21 \times (21 - 14) \times (21 - 13) \times (21 - 15)}$ $\Rightarrow \text{Area of } \Delta BCE = \sqrt{21 \times 7 \times 8 \times 6}$ $\Rightarrow \text{Area of } \Delta BCE = \sqrt{7^2 \times 3^2 \times 4^2} = 84 \text{ m}^2$ Also, Area of $\Delta BCE = \frac{1}{2}(BE \times CL)$ $\Rightarrow 84 = \frac{1}{2} \times 15 \times CL \Rightarrow CL = \frac{168}{15} = \frac{56}{5}$ $\Rightarrow \text{Height of parallelogram } ADCE = CL = \frac{56}{5} \text{ m}$ $\therefore \text{Area of parallelogram } ADCE = \text{Base } \times \text{Height}$ $= AE \times CL = 10 \times \frac{56}{5} = 112m^2$ Hence, Area of trapezium ABCE = Area of parallel

Hence, Area of trapezium ABCE = Area of parallelogram ABCE = Area of parallelogram ADCE + Area of $\Delta BCE = (112 + 84)m^2 = 196m^2$

(d):



Let ABCD be the rhombus of perimeter 80 m and diagonal AC = 24 m We have,

$$AB + BC + CD + DA = 80$$

$$\Rightarrow 4AB = 80 \qquad [:: A.B = BC = CD = DA]$$

$$\Rightarrow AB = 20 \text{ m}$$

In $\triangle ABC$, we have

$$2s = AB + BC + AC = 20 + 20 + +24 = 64$$

$$\Rightarrow s = 32$$

$$:: \Delta_1 = Area of \triangle ABC = \sqrt{32 \times 12 \times 12 \times 8} = 16 \times 12 = 192 m^2$$

Hence, area of rhombus $ABCD = 2 \times 192 m^2 = 384 m^2$.

- 5. (c): Let the breadth of floor be x metre. \therefore Length = (x + 20) metre

 - ∴Area of the floor
 - (x+20)x sq. metre

According to question,

$$(x+10)(x+5) = x(x+20)$$

$$\Rightarrow x^{2} + 15x + 50 = x^{2} + 20x$$

$$\Rightarrow 20x = 15x + 50$$

$$\Rightarrow 5x = 50$$

$$\Rightarrow x = 10 \text{ metre}$$

$$\therefore \text{ Length } = x + 20 = 10 + 20 = 30 \text{ metre}$$

$$\therefore \text{ Area of the floor } = 30 \times 10 = 300 \text{ sq. metre}$$

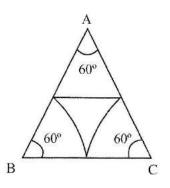
New effect on area of rectangle

$$= \left(20 + 10 + \frac{20 \times 10}{100}\right)\% = 32\% \qquad \left[:: \text{Net }\% \text{ change} = \frac{a + b + ab}{100}\right]\%$$

7. (c):
$$AB = AC = 10 \text{ cm}$$

 $Area = \frac{1}{2}bc \sin A = \frac{1}{2} \times 10 \times 10 \sin 45^{\circ}$
 $= \frac{50}{\sqrt{2}} = \frac{50 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 25\sqrt{2} \text{ cm}^2$

8. (c):

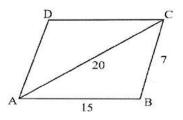


Each angle of the triangle = 60° Required area of the three sectors

$$= 3 \times \frac{60}{360} \times \pi (2)^2 = 2\pi cm^2$$

Area of triangle
$$= \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3}cm^2$$
$$\therefore \text{ Required Area} = (4\sqrt{3} - 2\pi)cm^2$$

9. (c): Let,
$$AB = AC = x$$
 units
 $BD = DC = 2$ unit [$\therefore BC = 4$ units]
Now, $AD = \sqrt{AB^2 - BD^2} = \sqrt{x^2 - 1}$
 $\therefore \frac{4}{2} \times BC \times AD = 8$
 $\Rightarrow \frac{1}{2} \times 4 \times \sqrt{x^2 - 1} = 8 \Rightarrow \sqrt{x^2 - 1} = 4$
 $\Rightarrow x^2 - 1 = 16 \Rightarrow x^2 = 17 \Rightarrow x = \sqrt{17}$ units



Area of parallelogram ABCD = Area of 2 $\triangle ABC$ Semi- perimeter of $\triangle ABC$, $S = \frac{20 + 7 + 15}{2} = \frac{42}{2} = 21cm$ \therefore area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{21(21-7)(21-20)(21-15)}$ $= \sqrt{21 \times 14 \times 6} = 42$ sq.cm. \therefore Area of parallelogram $= 2 \times 42 = 84$ sq.cm.

11. (b): Let the sides of parallelogram be 5x and 4xBase \times Height = Area of parallelogram

$$5x \times 20 = 1000 \Rightarrow x = \frac{1000}{5 \times 20} = 10$$
$$\Rightarrow \text{ sides} = 50 \text{ and } 40 \text{ units}$$

$$\therefore 40 \times h = 1000 \Longrightarrow h = \frac{1000}{40} = 25 \text{ units}$$
12. (b): $Side = \frac{16}{4} = 4cm$ but, $AB = AD = 4 \text{ cm}$
 $\angle DAB = \angle DCB = 60^{\circ}$
 $\therefore \text{ Area of the rhombus} = 2 \times \frac{\sqrt{3}}{4} \times (4)^2$
 $= 2 \times \frac{\sqrt{3}}{4} \times 4 \times 4 = 8\sqrt{3} \text{ cm}^2$

13. (c): Let the side of a square is increased x% by, its area is increased by $\left(2x + \frac{x^2}{100}\right)\%$

Here, x = 20%

 \therefore Effective increase in area

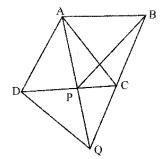
$$= \left(2 \times 20 + \frac{20 \times 20}{100}\right)\% = 44\%$$

14. (c): $PQ \parallel BC$

Also, $\angle APQ = \angle ABC = 60^{\circ}$ $\angle AQP = \angle ACB = 60^{\circ}$ \therefore Area of $\triangle APQ = \frac{\sqrt{3}}{4} \times (PQ)^2 = \frac{\sqrt{3}}{4} \times (3)^2 = \frac{9\sqrt{3}}{4}$ sq.cm

15. (a): Join $AC \& DQ \therefore \triangle APC$ and $\triangle BCP$ lie on the same base PC and between the same parallels AB and PC $\therefore ar(\triangle APC) = ar(\triangle BCP)$ (i) Now, $AD \parallel CQ$ and AD = CQ $\therefore \triangle DQC$ is a parallelogram,

Again $\triangle ADC$ and $\triangle DAQ$ are on the same base AD and between same parallels AD and CQ.



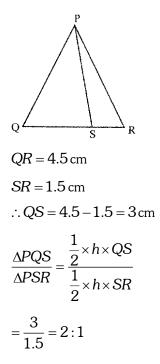
 $\therefore ar(\Delta ADC) = ar(\Delta ADQ)$

Subtracting ar (DAP) from both sides, we get

 $\operatorname{ar}(\Delta APC) = \operatorname{ar}(\Delta DPQ)$ (ii)

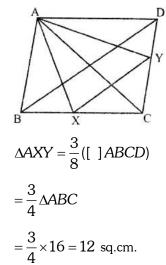
From (i) and (ii), we get ar $(\Delta BCP) = ar(\Delta DPQ)$

16. (d):

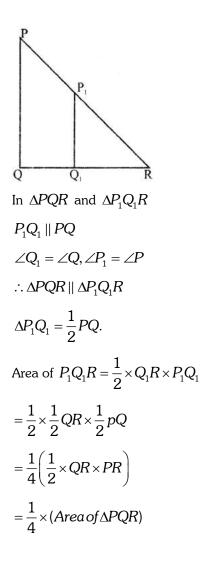




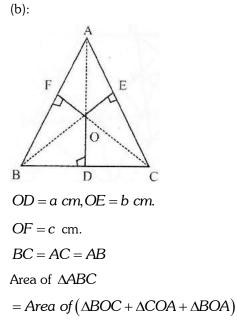
(a):











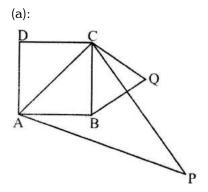
$$= \frac{1}{2} \times BC \times a + \frac{1}{2}AC \times b + \frac{1}{2} \times AB \times c$$
$$\frac{1}{2}BC(a+b+c) \qquad \dots (i)$$
$$(\therefore AB = BC = CA)$$
Again, Area of $\triangle ABC$

$$= \frac{\sqrt{3}}{4} \times BC^{2}$$
$$\therefore \frac{\sqrt{3}}{4} \times BC^{2} = \frac{1}{2}BC(a+b+c)$$
$$\Rightarrow BC = \frac{2}{\sqrt{3}}(a+b+c)$$

∴Required area

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}} (a+b+c)^2$$
$$= \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = (a+b+c)^2$$
$$= \frac{\sqrt{3}}{3} = (a+b+c)^2 \text{ sq. units}$$





From $\triangle ABC$

$$AC = \sqrt{AB^{2} + BC^{2}}$$
$$= \sqrt{AB^{2} + BC^{2}}$$
$$= \sqrt{2}BC$$
$$\Delta QBC \parallel \Delta PAC$$
$$\therefore \frac{Area \text{ of } \Delta QBC}{Area \text{ of } \Delta PAC} = \frac{BC^{2}}{AC^{2}}$$

$$= \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$
$$= \frac{BC^2}{2BC^2} = \frac{1}{2}$$

21.

(a):

	2. Å.

Area of rectangular park $= 60 \times 40 = 2400$ sq. metre

Let the width of cross road be *x* metre.

 \therefore Area of cross roads = $60x + 40x - x^2 = 100x - x^2$

According to the question.

$$100x - x^{2} = 2400 - 2109$$

$$\Rightarrow 100x - x^{2} = 291$$

$$\Rightarrow x^{2} - 100x + 291 = 0$$

$$\Rightarrow x^{2} - 3x - 97x + 291 = 0$$

$$\Rightarrow x(x - 3) - 97(x - 3) = 0$$

$$\Rightarrow (x - 3)(x - 97) = 0$$

$$\Rightarrow x = 3 \text{ because } x \neq 97$$

22. (b): I. $1.2(l+b) = 110 \Rightarrow l+b = 55$ II. $l = (b+5) \Rightarrow l-b = 5$. III. $\frac{l}{b} = \frac{6}{5} \Rightarrow 5l - 6b = 0$

These are three equations in I and b. We may solve then pair wise.

 \therefore Any two of the three will give the answer.

: Correct answer is (b).

23. (a): **III.** gives area of the path $=\frac{4416}{25} m^2$

II. gives width of path = 20 x (Length of the lawn).
I. gives length = 3x metres and breadth = x metres
Clearly, all the three will be required to find the width of the path.

.:. Correct answer is (a)

24. (b):- Area of rectangle lies between 40 cm^2 and 45 cm^2 Now one side = 5 cm Since, area cannot be less than 40 cm^2 \therefore Other side cannot be less than $=\frac{40}{5}=8$ cm Since, area cannot be greater than 45 cm^2 \therefore Other side cannot be greater than $=\frac{45}{5}=9$ cm \therefore Minimum value of diagonal $=\sqrt{8^2+5^2}$ Maximum value of diagonal $=\sqrt{9^2+5^2}$ $=\sqrt{106}=10.3$ cm So, diagonal lies between 9cm and 11 cm

25. (d):- In $\triangle AFD$ and $\triangle FEB$ $\angle AFD = \angle BFE$ (Vertically opposite angles) And $\angle ADC = \angle ABC$ $\therefore \ \triangle AFD \sim \triangle BFE$ So, $\frac{ar(\triangle FED)}{ar(\triangle AFD)} = \frac{EB^2}{AD^2} = \frac{m^2}{(mn)^2} = \left[\frac{m}{m+n}\right]^2$

In $\triangle ACD$. EG || DC and E and G are mid-points of AD and AC, respective

$$\therefore EG = \frac{1}{2}DC = \frac{3}{2}$$

Similarly, in $\triangle ABC$

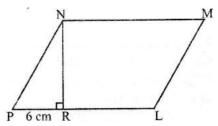
$$GF = \frac{1}{2}AB = 1$$

 $EF = EG + GF = 1 + \frac{3}{2} = \frac{5}{2}$

 $\therefore \text{Area if trapezium} = \frac{1}{2} \text{ (Sum of parallel sides} \times \text{Height)}$

Now, required ratio =
$$\frac{Area \ of \ ABFE}{Area \ of \ EFCD} = \frac{\frac{1}{2}\left(2+\frac{5}{2}\right) \times h}{\frac{1}{2}\left(3+\frac{5}{2}\right) \times h} = \frac{9}{11}$$

27. (b):- by given condition,



Area of parallelogram = $6 \times area$ of ΔNPR

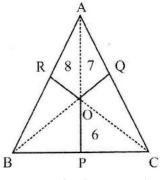
 $\therefore NR \times PL = 6 \times \frac{1}{2} \times NR \times pR$ $\Rightarrow PL = 3PR \qquad (here, PL = PR + RL)$ $\Rightarrow PR + RL = 3PR$ $\Rightarrow RL = 2PR = 2 \times 6 = 12 \text{ cm.}$

28. (a): $AB \parallel DC$ and $AD \parallel BC$

In $\triangle ABE$, $\angle EAB = \angle ABE = 60^{\circ}$ $\Rightarrow \angle AE = 60^{\circ}$ $\Rightarrow \triangle ABE$ is an equilateral triangle. Now, Perimeter of $\triangle ABE = 6$ $\Rightarrow AB + BE + EA = 2$ And in $\triangle ADE$, $AE^2 = AD^2 + ED^2$ $\Rightarrow 4 = AD^2 + 1$ (Since. E is mid-point of CD) $\Rightarrow AD = \sqrt{3}$ units Hence, area of quadrilateral $ABCD = AB \times AD$ $= 2 \times \sqrt{3} = 2\sqrt{3}$ sq units

29. (d):- We know that, ratio of the areas of two similar triangles is equal to the ratios of squares their corresponding sides.

$$\therefore \frac{Area(\Delta BPQ)}{Area(\Delta DPC)} = \frac{PB^{2}}{PC^{2}}$$
$$\Rightarrow \frac{20}{Area(\Delta DPC)} = \frac{1}{4}$$
$$\Rightarrow Area(\Delta DPC) = 80 \text{ sq units}$$



Area of $\triangle OBC + area of \triangle OAC + area of \triangle OAB$

= Area of $\triangle ABC$

$$\frac{1}{2} \times a \times 6 + \frac{1}{2} \times a \times 7 + \frac{1}{2} \times a \times 8 = \frac{\sqrt{3}}{4} a^{2}$$

$$\frac{1}{2} a \times 21 = \frac{\sqrt{3}}{4} a^{2}$$

$$a = \frac{42}{\sqrt{3}} = \frac{42 \times \sqrt{3}}{3} = 14\sqrt{3}$$

$$\therefore \text{ area of triangle } = \frac{\sqrt{3}}{4} \times a^{2}$$

$$= \frac{\sqrt{3}}{4} (14\sqrt{3})^{2} = \frac{\sqrt{3}}{4} \times 196 \times 3 = 147\sqrt{3} \text{ sq.cm}$$